

MATHEMATICS ONLINE CLASS X ON 09-08-2021

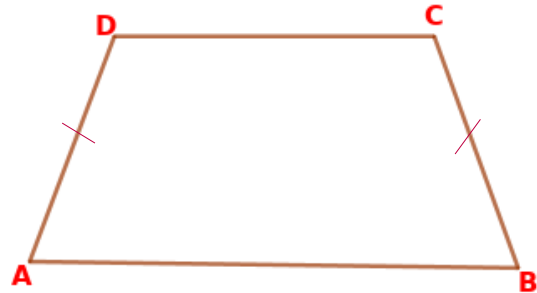
CIRCLES



Answer to assignment of previous class

Question

ABCD is an isosceles trapezium.
Check whether it is a cyclic quadrilateral.



Answer

ABCD is an isosceles trapezium.

$\therefore AD = BC$

Also, AB and CD are parallel.

We have to prove, $\angle A + \angle C = 180^\circ$ and $\angle B + \angle D = 180^\circ$

Since ABCD is an isosceles trapezium

$\angle A = \angle B$ (1)

Since $AB \parallel CD$ $\angle A + \angle D = 180^\circ$ (2)

From equations (1) and (2) $\angle B + \angle D = 180^\circ$

Sum of all angles of a quadrilateral is 360° .

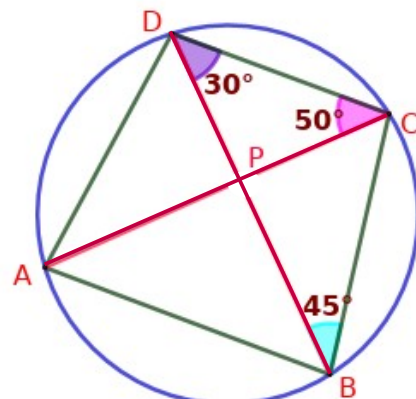
$\therefore \angle A + \angle C = 180^\circ$

That is, in the isosceles trapezium ABCD, opposite angles are supplementary.

Therefore it is cyclic.

Question

Calculate the angles of the quadrilateral in the picture and also the angles between their diagonals



Answer

Angles drawn from the end points of a chord to one part of a circle are equal.

That is, Angles in the same arc of a circle are equal.

$$\therefore \angle ACD = \angle ABD = 50^\circ$$

$$\angle BDC = \angle BAC = 30^\circ$$

$$\angle DBC = \angle DAC = 45^\circ$$

$$\text{In } \triangle ABC, \angle ABC = 50^\circ + 45^\circ = 95^\circ$$

Sum of angles of a triangle is 180°

$$\begin{aligned} \therefore \angle ACB &= 180^\circ - (30^\circ + 95^\circ) \\ &= 180^\circ - 125^\circ = 55^\circ \end{aligned}$$

Then,

$$\angle ACB = \angle ADB = 55^\circ$$

\therefore Angles of quadrilateral ABCD are

$$\angle A = 45^\circ + 30^\circ = 75^\circ$$

$$\angle B = 50^\circ + 45^\circ = 95^\circ$$

$$\angle C = 55^\circ + 50^\circ = 105^\circ$$

$$\angle D = 55^\circ + 30^\circ = 85^\circ$$

Let the chords AC and BD intersect at P.

$$\text{In } \triangle APD, \angle APD = 180^\circ - (45^\circ + 55^\circ) = 180^\circ - 100^\circ = 80^\circ$$

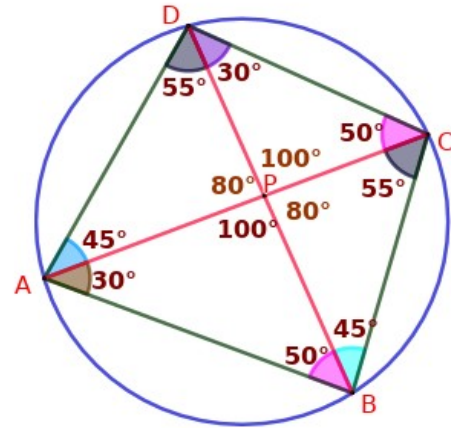
$\angle APD$ and $\angle CPD$ makes a linear pair.

$$\therefore \angle CPD = 180^\circ - 80^\circ = 100^\circ$$

When two lines intersect each other, opposite angles are equal.

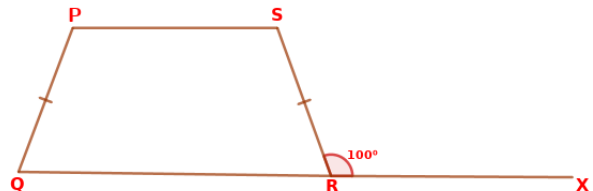
$$\therefore \angle APB = 100^\circ \text{ and } \angle CPB = 80^\circ$$

That is, Angles between the diagonals are 100° and 80°



Assignments

1. PQRS is an isosceles trapezium and QR extended to X. If $\angle SRX = 100^\circ$. Find all angles of PQRS.



2. Prove that any non-isosceles trapezium is not cyclic.