Mathematics Online Class X On 23-07-2021

CIRCLES

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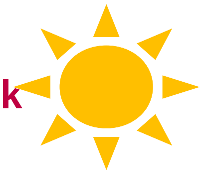
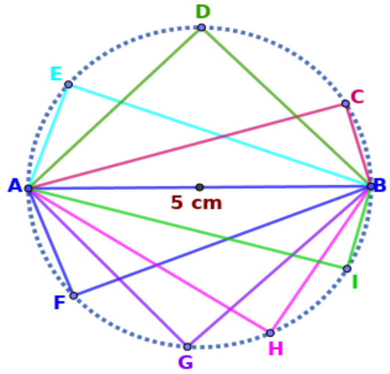
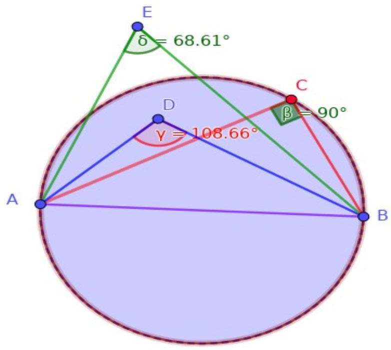


Points discussed in previous class



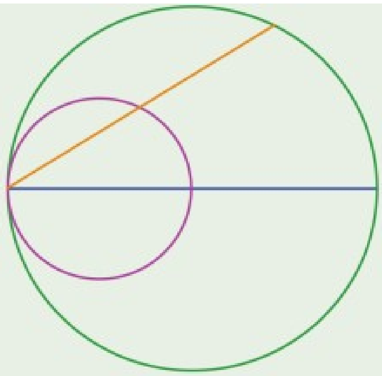
1. **If we join the ends of a diameter of a circle to any point on the circle , we get a right angle .**
   Angle in a semicircle is a right angle .



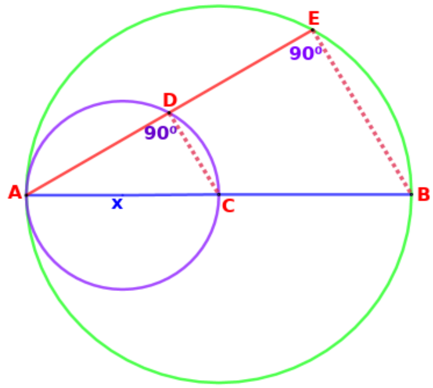
2. **If we join the ends of the diameter of a circle to any point on the circle, we get a right angle. If the point is inside the circle, the angle is obtuse. If the point is outside the circle, the angle is acute.**

Question

In the picture, a circle is drawn with a line as diameter and a smaller circle with half the line as diameter. Prove that any chord of the larger circle through the point where the circles meet is bisected by the small circle.



Answer



In the figure, C is the centre of larger circle. AB is the diameter of the larger circle and AC is the diameter of the smaller circle.

AE is a chord of the larger circle which meet the smaller circle at D.

Let AC = x ∴ AB = 2x . Join CD and BE

Consider ΔADC and ΔAEB.

Angle in a semicircle is 90°

Therefore, ∠ADC = ∠AEB = 90°

∠A is common angle of both triangles.

If two angles of ΔADC are equal to two angles of ΔAEB .

∴ their third angles are also equal

That is ΔADC and ΔAEB are similar

If two triangles are similar, then the sides opposite to equal angles are proportional

$$\frac{AC}{AB} = \frac{AD}{AE}$$

$$\frac{AD}{AE} = \frac{x}{2x} = \frac{1}{2}$$

Cross multiplyng, we get

AE = 2AD

That is, D is the midpoint of AE

That is, any chord of the larger circle through the point where the circles meet is bisected by the small circle.

OR

We know the perpendicular drawn from the centre of a circle to a chord bisects the chord .

∠ADC = 90° [Angle in a semicircle is 90°]

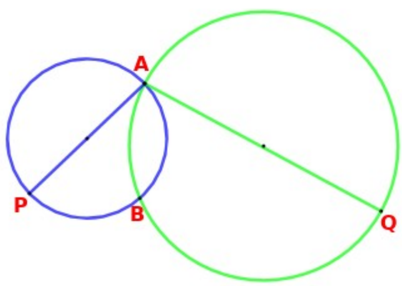
That is CD is perpendicular to AE

∴ we get CD bisects AE

That is, any chord of the larger circle through the point where the circles meet is bisected by the smaller circle.

Question

The circles in the picture cross each other at A and B .The points P and Q are the other ends of the diameters through A .



