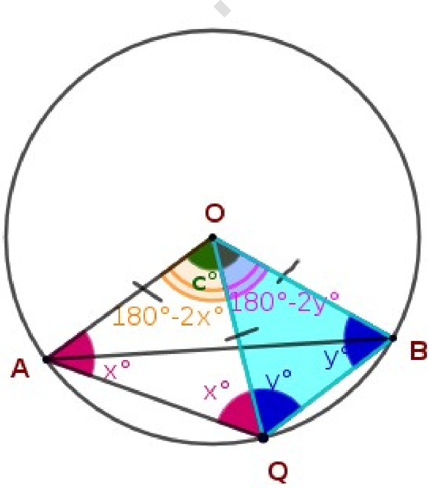
Online Class – X – 17 **29 / 07 / 2021**

2 . Circles – Class 5

To view class

Relation between the central angle of a non diametrical chord and the angles formed by joining the ends of the chord to the points on the smaller part of the circle .



Consider the given figure

Draw OA, OB, OQ

Let $\angle AQO = x°$ & $\angle BQO = y°$

$\angle Q = x° + y°$

Consider Δ AOQ,

ΔAOQ is an isosceles triangle (OA = OQ)

$\angle AQO = \angle QAO = x°$

$\angle AOQ = 180° - (x° + x°)$

$= 180° - 2x°$

Consider Δ BOQ,

ΔBOQ is an isosceles triangle (OB = OQ)

$\angle BQO = \angle QBO = y°$

$\angle BOQ = 180° - (y° + y°)$

$= 180° - 2y°$

$\angle AOB = \angle AOQ + \angle BOQ$

Let $\angle AOB = c°$

So, $c° = 180° - 2x° + 180° - 2y°$

$= 360° - 2(x° + y°)$

$= 360° - 2\angle Q$

$2\angle Q = 360° - c°$

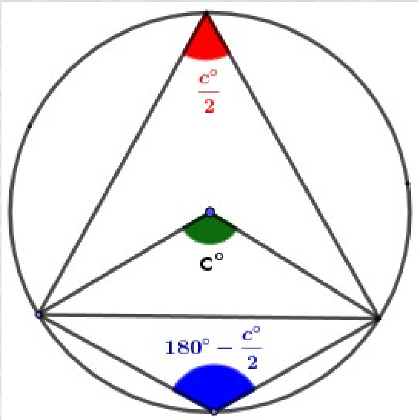
$$\angle Q = \frac{360° - c°}{2} = 180° - \frac{c°}{2}$$

Looking at the angles in the two parts of the circle and the angle at the centre together we have,

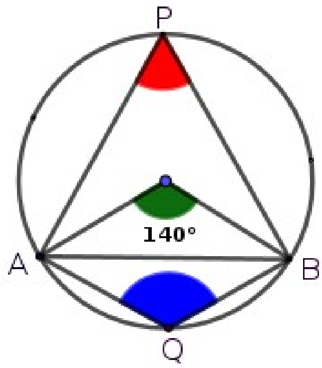
Any chord which is not a diameter splits the circle into unequal parts.

The angle got by joining any point on the larger part to the ends of the chord is half the angle got by joining the centre of the circle to these ends.

The angle got by joining any point on the smaller part to the ends of the chord is half the angle at the centre subtracted from 180 ° .



Q) If the chord AB makes an angle 140° at the centre of the circle find ∠APB & ∠ AQB.

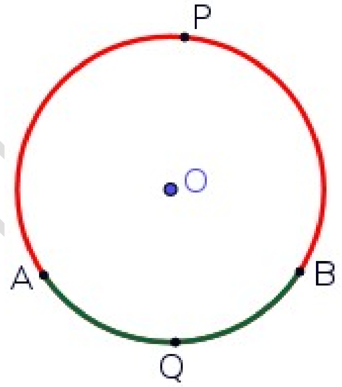


Ans)

