

# PHYSOL EXAMINATION SERIES

## CHAPTER 4- MOTION IN A PLANE


SUNDAY 04-07-2021 @ 7.00pm

**PES03**

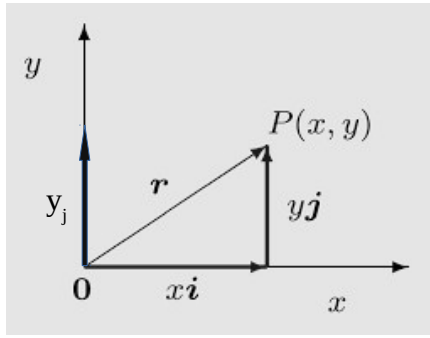
**TIME: 1 HOUR**

**MAXIMUM SCORE:30**

### ANSWER KEY

1	Vectors		1
2	90°		1
3	Zero Vector or Null vector		1
4	Acceleration		1
5	<p>For a projectile launched with velocity <math>v_0</math> at an angle <math>\theta_0</math>, the range is given by</p> <p>Range <math>R = \frac{u^2 \sin 2\theta_0}{g}</math></p> <p>Now, for angles, <math>(45^\circ + \alpha)</math> and <math>(45^\circ - \alpha)</math>, <math>2\theta_0</math> is <math>(90^\circ + 2\alpha)</math> and <math>(90^\circ - 2\alpha)</math>, respectively. The values of <math>\sin(90^\circ + 2\alpha)</math> and <math>\sin(90^\circ - 2\alpha)</math> are the same, equal to that of <math>\cos 2\alpha</math>. Therefore, ranges are equal for elevations which exceed or fall short of <math>45^\circ</math> by equal amounts <math>\alpha</math>.</p>		2
6	(a) At highest point the acceleration remains same as acceleration due to gravity		1
	(b) At highest point, velocity becomes zero		1
7	<p>Yes. When <math>u \cos \theta = \frac{u}{2}</math></p> <p><math>\cos \theta = 1/2</math></p> <p><math>\theta = 60^\circ</math>.</p>		2
8	<p><math>-h = -1/2 gt^2</math></p> <p><math>h = 1/2 \times 10 \times 100 = 500m</math></p>		2
9	<p>(a) The unit vector of <math>\vec{A}</math>,</p> $\hat{A} = \frac{\vec{A}}{ \vec{A} }$ <p>(b) <math>\vec{A} = 4\hat{i} - 3\hat{j} + \hat{k}</math></p> <p>Here <math> \vec{A}  = \sqrt{A_x^2 + A_y^2 + A_z^2}</math></p> $ \vec{A}  = \sqrt{4^2 + (-3)^2 + 1^2}$ $ \vec{A}  = \sqrt{16 + 9 + 1} = \sqrt{26}$ <p>Therefore <math>\hat{A} = \frac{\vec{A}}{ \vec{A} } = \frac{4\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{26}}</math></p>		1

10 a)



b)  $\vec{r} = x\hat{i} + y\hat{j}$   
 c)  $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

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We have  $H = \frac{u^2 \sin^2 \theta}{2g}$  &  $R = \frac{u^2 \sin 2\theta}{g}$

When  $H = \frac{R}{4}$

$$\frac{u^2 \sin^2 \theta}{2g} = \frac{1}{4} \frac{u^2 \sin 2\theta}{g}$$

$$\sin^2 \theta = \frac{\sin 2\theta}{2}$$

$$\sin^2 \theta = \frac{2 \sin \theta \cos \theta}{2}$$

$$\frac{\sin \theta}{\cos \theta} = 1$$

$$\tan \theta = 1$$

$$\theta = 45^\circ$$



**HSPTA**  
MALAPPURM

3

12

Given  $2u \sin \theta / g = 5$   
 $u \sin \theta = 25$

Now.  $H = \frac{u^2 \sin^2 \theta}{2g}$   
 $= \frac{25^2}{2} \times 10 = 31.25 \text{ m}$

3

13 a) **Expression for Maximum height(H):**

We have  $V^2 = u^2 + 2as$

Taking the vertical components;

$$V_y^2 = u_y^2 + 2a_y s_y$$

Here  $V_y = 0$ ,  $u_y = u \sin \theta$ ,  $a_y = -g$  and  $S_y = H$

Therefore  $0 = (u \sin \theta)^2 - 2gH$

$$2gH = u^2 \sin^2 \theta$$

Maximum Height,  $H = \frac{u^2 \sin^2 \theta}{2g}$

**Expression for Time of flight (T):**

We have  $S = ut + \frac{1}{2}at^2$

Taking vertical components;

3

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

Here  $S_y = 0$ ,  $u_y = u \sin \theta$ ,  $a_y = -g$  and  $t = T$ , time of flight.

Therefore 
$$0 = u \sin \theta T - \frac{1}{2} g T^2$$

$$\frac{1}{2} g T^2 = u \sin \theta T$$

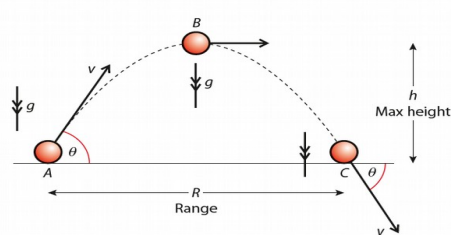
$$\frac{1}{2} g T = u \sin \theta$$

Time of flight 
$$T = \frac{2u \sin \theta}{g}$$

b) For Maximum horizontal range, angle of projection  $\theta = 45^\circ$ .

