CHAPTER FIFTEEN
WAVES

WAVE MOTION

- The motion of a disturbance from one point to another by the vibrations of the particles of the medium about their mean position is known as **wave motion**.
- It is a **mode of transfer of energy** from one point to another.
- The waves are mainly of three types:
  (a) ***mechanical waves,***
  (b) ***electromagnetic waves and***
  (c) matter waves.

Mechanical waves

- **Exist only within a material medium**, such as water, air, and rock
- examples : - water waves, sound waves, seismic waves, etc
- two types : - **1) transverse waves 2) longitudinal waves**

Electromagnetic waves

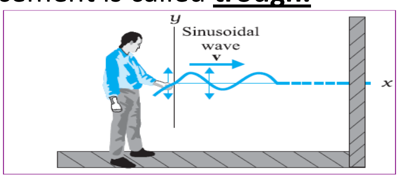
- The electromagnetic waves **do not require any medium** for their propagation.
- All electromagnetic waves travel through vacuum at the same speed *c,* given by **$c = 299, 792{,}458 \text{ m s}^{-1}$.**
- Examples of electromagnetic waves are **visible and ultraviolet light, radio waves, microwaves, x-rays** etc.

Matterwaves

- Matter **waves are associated with moving electrons, protons, neutrons and other fundamental particles**, and even atoms and molecules
- Matter waves associated with electrons are employed **in electron microscopes**

TRANSVERSE WAVES

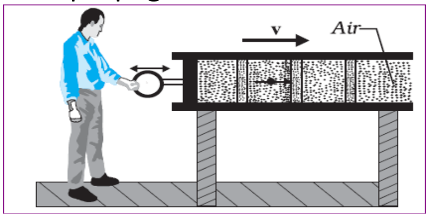
- In transverse waves, the **constituents of the medium oscillate perpendicular to the direction** of wave propagation.
- A point of maximum positive displacement in a wave is called **crest***, and a* point of maximum negative displacement is called **trough.**



- Transverse waves can be propagated **only through solids and strings**, and not in fluids.

LONGITUDINAL WAVES

- In longitudinal waves **the constituents of the medium oscillate along the direction** of wave propagation**.**



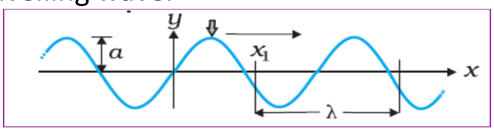
- Longitudinal sound waves propagates as **compressions(**high pressure region) and **rarefactions**(low pressure regions)
- longitudinal waves **can propagate in all elastic media** (solids and fluids)
- transverse and longitudinal waves travel with ***different speeds in the same medium.***

The waves on the surface of water

- The waves on the surface of water are of two kinds: **capillary waves and gravity waves.**
- **Capillary waves** are ripples of short wavelength.
- The restoring force that produces **capillary waves** is the surface tension of water.
- **Gravity waves** have wavelengths typically ranging from several metres to several hundred metres.
- The restoring force that produces gravity waves is the pull of gravity, which tends to keep the water surface at its lowest level.
- The waves in an ocean are a combination of both longitudinal and transverse waves.

Travelling or progressive wave

- A wave which **travels from one point of the medium to another** is called a travelling wave.



