ONLINE MATHS CLASS - X - 12 (15 / 07 /2021)

1. ARITHMETIC SEQUENCE - CLASS -10

What did we study in the last class ?

★ **The sum of any number of consecutive natural numbers , starting with one , is half the product of the last number and the next natural number .**

That is , $1 + 2 + 3 + . . . + n = \frac{n\ (n+1)}{2}$

★ **For the arithmetic sequence ,** $x_n = an + b$

the sum of the first n terms is $x_1 + x_2 + x_3 + . . . + x_n = a\frac{n\,(n+1)}{2} + bn$

Activity 1 **(Sum of the first n even numbers)**

Even numbers are got by multiplying the natural numbers by 2 .

Position	1	2	3	10	50	100	n
Even number	2 x 1 = 2	2 x 2 = 4	2 x 3 = 6	2 x 10 = 20	2 x 50 = 100	2 x 100 = 200	2 x n = 2 n

Sum of the first n natural numbers $= 1 + 2 + 3 + . . . + n = \frac{n\ (n+1)}{2}$

Sum of the first n even numbers $= 2 + 4 + 6 + . . . + 2n$

$= 2(\ 1 + 2 + 3 + . . . + n\ )$

$= 2 \times \frac{n\,(n+1)}{2} = n\,(n+1)$

Sum of the first n even numbers = $n\,(n+1)$

NOTE :

Sum of the first 10 even numbers = $10 \times 11 = 110$
Sum of the first 15 even numbers = $15 \times 16 = 240$
Sum of the first 20 even numbers = $20 \times 21 = 420$
Sum of the first 50 even numbers = $50 \times 51 = 2550$
Sum of the first 100 even numbers = $100 \times 101 = 10100$

Activity 2 **(Sum of the first n odd numbers)**

Odd numbers are got by subtracting 1 from the multiples of 2 .

Position	**1**	**2**	**3**	**10**	**50**	**100**	**n**
Odd number	**2 x 1 - 1** **= 2 – 1** **= 1**	**2 x 2 - 1** **= 4 – 1** **= 3**	**2 x 3 - 1** **= 6 – 1** **= 5**	**2 x 10 - 1** **= 20 – 1** **= 19**	**2 x 50 - 1** **= 100 – 1** **= 99**	**2 x 100-1** **= 200 – 1** **= 199**	**2 x n - 1** **= 2 n – 1**

Sum of the first n odd numbers $= 1 + 3 + 5 + . . . + 2n-1$

$= 2-1 + 4-1 + 6-1 + . . . + 2n-1$

n times

$= 2 + 4 + 6 + . . . + 2n - 1 - 1 - 1 - . . . - 1$

$= n(n+1) - 1 \times n = n^2 + n - n = n^2$

Sum of the first n odd numbers = n^2

NOTE :

a) $1 + 2 + 3 + . . . + n = \dfrac{n(n+1)}{2}$

b) $2 + 4 + 6 + . . . + 2n = 2(1 + 2 + 3 + . . . + n) = 2 \times \dfrac{n(n+1)}{2} = n(n+1)$

c) $1 + 3 + 5 + . . . + (2n-1) = n(n+1) - 1 \times n = n^2 + n - n = n^2$

Sum of the first 10 odd numbers	=	$10^2 = 100$
Sum of the first 15 odd numbers	=	$15^2 = 225$
Sum of the first 20 odd numbers	=	$20^2 = 400$
Sum of the first 50 odd numbers	=	$50^2 = 2500$
Sum of the first 100 odd numbers	=	$100^2 = 10000$

Activity 3 **(Sum of the first n multiples of 4)**

Multiples of 4 are got by multiplying the natural numbers by 4 .

Sum of the first n multiples of 4 $= 4 + 8 + 12 + \ldots + 4n$

$= 4(1 + 2 + 3 + \ldots + n)$

$= 4 \times \frac{n(n+1)}{2}$

$= 2 \times n(n+1) = 2(n^2 + n) = 2n^2 + 2n$

NOTE :

Arithmetic sequence	Algebraic form	Sum of the first n terms
1 , 2 , 3 , . . .	n	$\frac{n(n+1)}{2}$
2 , 4 , 6 , . . .	$2n$	$2 \times \frac{n(n+1)}{2} = n(n+1)$
1 , 3 , 5 , . . .	$2n - 1$	$2 \times \frac{n(n+1)}{2} - n = n^2$
4 , 8 , 12 , . . .	$4n$	$4 \times \frac{n(n+1)}{2} = 2n^2 + 2n$

