

CIRCLES



Two Chords

Draw two diameters of a circle.

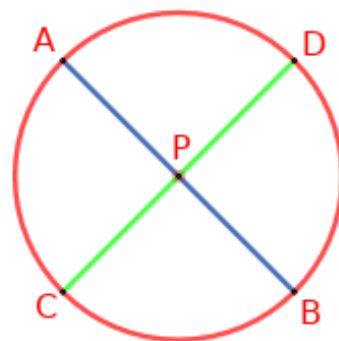
These two diameters intersect at the centre of the circle.

Diameters are chords of circle.

When they intersect we get four parts

PA, PB, PC and PD.

Here $PA = PB = PC = PD$ (Radii of the circle)



Draw two non diametrical chords AB and CD intersecting at a point P inside the circle.

Here also we get 4 parts PA, PB, PC and PD.

They are all not equal.

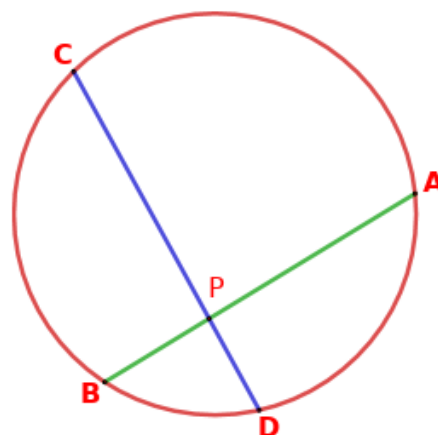
We have to find the relation between PA, PB, PC and PD.

Draw this picture in your notebook and measure the lengths of PA, PB, PC and PD.

Find the product of the parts of the chord AB (ie; $PA \times PB$)

Find the product of the parts of the chord CD (ie; $PC \times PD$)

We get $PA \times PB = PC \times PD$



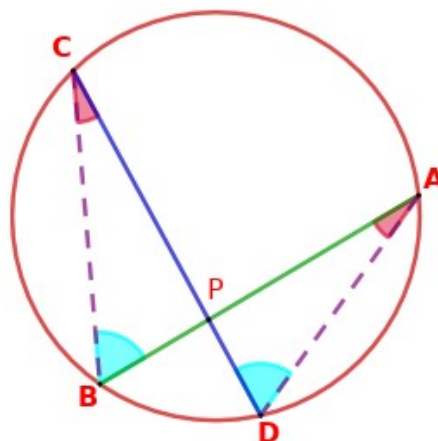
Proof :

In the figure, AB and CD are two chords of the circle intersecting at a point P inside the circle.

Join AD and BC. $\angle A$ and $\angle C$ are angles made by the arc BD at its alternate arc.

They are equal.

$\therefore \angle A = \angle C$



$\angle D$ and $\angle B$ are angles made by the arc AC at its alternate arc.
They are equal.

$$\therefore \angle B = \angle D$$

Consider $\triangle APD$ and $\triangle BPC$

All angles of $\triangle APD$ and $\triangle BPC$ are equal.

$\therefore \triangle APD$ and $\triangle BPC$ are similar.

In similar triangles, sides opposite to equal angles are proportional.

$$\frac{PC}{PA} = \frac{PB}{PD}$$

By cross multiplication, we get $PA \times PB = PC \times PD$

If two chords of a circle intersect within the circle then the products of the parts of the two chords are equal.

Question

In the figure, $PB = 2$ cm, $PC = 3$ cm, $PD = 4$ cm.

Find the length of AB.

Answer

$$PB = 2 \text{ cm}$$

$$PC = 3 \text{ cm}$$

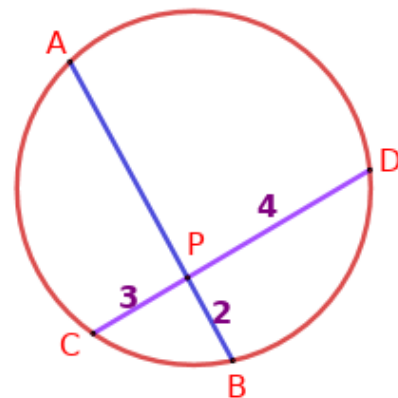
$$PD = 4 \text{ cm}$$

$$\text{We have } PA \times PB = PC \times PD$$

$$PA \times 2 = 3 \times 4 = 12$$

$$PA = \frac{12}{2} = 6 \text{ cm}$$

$$\therefore AB = PA + PB = 6 + 2 = 8 \text{ cm}$$

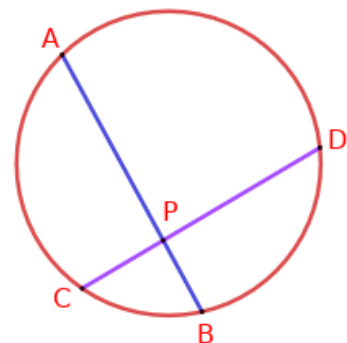


In the figure, $PA \times PB = PC \times PD$

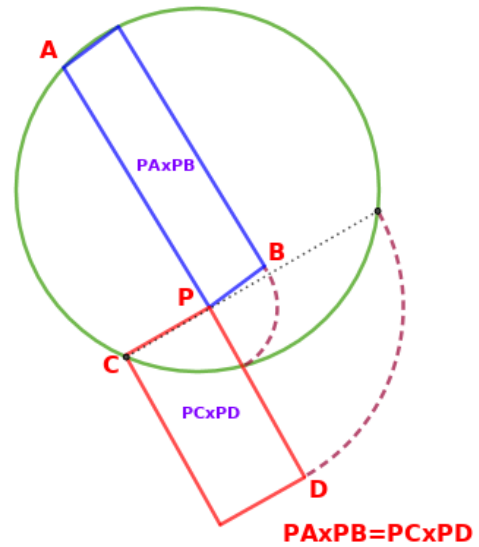
PA, PB, PC and PD are the lengths of parts of the chords AB and CD. Therefore, PA, PB, PC and PD are numbers.

$PA \times PB$ is the area of a rectangle of sides PA and PB

Similarly, $PC \times PD$ is the area of a rectangle of sides PC and PD



$PA \times PB = PC \times PD$ means area of these two rectangles are equal.



If two chords of a circle intersect within a circle, then the rectangles formed by the parts of the same chord have equal area.

ASSIGNMENT

The chords AB and CD intersect at a point P. If $PA = 9$ cm, $PD = 12$ cm, $AB = 13$ cm. Find the lengths of PB, PC and CD.

