First Year Higher Secondary Model Examination 202

Mathematics (Commerce) - Answer key.

1) (i) B 4^{th}	1	3
(ii) $A(2,3,5)$, $B(4,3,1)$		
$AB = \sqrt{(4-2)^2+(3-3)^2+(1-5)^2}$	1	
$= \sqrt{4+0+16} = \sqrt{20}$	1	
2) $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$		3
$\sin 5x + \sin 3x = 2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)$		
$= 2\sin 4x \cos x$	1	
$\cos 5x + \cos 3x = 2\cos\frac{5x+3x}{2}\cos\frac{5x-3x}{2}$		
$= 2\cos 4x \cos x$.	1	
$\therefore \dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \dfrac{2\sin 4x\cos x}{2\cos 4x \cos x}$		
$= \tan 4x$	1	
3) (i) $a_3 = 24$, $a_6 = 192$		
$ar^2 = 24$, $ar^5 = 192$	1/2	
$\dfrac{ar^5}{ar^2} = \dfrac{192}{24}$		
$\Rightarrow r^3 = 8 \Rightarrow r = 2$	1/2	
(ii) 10^{th} term, $a_{10} = ar^9$.		

	$ar^2 = 24$, $8 = 2$		
	$\Rightarrow 4a = 24 \Rightarrow a = 6$	1	3
	$\therefore ar^9 = 6 \times 2^9 = 6 \times 512$		
	$= 3072$	1	
4.	(i) (c) 6	1	
	(ii) $(a, 0) = (6, 0) \Rightarrow a = 6.$	1	3
	axis - x axis		
	Equation is $y^2 = 4ax.$	½	
	ie $y^2 = 24x$	½	
5.	$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{x-2}$	1	3
	$= \lim_{x \to 2} x + 2$	1	
	$= 2 + 2 = 4$	1	
6.	(i) 5 is not a prime numbes	1	
	(ii) Contrapositive : If a triangle is not isosceles then it is not equilateral	1	3
	Converse : If a triangle is isocless then it is equilateral	1	

7. 4, 7, 8, 9, 10, 12, 13, 17.

Mean $\bar{x} = \dfrac{4+7+8+9+10+12+13+17}{8} = \dfrac{80}{8} = 10$ — 1

Deviations about mean $|x_i - \bar{x}|$ are

6, 3, 2, 1, 0, 2, 3, 7. — 1 — 3

Mean deviation about mean

$= \dfrac{6+3+2+1+0+2+3+7}{8} = \dfrac{24}{8} = 3$ — 1

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8. $4x + 3 < 5x + 7$

$\Rightarrow 4x - 5x < 7 - 3$ — 1 — 3

$\Rightarrow -x < 4$ — 1

$\Rightarrow x > -4$ — 1

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9. (i) ${}^nC_n = 1$ option (c). — 1

(ii) ${}^nC_9 = {}^nC_8 \Rightarrow n = 9 + 8 = 17$ — 1 — 3

${}^nC_{17} = {}^{17}C_{17} = 1$ — 1

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10. $(x+2y)^4 = {}^4C_0\, x^4 + {}^4C_1\, x^{4-1}(2y)^1 +$

${}^4C_2\, x^{4-2}(2y)^2 + {}^4C_3\, x^{4-3}(2y)^3 + {}^4C_4(2y)^4$ — 1

$= x^4 + 4x^3\, 2y + 6x^2\, 4y^2 + 4x\, 8y^3 + 16y^4$ — 1 — 3

$= x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4$ — 1

11 $\left(\frac{x}{3} + 9y\right)^{10}$

(i) $T_{r+1} = {}^{n}C_{r}\, a^{n-r}\, b^{r}$ 1

$= {}^{10}C_{r} \left(\frac{x}{3}\right)^{10-r} (9y)^{r}$ 1 | 3

(ii) $T_3 = T_{2+1} = {}^{10}C_{2} \left(\frac{x}{3}\right)^{10-2} (9y)^{2}$ 1

$= \cancel{\frac{10\times 9}{1\times 2}}\ \cancel{\frac{x^8}{3^8}}\ \cancel{81y^2}$

