

Second Degree Equations

- 1) Form the equations in the following cases.
 - a) The sum of a number and its square is 12
 - b) When a number is subtracted from its square results 20
 - c) The sum of the square of a number and two times that number is 63
 - d) Product of two consecutive odd numbers is 63.
 - e) The sum of a number and its reciprocal is $\frac{10}{3}$.

Answers

- a) If the number is x then $x^2 + x = 12$
- b) If the number is x then $x^2 - x = 20$
- c) If the number is x then $x^2 + 2x = 63$
- d) Numbers are $x, x + 2$ then $x(x + 2) = 63, x^2 + 2x = 63$
- e) If the number is x then $x + \frac{1}{x} = \frac{10}{3}$
 $\frac{x^2+1}{x} = \frac{10}{3}$
 $3(x^2 + 1) = 10x,$
 $3x^2 - 10x + 3 = 0$

- 2) The square of a number is 16.
 - a) What are the numbers ?
 - b) Take the number as x and form an equation
 - c) Can the square of a real number -16 ? Explain.

Answers

- a) Numbers are 4, -4
- b) If the number is x then $x^2 = 16$
- c) No real number exist with its square a negative number. The square of -4 and the square of $+4$ is 16.

- 3) The sum of a number and its square is 30.
 - a) If the number is x , form an equation.
 - b) What is the positive number x ?
 - c) Can more than one number satisfying this condition?

Answers

- a) If the number is x then $x^2 + x = 30$
- b) $x = 5$
- c) $5^2 + 5 = 30, (-6)^2 + (-6) = 30$. Numbers are 5, -6

- 4) x is an odd number greater than 1.
- a) What are the odd numbers nearer to x
 - b) If the product of those numbers is 45, form an equation.
 - c) Find the numbers.

Answers

- a) Odd number is x . The numbers nearer to it are $x - 2, x + 2$
- b) $(x - 2)(x + 2) = 45$
 $x^2 - 4 = 45, x^2 = 49$
- c) $x = 7$
Numbers are 5, 9

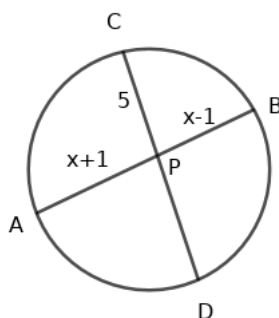
- 5) If the sides of a square are reduced by 1, the area becomes 100.
- a) If the side of the first square before reducing is x , form an equation.
 - b) Find the side of the square.
 - c) What will be the perimeter of the new square.

Answers

- a) If one side is x then $(x - 1)^2 = 100$
- b) $x - 1 = \sqrt{100} = 10, x = 11$
- c) Perimeter decreases by 4

Second Degree Equations

- 1) The chords AB and CD meet at a point P inside the circle.
 $CD = 21\text{cm}$, $PC = 5\text{cm}$.



- What is PD ?
- If $PA = x + 1$ and $PB = x - 1$ then form an equation
- Find the length PA and PB .

Answers

- $PD = 21 - 5 = 16\text{cm}$
- $PA \times PB = PC \times PD$
 $(x + 1)(x - 1) = 5 \times 16 = 80$
 $x^2 - 1^2 = 80, x^2 - 1 = 80$
- $x^2 - 1 = 80 \rightarrow x^2 = 81, x = 9$
- $PA = 9 + 1 = 10\text{cm}, PB = 9 - 1 = 8\text{cm}$

- 2) The product of two consecutive even numbers is 360

- If the odd number in between these numbers is x then write the numbers .
- Form an equation using the given condition.
- Find the numbers.

