ONLINE MATHS CLASS - X - 12 (15 / 07 /2021)

1. ARITHMETIC SEQUENCE - CLASS-10 -WORK SHEET - ANSWER

Important points

- **The sum of any number of consecutive terms of an arithmetic sequence is half the product of the number of terms and the sum of the first and last terms .**

$$x_1 + x_2 + x_3 + \ldots + x_n = \frac{n}{2}\left(x_1 + x_n\right)$$

- **For the arithmetic sequence ,** $x_n = an + b$

the sum of the first n terms is $x_1 + x_2 + x_3 + \ldots + x_n = a\,\frac{n(n+1)}{2} + bn$

- **The algebraic form of the sum of an arithmetic sequence is** $pn^2 + qn$

$$\left(p = \frac{a}{2}\ ,\ p + q = f\right)$$

1) Consider the arithmetic sequence 7 , 11 , 15 , . . .

a) What is the common difference of the sequence ?

b) What is the 30^{th} term of the sequence ?

c) Find the sum of the first 30 terms of the sequence .

Answer

a) $d = 11 - 7 = 4$

b) $x_{30} = x_1 + 29d = 7 + 29 \times 4 = 7 + 116 = 123$

c) Sum of the first 30 terms = $\frac{30}{2} \times (x_1 + x_{30}) = \frac{30}{2} \times (7 + 123) = \frac{30}{2} \times 130$

$= 1950$

2) Consider the arithmetic sequence 8 , 13 , 18 , . . .

a) What is the common difference of the sequence ?

b) Write the algebraic form of the sequence .

c) Find the sum of the first n terms of the sequence .

Answer

a) $d = 13 - 8 = 5$

b) $x_n = dn + f - d = 5n + 8 - 5 = 5n + 3$

c)

Sum of the first n terms $= pn^2 + qn$

$= \frac{5}{2}n^2 + \frac{11}{2}n$

$p = \frac{d}{2} = \frac{5}{2}$

$p + q = f$

$\frac{5}{2} + q = 8$

$q = 8 - \frac{5}{2} = \frac{16-5}{2} = \frac{11}{2}$

OR

$x_n = 5n + 3$

Sum of the first n terms $= 5 \times \frac{n(n+1)}{2} + 3n = \frac{5}{2}n(n+1) + 3n$

$= \frac{5}{2}(n^2 + n) + 3n = \frac{5}{2}n^2 + \frac{5}{2}n + 3n = \frac{5}{2}n^2 + \frac{11}{2}n$

3) The sum of the first n terms of an arithmetic sequence is $4n^2 + 3n$.

a) What is the first term of the sequence ?

b) What is the common difference of the sequence ?

c) Write the algebraic form of the sequence .

Answer

a) $p + q = f$ ==> $f = 4 + 3 = 7$ $\quad (p = 4 \ , \ q = 3)$

b) $p = \frac{d}{2}$ ==> $\frac{d}{2} = 4$ ==> $d = 4 \times 2 = 8$

c) $x_n = dn + f - d$ ==> $= 8n + 7 - 8 = 8n - 1$

OR

Sum of the first n terms $= 4n^2 + 3n$

a) **First term** $= 4 \times 1^2 + 3 \times 1 = 4 \times 1 + 3 = 4 + 3 = 7$

Sum of the first 2 terms $= 4 \times 2^2 + 3 \times 2 = 4 \times 4 + 6 = 16 + 6 = 22$

==> $x_1 + x_2 = 22$

$7 + x_2 = 22$ ==> $x_2 = 22 - 7 = 15$

b) $d = 15 - 7 = 8$

c) $x_n = dn + f - d$

$= 8n + 7 - 8 = 8n - 1$

4) Look at the following number pattern given below .

1

2 3

4 5 6

7 8 9 10

.....

.....

a) Write the next two lines of the pattern above .

b) How many numbers are there in the 20th line ?

c) Write the last term of the 19^{th} line .

d) Write the First number of the 20^{th} line .

e) Write the Last number of the 20^{th} line .

f) Find the sum of the numbers in the 20^{th} line .

Answer

a) 11 12 13 14 15

16 17 18 19 20 21

b) Total numbers in the 20^{th} line = 20

c) Last number of the 19^{th} line = $\frac{19 \times 20}{2} = 190$

d) First number of the 20^{th} line = 190 + 1 = 191

e) Last number of the 20^{th} line = $\frac{20 \times 21}{2} = 210$

f) Sum of the numbers in the 20^{th} line = $\frac{20}{2} \times (x_1 + x_{20})$

$= \frac{20}{2} \times (191 + 210) = \frac{20}{2} \times 401 = 4010$

5) Look at the following number patterns given below .

Pattern - 1	Pattern - 2	Pattern - 3
1	2	3
2 3	4 6	5 7
4 5 6	8 10 12	9 11 13
7 8 9 10	14 16 18 20	15 17 19 21
11 12 13 14 15	22 24 26 28 30	23 25 27 29 31
.....	.....	.....
.....	.....	.....

Complete the following table .

	Pattern - 1	Pattern -2	Pattern - 3
Next two lines			
Number of terms in the 10th line			
Last number of the 9th line			
First number of the 9th line			
Last number of the 10th line			
Sum of the numbers in the 10th line			

Answer

	Next two lines
Pattern - 1	16 17 18 19 20 21 22 23 24 25 26 27 28
Pattern -2	32 34 36 38 40 42 44 46 48 50 52 54 56
Pattern - 3	33 35 37 39 41 43 45 47 49 51 53 55 57

	Pattern - 1	Pattern -2	Pattern - 3
Number of terms in the 10^{th} line	10	10	10
Last number of the 9^{th} line	$\frac{9 \times 10}{2}$ $= 45$	45×2 $= 90$	$90+1$ $= 91$
First number of the 9^{th} line	46	92	$92+1 = 93$
Last number of the 10^{th} line	$\frac{10 \times 11}{2}$ $= 55$	55×2 $= 110$	$110+1$ $= 111$
Sum of the numbers in the 10^{th} line	$\frac{10}{2} \times (46+55)$ $= \frac{10}{2} \times 101$ $= 505$	2×505 $= 1010$	$1010+1 \times 10$ $=$ $1010 + 10$ $= 1020$

NOTE :

Second question contains fractions .So another question contains only natural numbers is given below .

6) Consider the arithmetic sequence 10 , 16 , 22 , . . .

a) What is the common difference of the sequence ?

b) Write the algebraic form of the sequence .

c) Find the sum of the first n terms of the sequence .

Answer

a) $d = 16 - 10 = 6$

b) $x_n = dn + f - d = 6n + 10 - 6 = 6n + 4$

c)

$p = \frac{d}{2} = \frac{6}{2} = 3$

$p + q = f$

$3 + q = 10$

$q = 10 - 3 = 7$

Sum of the first n terms $= pn^2 + qn$

$= 3n^2 + 7n$

OR

Sum of the first n terms $= 6 \times \frac{n(n+1)}{2} + 4n$

$= 3n(n+1) + 4n$

$= 3(n^2 + n) + 4n$

$= 3n^2 + 3n + 4n$

$= 3n^2 + 7n$