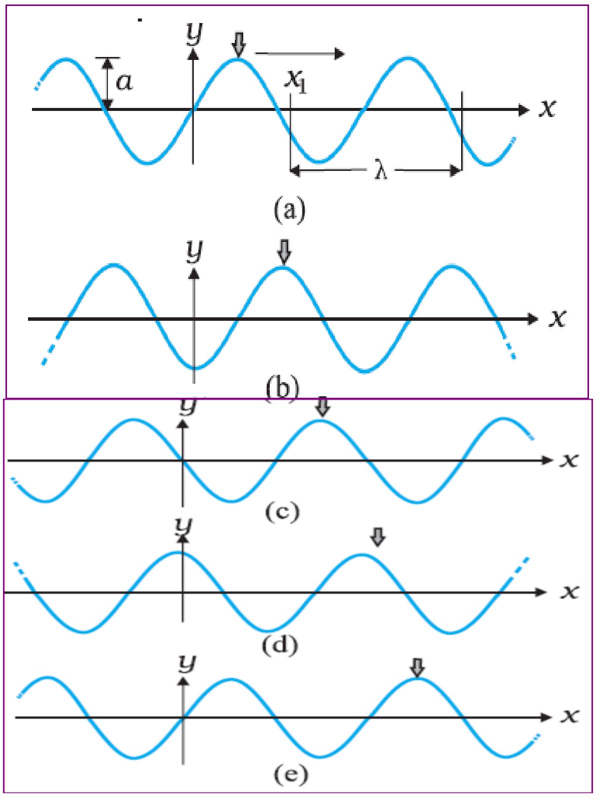
DISPLACEMENT RELATION IN A PROGRESSIVE WAVE

- *At any time t, the* displacement of a wave travelling in positive x-axis is given by

$$y(x, t) = a \sin(kx - \omega t + \phi)$$

- Where , **a- amplitude , k- angular wave number or propagation constant , ω- angular frequency , ϕ- initial phase angle and (kx- ωt+ ϕ) - phase**

Plots for a wave travelling in the positive direction of an x-axis at different values of time t.



- A **wave travelling in the negative direction of *x-axis can be represented by***

$$y(x, t) = a \sin (kx + \omega t + \phi)$$

Amplitude

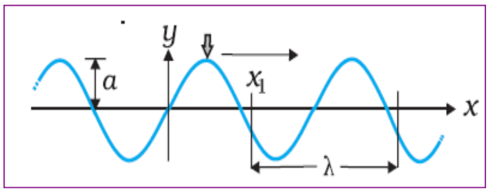
- The **amplitude *a of a wave*** is the **magnitude of the maximum displacement** of the elements from their equilibrium positions as the wave passes through them.
- It is a positive quantity, even if the displacement is negative.

Phase

- It describes the state of motion as the wave sweeps through a string element at a particular position *x*
- The constant ϕ is called the **initial phase angle.**
- **The** value of ϕ is determined by the initial (*t = 0)* displacement and velocity of the element (say, at *x = 0).*

Wavelength (λ)

- It is the minimum distance between two consecutive troughs or crests or two consecutive points in the same phase of wave motion.



Propagation constant *or the* angular wave number (k)

- For *t = 0* and ϕ = 0

$$y(x, 0) = a \sin kx$$

- By definition, the displacement *y is same at* both ends of this wavelength, that is at $x = x_1$ and at $x = x_1 + \lambda$.
- *Thus*

$$\begin{aligned} a \sin kx_1 &= a \sin k(x_1 + \lambda) \\ &= a \sin (kx_1 + k\lambda) \end{aligned}$$

- This condition can be satisfied only when,

$$k\lambda = 2\pi n$$

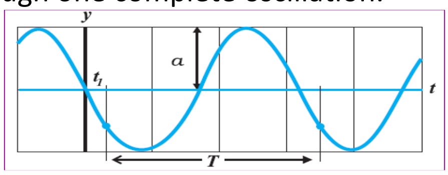
- where *n = 1, 2, 3... Since λ is defined as the least* distance between points with the same phase, *n =1 and therefore*

$$k = \frac{2\pi}{\lambda}$$

- *k is called the **propagation constant or the*** **angular wave number ; its SI unit is radian per** metre or rad m^{-1}

Period

- The **period of oscillation *T of a wave is*** the time any string element takes to move through one complete oscillation.



Angular Frequency

- The **angular frequency of the wave is given by**

$$\omega = 2\pi / T$$

- Its SI unit is rad s^{-1}.

Frequency

- It is the number of oscillations per unit time made by a string element as the wave passes through it
- The **frequency *v of a wave is defined as 1/T*** and is related to the angular frequency ω by

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi}$$

- It is usually measured in **hertz**

Displacement relation of a longitudinal wave

- In a longitudinal wave, the displacement of an element of the medium is parallel to the direction of propagation of the wave.
- The displacement function for a longitudinal wave is written as,

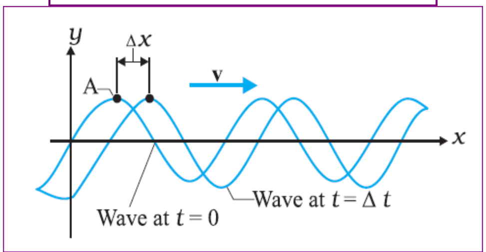
$$s(x, t) = a \sin(kx - \omega t + \phi)$$