Answers

- Numbers are $x - 1, x + 1$
- $(x - 1)(x + 1) = 360, x^2 - 1 = 360$
- $x^2 = 361, x = \sqrt{361} = 19$. Numbers $19 - 1 = 18, 19 + 1 = 20$

- 3) Consider the arithmetic sequence $5, 9, 13, 17, 21 \dots$

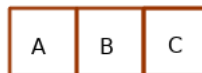
- Write the algebraic form of this sequence.
- What is the position of the term in the sequence whose square is 625?
- Is 36 a term of this sequence . How can you realize it ?

d) What is the position of 49 in this sequence ?

Answers

- a) $x_n = dn + (f - d) = 4n + (5 - 4) = 4n + 1$
- b) $(4n + 1)^2 = 625, 4n + 1 = \sqrt{625} = 25, 4n = 24, n = 6$
- c) All terms are odd numbers . The even number 36 cannot be a term of this sequence
- d) $4n + 1 = 49, 4n = 48, n = 12.$
12 th term is 49

4) Three boxes in which dates of a calendar are given.



- a) If $B = x$ find A, C
- b) If $A \times C = 120$ form an equation.
- c) Find B
- d) Find A and C

Answers

- a) $A = x - 1, C = x + 1$
- b) $(x - 1)(x + 1) = 120, x^2 - 1 = 120, x^2 = 121$
- c) $x = \sqrt{121} = 11, B = 11$
- d) $A = 10, C = 12$

5) Sum of the areas of two squares is 130. Side of one square is 2 more than the side of the other square .

- a) If the side of the small square is x then what is the side of the big square ?
- b) Form a second degree equation using the condition.

Answers

- a) Side of the big square is $x + 2$
- b) $x^2 + (x + 2)^2 = 130$
 $x^2 + x^2 + 4x + 4 = 130, 2x^2 + 4x + 4 - 130 = 0, 2x^2 + 4x - 126 = 0$

Second Degree Equations

- 1) Consider two adjacent even numbers
 - a) If one of them is x then what is the other?
 - b) If the product is 120 then write a second degree equation.
 - c) Convert this equation as a completed square by suitable changes
 - d) Find the numbers .

Answers

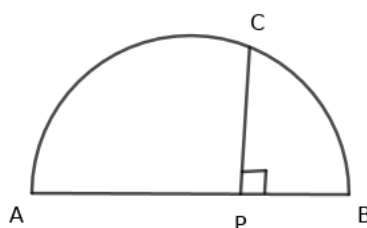
- a) $x + 2$
- b) $x(x + 2) = 120$
 $x^2 + 2x = 120$
- c) Add 1 on both sides $x^2 + 2x + 1 = 120 + 1$
 $(x + 1)^2 = 121$
- d) $x + 1 = \sqrt{121} = 11, 11, x + 1 = 11, x = 10$
Even numbers are 10, 12

- 2) Length of a rectangle is 8 more then its breadth.
 - a) If the breadth is x then what is its length?
 - b) If the area is 240 sq.cm form a second degree equation.
 - c) Calculate the lenth and breadth

Answers

- a) Length= $x + 8$
- b) $x(x + 8) = 240, x^2 + 8x = 240$
- c) Add $(\frac{8}{2})^2$ on both sides. It is 16
 $x^2 + 8x + 16 = 240 + 16$
 $(x + 4)^2 = 256, x + 4 = \sqrt{256} = 16, x = 16 - 4 = 12$
Breadth 12 cm,Length $12 + 8 = 20$ cm

- 3) In the figure AB is the diametre of the semicircle. AB is perpendicular to PC .Also, $AP = BP + 5$,
 $PC = 6$.



- Write the relation between the lengths PA , PB and PC
- If $PB = x$ then write an equation connecting the lengths PA , PB and PC
- What is the length of PB ?
- What is the radius of this circle. $\text{ഘൃത്തത്തിന്റെ ആരമെത്ര?}$

Answers

- $PA \times PB = PC^2$
- $$(x + 5) \times x = 6^2, x^2 + 5x = 36$$

$$x^2 + 5x + \left(\frac{5}{2}\right)^2 = 36 + \left(\frac{5}{2}\right)^2$$

$$\left(x + \frac{5}{2}\right)^2 = 36 + \frac{25}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{169}{4}$$

$$\left(x + \frac{5}{2}\right) = \sqrt{\frac{169}{4}} = \frac{13}{2}$$

$$x = \frac{13}{2} - \frac{5}{2} = 4$$
- $PB = 4$
 $AP = 4 + 5 = 9, AB = 9 + 4 = 13$
 Radius = 12cm

- 4) Consider the sequence of even numbers 2, 4, 6, 8...