(i) Prove that P,B,Q lie on a line

(ii) Prove that PQ is parallel to the line joining the centres of the circles and is twice as long as this line .

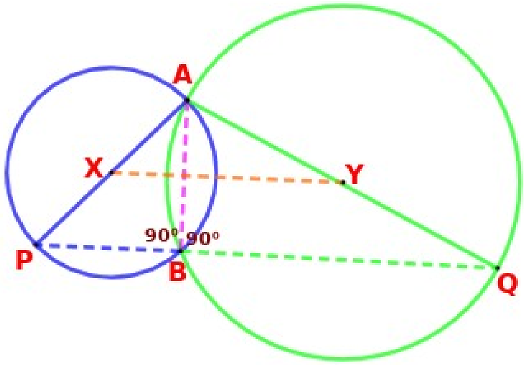
Answer

i) In the figure, AP and AQ are diameters of circles with centres X and Y.

Angle in a semicircle is 90°

Therefore,

∠ABP = ∠ABQ = 90°



AB is perpendicular to PQ.

Therefore,

P, B, Q are on the same line.

ii) X and Y are centres of circles.

That is, X is the midpoint of AP and Y is the midpoint of AQ.

Join XY.

We know ,

The line joining the midpoints of any two sides of a triangle is parallel to the third side and half of the third side.

Therefore,

PQ is parallel to XY

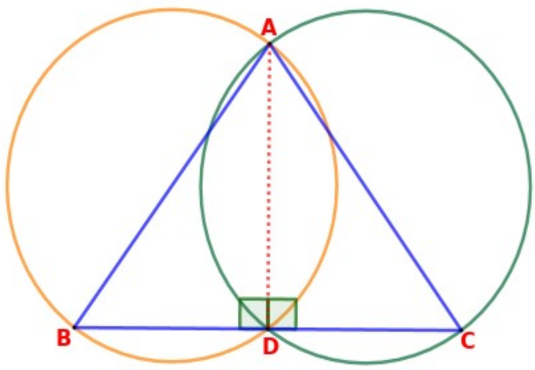
and PQ = 2XY

Question

Prove that the two circles drawn on the two equal sides of an isosceles triangle as diameters pass through the mid point of the third side.

Answer

In the figure, ΔABC is an isosceles triangle. In ΔABC, AB = AC



The perpendicular drawn from the vertex joining equal sides of an isosceles triangle pass trough the midpoint of the third side.

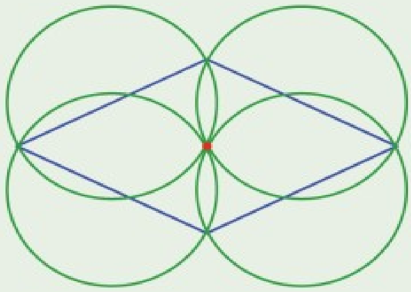
That is D is the midpoint of BC and AD is perpendicular to BC.

∴ ∠ADB = ∠ADC = 90°

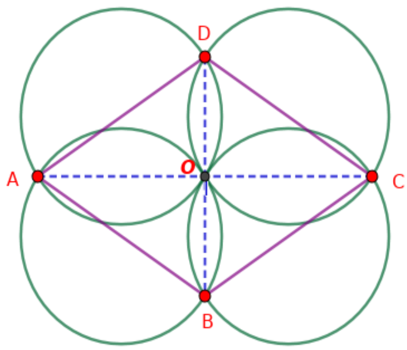
Therefore, If we draw a circle with AB and AC as diameter, that circle passes through D.
That is, the two circles drawn on the two equal sides of an isosceles triangle as diameters pass through the mid point of the third side.

Question

Prove that all four circles drawn with the sides of a rhombus as diameters pass through a common point.



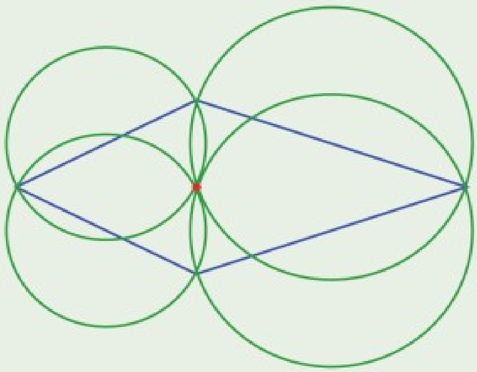
Answer



In the figure, ABCD is a rhombus.
Draw diagonals AC and BD.
In a rhombus; all sides are equal, opposite angles are equal and diagonals are perpendicular bisectors.
∴ AB = AD = BC = CD
'O' is the midpoint of AC and BD.
ΔABD and ΔCBD are isosceles triangles.
We know that the two circles drawn on the two equal sides of an isosceles triangle as diameters pass through the mid point of the third side.
Therefore, if we draw circles with diameters AB, AD, BC and CD; that circles pass through 'O' a common point.