$=$

12. $\cancel{A}$ $S = \{1, 2, 3, 4, 5, 6\}$

(i) $A = \{2, 3, 5\}$, $B = \{1, 3, 5\}$ 1 | 3

A or B $= A \cup B = \{1, 2, 3, 5\}$ 1

(ii) A but not B $= A - B = \{2\}$ 1

13. (i) (B) $\{x \mid x \in R,\ 6 < x \leq 12\}$ 1

(ii) $A = \{a, b, c\}$ | 4

$\{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\},$
$\{a, b, c\}\}$ 3

14. (i) $R = \{(5, 3), (6, 4), (7, 5)\}$ 2

(ii) Domain $= \{5, 6, 7\}$ 1 | 4

Range $= \{3, 4, 5\}$ 1

15. (i) $\sin 30^\circ = \frac{1}{2}$, option (c) . 1

(ii) $\sin 75^\circ = \sin(45^\circ + 30^\circ)$ ½ 4

$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 20^\circ$ 1

$= \frac{1}{\sqrt{2}} \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$ 1

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$ ½

16. $P(n) : 1 + 3 + 3^2 + \cdots + 3^{n-1} = \frac{3^n - 1}{2}$

(i) P(1) : LHS = 1

RHS = $\frac{3^1 - 1}{2} = \frac{2}{2} = 1$

$\therefore$ P(1) is true 1

(ii) $P(k) : 1 + 3 + 3^2 + \cdots + 3^{k-1} = \frac{3^k - 1}{2}$ — (1) 1 4

$P(k+1) : 1 + 3 + 3^2 + \cdots + 3^{k-1} + 3^k = \frac{3^{k+1} - 1}{2}$ 1

Proof :-

$LHS = \underbrace{1 + 3 + \cdots + 3^{k-1}} + 3^k$

$= \frac{3^k - 1}{2} + 3^k$ (using (1)) ½

$= \frac{3^k - 1 + 2 \times 3^k}{2} = \frac{3 \times 3^k - 1}{2} = \frac{3^{k+1} - 1}{2}$ ½

$= RHS$

17. (i) $i^2 = -1$ option D 1

(ii) $(2+3i)(1-i) = 2 - 2i + 3i - 3i^2$ 1 4

$= 2 + i + 3$ 1

$= 5 + i$ 1

18. Let $A_1, A_2, A_3, A_4, A_5, A_6$ be the 6 nos. ½

Then $3, A_1, A_2, A_3, A_4, A_5, A_6, 24$ is an AP ½

$a = 3$

$a_8 = 24 \Rightarrow a + 7d = 24$ 1

$7d = 24 - 3 = 21$ 4

$d = 3$ 1

$\therefore$ The AP is.

$3, 6, 9, 12, 15, 18, 21, 24$ 1

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19. $3x - 4y + 10 = 0$

(i) ~~now~~. $3x - 4y = -10$ ½

$\frac{3x - 4y}{-10} = 1 \Rightarrow \frac{3x}{-10} - \frac{4y}{-10} = 1$ 1

$\frac{x}{-10/3} + \frac{y}{10/4} = 1$ ½ 4

(ii) x intercept $= \frac{-10}{3}$ 1

y intercept $= 5/2$ 1

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20. (i) 0. 1

(ii) $A(4, 8, 10)$, $B(6, 10, -8)$

Let C be the point on YZ plane which divides AB in the ratio $k:1$ 4

then x coordinate of C is 0. 1

ie $\frac{6k + 4}{k+1} = 0 \Rightarrow 6k = -4 \Rightarrow k = \frac{-4}{6} = \frac{-2}{3}$ 1

$K:1 = -2:3$ $\frac{1}{2}$

$\therefore$ YZ plane divide AB in the ratio $2:3$ externally. $\frac{1}{2}$

21\. (a) $\frac{d}{dx}(5) = 0$ (iii) 1

(b) $\frac{d}{dx}(x^3) = 3x^2$ (iv) 1

(c) $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$ (i) 1 4

(d) $\frac{d}{dx}(x^2) = 2x$ (ii) 1

22\. Assume that $\sqrt{3}$ is rational. 1

Then let $\sqrt{3} = \frac{p}{q}$ where $p, q \in Z$ and p and q have no common factors.

$3 = \frac{p^2}{q^2} \Rightarrow p^2 = 3q^2$

$\Rightarrow p = 3m$. 1

Then $(3m)^2 = 3q^2 = 9m^2 = 3q^2$ 4

$\Rightarrow q^2 = 3m^2 \Rightarrow q = 3n$. 1

Then p and q have common factor 3

which is a contradiction.

$\therefore \sqrt{3}$ is irrational 1

3\. $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{8}$

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ 1

$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8}$ ½

$= \frac{5}{8}$ ½

(ii) $P(A') = 1 - P(A)$ ½

$= 1 - \frac{1}{4} = \frac{3}{4}$ ½

4

(iii) $P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B)$ ½

$= 1 - \frac{5}{8} = \frac{3}{8}$ ½

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24. $x - 2y + 3 = 0$

(i) slope $= -\frac{a}{b} = \frac{-1}{-2} = \frac{1}{2}$ 1

(ii) ∴ slope of perpendicular line $= -2$ 1

It passes thrgh $(1,-2)$.

