

MATHEMATICS ONLINE CLASS X ON 11-08-2021

CIRCLES

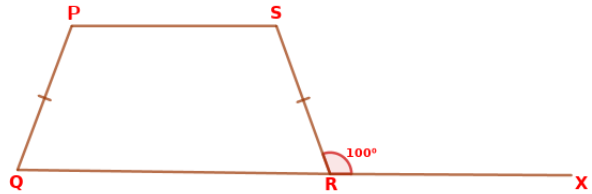
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Answer to assignment of previous class

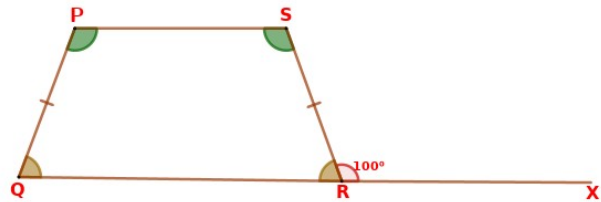
Question

- 1) PQRS is an isosceles trapezium and QR extended to X. If $\angle SRX = 100^\circ$. Find all angles of PQRS.



Answer

Given $\angle SRX = 100^\circ$
 $\angle SRX + \angle SRQ = 180^\circ$
[linear pair]
 $100^\circ + \angle SRQ = 180^\circ$
 $\angle SRQ = 180^\circ - 100^\circ = 80^\circ$



Since PQRS is an isosceles trapezium in which $PQ = SR$
 $\angle SRQ = \angle PQR = 80^\circ$
All isosceles trapeziums are cyclic
 $\angle QPS = 180^\circ - \angle QRS = 180^\circ - 80^\circ = 100^\circ$
Also $\angle PSR = 180^\circ - \angle PQR = 180^\circ - 80^\circ = 100^\circ$
Angles of PQRS are $\angle P = 100^\circ$, $\angle Q = 80^\circ$, $\angle R = 80^\circ$, $\angle S = 100^\circ$

Question

- 2) Prove that any non-isosceles trapezium is not cyclic.

Answer

In a trapezium, a pair of opposite sides are parallel.
Here, $AB \parallel CD$.
ABCD is a non-isosceles trapezium.

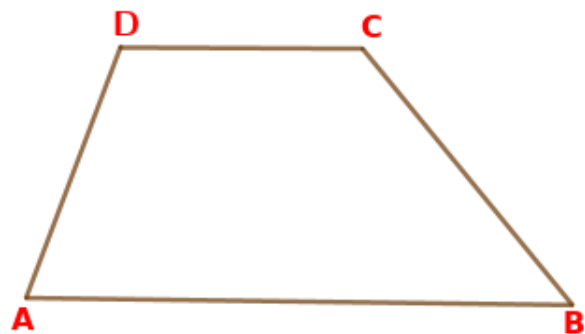
$\therefore \angle A \neq \angle B$ (1)

Since $AB \parallel CD$,

$\angle A + \angle D = 180^\circ$ (2)

From equations (1) and (2)

$\angle B + \angle D \neq 180^\circ$



$$\therefore \angle A + \angle C \neq 180^\circ$$

Since opposite angles are not supplementary, a non-isosceles trapezium ABCD is not cyclic.

Question

Prove that any outer angle of a cyclic quadrilateral is equal to the inner angle at the opposite vertex

Answer

We have to prove $\angle CBE = \angle ADC$.

ABCD is a cyclic quadrilateral, its opposite angles are supplementary.

$$\therefore \angle ABC + \angle ADC = 180^\circ \dots (1)$$

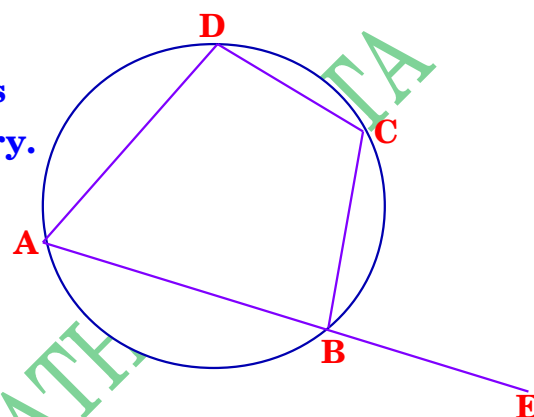
$\angle ABC$ and $\angle CBE$ are linear pairs.