- What is its algebraic form?
- How many terms from the beginning in the order makes the sum 210?

Answers

- $x_n = 2n$
- $$n(n + 1) = 210, n^2 + n = 210$$

$$n^2 + n + \frac{1}{4} = 210 + \frac{1}{4}$$

$$\left(n + \frac{1}{2}\right)^2 = \frac{841}{4}$$

$$n + \frac{1}{2} = \sqrt{\frac{841}{4}} = \frac{29}{2}$$

$$n = \frac{29}{2} - \frac{1}{2} = 14$$
 The sum of the first 14 even numbers is 210

- 5) The smallest side of a right angled triangle is 4 less than its hypotenuse. Third side is 2 more than the smallest side.

- If the smallest side is x what are the other two sides.
- Write an equation connecting the length of the sides .
- What is the length of the smallest side?
- Find the length of other sides of the triangle.

Answers

- a) If the smallest side is x then hypotenuse is $x + 4$, third side is $x + 2$
- b) $(x + 4)^2 = (x + 2)^2 + x^2$, $x^2 + 8x + 16 = x^2 + 4x + 4 + x^2$
 $x^2 - 4x - 12 = 0$
- c) $x^2 - 4x = 12$, $x^2 - 4x + 4 = 12 + 4$
 $(x - 2)^2 = 16$, $x - 2 = 4$, $x = 6$
Smallest side is 6
- d) Sides are 6, 8, 10

Second Degree Equations

1) 1 is added to the product of two cosecutive even numbers gives 289

- If one number is x then what is the other number?
- Form an equation
- Find the numbers by solving the equation

Answers

- $x + 2$
- $x(x + 2) + 1 = 289$
 $x^2 + 2x + 1 = 289$
- $(x + 1)^2 = 289, X + 1 = \sqrt{289} = 17, x = 16$
Numbers are 16, 18

2) 9 added to the product of two cosecutive multiples of 6 gives 729.

- If n is a multiple of 6 then what is the next multiple of 6
- Write an equation
- Find the numbers

Answers

- $n + 6$
- $n(n + 6) + 9 = 729$
 $n^2 + 6n + 9 = 729$
- $(n + 3)^2 = 729, n + 3 = \sqrt{729} = 27, n = 27 - 3 = 24$
Numbers are 24, 28

3) Consider the arithmetic sequence 5, 7, 9... ?

- Write the algebraic form of this sequence ?
- Form an equation to find the number of terms which make the sum 140
- How many terms make the sum 140?

Answers

- $x_n = dn + (f - d) = 2n + 3$
- Suppose that n terms make the sum 140. $(x_1 + x_n) \times \frac{n}{2} = 140$
 $(5 + 2n + 3) \times \frac{n}{2} = 140, (8 + 2n) \times \frac{n}{2} = 140$
 $n^2 + 4n = 140, n^2 + 4n + 2^2 = 144, (n + 2)^2 = 144, n + 2 = \sqrt{144} = 12,$
 $n = 12 - 2 = 10.$
- Sum of first 10 terms is 140

4) 16 added to the sum of first few terms of the arithmetic sequence 9, 11, 13... gave 256

- a) Form an equation to the number of terms
- b) Calculate the number of terms which make the sum 256

Answers

a) Suppose that the sum of the first n terms is 256.

n th term is $2n + 7$

b) $(x_n + x_1) \times \frac{n}{2} + 16 = 256, (9 + 2n + 7) \times \frac{n}{2} + 16 = 256$

$(16 + 2n) \times \frac{n}{2} + 16 = 256$

$n^2 + 8n + 16 = 256$

$(n + 4)^2 = 256, n + 4 = \sqrt{256} = 16, n = 12.$

c) First 12 terms and 16 added make the sum 256

Second Degree Equations

1) One side of a rectangle is 2 more than other side. Area of the rectangle is 120 square metre .