4

Eqn is $y - y_1 = m(x - x_1)$ 1

$\Rightarrow y + 2 = -2(x - 1)$

$\Rightarrow y + 2 = -2x + 2$ 1

$\Rightarrow 2x + y = 0$.

25. (i) $A' = \{1, 4, 5, 6\}$ 1

$B' = \{1, 2, 6\}$ 1

(ii) $A \cup B = \{2, 3, 4, 5\}$ 1 6

$A \cap B = \{3\}$ 1

(iii) $(A \cup B)' = \{1, 6\}$ 1

$A' \cap B' = \{1, 6\}$. 1

$\therefore (A \cup B)' = A' \cap B'$.

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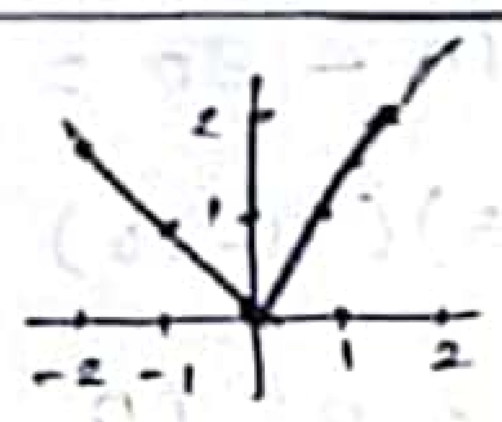
26. $f(x) = |x|, \ x \in \mathbb{R}$.

(i) $f(2) = |2| = 2$ ½

$f(-2) = |-2| = 2$ ½

(ii)

