Question of the day - 2

The sum of the s first 11 terms of an arithmetic sequence is 506 and the sum of its first 12 terms is 600 .

a) What is the 6^{th} term of this sequence ?

b) What is the 12^{th} term of this sequence ?

c) What is the sum of the first 17 terms of this sequence ?

Answer

a) $x_6 = \frac{506}{11} = 46$

b) $S_{12} - S_{11} = x_{12} = 600 - 506 = 94$

c) $S_{17} = 17 \times middle\ \ term = 17 \times \frac{(x_6 + x_{12})}{2} = 17 \times \frac{(46 + 94)}{2} = 17 \times 70 = 1190$

Question of the day - 3

10^{th} term of an arithmetic sequence is 30 and its 30^{th} term is 10 .

a) What is the common difference of this sequence ?

b) What is the 40^{th} term of this sequence ?

c) What is the 80^{th} term of this sequence ?

d) Sum of how many terms , starting from the first term of this sequence is zero ?

Answer

a) $d = \frac{10 - 30}{30 - 10} = -1$

b) $x_{40} = x_{10} + 30d = 30 + 30 \times (-1) = 0$

c) $x_{80} = x_{40} + 40d = 0 + 40 \times (-1) = -40$

d) $x_{40} = 0$ ==> $79 \times x_{40} = 0$ ==> $S_{79} = 0$ ==> **Sum of the first 79 terms is zero .**

Or

$x_1 = x_{40} - 39\ d = 0 - 39 \times (-1) = 39$, $x_{80} = -40$ ==> $x_{79} = -39$

$x_1 + x_{79} = 0$ ==> $S_{79} = 0$ ==> **Sum of the first 79 terms is zero .**

Question of the day - 4

The sum of the first 13 terms of an arithmetic sequence is 208 and the sum of the first 16 terms is 304 .

a) What is the 7th term of this sequence ?

b) What is the 15th term of this sequence ?

c) Find the sum of the terms from the 14th term to the 29th term of this sequence ?

Answer

a) $x_7 = \frac{208}{13} = 16$

b) $S_{16} - S_{13} = 304 - 208 = 96$ ===> $x_{14} + x_{15} + x_{16} = 96$ ===> $x_{15} = \frac{96}{3} = 32$

c) $S_{29} - S_{13} = 29 \times x_{13} - 13 \times x_7 = 29 \times 32 - 13 \times 16 = 720$

Question of the day - 5

The sum of the first 8 terms of an arithmetic sequence is 136 and the sum of the first 12 terms is 300 .

a) What is the sum of the first and the 8th terms ?

b) What is the sum of the first and the 12th terms ?

c) What is the number got by adding three times the first term to the 19th term ?

Answer

a) $x_1 + x_8 = \frac{136}{4} = 34$

b) $x_1 + x_{12} = \frac{300}{6} = 50$

c) $x_1 + x_{12} = 50 +$

$x_1 + x_8 = 34$

$2x_1 + x_{12} + x_8 = 84$ ==> $2x_1 + x_1 + x_{19} = 84$ ==> $3x_1 + x_{19} = 84$

Question of the day - 6

Consider the arithmetic sequence 4 , 12 , 20 , . . .

a) Prove that the sum of consecutive terms of this sequence (starting from the first term) is always a perfect square .

b) What is the difference between the sum of the first 20 terms and the next 20 terms of this sequence ?

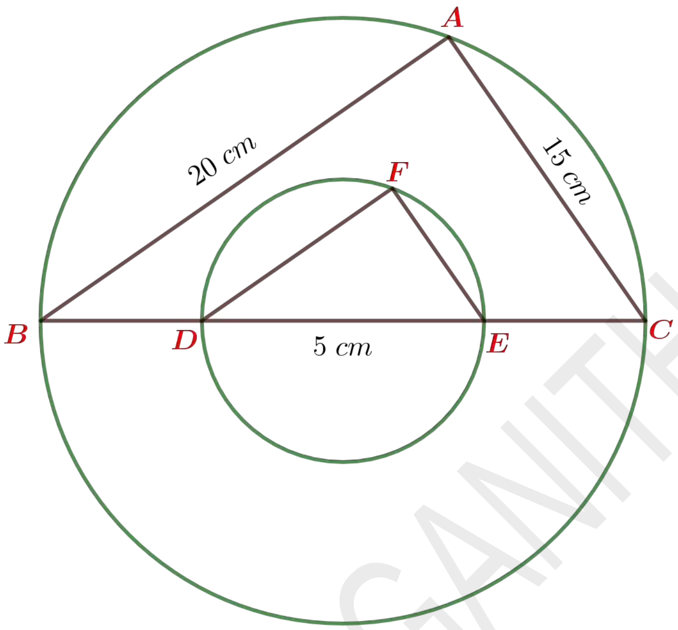
Answer

a) $x_n = 8n + 4 - 8 = 8n - 4$

Sum of the first n terms $= 8 \times \frac{n(n+1)}{2} - 4n = 4n^2 = (2n)^2$

b) $20 \times 20d = 20 \times 20 \times 8 = 3200$

Question of the day – 7



In the figure BC is the diameter of the larger circle and DE is the diameter of the smaller circle . AB is parallel to FD . AB = 20 cm . , AC = 15 cm , DE = 5 cm .