1

14 a)



1

b) **Expression for the Range of a Projectile(R):**

The product of horizontal component of velocity and time of flight gives the range of a projectile.

$$R = u_x \times T$$

$$R = u \cos \theta \times \frac{2u \sin \theta}{g}$$

$$R = \frac{u^2}{g} 2 \sin \theta \cos \theta$$

Therefore Range, 
$$R = \frac{u^2}{g} \sin 2\theta$$

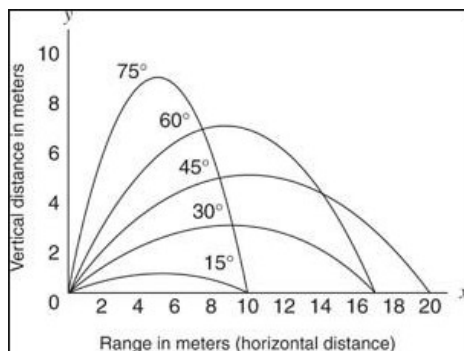
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When  $\theta = 45^\circ$ ,  $2\theta = 90^\circ$ , then the range is maximum.

$$\therefore R_{\max} = \frac{u^2}{g}$$

An athlete throws javelin at an angle of  $45^\circ$  to get maximum range.

For a projectile the range will be same at an angle  $\theta$  and  $(90-\theta)$ . (for example at  $30^\circ$  and  $60^\circ$ , the range will be same for a projectile).



15 Given  $H=25\text{m}$ ,  $u=40\text{ m/s}$ .

$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{40^2 \sin^2 \theta}{2 \times 9.8} = 25\text{ m}$$

$$\sin^2 \theta = \frac{25 \times 9.8 \times 2}{40^2} = .30625$$

Therefore  $\theta=33.6^\circ$

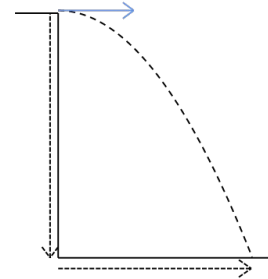
$$\text{Range } R = \frac{u^2 \sin 2\theta}{g} = \frac{40^2 \sin 2(33.6)}{9.8} = 150.5\text{ m}$$

4

16 (a) Horizontal displacement  $S_x = u_x t$  because  $a_x=0$

Vertical displacement  $S_y = \frac{1}{2} g t^2$  because  $u_y=0$   $a_y=g$

<b>t</b>	1	2	3	4	5
<b>S<sub>x</sub></b>	40	80	120	160	200
<b>S<sub>y</sub></b>	5	20	45	80	125



(b) Height of the tower,  $H = \frac{1}{2} g t^2 = \frac{1}{2} \times 10 \times 4^2$

$H=80\text{m}$ .

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2

17 a) Two Dimensional Motion

b) Given  $y = x - \frac{x^2}{80}$  and We have  $y = (\tan \theta) x - \left(\frac{g}{2u^2 \cos^2 \theta}\right) x^2$

Comparing, we get

$\tan \theta = 1$  Therefore  $\theta = 45^\circ$ .

$$\text{And } \frac{g}{2u^2 \cos^2 \theta} = \frac{1}{80}$$

$$\text{That is } \frac{u^2}{g} = 80$$

$$\text{We have Maximum height } H = \frac{u^2 \sin^2 \theta}{2g}$$

$$H = \frac{80}{2 \times 2} = 20\text{ m.}$$

c) Here  $u = 37\text{ m/s}$   $\theta = 53.1^\circ$   $g = 9.8\text{ m/s}^2$   
 $u_x = u \cos \theta = 37 \cos (53.1) = 37 \times 0.6 = 22.2\text{ m/s}$   
 $u_y = u \sin \theta = 37 \sin (53.1) = 37 \times 0.79 = 29.59\text{ m/s}$

The x-coordinate is given by

$$S_x = u_x t + \frac{1}{2} a_x t^2 = u_x t \quad (a_x = 0)$$

Therefore at  $t=2\text{s}$ ,  $x = 22.2 \times 2 = 44.4\text{ m}$

The y-coordinate is given by

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$y = 29.59 \times 2 - \frac{1}{2} \times 9.8 \times 2^2 \quad (a_y = -g = -9.8\text{ m/s}^2)$$

$$y = 39.6\text{ m}$$

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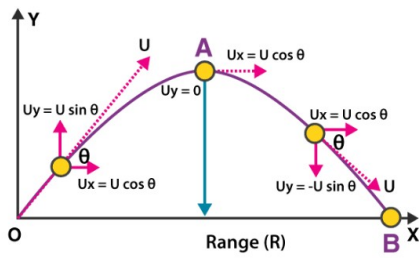
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Therefore the position of the ball when  $t = 2s$  is given by (44.4, 39.6)

- 18 a) Parabola  
b) at its highest point

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c) On projection, the entire energy is kinetic energy (so P.E = 0) =  $\frac{1}{2} m v^2 = E$

$$\begin{aligned} \text{At the highest point KE} &= \frac{1}{2} m v^2 \cos^2(\theta) \\ &= E \cos^2(\theta) \end{aligned}$$

$$\begin{aligned} \text{PE at highest point} &= \text{Total energy} - \text{KE} \\ &= E - E \cos^2 \theta \\ &= E(1 - \cos^2 \theta) \\ &= E \sin^2 \theta \end{aligned}$$

