Online Class – X – 11 14 / 07 / 2021

1. Arithmetic Sequences – Class 9

To view class

If the number of terms of an arithmetic sequence is **odd**

$$\text{Sum} = \text{Number of terms} \times \text{Middle term}$$

$$\text{Middle Term} = \frac{\text{Sum}}{\text{Number of terms}} \quad \text{or} \quad \text{Middle Term} = \frac{\text{Pair Sum}}{2}$$

If the number of terms of an arithmetic sequence is **even**

$$\text{Sum} = \text{Number of pairs} \times \text{Pair sum}$$

$$\text{Pair Sum} = \frac{\text{Sum}}{\text{Number of pairs}}$$

Sum of consecutive natural numbers starting with 1

- When the number of terms are **odd**

Odd number of terms	No of terms (n)	Middle Term	Sum = n × middle term
1 + 2 + 3	3	$\frac{3+1}{2} = \frac{4}{2} = 2$	$3 \times 2 = 6$
1 + 2 + 3 + 4 + 5	5	$\frac{5+1}{2} = \frac{6}{2} = 3$	$5 \times 3 = 15$
1 + 2 + 3 + 4 + 5 + 6 + 7	7	$\frac{7+1}{2} = \frac{8}{2} = 4$	$7 \times 4 = 28$
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9	9	$\frac{9+1}{2} = \frac{10}{2} = 5$	$9 \times 5 = 45$
1 +2 + 3 + ...+ n	n	$\frac{n+1}{2}$	$n \times \left(\frac{n+1}{2}\right)$

If there are odd number of terms

Middle term = $\frac{n+1}{2}$

Sum = No of terms × Middle term

= $n \times \left(\frac{n+1}{2}\right)$

Sum = $\frac{n(n+1)}{2}$

- **When the number of terms are even**

Even number of terms	No of terms (n)	No of pairs $\frac{n}{2}$	Sum of a pair	Sum = No of pairs × Pair sum
1 + 2 + 3 + 4	4	$\frac{4}{2} = 2$	4 + 1 = 5	2 × 5 = 10
1 + 2 + 3 + 4 + 5 + 6	6	$\frac{6}{2} = 3$	6 + 1 = 7	3 × 7 = 21
1 + 2 + 3 + 4 + 5 + 6 + 7+ 8	8	$\frac{8}{2} = 4$	8 + 1 = 9	4 × 9 = 36
1 + 2 + 3 + ...+ n	n	$\frac{n}{2}$	n + 1	$\frac{n}{2} \times (n + 1)$

If there are even number of terms

Number of pairs = $\frac{n}{2}$

Pair Sum = n + 1

Sum = No of pairs × Pair sum

= $\frac{n}{2} \times (n + 1)$

Sum = $\frac{n(n+1)}{2}$

Conclusion :

The sum of any number of consecutive natural numbers , starting with one , is half the product of the last number and the next natural number .

That is ,

$$1 + 2 + 3 + \ldots\ldots\ldots\ldots + n = \frac{n(n+1)}{2}$$

Activity 1

Find the following sums.

a) 1 + 2 + 3 + . . . + 100

b) 2 + 4 + 6 + . . . + 200

c) 3 + 6 + 9 + . . . + 300

d) 5 + 10 + 15 + . . . + 500

Ans)

a) $1 + 2 + 3 + \ldots\ldots\ldots + 100 = \frac{100(100+1)}{2}$

$= \frac{100 \times 101}{2}$

$= 50 \times 101 = 5050$

$1 + 2 + 3 + \ldots\ldots\ldots + 100 = 5050$

b) $2 + 4 + 6 + \ldots\ldots + 200 = 2(1 + 2 + 3 + \ldots\ldots + 100)$

$= 2 \times 5050$

$= 10100$

c) $3 + 6 + 9 + \ldots\ldots + 300 = 3(1 + 2 + 3 + \ldots\ldots + 100)$

$= 3 \times 5050$

$= 15150$

d) $5 + 10 + 15 + \ldots\ldots + 500 = 5(1 + 2 + 3 + \ldots\ldots + 100)$

$= 5 \times 5050$

$= 25250$

Activity 2

Consider the sequence 6 , 11 ,16 , ...

a) Write the algebraic form of the sequence.

b) Find the sum of first 100 terms of this sequence.