Calculate the area of triangle DFE .

Answer

∠ A = ∠ F = 90^0 (Angle in a semicircle)

$$BC = \sqrt{(20^2 + 15^2)} = 25 \ cm$$

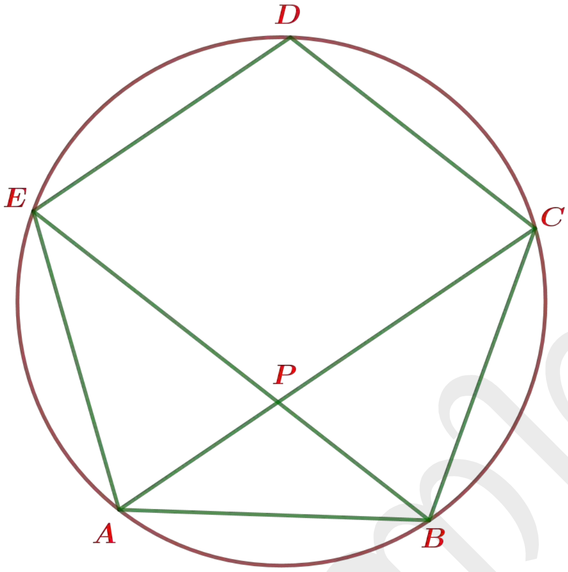
∠ B = ∠EDF (AB is parallel to FD , corresponding angles)

ABC and DEF are similar triangles .

$\frac{20}{DF} = \frac{15}{EF} = \frac{25}{5}$ **==> DF = 4 cm , EF = 3 cm .**

Area of triangle DFE = $\frac{1}{2} \times 4 \times 3 = 6 \ sq.cm$

Question of the day – 8

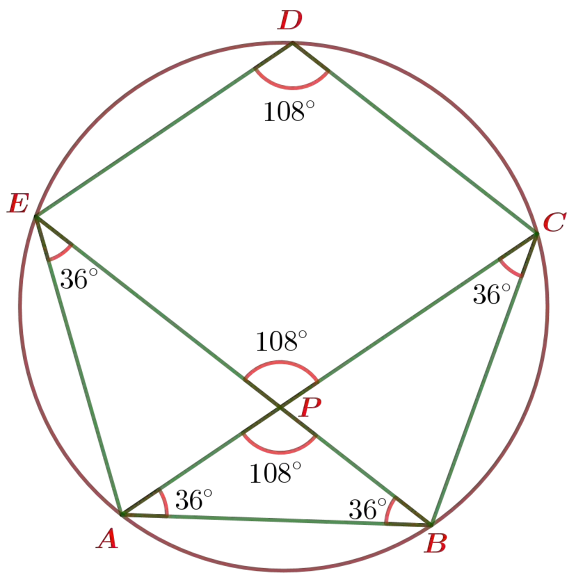


In the figure ABCDE is a regular pentagon . The diagonals AC and BE intersect at P .

a) What is the measure of ∠ APE ?

b) Check whether PCDE is a cyclic quadrilateral or not .

Answer



∠ BAE = ∠ ABC = ∠ CDE = $\frac{540}{5} = 108^0$

In isosceles triangle BAE ,

∠ AEB = ∠ ABE = $\frac{180-108}{2} = 36^0$

In isosceles triangle ABC ,

∠ BAC = ∠ ACB = $\frac{180-108}{2} = 36^0$

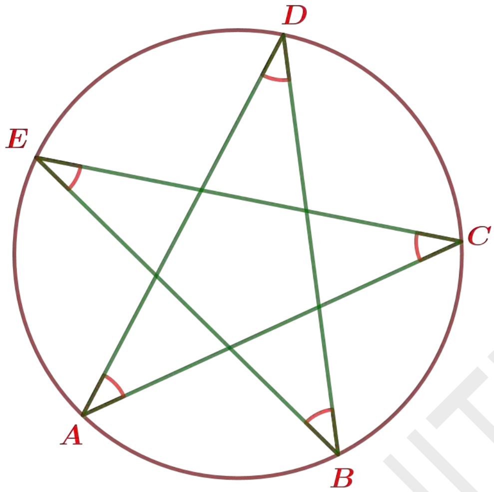
In triangle APB ,

∠ APB = $180 - (36 + 36) = 108^0$ **===> ∠ CPE** $= 108^0$

In quadrilateral PCDE , ∠ CDE + ∠ CPE $= 108 + 108 = 216^0$

Since the opposite angles are not supplementary , PCDE is not cyclic .