- Take one side as x then form a second degree equation.
- Find the sides of the rectangle

Answers

- Sides are x and $x + 2$
 $x(x + 2) = 120, x^2 + 2x = 120$
- $x^2 + 2x + 1 = 121, (x + 1)^2 = 121, x + 1 = \sqrt{121} = 11, x = 10$
 Sides are 10cm and 12cm

2) One side of a triangle is 6 more than altitude to the side. Area of the triangle is 216

- Form a second degree equation by taking one measure as x
- Find the side and altitude to that side of the triangle

Answers

- Altitude is x and one side is $x + 6$
 $\frac{1}{2} \times x(x + 6) = 216, x(x + 6) = 2 \times 216 = 432$
- $x^2 + 6x = 432, x^2 + 6x + 9 = 432 + 9 = 441, (x + 3)^2 = 441, x + 3 = \sqrt{441} = 21, x = 21 - 3 = 18$
 Altitude = 18cm and side 24cm.

3) The picture given below is the section of a calendar page. A, B, C, D are the numbers representing the days.

- If the smallest number is x then what are other numbers
- If $A \times C = 105$ then form a second degree equation
- Find the numbers in the squares

Answers

- $A = x, B = x + 1, D = x + 7, C = x + 8$
- $A \times C = 105, x(x + 8) = 105, x^2 + 8x = 105$
- $x^2 + 8x + 16 = 105 + 16 = 121, (x + 4)^2 = 121, x + 4 = \sqrt{121} = 11, x + 4 = 11 - 4 = 7$
 $A = 7, B = 8, D = 14, C = 15$

4) Hypotenuse of a right triangle is 4 more than its smallest side. Third side is 2 less than hypotenuse.

- If the smallest side is x then what are the other sides?
- Form a second degree equation connecting the sides

c) Calculate the sides of the triangle

Answers

a) Hypotenuse is $x + 4$, other side is $x + 2$

b) Using Pythagoras theorem $(x + 4)^2 = x^2 + (x + 2)^2$

$$x^2 + 8x + 16 = x^2 + x^2 + 4x + 4$$

$$x^2 - 4x = 12$$

c) $x^2 - 4x + 4 = 16$, $(x - 2)^2 = 16$, $x - 2 = 4$, $x = 6$

Sides are 6, 8, 10

Concepts

We are discussing the process of solving a second degree equation using the completing the square method. As the generalization of this method we can establish a formula to solve the second degree equation. It is not necessary to use the formula for solving the second degree equation.

In the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- 1) The difference between the length of the perpendicular sides of a right triangle is 10, area of the triangle is 600 square cm. ആണം .
- One of the perpendicular sides is x then what is the length of the other?
 - Form an equation using the given condition.
 - What is the length of the perpendicular sides?
 - Calculate the perimeter of the triangle.

Answers

a) $x + 10$

b) $\frac{1}{2} \times x \times (x + 10) = 600$
 $x^2 + 10x = 1200$
 $x^2 + 10x - 1200 = 0$

c) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-10 \pm \sqrt{10^2 - 4 \times 1 \times -1200}}{2 \times 1}$
 $x = \frac{-10 \pm \sqrt{4900}}{2 \times 1}$
 $x = 30, -40$. Length of the side cannot be a negative quantity. $x = 30$ cm.
 Perpendicular sides are 30cm, 40cm.