Activity 4 **(Another way of finding the sum of an arithmetic sequence)**

We have seen that , for the arithmetic sequence , $x_n = an + b$

the sum of the first n **terms is** $x_1 + x_2 + x_3 + \ldots + x_n = a\frac{n(n+1)}{2} + bn$

$$a\frac{n(n+1)}{2} + bn = n\left(\frac{a(n+1)}{2} + b\right)$$

$$= n\left(\frac{a(n+1)}{2} + \frac{2b}{2}\right)$$

$$= \frac{n}{2}\left[\left(a(n+1) + 2b\right)\right]$$

$$= \frac{n}{2}\left[an + a + 2b\right]$$

$$= \frac{n}{2}\left[an + a + b + b\right]$$

$$= \frac{n}{2}\left[(an + b) + (a + b)\right]$$

$$= \frac{n}{2}\left[(an + b) + (a + b)\right]$$

$$= \frac{n}{2}\left[x_n + x_1\right]$$

(NOTE: $x_n = an + b$, $x_1 = a + b$)

Finding

The sum of any number of consecutive terms of an arithmetic sequence is half the product of the number of terms and the sum of the first and last terms .

$$x_1 + x_2 + x_3 + \ldots + x_n = \frac{n}{2}(x_1 + x_n)$$

Activity 5

Consider the arithmetic sequence 5 , 8 , 11 , . . .

a) What is the common difference of this sequence ?

b) What is the 20^{th} term of this sequence ?

c) Find the sum of the first 20 terms of this sequence ?

Answer

a) Common difference $= 8 - 5 = 3$

b) $x_{20} = x_1 + 19d = 5 + (19 \times 3) = 5 + 57 = 62$

c) Sum of the first 20 terms $= \frac{20}{2}(x_1 + x_{20})$

$$= \frac{20}{2} \times (5 + 62)$$

$$= \frac{20}{2} \times 67 = 670$$

Activity 6 (Algebraic form of the sum)

We have seen that , for the arithmetic sequence , $x_n = an + b$

the sum of the first n terms is $x_1 + x_2 + x_3 + . . . + x_n = a\frac{n(n+1)}{2} + bn$

$$a\frac{n(n+1)}{2} + bn = \frac{a}{2}(n(n+1)) + bn$$

$$= \frac{a}{2}(n^2 + n) + bn$$

$$= \frac{a}{2}n^2 + \frac{a}{2}n + bn$$

$$= \frac{a}{2}n^2 + (\frac{a}{2} + b)n$$

In this $\frac{a}{2}$ *and* $\frac{a}{2} + b$ **are constants associated with the sequence .**

(We have seen earlier that $a = d$ *and* $b = f - d$ **)**

Thus the sum is the sum of products of n^2 and n with definite numbers .

That is ,

The algebraic form of the sum of an arithmetic sequence is $pn^2 + qn$

$$\left(p = \frac{a}{2} \quad , \quad q = \frac{a}{2} + b \right)$$

Here $p + q = \frac{a}{2} + \left(\frac{a}{2} + b\right) = a + b = f$

That is p **is half the common difference and** $p + q$ **is the first term .**

NOTE :

For the arithmetic sequence , $x_n = an + b$

a) $a = d$ **(*d* = Common difference)**

b) $b = f - d$ **(*f* = first term)**

c) Algebraic form of the sum = $pn^2 + qn$

d) $p = \frac{d}{2}$

e) $p + q = f$

Activity 7

Consider the arithmetic sequence 5 , 9 , 13 , . . .

a) What is its algebraic form ?

b) What is the sum of the first n terms of this sequence ?

Answer

a) $x_n = dn + f - d = 4n + 5 - 4 = 4n + 1$ $\quad (f = 5 \; ; \; d = 9 - 5 = 4)$

b)

Sum of the first n terms $= pn^2 + qn$

$= 2n^2 + 3n$

$p = \frac{d}{2} = \frac{4}{2} = 2$

$p + q = f$

$2 + q = 5$

$q = 5 - 2 = 3$

OR

Sum of the first n terms $= 4 \times \frac{n(n+1)}{2} + n = 2n(n+1) + n$

$= 2n^2 + 2n + n = 2n^2 + 3n$

Activity 8

The sum of the first n terms of an arithmetic sequence is $3n^2 + 2n$. Write the algebraic form of the sequence .

Answer

Sum of the first n terms $= 3n^2 + 2n$

$x_n = dn + f - d$

$= 6n + 5 - 6 = 6n - 1$

$p = \frac{d}{2}$, $p + q = f$

$\frac{d}{2} = 3 \rightarrow d = 3 \times 2 = 6$

$f = 3 + 2 = 5$

OR

Sum of the first n terms $= 3n^2 + 2n$

First term $= 3 \times 1^2 + 2 \times 1 = 3 \times 1 + 2 = 3 + 2 = 5$

Sum of the first two terms $= 3 \times 2^2 + 2 \times 2 = 3 \times 4 + 4 = 12 + 4 = 16$

$==> x_1 + x_2 = 16$

$5 + x_2 = 16 ==> x_2 = 16 - 5 = 11$

$d = 11 - 5 = 6$

$x_n = dn + f - d = 6n + 5 - 6 = 6n - 1$

Activity 9 (Number pattern - 1)

Look at the number pattern given below .