Ans)

a) d = 11 – 6 = 5

Here each term of the sequence is one more than the multiples of 5 . So, algebraic form is 5n + 1

b) 6 + 11 + 16 + ... (100 terms)

=(5 + 1) + (10 + 1) + (15 + 1) + ...

= (5 + 10 + 15 +. . . . . .) + (1 + 1 + 1+ . . . . .)

100 terms **100 times**

= 5 (1 + 2 + 3 + ... + 100) + 1 ×100

= 5 × 5050 + 100

= 25250 + 100 = 25350

Activity 3

Consider the sequence 4 , 9 , 14 , ...

a) Write the algebraic form of the sequence.

b) Find the sum of first 100 terms of this sequence.

Ans)

a) d = 9 – 4 = 5

Here each term of the sequence is one less than the multiples of 5 . So, algebraic form is 5n – 1

b) 4 + 9 + 14 + ... (100 terms)

=(5 – 1) + (10 –1) + (15 –1) + ...

=(5 + 10 + 15 +. . . . . .) – (1 + 1 + 1+ . . . . .)

100 terms **100 times**

= 5(1 + 2 + 3 + ...+ 100) – 1 ×100

= 5 × 5050 – 1 ×100

= 25250 – 100 = 25150

Activity 4

Consider the sequence 4 , 11 , 18 , ...

a) Write the algebraic form of the sequence.

b) Find the sum of first 100 terms of this sequence.

Ans)

a) $d = 11 - 4 = 7$

Here each term of the sequence is 3 less than the multiples of 7 . So, algebraic form is $7n - 3$

b) $4 + 11 + 18 + \ldots\ldots$ (100 terms)

$= (7 - 3) + (14 - 3) + (21 - 3) + \ldots\ldots$

$= (7 + 14 + 21 + \ldots\ldots\ldots) - (3 + 3 + 3 + \ldots\ldots\ldots)$

(100 terms) (100 times)

$= 7(1 + 2 + 3 + \ldots\ldots + 100) - 3 \times 100$

$= 7 \times 5050 - 3 \times 100$

$= 35350 - 300 = 35050$

Activity 5 :

The algebraic form of an arithmetic sequence is 10n – 4

Find the sum of first 20 terms of this sequence.

Ans)

$X_n = 10n - 4$

$d = 10$

1^{st} term , $X_1 = 10 \times 1 - 4 = 10 - 4 = 6$

Sequence is 6, 16, 26 , ...

Sum = $6 + 16 + 26 + \ldots\ldots\ldots\ldots$.(20terms)

$= (10 - 4) + (20 - 4) + (30 - 4) + \ldots\ldots$

$= (10 + 20 + 30 + \ldots\ldots\ldots) - (4 + 4 + 4 + \ldots\ldots\ldots\ldots)$

(20 terms) (20 times)

$= 10(1 + 2 + 3 + \ldots\ldots + 20) - 4 \times 20$

$= 10 \times \frac{20 \times 21}{2} - 80 = 2100 - 80 = 2020$

or

1^{st} term , $X_1 = 10 \times 1 - 4 = 10 - 4 = 6$

20^{th} term , $X_{20} = 10 \times 20 - 4 = 200 - 4 = 196$

20 terms of sequence are 6, 16, ... 196

Sum = No of pairs × Pair sum

$= \frac{20}{2} \times (196 + 6)$

$= 10 \times (196 + 6) = 10 \times 202 = 2020$

Activity 6 :