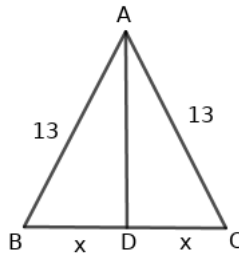
d) Hypotenuse = $\sqrt{30^2 + 40^2} = \sqrt{2500} = 50$ cm Perimetre = $30 + 40 + 50 = 120$ cm.

- 2) Perimetre of a rectangle is 82cm, area 400 sq.cm
- What is the total length of the adjacent sides?
 - If one side is x then what is the length of the other side?
 - Form a second degree equation using the given condition.
 - Calculate the length of the sides.

Answers

- a) $2(\text{length} + \text{breadth}) = 82, l + b = 41$
- b) If one side is x , the other side will be $41 - x$
- c) $x(41 - x) = 400, -x^2 + 41x = 400, x^2 - 41x + 400 = 0$
- d) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-(-41) \pm \sqrt{(-41)^2 - 4 \times 1 \times 400}}{2 \times 1}$
 $x = \frac{41 \pm \sqrt{1681 - 1600}}{2 \times 1}$
 $x = \frac{41 \pm \sqrt{81}}{2 \times 1}$
 $\frac{41 \pm 9}{2} = 25, 16$
 If one side is 16 cm and other side will be $41 - 16 = 25$ cm

- 3) In triangle ABC , $AB = AC = 13$ cm, area of the triangle is 60sq.cm .The perpendicular distance from A to BC is AD .



- a) If $BD = x$ then what is AD ?
- b) Form a second degree equation using BC , AD , and area .
- c) What is the length of BC ?
- d) What is the perimeter of the triangle?

Answers

- a) $AD = \sqrt{13^2 - x^2}$
- b) $\frac{1}{2} \times BC \times AD = 60$
 $\frac{1}{2} \times 2x \times \sqrt{13^2 - x^2} = 60$
 $x\sqrt{13^2 - x^2} = 60$
 Squaring on both sides, $x^2(169 - x^2) = 3600$
 If $x^2 = y, y(169 - y) = 3600, y^2 - 169y + 3600 = 0$.
- c) Solving $y = 144, 25$.
 If $x^2 = 144, x = 12, -12$.
 If $x^2 = 25, x = 5, -5$
 If $x = 12, BC = 24$ cm .
 If $x = 5, BC = 10$ cm
- d) Perimetre of the triangle $13 + 13 + 10 = 36$ cm or $13 + 13 + 24 = 50$ cm

- 4) The perimeter of a right triangle is 60 cm, hypotenuse is 25cm
- a) What is the total length of the perpendicular sides ?

b) If the length of one perpendicular side is x then what will be the length of the other ?

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c) Form an equation using the length of the sides

d) Calculate the area of the triangle.

Answers

a) Length of the perpendicular sides is $60 - 25 = 35\text{cm}$

b) If one perpendicular side is x then the other perpendicular side is $35 - x$

$$\begin{aligned} \text{c) } 25^2 &= x^2 + (35 - x)^2 \\ x^2 - 35x + 300 &= 0 \end{aligned}$$

d) Solving , $x = 20, 15$.

If $x = 20$ other perpendicular side is $35 - 20 = 15$. Area = $\frac{1}{2} \times 20 \times 15 = 150\text{sq.cm}$

5) The difference between the length of the sides of two squares is 4cm. The sum of the areas is 400sq.cm

a) If the side of the small square is x then what is the side of the other square?

b) Form an equation using the given condition.

c) Calculate the side of the squares.

Answers

a) $x + 4$

$$\begin{aligned} \text{b) } x^2 + (x + 4)^2 &= 400 \\ x^2 + x^2 + 8x + 16 &= 400 \\ 2x^2 + 8x - 384 &= 0 \\ x^2 + 4x - 192 &= 0 \end{aligned}$$

c) Solving $x = 12, -16$.

