MATHEMATICS ONLINE CLASS X ON 06-08-2021

CIRCLES

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Answer to assignment of previous class

Question

Draw a triangle of circumradius 3cm and two of its angles are $(32\frac{1}{2})^0$ and $(37\frac{1}{2})^0$

Answer

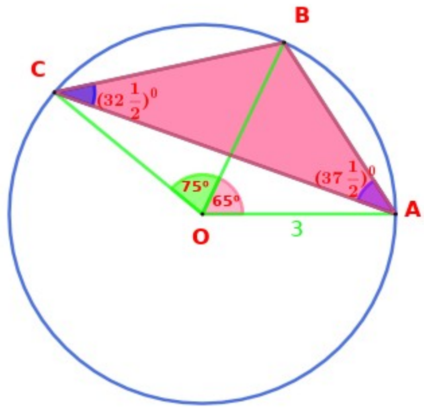
Given angles of triangle are $(32\frac{1}{2})^0$ and $(37\frac{1}{2})^0$

∴ Central angles of two arcs of the circle are

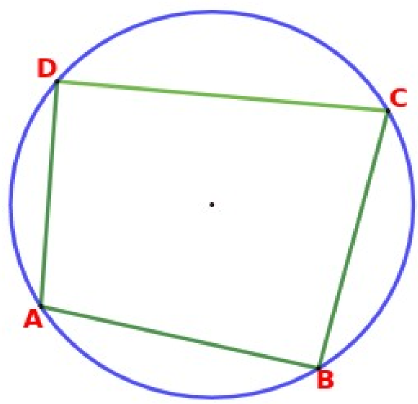
$2 \times (32\frac{1}{2})^0 = 65^0$ **and** $2 \times (37\frac{1}{2})^0 = 75^0$

$\angle ACB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 65^0 = (32\frac{1}{2})^0$

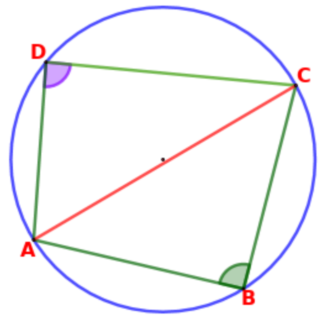
$\angle BAC = \frac{1}{2} \angle BOC = \frac{1}{2} \times 75^0 = (37\frac{1}{2})^0$



In the figure A, B, C, D are four points on the circle. Join ABCD to form a quadrilateral.
We have to find the properties of angles of this quadrilateral.

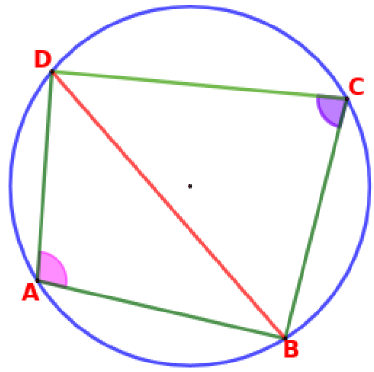


A quadrilateral have two diagonals.
Draw the diagonal AC. AC is a chord of the circle. Chord AC divides the circle into two parts. End points of chord AC makes angles ∠B and∠D on both parts of the circle.



$\angle B + \angle D = 180^0$

Angles formed at the opposite arcs are supplementary
That is,
$\angle B + \angle D = 180°$
Draw the diagonal BD. BD is a chord of the circle.
Angles formed at the opposite arcs are supplementary
∴ ∠A and ∠C are supplementary.
$\angle A + \angle C = 180°$



$\angle A + \angle C = 180^0$

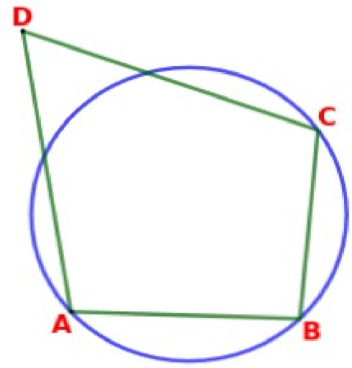
In the quadrilateral ABCD, four vertices are on the circle. Then, $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$.

If all the four vertices of a quadrilateral are on a circle, its opposite angles are supplementary

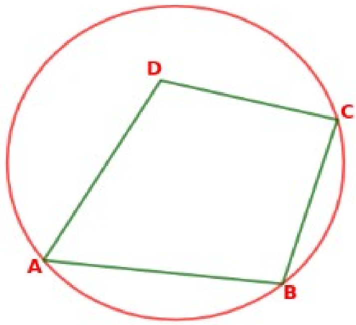
The converse of this statement is :
"If the opposite angles of a quadrilateral are supplementary, then all of its vertices are on a circle"

We know that, we can draw a circle passing through 3 points which are not in a straight line. That circle is the circumcircle of triangle formed by joining these points.