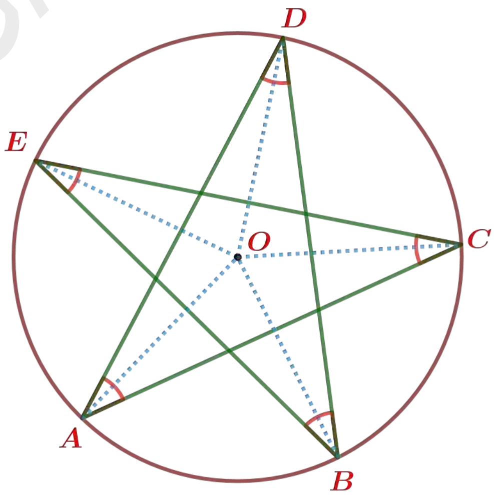
Question of the day – 9



What is the sum of the angles marked in the figure ? Justify your answer .

Answer

In the figure O is the centre of the circle .



$\angle ADB = \frac{1}{2}\angle AOB$

$\angle BEC = \frac{1}{2}\angle BOC$

$\angle CAD = \frac{1}{2}\angle COD$

$\angle DBE = \frac{1}{2}\angle DOE$

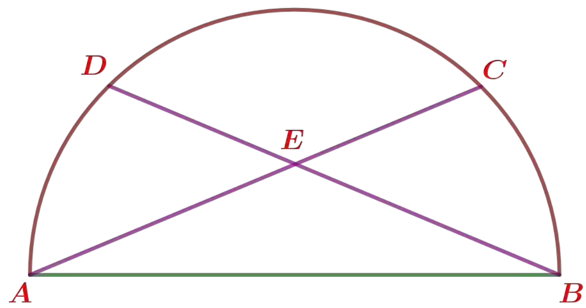
$\angle ACE = \frac{1}{2}\angle AOE$

$\angle ADB + \angle BEC + \angle CAD + \angle DBE + \angle ACE$

$$= \frac{1}{2}\angle AOB + \frac{1}{2}\angle BOC + \frac{1}{2}\angle COD + \frac{1}{2}\angle DOE + \frac{1}{2}\angle AOE$$

$$= \frac{1}{2}(\angle AOB + \angle BOC + \angle COD + \angle DOE + \angle AOE) = \frac{1}{2} \times 360 = 180^0$$

Question of the day – 10

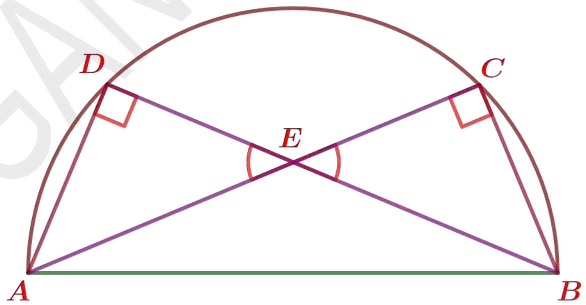


In the figure AB is the diameter of the semicircle . Two chords AC and BD intersect at E .

Prove that $(AC \times AE) + (BD \times BE) = AB^2$

Answer

∠ D = ∠ E = 90^0



In right triangle ABC , $AC^2 + BC^2 = AB^2$

In right triangle ADC , $AD^2 + BD^2 = AB^2$

In right triangle ADE , $AD^2 + DE^2 = AE^2$

In right triangle BCE , $BC^2 + CE^2 = BE^2$

Triangle ADE and triangle BCE are similar ==> $\frac{AE}{BE} = \frac{DE}{CE}$ **==>** $AE \times CE = BE \times DE$

$(AC \times AE) = (AE + CE) \times AE = AE^2 + AE \times CE$

$(BD \times BE) = (BE + DE) \times BE = BE^2 + BE \times DE$

$(AC \times AE) + (BD \times BE) = AE^2 + AE \times CE + BE^2 + BE \times DE$

$= AE^2 + AE \times CE + BE^2 + AE \times CE$

$= AE^2 + 2AE \times CE + BE^2 = AE^2 + 2AE \times CE + (BC^2 + CE^2)$

$= (AE^2 + 2AE \times CE + CE^2) + BC^2 = (AE + CE)^2 + BC^2$

$= AC^2 + BC^2 = AB^2$