$$\therefore \angle ABC + \angle CBE = 180^\circ \dots (2)$$

From equations (1) and (2)

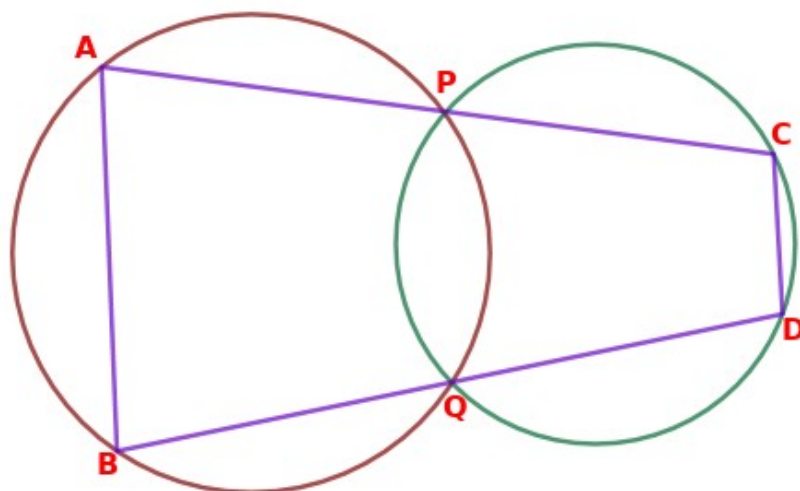
$$\cancel{\angle ABC} + \angle CBE = \cancel{\angle ABC} + \angle ADC$$

We get $\angle CBE = \angle ADC$

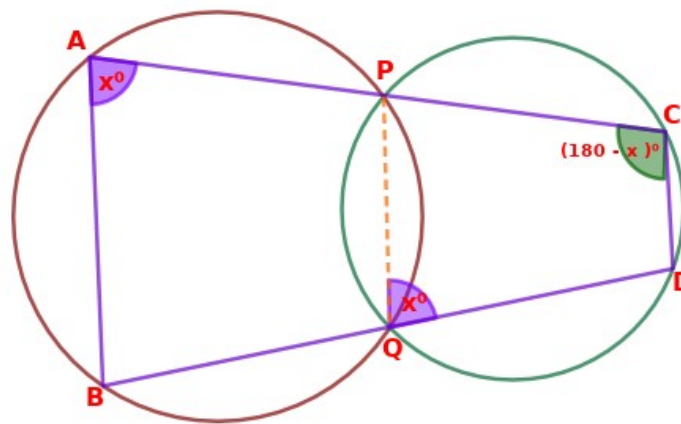


Question

The two circles below intersect at P, Q and lines through these points meet the circles at A, B, C, D. The lines AC and BD are not parallel. Prove that if these lines are of equal length, then ABDC is a cyclic quadrilateral.



Answer



In the figure, Join PQ.

ABQP and QDCP are cyclic quadrilaterals.

Let $\angle A = x^\circ$, then

$\angle P Q D = x^\circ$ (Any outer angle of a cyclic quadrilateral is equal to the inner angle at the opposite vertex)

$\angle C = 180 - x^\circ$ (Opposite angles of a cyclic quadrilateral are supplementary)

$\angle A + \angle C = x^\circ + 180 - x^\circ = 180^\circ$.

AB and CD are parallel. ($\angle A$ and $\angle C$ are co-interior angles)

If $AC = BD$, ABDC become an isosceles trapezium.

We know that an isosceles trapezium is always cyclic.

\therefore ABDC is a cyclic quadrilateral.

ASSIGNMENT

In the picture, the circles on the left and right intersect the middle circle at P, Q, R, S ; the lines joining them meet the left and right circles at A, B, C, D . Prove that $ABDC$ is a cyclic quadrilateral.

