ONLINE MATHS CLASS- X - 15 (23 / 07 /2021)

2 . CIRCLES – CLASS - 3 part 2

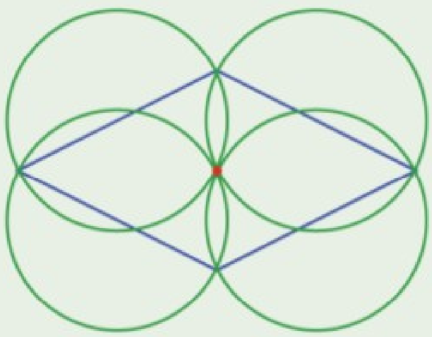
What did we study in the last classes ?

Important points

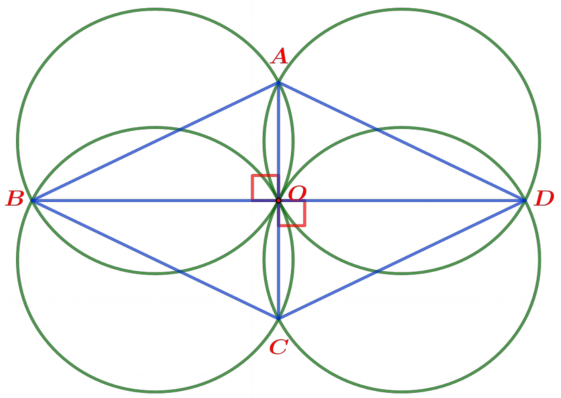
- **If we join the ends of a diameter of a circle to a point on the circle , we get a right angle .**
- **Angle in a semicircle is right**
- **If a pair of lines drawn from the ends of a diameter of a circle are perpendicular to each other , then they meet on the circle .**
- **The angle formed by joining the end points of the diameter of a circle to a point inside the circle is greater than 90º , on the circle is 90 º and outside the circle is less than 90º**

Activity 1

Prove that all four circles drawn with the sides of a rhombus as diameters pass through a common point.



Answer .



Answer .

In the figure ABCD is a rhombus and its diagonals intersect at O .

∠ AOB = ∠ BOC = ∠ COD = ∠ AOD = 90º

(The diagonals of a rhombus are perpendicular to each other)

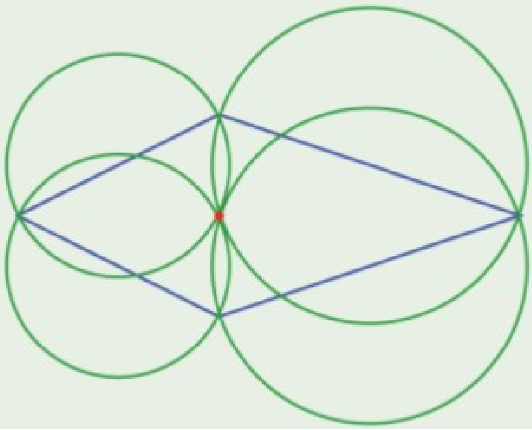
The circle drawn with AB as diameter passes through the point O . (If a pair of lines drawn from the ends of a diameter of a circle are perpendicular to each other , then they meet on the circle)

Similarly circles drawn with BC , CD and AD as diameters pass through the point O

Hence all four circles drawn with the sides of a rhombus as diameters pass through a common point .

Activity 2

Prove that this is true for any quadrilateral with adjacent sides equal, as in the picture.

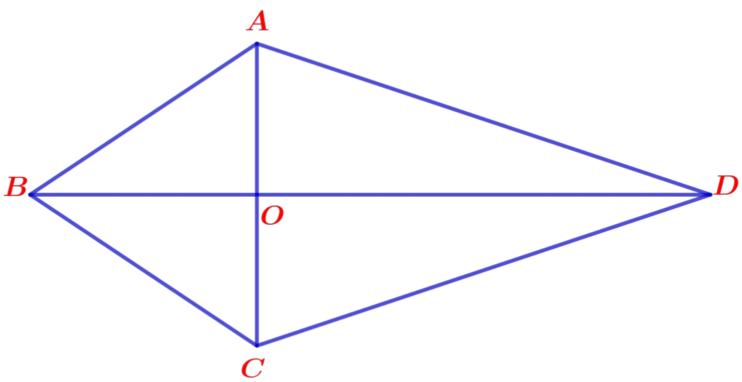


Answer .

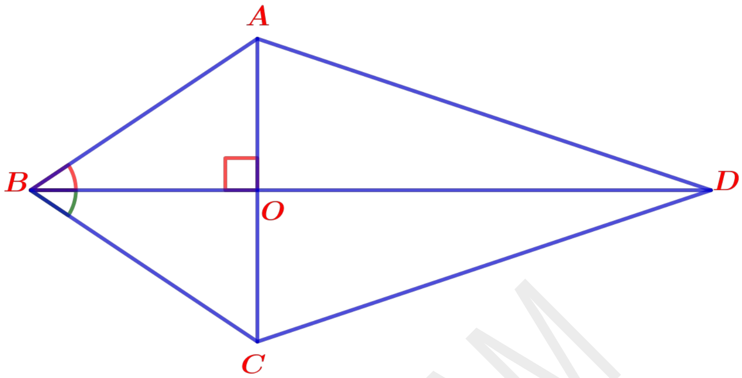
In the figure AB = BC , AD = CD

The triangles ABD and BCD are equal

(AB = BC , AD = CD , BD = BD)