- where *s(x, t) is the displacement of an element* of the medium in the direction of propagation of the wave at position *x and time t.*

THE SPEED OF A TRAVELLING WAVE

- The speed of a wave is related to its wavelength and frequency by the relation

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda \nu$$



- The speed **is determined by the properties of the medium.**

Speed of a Transverse Wave on Stretched String

- The speed of transverse waves on a string is determined by two factors,
  (i) the linear mass density or mass per unit length, μ, and
  (ii) (ii) the tension *T*.
- The linear mass density, μ, of a string is the mass *m of the string divided by its length l.* therefore its dimension is $[ML^{-1}]$.
- The tension *T* has the dimension of force $[M L T^{-2}]$.
- Let the speed $v = C \mu^a T^b$*, where c is a dimensionless constant.*
- *Taking dimensions on both sides*
  $[M^0L^1T^{-1}] = [M^1L^{-1}]^a[M L T^{-2}]^b = [M^{a+b}L^{-a+b}T^{-2b}]$
- Equating the dimensions on both sides we get
  a+b = 0 , therefore a=-b, -a+b = 1, therefore 2b=1 or b= ½ and a= - ½
- Thus $v = C \mu^{-1/2} T^{1/2}$ **, or**

$$v = C\sqrt{\frac{T}{\mu}}$$

- It can be shown that C=1, therefore the speed of transverse waves on a stretched string is

$$v = \sqrt{\frac{T}{\mu}}$$

- The **speed** of a wave along a stretched ideal string **depends** only on the **tension and the linear mass** density of the string and does not depend on the frequency of the wave.

Speed of a Longitudinal Wave - Speed of Sound

- In a longitudinal wave the constituents of the medium oscillate forward and backward in the direction of propagation of the wave.
- The sound waves travel in the form of compressions and rarefactions of small volume elements of air.
- The speed of sound waves depends on
  i) Bulk modulus , B and
  ii) Density of the medium, ρ
- Using dimensional analysis we may write
  $v = C B^a \rho^b$
- Taking dimensions $[M^0L^1T^{-1}] = [ML^{-1}T^{-2}]^a[M L^{-3}]^b = [M^{a+b}L^{-a-3b}T^{-2a}]$
- Equating the dimensions on both sides we get
  a+b = 0 , therefore a=-b, -2a=-1, a=1/2 , therefore b=-1/2
- Therefore

$$v = C\sqrt{\frac{B}{\rho}}$$

- where *C is a dimensionless constant and can* be shown to be unity.
- Thus the speed of longitudinal waves in a medium is given by,

$$v = \sqrt{\frac{B}{\rho}}$$

- The **speed of propagation of a longitudinal wave** in a fluid therefore **depends only on the bulk modulus and the density of the medium.**
- The bulk modulus is given by

$$B = -\frac{\Delta P}{\Delta V/V}$$

- Here *ΔV/V is the fractional change in volume* produced by a change in pressure *ΔP.*

Speed of sound wave in a material of a bar

- The speed of a longitudinal wave in the bar is given by,

$$v = \sqrt{\frac{Y}{\rho}}$$

- where ***Y** is the **Young's modulus of the material*** of the bar.

Speed of sound in different media

Medium	Speed (m s^{-1})
Gases	
Air (0 °C)	331
Air (20 °C)	343
Helium	965
Hydrogen	1284
Liquids	
Water (0 °C)	1402
Water (20 °C)	1482
Seawater	1522
Solids	
Aluminium	6420
Copper	3560
Steel	5941
Granite	6000
Vulcanised Rubber	54

Newton's Formula

- In the case of an ideal gas, the relation between pressure *P and volume V is given by*

$$PV = Nk_BT$$

- Therefore, **for an isothermal change** it follows that

$$V\Delta P + P\Delta V = 0$$

$$-\frac{\Delta P}{\Delta V/V} = P$$

- Thus **B=P**
- Therefore, the speed of a longitudinal wave in an ideal gas is given by,

$$v = \sqrt{\frac{P}{\rho}}$$

- This relation was first given by Newton and is known as **Newton's formula.**

Laplace correction

- According to Newton's formula for the speed of sound in a medium, we get for the speed of sound in air at STP,

$$v = \left[\frac{1.01\times10^5\ \mathrm{N\ m^{-2}}}{1.29\ \mathrm{kg\ m^{-3}}}\right]^{1/2} = 280\ \mathrm{m\ s^{-1}}$$

