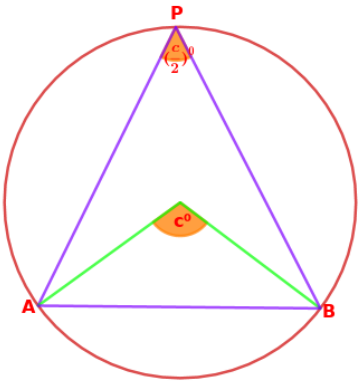
Mathematics Online Class X On 29-07-2021

CIRCLES

click

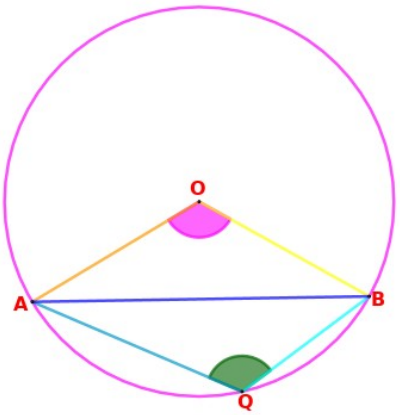
Discussed in previous class

If we joining the ends of a non diametrical chord to any point on the larger part of the circle, we get an angle which is half the size of the angle we get by joining them to the centre of the circle.



QUESTION

In the figure, what is the relation between the angle got by joining any point on the smaller part to the ends of the chord (∠AQB) and the angle made at the centre by the chord (∠AOB) ?



ANSWER

In the figure, AB is a non diametrical chord of the circle with centre O and Q is a point on the smaller part of the circle.

Draw AQ and BQ. Join OA, OB and OQ.

Let ∠AQO = x° and∠BQO = y°

∴∠AQB = x+y

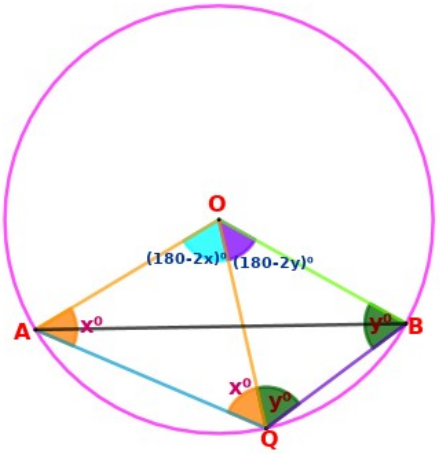
Consider ΔAOQ.

AO=QO (radii of the same circle)

ΔAOQ is an isosceles triangle

∠AQO =∠QAO = x

∠AOQ = 180-2x



Consider ΔBOQ.

BO=QO (radii of the same circle)

ΔBOQ is an isosceles triangle

∠BQO = ∠QBO = y

∠BOQ = 180-2y

∠AOB = ∠AOQ +∠BOQ

Let ∠AOB = c°

That is,

c = 180-2x+ 180-2y

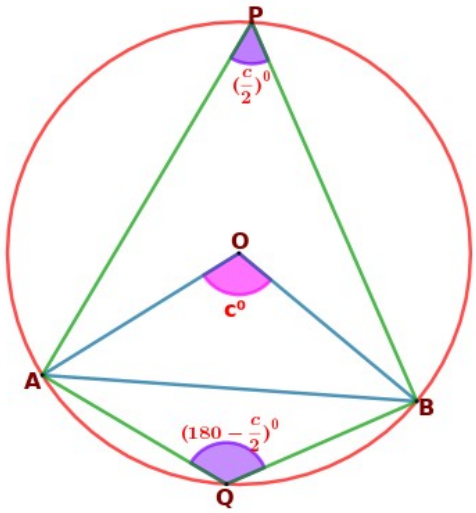
= 360 -2x – 2y

= 360 – 2(x+y)

= 360 – 2 × ∠AQB

2 × ∠AQB = 360 – c

$\angle AQB = \frac{360-c}{2} = 180 - \frac{c}{2}$



The angle got by joining any point on the smaller part to the ends of the chord is half the angle at the centre subtracted from 180° .

NOTE

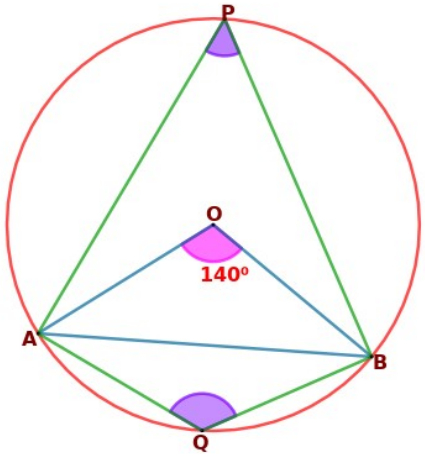
Any chord which is not a diameter splits the circle into two unequal parts.