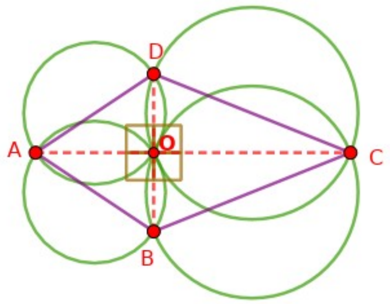
Question

Prove that for any quadrilateral with adjacent sides equal , the circles drawn with sides as diameter will pass through a common point .



Answer

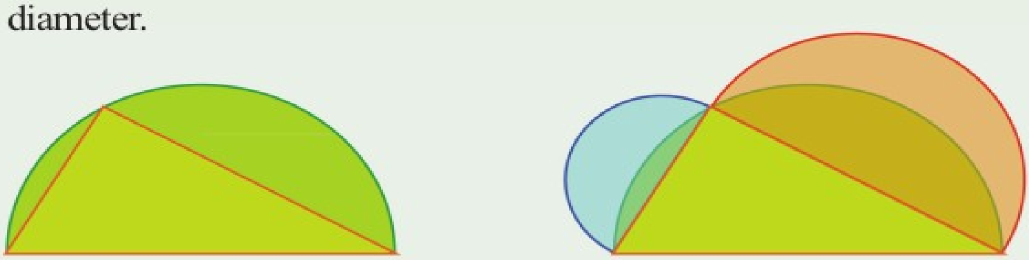
In the quadrilateral ABCD, adjacent sides are equal.
That is AB = AD and CB = CD.
Draw diagonals AC and BD.
ΔABD and ΔCBD are isosceles triangles.



We know that the two circles drawn on the two equal sides of an isosceles triangle as diameters pass through the mid point of the third side.
Therefore, if we draw circles with diameters AB and AD; that circles pass through 'O'. If we draw circles with diameters CB and CD; that circles pass through 'O'.

Question

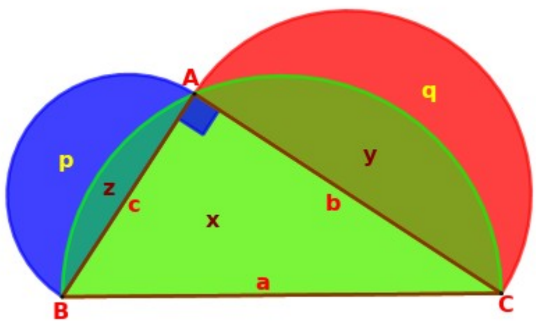
A triangle is drawn by joining a point on a semicircle to the ends of the diameter. Then semicircles are drawn with the other two sides as diameter.



Prove that the sum of the areas of the blue and red crescents in the second picture is equal to the area of the triangle.

Answer

Angle in a semicircle is 90°.
So ∠BAC = 90°
Therefore, Δ ABC is a right angled triangle.
Let the sides AC = b, AB = c and BC = a



Area of semicircle with diameter AC = $\frac{1}{2}\pi\left(\frac{b}{2}\right)^2 = \frac{1}{8}\pi b^2$

Area of semicircle with diameter AB = $\frac{1}{2}\pi\left(\frac{c}{2}\right)^2 = \frac{1}{8}\pi c^2$

Area of semicircle with diameter BC = $\frac{1}{2}\pi\left(\frac{a}{2}\right)^2 = \frac{1}{8}\pi a^2$

In right triangle ABC , Using Pythagoras theorem , we have

$$AC^2 + AB^2 = BC^2$$

$$b^2 + c^2 = a^2$$

$$\therefore \frac{1}{8}\pi b^2 + \frac{1}{8}\pi c^2 = \frac{1}{8}\pi a^2$$

That is, Sum of the areas of semicircles with diameters as perpendicular sides (b and c) is equal to the area of semicircle with diameter as hypotenuse (a)

That is, $(q+y) + (p+z) = x+y+z$

$$q + \cancel{y} + p + \cancel{z} = x + \cancel{y} + \cancel{z}$$

We get $p+q = x$

Sum of the areas of the blue and red crescents in the picture is equal to the area of the triangle.