- This is about 15% smaller as compared to the experimental value of **331 m s**$^{-1}$
- Laplace pointed out that the **pressure variations in the propagation of sound waves are adiabatic** and not isothermal.
- For adiabatic processes the ideal gas satisfies the relation,

$$PV^{\gamma} = \text{constant} \qquad \text{i.e.} \qquad \Delta(PV^{\gamma}) = 0$$

$$P\gamma V^{\gamma-1}\Delta V + V^{\gamma}\Delta P = 0$$

- Thus for an ideal gas the adiabatic bulk modulus is given by,

$$B_{ad} = -\frac{\Delta P}{\Delta V/V} = \gamma P$$

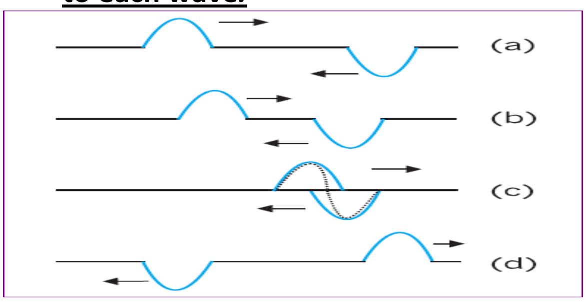
- where γ is the ratio of two specific heats, Cp/Cv.
- The speed of sound is, therefore, given by,

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

- This modification of Newton's formula is referred to as the **Laplace correction.**
- For **air** γ = 7/5,therefore the speed of sound in air at STP, we get a value ***331.3 m s***$^{-1}$, which agrees with the measured speed.

THE PRINCIPLE OF SUPERPOSITION OF WAVES

- The principle of super position of waves states that the **net displacement at a given time of a number of waves is the algebraic sum of the displacements due to each wave.**



- Let ***y**$_1$**(x, t)** and **y**$_2$**(x, t)** be the* displacements that any element of the string would experience if each wave travelled alone.
- The displacement ***y (x,t)** of an element of the* string when the waves overlap is then given by,

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

- Let a wave travelling along a stretched string be given by,

$$y_1(x, t) = a\sin(kx - \omega t)$$

- And another wave, shifted from the first by a phase ϕ,

$$y_2(x, t) = a\sin(kx - \omega t + \phi)$$

- Both the waves have the same angular frequency, same angular wave number *k* (same wavelength) and the same amplitude *a.*
- Applying the superposition principle

$$y\ (x, t) = a \sin (kx - \omega t) + a \sin (kx - \omega t + \phi)$$

- Using the trigonometric relation

$$\sin \alpha + \sin \beta = 2 \sin \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

$$y\ (x, t) = [2a \cos \frac{1}{2}\phi\ ] \sin (kx - \omega t + \frac{1}{2}\phi\ )$$

- Thus the resultant wave is also a sinusoidal wave, travelling in the positive direction of *x-axis.*
- The resultant wave differs from the constituent waves in two respects:

i) its phase angle is (½)ϕ and

II) its amplitude is the quantity given by

$$A(\phi) = 2a \cos (½)\phi$$

- If ϕ *= 0,the amplitude of the resultant wave is 2a, which is the largest possible value of A(φ).*
- *If φ = π, the two waves are completely out of phase, the amplitude of the resultant reduces to zero.*

REFLECTION OF WAVES

- When a pulse or a travelling wave encounters a rigid boundary it gets reflected.
- If the boundary is not completely rigid or is an interface between two different elastic media, a part of the wave is reflected and a part is transmitted into the second medium.
- ***The incident*** and refracted waves obey **Snell's law** of refraction, and the incident and reflected waves obey the laws of reflection.
- **A travelling wave, *at a rigid boundary or a closed end*, is reflected with a *phase reversal.***
- **A travelling wave ,at an *open boundary* is reflected *without any phase change.***
- Let the incident wave be represented by

$$y_i(x, t) = a \sin (kx - \omega t)$$

- then, for reflection at a **rigid boundary** the reflected wave is represented by,