1

2 3

4 5 6

7 8 9 10

....

....

Here , one number in the first line , 2 numbers in the second line , 3 numbers in the third line , 4 numbers in the fourth line and so on . If we continue so , there are n numbers in the n^{th} line .

In this number pattern n^{th} line contains n numbers

Last number of the first line	1	1
Last number of the second line	3	1 + 2
Last number of the third line	6	1 + 2 + 3
Last number of the fourth line	10	1 + 2 + 3 + 4

1

2 3 = 1 + 2

4 5 6 = 1 + 2 + 3

7 8 9 10 = 1 + 2 + 3 + 4

....

....

	Number of terms	Last number	
First line	**1**	**1**	**1**
Second line	**2**	**1 + 2**	**3**
Third line	**3**	**1 + 2 + 3**	**6**
Fourth line	**4**	**1 + 2 + 3 + 4**	**10**
Fifth line	**5**	**1 + 2 + 3 + 4 + 5**	**15**
Sixth line	**6**	**1 + 2 + 3 + 4 + 5 + 6**	**21**
Seventh line	**7**	**1 + 2 + 3 + 4 + 5 + 6 + 7**	**28**
Eighth line	**8**	**1 + 2 + 3 + 4 + 5 + 6 + 7 + 8**	**36**
Ninth line	**9**	**1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9**	**45**
Tenth line	**10**	**1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10**	**55**

Last number in the n^{th} line $= 1 + 2 + 3 + \ldots + n$

$$\left(1 + 2 + 3 + \ldots + n = \frac{n\,(n+1)}{2} \right)$$

Findings

In this number pattern ,

a) n^{th} **line contains** n **numbers .**

b) **Last number of the** n^{th} **line** $= 1 + 2 + 3 + \ldots + n = \frac{n\,(n+1)}{2}$

Activity 10

Look at the number pattern given below .

1

2 3

4 5 6

7 8 9 10

....

..

a) Write the next two lines of the pattern above .

b) How many numbers are there in the 10^{th} line ?

c) Write the last term of the 9^{th} line .

d) Write the First number of the 10^{th} line .

e) Write the Last number of the 10^{th} line .

f) Find the sum of the numbers in the 10^{th} line .

Answer

a) 11 12 13 14 15

16 17 18 19 20 21

b) Total numbers in the 10^{th} line = 10

c) Last number of the 9^{th} line = $\frac{9 \times 10}{2} = 45$

d) First number of the 10^{th} line = 46

e) Last number of the 10^{th} line . = $\frac{10 \times 11}{2} = 55$

f) Sum of the numbers in the 10^{th} line = $\frac{10}{2} \times (46 + 55) = \frac{10}{2} \times 91 = 455$

Activity 11 (**Number pattern - 2**)

Look at the number pattern given below .

5			
8	11		
14	17	20	
23	26	29	32

....

..

Algebraic form of the arithmetic sequence 5 , 8 , 11 , ... $= d\,n + f - d$

$= 3\,n + 5 - 3 = 3\,n + 2$

The terms of the arithmetic sequence 5 , 8 , 11 , . . . are got by multiplying the natural numbers by 3 and adding 2 .That is , the terms the arithmetic sequence 5 , 8 , 11 , . . . are got by multiplying the terms of the arithmetic sequence by 3 and adding 2 .

That is , the terms of the above pattern are got by multiplying the terms of the pattern

1			
2	3		
4	5	6	
7	8	9	10

....

.. , by 3 and add 2 .

Eg:

Last number of the fifth line of the first pattern = $\frac{5 \times 6}{2} = 15$

Last number of the fifth line of the second pattern = $3 \times 5 + 2 = 15 + 2 = 17$

Last number of the 10th line of the first pattern = $\frac{10 \times 11}{2} = 55$

Last number of the 10th line of the second pattern = $3 \times 55 + 2 = 165 + 2 = 167$

Activity 12

Look at the number pattern given below .

5			
8	**11**		
14	**17**	**20**	
23	**26**	**29**	**32**

...

..

a) Write the next two lines of the pattern above .

b) How many numbers are there in the 10th line ?

c) Write the last term of the 9th line .

d) Write the First number of the 10th line .

e) Write the Last number of the 10th line .

f) Find the sum of the numbers in the 10th line .

Answer

a) 35 38 41 44 47

50 53 56 59 62 65

b) Total numbers in the 10th line = 10

c) Last number of the 9th line = $3 \times \left(\frac{9 \times 10}{2}\right) + 2$

$= 3 \times 45 + 2 = 135 + 2 = 137$

d) First number of the 10th line = $137 + 3 = 140$

e) Last number of the 10th line . = $3 \times \left(\frac{10 \times 11}{2}\right) + 2$

$= 3 \times 55 + 2 = 165 + 2 = 167$

f) Sum of the numbers in the 10th line = $\frac{10}{2} \times (140 + 167) = \frac{10}{2} \times 307 = 1535$