

ONLINE MATHS CLASS - X - 11 (14 / 07 /2021)

1. ARITHMETIC SEQUENCE - CLASS 9 – WORK SHEET-ANSWER

Important points .

➤ In an arithmetic sequence , the sums of the pairs of the terms are equal if the sums of their positions are equal .

➤ $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

➤ For the arithmetic sequence , $x_n = an + b$

the sum of first n terms is $x_1 + x_2 + x_3 + \dots + x_n = a \frac{n(n+1)}{2} + bn$

1. Compute the following sums .

a) $1 + 2 + 3 + \dots + 40$

b) $2 + 4 + 6 + \dots + 80$

c) $3 + 5 + 7 + \dots + 81$

d) $6 + 11 + 16 + \dots + 201$

Answer

a) $1 + 2 + 3 + \dots + 40 = \frac{40 \times 41}{2} = 820$

b) $2 + 4 + 6 + \dots + 80 = 2(1 + 2 + 3 + \dots + 40) = 2 \times 820 = 1640$

c) $3 + 5 + 7 + \dots + 81 = 1640 + 40 \times 1 = 1640 + 40 = 1680$

(Here the terms of the arithmetic sequence $2 , 4 , 6 , \dots , 80$ are got by adding 1 to the terms of the arithmetic sequence $3 , 5 , 7 , \dots , 81$)

d) $6 + 11 + 16 + \dots + 201 = 820 + 1640 + 1680 = 4140$ ($a + b + c$)

2. Compute the following sums .

- a) $1 + 2 + 3 + \dots + 50$
- b) $6 + 12 + 18 + \dots + 300$
- c) $1 + 7 + 13 + \dots + 295$
- d) $7 + 19 + 31 + \dots + 595$

Answer

a) $1 + 2 + 3 + \dots + 50 = \frac{50 \times 51}{2} = 1275$

b) $6 + 12 + 18 + \dots + 300 = 6(1 + 2 + 3 + \dots + 50) = 6 \times 1275 = 7650$

c) $1 + 7 + 13 + \dots + 295 = 7650 - 50 \times 5 = 7650 - 250 = 7400$

(Here the terms of the arithmetic sequence $6, 12, 18, \dots, 300$ are got by subtracting 5 from the terms of the arithmetic sequence $3, 5, 7, \dots, 81$)

d) $7 + 19 + 31 + \dots + 595 = 7650 + 7400 = 15050$ ($b + c$)

3) Consider the arithmetic sequences $9, 14, 19, \dots$ and $7, 12, 17, \dots$

- a) Find the common difference of these sequences .
- b) What is the difference between the first terms of these sequences ?
- c) Calculate the difference between the sums of the first 30 terms of these sequences .

Answer

a) Common difference of the first sequence = $14 - 9 = 5$

Common difference of the second sequence = $12 - 7 = 5$

b) $9 - 7 = 2$

c)

$$\begin{array}{r} 9 + 14 + 19 + \dots + x_{30} - \\ 7 + 12 + 17 + \dots + y_{30} \\ \hline 2 + 2 + 2 + \dots + 2 = 2 \times 30 = 60 \end{array}$$

4) Consider the arithmetic sequence 5, 8, 11,

- a) What is the common difference of the sequence ?
- b) What is the difference between the 21st and first terms of this sequence ?
- c) What is the difference between the 40th and 20th terms of this sequence ?
- d) What is the difference between the sum of the first 20 terms and the next 20 terms of this sequence ?

Answer

a) **Common difference = 8 – 5 = 3**

b) $x_{21} - x_1 = 20 \times \text{common difference} = 20 \times 3 = 60$

c) $x_{40} - x_{20} = 20 \times \text{common difference} = 20 \times 3 = 60$

d)

$$\begin{array}{r} x_{21} + x_{22} + x_{23} + \dots + x_{40} - \\ x_1 + x_2 + x_3 + \dots + x_{20} \\ \hline 20d + 20d + 20d + \dots + 20d = 20 \times 20d \\ = 20 \times 20 \times 3 = 1200 \end{array}$$

5. Common difference of an arithmetic sequence is 8 and the sum of the first 20 terms is 636 .

- a) What is the sum of the first and 12th terms of this sequence ?
- b) What is the common difference of this sequence ?

c) Write down the sequence .

Answer

$$\text{a) } x_1 + x_{12} = \frac{636}{6} = 106 \quad (12 \text{ terms } \Rightarrow \text{ total } 6 \text{ pairs })$$

$$x_1 + (x_1 + 11d) = 106$$

$$2x_1 + 11d = 106$$

$$2x_1 + 11 \times 8 = 106$$

$$2x_1 + 88 = 106$$

$$2x_1 = 106 - 88 = 18 \implies x_1 = \frac{18}{2} = 9$$

Sequence = 9 , 17 , 25 , . . .

NOTE : (Another method)

The algebraic form any arithmetic sequence of common difference 8 can be taken as

$$8n + b$$

$$\text{Sum of first 12 terms} = 636 \implies 8 \times \frac{12 \times 13}{2} + b \times 12 = 636$$

$$8 \times 78 + 12b = 636$$

$$624 + 12b = 636$$

$$12b = 636 - 624 = 12$$

$$b = \frac{12}{12} = 1$$

$$x_n = 8n + b = 8n + 1$$

$$x_1 = 8 \times 1 + 1 = 8 + 1 = 9$$

Sequence = 9 , 17 , 25 , . . .