$$y_r(x, t) = a \sin (kx + \omega t + \pi)$$
$$= - a \sin (kx + \omega t)$$

- For reflection at an open boundary, the reflected wave is represented by

$$y_r (x, t) = a \sin (kx + \omega t)$$

Standing Waves and Normal Modes

- The waveform or the disturbance does not move to either side is known as stationary wave or standing wave.
- Let the wave travelling in the positive direction of *x-axis be*

$$y_1(x, t) = a \sin (kx - \omega t)$$

- And the wave travelling in the negative direction of *x-axis*

$$y_2(x, t) = a \sin (kx + \omega t)$$

- The principle of superposition gives, for the combined wave

$$y\ (x, t) = y_1(x, t) + y_2(x, t)$$
$$= a \sin (kx - \omega t) + a \sin (kx + \omega t)$$
$$= (2a \sin kx) \cos \omega t$$

- The amplitude is zero for values of *kx that* give sin *kx = 0 . Those values are given by*

$$kx = n\pi, \quad \text{for } n = 0, 1, 2, 3, \ldots$$

- Substituting *k = 2π/λ in this equation, we get*

$$x = n\,\frac{\lambda}{2}\ , \text{for } n = 0, 1, 2, 3, \ldots$$

NODES

- The positions of zero amplitude in a standing wave are called **nodes.**
- A distance of λ/2 or half a wavelength separates two consecutive nodes.
- The amplitude has a maximum value of 2*a,* which occurs for the values of *kx that give* $|\sin kx|$ *= 1.*
- *The values are*

$$kx = (n + ½)\,\pi \text{ for } n = 0, 1, 2, 3, \ldots$$

- Substituting *k = 2π/λ in this equation, we get*

$$x = (n + ½)\frac{\lambda}{2} \text{ for } n = 0, 1, 2, 3, \ldots$$

ANTINODES

- The positions of maximum amplitude are called **antinodes.**
- ***The antinodes are* separated by λ/2 *and are located half way* between pairs of nodes.**

STANDING WAVES ON A STRETCHED STRING

- For a stretched string of length L, fixed at both ends, the **two ends of the string have to be nodes.**
- If one of the ends is chosen as position *x = 0, then the other end is x = L. In order that* this end is a node; the length *L must satisfy the* condition

$$L = n\,\frac{\lambda}{2},\ \text{for } n = 1, 2, 3, \ldots$$

❖ The standing waves on a string of length *L have restricted wavelength* given by

$$\lambda = \frac{2L}{n},\ \text{for } n = 1, 2, 3, \ldots \text{ etc.}$$

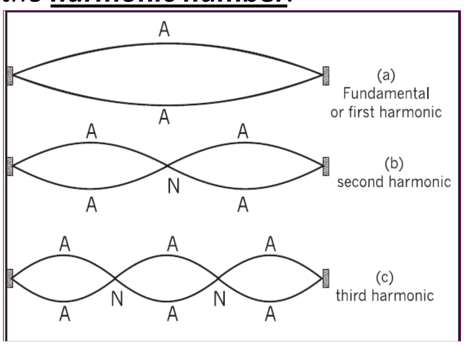
- The frequencies corresponding to these wavelengths is given by

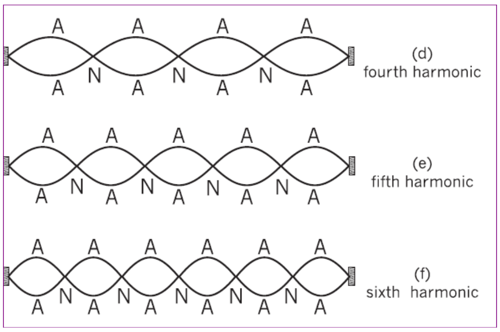
$$\nu = n\,\frac{v}{2L},\ \text{for } n = 1, 2, 3, \ldots \text{ etc.}$$

- where *v is the speed of travelling waves on the* string.
- **The set of frequencies possible in a standing wave are called the <u>natural frequencies or modes</u> of oscillation of the system.**
- The frequency corresponding to n=1 is

$$\nu = \frac{v}{2L}$$

- The oscillation mode with this lowest frequency (n=1) is called the **<u>fundamental mode or the first harmonic.</u>**
- The **<u>second harmonic</u>** is the oscillation mode with ***n = 2.*** *The **<u>third harmonic</u>*** corresponds to ***n = 3*** *and so on.*
- *The frequencies* associated with these modes are often labelled as *v1, v2, v3 and so on.*
- The **<u>collection of all possible modes</u>** is called the **<u>harmonic series</u>** and *n is called the **<u>harmonic number</u>**.*