The angle got by joining any point on the larger part to the ends of the chord is half the angle got by the centre of the circle to these end points.

The angle got by joining any point on the smaller part to the ends of the chord is half the angle at the centre subtracted from 180° .

QUESTION

If the chord AB makes an angle 140° at the centre of the circle, find ∠APB and ∠AQB ?



ANSWER

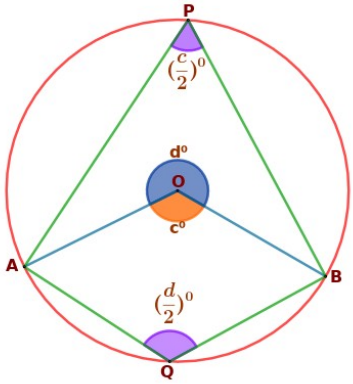
$\angle AOB = 140^0$

$\angle APB = \frac{\angle AOB}{2} = \frac{140}{2} = 70^0$

$\angle AQB = 180^0 - \frac{\angle AOB}{2} = 180^0 - \frac{140}{2}$

$= 180 - 70 = 110^0$

Any two points on a circle, divides the circle into two parts.
Each part is called the arc of the circle .
In the figure here two arcs are arc APB and arc AQB.
Arc APB is the alternate arc or complementary arc of arc AQB.
Arc AQB is the alternate arc or complementary arc of arc APB.
Central angle of an arc is the angle made by the arc at its centre.
Central angle of arc AQB = c°
Let central angle of arc APB = d°
We know angle around a point is 360°



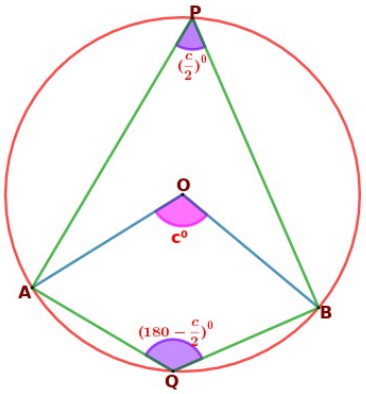
∴ c° + d° = 360°

$$d = 360 - c$$

$$\frac{d}{2} = \frac{360-c}{2} = 180 - \frac{c}{2}$$

∠APB is the angle made by the arc AQB at its alternate arc APB.
∠APB = Half of central angle of arc AQB.
Similarly,
∠AQB is the angle made by the arc APB at its alternate arc AQB.
∠AQB = Half of central angle of arc APB.
That is,

The angle made by any arc of a circle on the alternate arc is half the angle made at the centre of the arc.



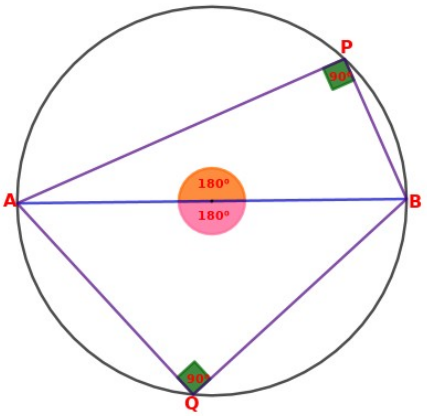
In the figure,

$$\angle P + \angle Q = \frac{c}{2} + 180 - \frac{c}{2} = 180^0$$

That is, Sum of angles in the alternate arc is 180°.
Angles in the alternate arcs is supplementary.

All angles made by an arc on the alternate arc are equal.
A pair of angles on an arc and its alternate arc are supplementary

In the figure, AB is the diameter of the circle.
Arc APB and arc AQB are semicircles.
Central angle of a semicircle is 180°.
∠P is the angle made by the arc AQB at its alternate arc APB and
∠Q is the angle made by the arc APB at its alternate arc AQB



$$\therefore \angle P = \frac{180}{2} = 90^0$$

$$\angle Q = \frac{180}{2} = 90^0$$

Angles in semicircle are right or 90°

ASSIGNMENT

In all the pictures given below, O is the centre of the circle and A , B , C are points on it . Calculate all angles of △ABC and △ OBC in each .