x	0	2	-2
y	0	2	2



3 6

(iii) Domain $= \mathbb{R}$. 1

Range $= [0, \infty)$. 1

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27. $x + 2y = 8$

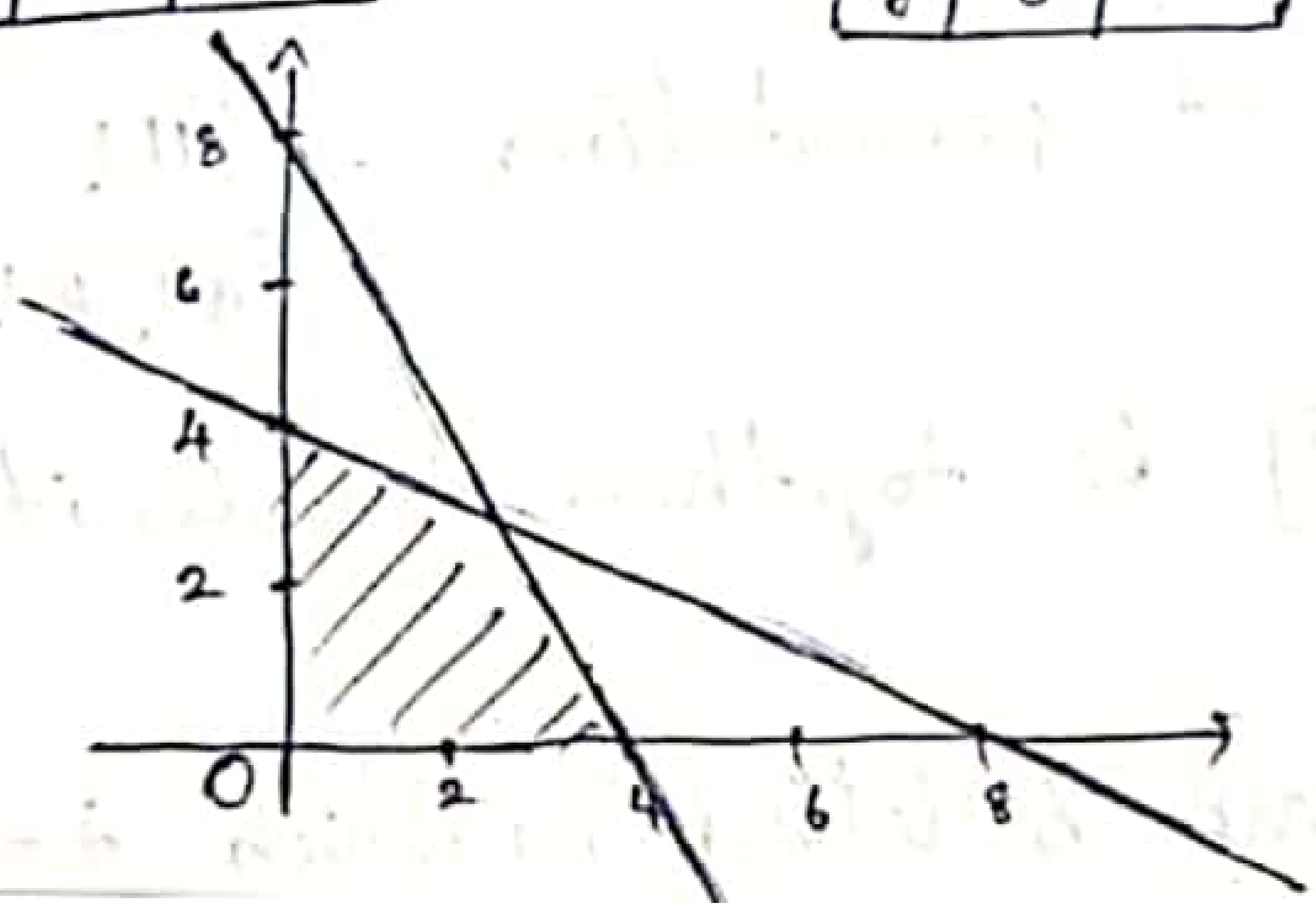
x	0	8
y	4	0

$2x + y = 8$

x	0	4
y	8	0

2

6



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28. (i) ~~$^nP_5$~~ $\qquad {}^nP_r = \dfrac{n!}{(n-r)!}$

$^nP_5 = 42\ {}^nP_3$

$\Rightarrow \dfrac{n!}{(n-5)!} = 42 \times \dfrac{n!}{(n-3)!}$ 1

$\Rightarrow \dfrac{1}{(n-\cancel{5})!} = 42 \times \dfrac{1}{(n-3)(n-4)(n-\cancel{5})!}$ 1

$\Rightarrow (n-3)(n-4) = 42 \quad (= 6 \times 7)$

$\Rightarrow n-3 = 7, \quad n-4 = 6$ ½

$\Rightarrow n = 10.$ ½

OR $\quad n^2 - 7n + 12 = 42$ ½

$\Rightarrow n^2 - 7n - 30 = 0$

$\Rightarrow (n-3)(n-10) = 0$ ½

$\Rightarrow n = 3$ or 10.

Since $n > 8$, $n \neq 3$. $\therefore n = 10$.

(ii) MISSISSIPPI

M – 1, I – 4, S – 4, P – 2 → Total 11 letters

Total no of permutations $= \dfrac{11!}{4!\,4!\,2!}$ 1

Let $\boxed{IIII}$ be together. Consider it as one letter.

Then there are 8 letters in which 4 – S and 2 – P.

∴ no of permutations in which the 4 Is come together $= \frac{8!}{4!\,2!}$ 1

∴ no of permutations in which the 4 Is donot come together

$$= \frac{11!}{4!\,4!\,2!} - \frac{8!}{4!\,2!}$$ 1

6

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29. $\frac{x^2}{36} + \frac{y^2}{16} = 1$

$a^2 = 36, \quad b^2 = 16$

$c^2 = a^2 - b^2 = 20.$

$a = 6, \; b = 4, \; c = \sqrt{20} = 2\sqrt{5}$

foci $(\pm c, 0) = (\pm 2\sqrt{5}, 0)$ 1

Vertices $(\pm a, 0) = (\pm 6, 0)$ 1

length of major axis $= 2a = 12$ 1

Length of minor axis $= 2b = 8$ 1

6

$e = \frac{c}{a} = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$ 1

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 16}{6}$

$= \frac{16}{3}$ 1

30.

x_i	f_i	$x_i f_i$	$x_i^2 f_i$
4	3	12	48
8	5	40	320
11	9	99	1089
17	5	85	1445
20	4	80	1600
24	3	72	1728
32	1	32	1024
	30	420	7254

$N = \Sigma f_i = 30$ 2

(i) Mean $\bar{x} = \dfrac{\Sigma x_i f_i}{N} = \dfrac{420}{30} = 14$ 1

Variance $\sigma^2 = \dfrac{\Sigma x_i^2 f_i}{N} - \bar{x}^2$ ½

$= \dfrac{7254}{30} - 14^2$

$= 241.8 - 196 = 45.8$ ½

(ii). standard deviation $= \sqrt{\text{variance}}$ 1

$= \sqrt{45.8} = 6.77$ 1