Modes of vibration of a pipe closed at one end

- In a closed pipe standing waves are formed such that a node at the closed end and antinode at open end.
- Now if the length of the air column is *L, then the open end, x = L, is an* antinode and therefore,

$$L = (n + \frac{1}{2})\frac{\lambda}{2}$$

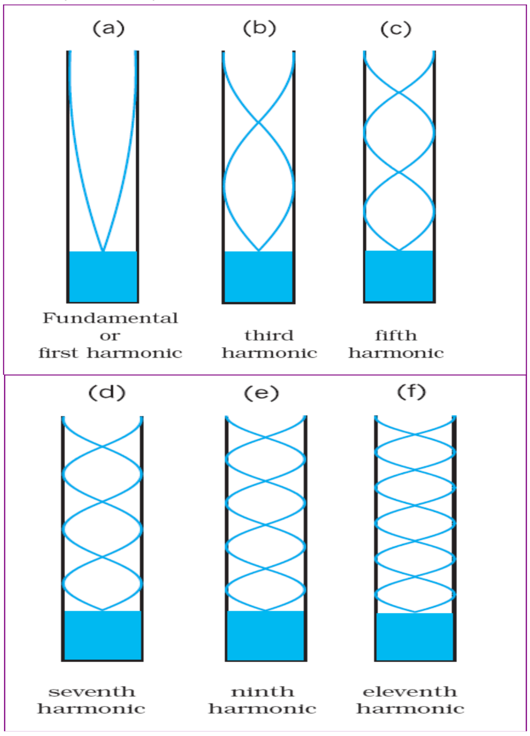
- Where n=0,1,2,3….
- The modes, which satisfy the condition

$$\lambda = \frac{2L}{(n + 1/2)},\ \text{for } n = 0, 1, 2, 3, \ldots$$

- The corresponding frequencies of various modes of such an air column are given by,

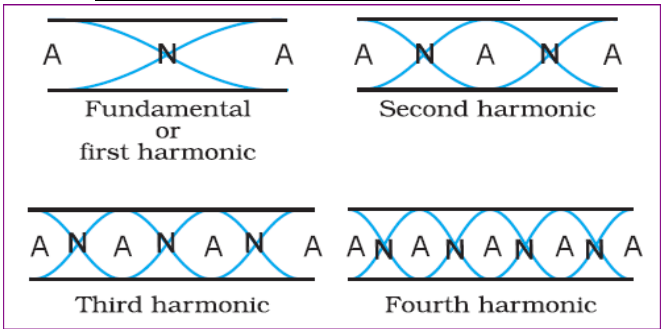
$$\nu = (n + \frac{1}{2})\frac{v}{2L},\ \text{for } n = 0, 1, 2, 3, \ldots$$

- The **<u>fundamental frequency is *v/4L*</u>** and the higher frequencies are **<u>odd harmonics</u>** of the fundamental frequency, i.e. 3 *v/4L,5 v/4L,…*



Pipe open at both ends

- In the case of a pipe open at both ends, there will be antinodes at both ends, and **all harmonics will be generated.**

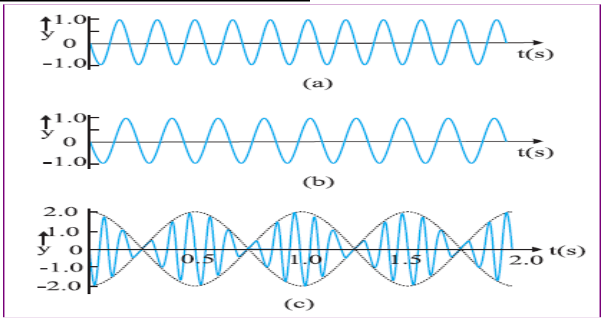


BEATS

- **The phenomenon of wavering of sound intensity when two waves of nearly same frequencies and amplitudes travelling in the same direction, are superimposed on each other is called beats.**
- **The beat frequency,** is given by

$$\nu_{beat} = \nu_1 - \nu_2$$

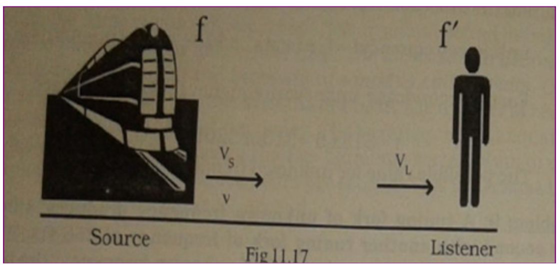
The time-displacement graphs of two waves of frequency 11 Hz and 9 Hz



