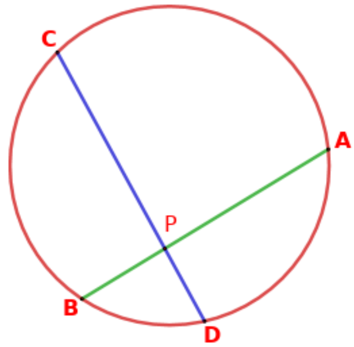
MATHEMATICS ONLINE CLASS X ON 26-08-2021

CIRCLES

click

Discussed in the previous class

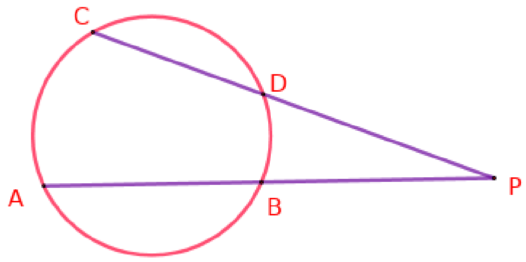
If two chords AB and CD intersecting at a point P inside the circle, then PA × PB = PC × PD



If two chords of a circle intersect within the circle then the product of the parts of the two chords are equal

Question

In the picture , chords AB and CD of the circle are extended to meet at P.



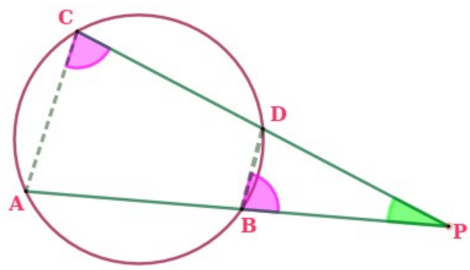
i) Prove that the angles of △APC and △ PBD formed by joining AC and BD, are the same.

ii) Prove that PA × PB = PC × PD.

iii) Prove that if PB = PD then ABDC is an isosceles trapezium.

Answer

i)



In the figure, join AC and BD.

Consider ΔPBD and ΔAPC.

∠P is common for both triangles.

∠PBD = ∠C (Outer angle of a cyclic quadrilateral is equal to the inner angle at the opposite vertex)

$\angle PDB = \angle A$ (Outer angle of a cyclic quadrilateral is equal to the inner angle at the opposite vertex)

ie; Angles of ΔPBD and ΔAPC are same.

ii) Angles of ΔPBD and ΔAPC are same.

$\therefore$ ΔAPC and ΔPBD are similar triangles.

In similar triangles sides opposite to equal angles are proportional

$$\therefore \frac{PB}{PC} = \frac{PD}{PA}$$

By cross multiplication, we get $PA \times PB = PC \times PD$

iii) We have to prove ABDC is an isosceles trapezium.

That is to prove a pair of opposite sides are parallel and non parallel sides are equal.

$PA \times PB = PC \times PD$

Given that $PB = PD$

$\therefore PA \times PB = PC \times PB$

we get $PA = PC$

$\therefore$ ΔPAC is an isosceles triangle.

In isosceles triangles, angles opposite to equal sides are equal.

$\therefore \angle A = \angle C$

Since ABDC is a cyclic quadrilateral, $\angle C + \angle ABD = 180°$

Since $\angle A = \angle C$

$\angle A + \angle ABD = 180°$

Since co-interior angles are supplementary AC and BD are parallel.

$AB = PA - PB$

$= PC - PD$ (Since $PA = PC$ and $PB = PD$)

$= CD$

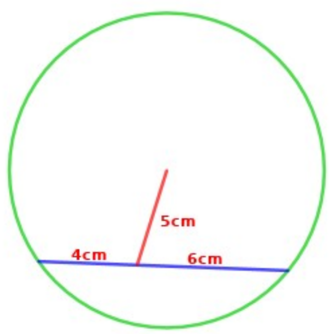
ie; $AB = CD$ and $AC \parallel BD$

$\therefore$ ABDC is an isosceles trapezium.

Question

In the picture , a line through the centre of a circle cuts a chord in to two parts:

What is the radius of the circle ?



Answer

In the picture, extend both ends of OP to meet the circle at C and D.

Let the radius of the circle be r

∴ PC = r + 5 and PD = r – 5

From the figure,

PA = 4 cm, PB = 6 cm

Now we have PA × PB = PC × PD

$(r + 5) \times (r - 5) = 4 \times 6$

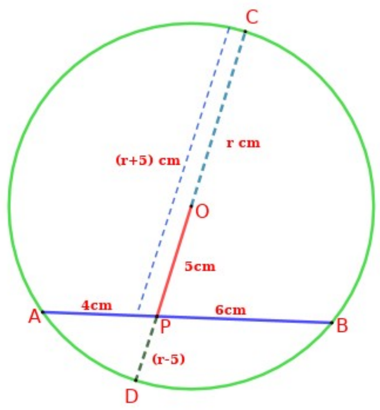
$r^2 - 5^2 = 24$

$r^2 - 25 = 24$

$r^2 = 24 + 25 = 49$

$\therefore r = \sqrt{49} = 7$ cm

Radius of the circle = 7 cm



Assignment

In the picture , a line through the centre of a circle meets a chord of the circle:

What are the lengths of the two pieces of the chord ?