The sum of first 'n' terms of an arithmetic sequence

Algebraic form of arithmetic sequence is $x_n = an + b$

1^{st} term = $x_1 = a \times 1 + b = a + b$

2^{nd} term = $x_2 = a \times 2 + b = 2a + b$

3^{rd} term = $x_3 = a \times 3 + b = 3a + b$

4^{th} term = $x_4 = a \times 4 + b = 4a + b$

5^{th} term = $x_5 = a \times 5 + b = 5a + b$

⋮

n^{th} term = $x_n = an + b$

Sum of first n terms = $(a\times1+b)+(a\times2+b)+(a\times3+b) + \ldots . .+(a\times n+b)$

$= a(1 + 2 + 3 + \ldots + n) + \underbrace{(b + b + b + \ldots . .+ b)}_{\text{n times}}$

$= a \times \frac{n(n+1)}{2} + b \times n$

$= a \frac{n(n+1)}{2} + bn$

For the arithmetic sequence

$x_n = an + b$

the sum of the first 'n' terms is

$$x_1 + x_2 + x_3 + \ldots + x_n = a\frac{n(n+1)}{2} + bn$$

Activity 7 :

Calculate the difference between the sums of the first 20 terms of the arithmetic sequences 2 , 9 , 16 , ... and 5 , 12 , 19 , ...

Ans) 2 , 9 , 16 , ... d = 9 – 2 = 7

5 , 12 , 19 , ... d = 12 – 5 = 7

Here both the arithmetic sequences have same common difference 7.

Difference between corresponding terms →

5 , 12 , 19 , ...

3 3 3 . . . . . (20 times)

2 , 9 , 16 , ...

∴ Difference between the sums of the first 20 terms = 20 × 3

= 60

Activity 8 :

What is the difference between the sum of the first 10 terms and the next 10 terms of the arithmetic sequence 7 , 11 , 15 , ...

Ans)

Sequence is 7 , 11 , 15 , ...

d = 11 – 7 = 4

First 10 terms X_1 , X_2 , X_3 , ... X_{10}

Next 10 terms X_{11} , X_{12} , X_{13} , ... X_{20}

Difference between corresponding terms → 10d , 10d , 10d ,. 10d

∴ Difference between sums = 10d + 10d+ 10d + ...(10times)

=10 ×10d

= 10 × 10 × 4

= 400

Activity 9 :

Common difference of an arithmetic sequence is 6 and the sum of the first 20 terms is 1300 . Write down the sequence .

Ans) **Sum of first 20 natural numbers is**

$$1 + 2 + 3 + \ldots\ldots\ldots + 20 = \frac{20 \times 21}{2} = 210$$

Any arithmetic sequence with common difference 6 is obtained by multiplying the sequence of natural numbers by 6 and adding or subtracting a fixed number.

$$6 \times (1 + 2 + 3 + \ldots\ldots\ldots + 20) = 6 + 12 + 18 + \ldots\ldots\ldots$$

$$6 + 12 + 18 + \ldots\ldots\ldots = 6 \times 210 = 1260$$

Sum of the first 20 terms of the arithmetic sequence given in the question is 1300 and this is more than 1260.

Difference in sum = 1300 – 1260

= 40

Divide this 40 among 20 terms of the required sequence,

ie, $\frac{40}{20} = 2$ **,**

Add this 2 to each term of the sequence 6, 12, 18, ...

∴ Required sequence is 8, 14, 20, ...

Assignment
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(1) Find the sum of the first 25 terms of each of the arithmetic sequences below.

i) 11, 22, 33, ... ii) 12, 23, 34, ... iii) 21, 32, 43, ...

iv) 19, 28, 37, ... v) 1, 6, 11, ...

(2) What is the difference between the sum of the first 20 terms and the next 20 terms of the arithmetic sequence 6, 10, 14, ...?

(3) Calculate the difference between the sums of the first 20 terms of the arithmetic sequences 6, 10, 14, ... and 15, 19, 23, ...

(4) Find the sum of all three digit numbers, which are multiples of 9.