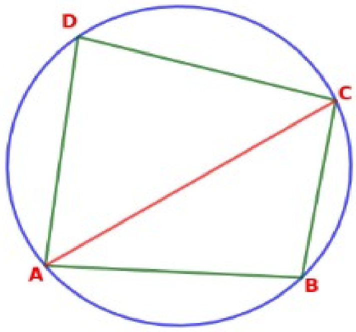
In quadrilateral ABCD,
Draw the circumcircle of ΔABC. The forth vertex D may be either outside the circle or inside the circle or on the circle.



case:1



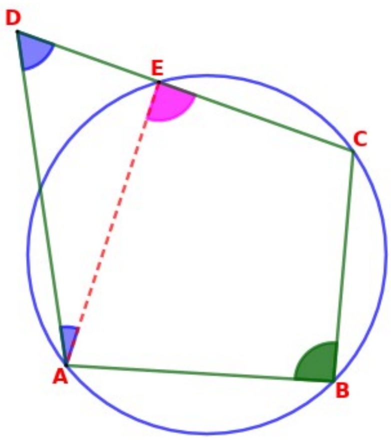
case:2



case:3

Case:1 Forth vertex D is outside the circle

We have to find the relation of $\angle B$ and $\angle D$.
Mark the point E which meets CD on the circle.
Join AE.
Consider quadrilateral ABCE.
A, B, C, E are points on the circle.
∴ Opposite angles of quadrilateral ABCE are supplementary.
That is, $\angle B + \angle AEC = 180°$ .... (1)



Consider ΔAED
$\angle AEC$ is an outer angle of ΔAED.
We know that, the outer angle of a triangle at one vertex is the sum of inner angles at the other two vertices.
∴ $\angle AEC = \angle EAD + \angle D$
which means that $\angle D$ is less than $\angle AEC$
That is, $\angle D < \angle AEC$ .... (2)
Apply equation (2) in equation (1)
Then we get $\angle B + \angle D < 180°$

Case:1I Forth vertex D is inside the circle

Extend CD to meet at E, a pont on the circle.
Join AE
Consider quadrilateral ABCE.
A, B, C, E are points on the circle.
∴ Opposite angles of quadrilateral ABCE are supplementary.

That is, ∠B +∠E = 180° ....(1)

Consider ΔAED,

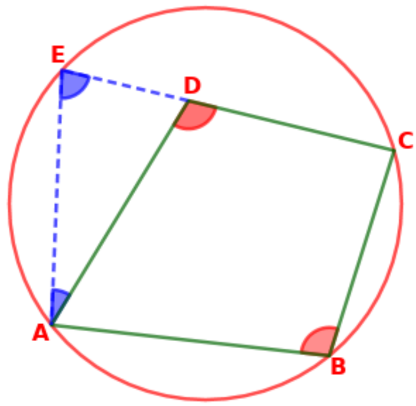
∠ADC is an outer angle of ΔAED.

∠ADC = ∠E +∠EAD

That is ,∠ADC > ∠E ....(2)

Apply equation (2) in equation (1)

Then we get ∠B +∠D > 180°

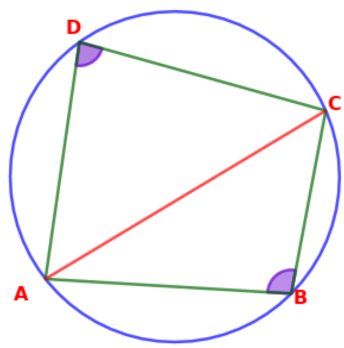


Case:III Forth vertex D is on the circle

If ∠B +∠D = 180°, D is on the circle.

In a quadrilateral, if the opposite angles are supplementary, we can draw a circle passing through the four vertices.

Such quadrilaterals are called **cyclic quadrilaterals.**



NOTE

If the four vertices of a quadrilateral are on a circle, that quadrilateral is called cyclic quadrilateral.

Cyclic quadrilaterals are those quadrilaterals with opposite angles are supplementary.

Examples:

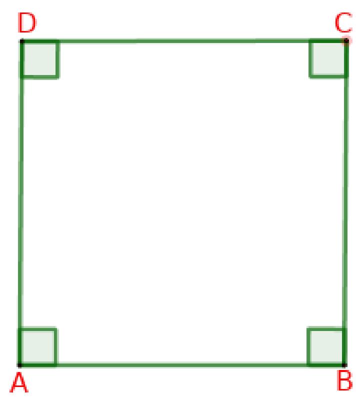
1. All squares are cyclic quadrilaterals

   All angles of a square are 90°.

   Since opposite angles are supplementary.

   ∴ Squares are always cyclic.

   (That means we can draw a circle passing through four vertices of a square)



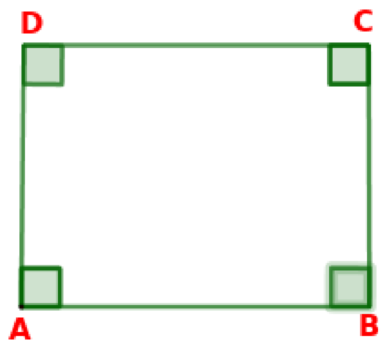
2. All rectangles are cyclic quadrilaterals

   All angles of a rectangle are 90°.

   Since opposite angles are supplementary.

   ∴ Rectangles are always cyclic.

   (That means we can draw a circle passing through four vertices of a rectangle)



ASSIGNMENT

ABCD is an isosceles trapezium. Check whether it is a cyclic quadrilateral.