Therefore ∠ ABD = ∠ CBD (Angles opposite to equal sides of equal triangles are equal)



Also the triangles AOB and BOC are equal

(AB = BC , BO = BO , ∠ABO = ∠CBO)

Therefore ∠ AOB = ∠ BOC

Also , ∠ AOB + ∠ BOC = 180º (linear pair)

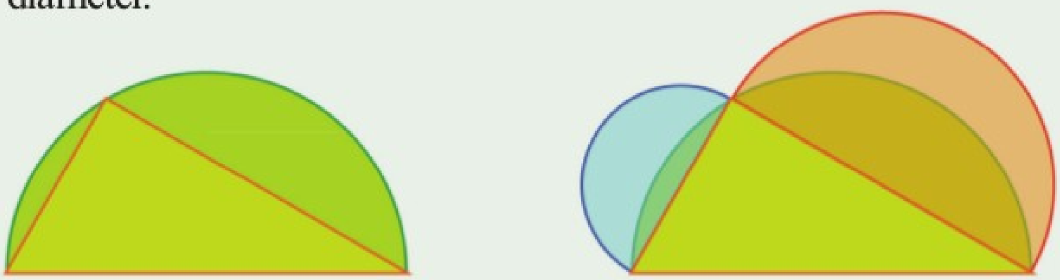
Therefore ∠ AOB = ∠ BOC = 90 º

The circle drawn with AB as diameter passes through the point O . (If a pair of lines drawn from the ends of a diameter of a circle are perpendicular to each other , then they meet on the circle)

Similarly circles drawn with BC , CD and AD as diameters pass through the point O .

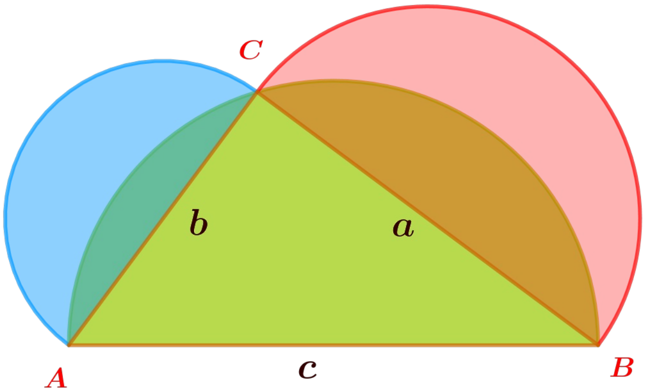
Activity 3

A triangle is drawn by joining a point on a semicircle to the ends of the diameter. Then semicircles are drawn with the other two sides as diameter.



Prove that the sum of the areas of the blue and red crescents in the second picture is equal to the area of the triangle.

Answer .



In the figure C is a point on the with diameter AB .

∠ACB = 90º (Angle in a semicircle is right)

If we take , BC = a , AC = b , AB = c , in triangle ABC

$BC^2 + AC^2 = AB^2 \implies a^2 + b^2 = c^2$ **(Pythagoras theorem)**

Area of the semicircle with diameter BC $= \frac{1}{2}\pi \times \left(\frac{a}{2}\right)^2 = \frac{1}{2}\pi \times \frac{a^2}{4} = \frac{\pi a^2}{8}$

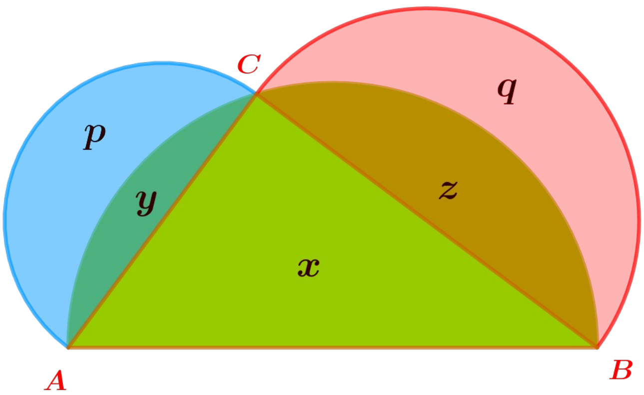
Area of the semicircle with diameter AC $= \frac{1}{2}\pi \times \left(\frac{b}{2}\right)^2 = \frac{1}{2}\pi \times \frac{b^2}{4} = \frac{\pi b^2}{8}$

Area of the semicircle with diameter AB $= \frac{1}{2}\pi \times \left(\frac{c}{2}\right)^2 = \frac{1}{2}\pi \times \frac{c^2}{4} = \frac{\pi c^2}{8}$

Area of the semicircle with diameter BC + Area of the semicircle with diameter AC

$$= \frac{\pi a^2}{8} + \frac{\pi b^2}{8} = \frac{\pi a^2 + \pi b^2}{8} = \frac{\pi (a^2 + b^2)}{8} = \frac{\pi c^2}{8}$$

= Area of the semicircle with diameter AB



Let's take the area of the triangle as x , area of the blue crescent as p , area of the red crescent as q and the areas of the region common to the semicircles as y and z

Area of the semicircle with diameter AC + Area of the semicircle with diameter BC

= Area of the semicircle with diameter AC

$$\Rightarrow \quad (p + y) + (q + z) = x + y + z$$

$$p + q + y + z = x + y + z$$

$$p + q = x$$

That is ,

Area of the blue crescent + Area of the red crescent = Area of the triangle