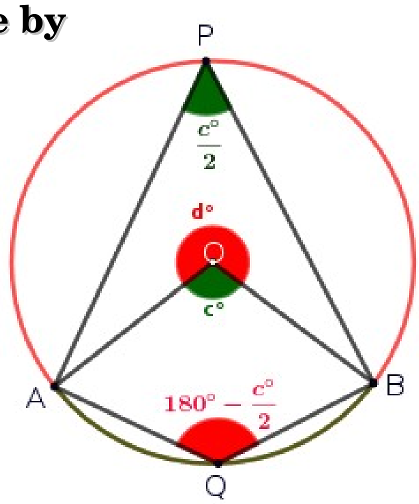
$$\angle APB = \frac{\angle AOB}{2} = \frac{140^\circ}{2} = 70^\circ$$

$$\angle AQB = 180° - \frac{\angle AOB}{2}$$
$$= 180° - \frac{140°}{2}$$
$$= 180° - 70°$$
$$= 110°$$

Putting the results obtained in terms of arcs and their central angles



Any two points on a circle divide it into two arcs.
Each of these two arcs can be called the **alternate arc or complementary arc** of the other.
In the figure the two arcs are **arc APB** and **arc AQB.**
Alternate arc or complementary arc of **arc APB** is **arc AQB**
Alternate arc or complementary arc of **arc AQB** is **arc APB**



Central angle of an arc is the angle made by the arc at the centre of the circle .
Let central angle of arc AQB = c°
and central angle of arc APB = d°

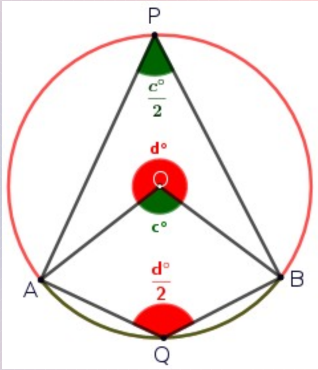
We know angle around a point is 360°

So, $c° + d° = 360°$
$$d° = 360° - c°$$
$$\frac{d°}{2} = \frac{360° - c°}{2}$$

$$\boxed{\frac{d°}{2} = 180° - \frac{c°}{2}}$$

Conclusion

The angle made by any arc of a circle on the alternate arc is half the angle made at the centre



Note:

Sum of the angles on an arc and its alternate arc $= \angle P + \angle Q$

$$= \cancel{\frac{c^\circ}{2}} + 180^\circ - \cancel{\frac{c^\circ}{2}}$$

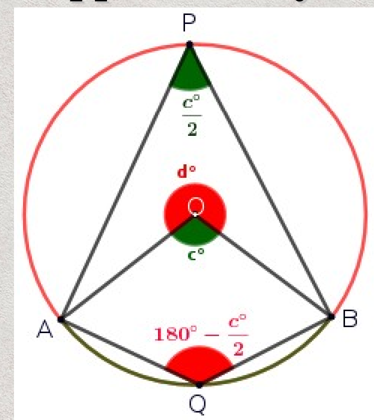
$$= 180^\circ$$

Sum of the angles on an arc and its alternate arc is 180 °.

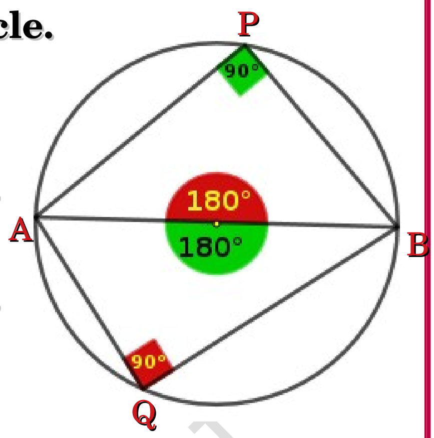
Pairs of angles of sum 180 ° are usually called **supplementary angles.**

Conclusion

All angles made by an arc on the alternate arc are equal and a pair of angles on an arc and its alternate arc are supplementary



In the figure, AB is the diameter of the circle.
Arc APB and arc AQB are semicircles.
Central angle of a semicircle is 180°.
∠P is the angle made by the arc AQB at its alternate arc APB and
∠Q is the angle made by the arc APB at its alternate arc AQB



$$\angle P = \frac{180^\circ}{2} = 90^\circ$$

$$\angle Q = \frac{180^\circ}{2} = 90^\circ$$

Angles in semicircle are right or 90°

Assignment
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(1) In all the pictures given below, O is the centre of the circle and A, B, C are points on it. Calculate all angles of ΔABC and ΔOBC in each.