Mathematics Online Class X On 27-07-2021

CIRCLES

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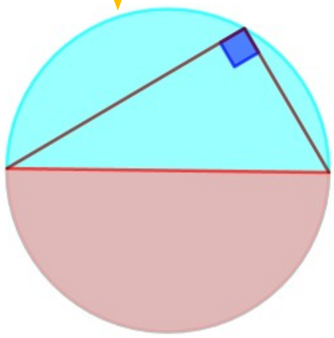
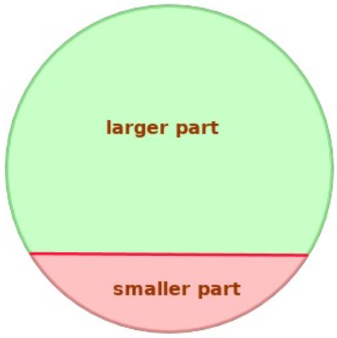


If we draw the diameter of a circle, the circle is divided into two equal parts. Each part is a semicircle .
If we join the end points of the diameter to any other point on the circle we get a right angle.



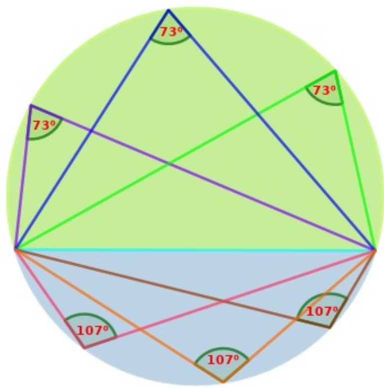
If we draw a chord other than the diameter then also the circle is divided in to two parts, one part is larger and other part is smaller

Activity

Draw a circle of radius 5 cm. Draw a chord other than diameter. Mark 3 points on the larger part of the circle and other 3 points on the smaller part of the circle. Join the end points of the chord to the points marked in larger part and measure the angles so get. Similarly, join the end points of the chord to the points marked in smaller part and measure the angles so get.

Answer

We can find that
(a) These angles are not right angles.
(b) All angles are not equal.
(b) Angles in the larger part are same.
(c) Angles in the smaller part are same.



If we join the endpoints of a chord other than the diameter, to any point on the larger part of the circle, what is the relation between the angle so formed on the circle and central angle of the chord.

Proof:

We can prove this in 3 different situations

(1)

If the lines joined from the endpoints of a chord to a point on the circle, is on both sides of the centre of the circle.

Draw OA, OB and OP.

We have to find $\angle P$.

Let $\angle APO = x°$ and $\angle BPO = y°$

$\angle P = (x+y)°$

Consider ΔAPO.

OA = OP (radii of the same circle)

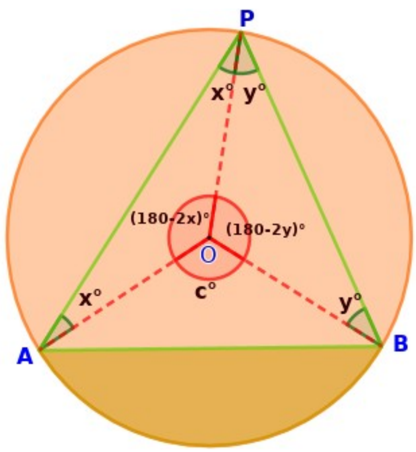
ΔAPO is an isosceles triangle .

In isosceles triangles, angles opposite to equal sides are equal.

$\therefore \angle APO = \angle PAO = x°$

Sum of angles of a triangle is 180°

$\therefore \angle AOP = 180 - (x°+x°) = (180 - 2x)°$



Consider ΔBPO.

OB = OP (radii of the same circle)

ΔBPO is an isosceles triangle .

$\therefore \angle BPO = \angle PBO = y°$

Sum of angles of a triangle is 180°

$\therefore \angle BOP = 180 - (y°+y°) = (180 - 2y)°$

Let $\angle AOB = c°$

We know angles around a point is 360°

$$180 - 2x + 180 - 2y + c = 360$$

$$360 - 2x - 2y + c = 360$$

$$-2x - 2y + c = 0$$

$$c = 2x + 2y = 2(x+y) = 2 \times \angle P$$

$\therefore$ We get $\angle P = \frac{1}{2}\ c^0 = \frac{1}{2}\ \angle AOB$

(2)

If one of the lines joined from the endpoints of the chord to the point on the circle, is the diameter of the circle.

Let ∠APB = x°

Draw OB.

Let ∠AOB = c°

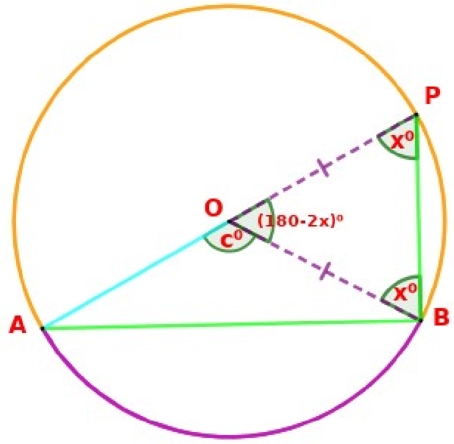
Consider ΔOPB.

OP = OB(radii of the same circle)

ΔOPB is isosceles.

∴ ∠PBO = x°

∠BOP = (180 – 2x)°



∠AOB and ∠BOP are linear pairs

$$\therefore 180 - 2x^\circ + c = 180^\circ$$

$$-2x^\circ + c = 0$$

$$c = 2x$$

$$c = 2 \times \angle P$$

$$\angle P = \frac{1}{2}\, c^0 = \frac{1}{2}\, \angle AOB$$

(3)

If the lines joined from the endpoints of a chord to a point on the circle, is on one side of the centre of the circle.

Draw OA, OB and OP.

Let ∠APO = x° and ∠BPO = y°

∴ ∠APB = (y-x)^0

Consider ΔAPO.

OA = OP(radii of the same circle)

ΔAPO is isosceles.

∠APO = ∠PAO = x°

∠AOP = (180-2x)^0

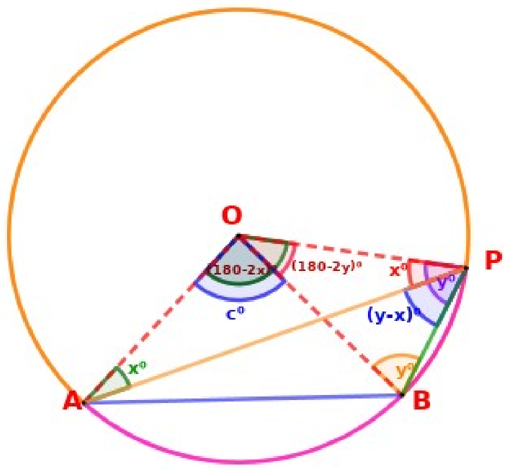
Consider ΔBPO.

OB = OP(radii of the same circle)

ΔBPO is isosceles.

∠BPO = ∠PBO = y°

∠BOP = (180-2y)^0



Let $\angle AOB = c°$

$c = \angle AOP - \angle BOP$

$= (180-2x) - (180-2y)$

$= 180-2x-180+2y$

$= 2y-2x = 2(y-x)$

$= 2 \times \angle APB$

$\angle APB = \frac{1}{2} c^0 = \frac{1}{2} \angle AOB$

If we join the ends of a non diametrical chord to any point on the larger part of the circle, we get an angle which is half the size of the angle we get by joining them to the centre of the circle.

Assignment :

Is any relation between angle made on the larger part and the angle made on the smaller part of the circle