- Musicians use the beat phenomenon in **tuning** their instruments.
- If an instrument is sounded against a standard frequency and tuned until the beat disappears, then the instrument is in tune with that standard.

DOPPLER EFFECT

- ***The apparent change in the pitch or the frequency of sound produced by a source due to relative motion of the source, listener or the medium is called Doppler effect.***



- It was proposed by Christian Doppler and tested experimentally by Buys Ballot
- All types of waves shows Doppler effect.
  - S- source
  - f – frequency of sound from source
  - V – velocity of sound
  - λ- wavelength

When Source and Listener at Rest

- When the source and the listener are at rest , the frequency of sound heard by the listener

$$f = \frac{V}{\lambda} \quad \text{or} \quad \lambda = \frac{V}{f}$$

When source and listener moving in the direction of sound

- The relative velocity of sound wave with respect to source is V – Vs
- Vs - velocity of source
- Thus apparent wavelength is

$$\lambda' = \frac{V - V_S}{f}$$

- The relative velocity of sound with respect to listener is $V' = V - V_L$
- The apparent frequency of sound heard by the listener is

$$f' = \frac{V'}{\lambda'}$$

- Thus

$$f' = \frac{V - V_L}{\frac{V - V_S}{f}} = f\frac{(V - V_L)}{(V - V_S)}$$

Special cases

Source moving and listener stationary

a) **Source moves towards the listener**

- Now V_S = +ve , V_L= 0
- Thus $f' = f\left(\frac{V}{V - V_S}\right)$

b) **Source moves away from the listener**

- Now V_S = - ve , V_L= 0
- Thus $f' = f\left(\frac{V}{V + V_S}\right)$

Source stationary , listener moving

a) **Listener moves towards the source**

- Now V_L = - ve , V_S= 0
- Thus $f' = f\left(\frac{V + V_L}{V}\right)$

b) <u>Listener moves away from the source</u>

- Now V_L = + ve , V_S= 0
- Thus $f' = f\left(\frac{V - V_L}{V}\right)$

<u>Both source and listener moving</u>

a) <u>Source and listener move towards each other</u>

- **Now V_S = +ve, V_L = -ve**
- **Thus** $f' = f\left(\frac{V + V_L}{V - V_S}\right)$

b) <u>Source and listener move away from each other</u>

- **Now V_S = -ve, V_L = +ve**
- **Thus** $f' = f\left(\frac{V - V_L}{V + V_S}\right)$

<u>c) Source moves towards the listener and listener moves away</u>

- **Now V_S = +ve, V_L = +ve**
- Thus $f' = f\left(\frac{V - V_L}{V - V_S}\right)$

d) <u>Source moves away from the listener and listener moves towards the source</u>

- **Now V_S = -ve , V_L = -ve**
- **Thus** $f' = f\left(\frac{V + V_L}{V + V_S}\right)$

<u>Effect of motion of the medium</u>

- When the wind blows the air medium will moves with a velocity **w**
- When wind moves towards the listener the velocity of sound is **V+ w**
- Thus the apparent frequency

$$f' = f\left(\frac{(V + w) - V_L}{(V + w) - V_S}\right)$$

- If the wind is blowing from listener to the source , velocity of sound is **V – w**
- Thus $f' = f\left(\frac{(V - w) - V_L}{(V - w) - V_S}\right)$

<u>Uses of Doppler Effect</u>

- To estimate the speed of submarine , aero plane, automobile , etc
- To track artificial satellite
- To estimate velocity and rotation of star
- Doctors use it to study heart beats and blood flow in different part of the body. Here they use ulltrasonic waves, and in common practice, it is called **<u>sonography.</u>**
- In the case of heart, the picture generated is called **<u>echocardiogram.</u>**

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