Length of the side of one square is 12, length of the side of other square is $12 + 4 = 16\text{cm}$

Concepts

We are discussing the process of solving a second degree equation using the completing the square method. As the generalization of this method we can establish a formula to solve the second degree equation. It is not necessary to use the formula for solving the second degree equation.

In the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- 1) The age of a man after 15 years would be the square of his age before 15 years.
 - a) If the present age is x , form a second degree equation
 - b) Find the present age.
 - c) Without using algebra find the present age.

Answers

$$\begin{aligned} \text{a) } (x - 15)^2 &= (x + 15) \\ x^2 - 30x + 225 &= x + 15 \\ x^2 - 31x + 210 &= 0 \end{aligned}$$

$$\begin{aligned} \text{b) } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-(-31) \pm \sqrt{(-31)^2 - 4 \times 1 \times 210}}{2 \times 1} \\ x &= \frac{31 \pm \sqrt{961 - 840}}{2} \\ \frac{31 \pm 11}{2} &= 21, 10 \end{aligned}$$

- c) x values are 21, 10. The value of $x = 10$ cannot be admissible. Present age is 21 years.
- d) While considering the age before and after 15 years there will be a gap of 30 years. The perfect square just above 30 is 36. 15 years below 36 is 21, 15 years below 21 is 6. The square of 6 is 36. The present age is 21

- 2) Manju's present age is the square of Laya's age. After 5 years Manju's age become 3 times Laya's age.
 - a) If Laya's present age is x form a second degree equation.
 - b) Find the present age of both.
 - c) How many years later the age of Manju become two times Laya's age?

Answers

- a) Laya's present age is $= x$. Manju's present age is $= x^2$.
 Laya's age after five years is $= x + 5$, Manju's age after five years is $= x^2 + 5$.
 $x^2 + 5 = 3(x + 5)$, $x^2 + 5 = 3x + 15$, $x^2 - 3x - 10 = 0$
- b) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times -10}}{2 \times 1}$
 $x = \frac{3 \pm 7}{2} = 5, -2$
 Laya's present age is 5 years. Manju's present age is 25 years
- c) Suppose that after n years Manju's age become two times Laya's age .
 $(5 + n) \times 2 = 25 + n$, $n = 15$
 After 15 years Manju's age become two times Laya's age .

- 3) One year ago, Ajayan's age was 8 times his son's age. Present age of Ajayan is the square of his son's present age.
- If son's age before 1 year is x what was Ajayan's age one year ago.
 - Form a second degree equation using the given condition.
 - Calculate their present age.

Answers

- a) Son's age before one year is x , Ajay's age is $8x$.
- b) Present age of son $= x + 1$, Ajay's present age $= 8x + 1$
 $(x + 1)^2 = 8x + 1$, $x^2 + 2x + 1 - 8x - 1 = 0$, $x^2 - 6x = 0$,
- c) $x(x - 6) = 0$, $x = 6, 0$
 We can take x value 6. Son's present age $x + 1 = 7$, Ajay's present age is $8 \times 6 + 1 = 49$

- 4) The sum of the ages of a father and son is 45. 5 years ago the product of their ages was 124.
- If father's present age is x what is son's present age?
 - Form a second degree equation using the given condition.
 - Find the their present age.

Answers

- a) Son's present age $= 45 - x$
- b) Father's age before five years $= x - 5$, Son's age before five years $= 40 - x$
 $(x - 5)(40 - x) = 124$,
 $x^2 - 45x + 324 = 0$
- c) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-(-45) \pm \sqrt{(-45)^2 - 4 \times 1 \times 324}}{2 \times 1}$
 $x = \frac{45 \pm \sqrt{729}}{2}$
 $x = 36, 9$
 Father's age $= 36$, Son's age $= 9$

- 5) Nasrin's age is two times Riswan's age. Four years hence the product of their ages become 160.

- a) If Riswan's present age is x what is Nasrin's present age?
b) Form a second degree equation using the given condition.
c) Calculate their present age.

3

Answers

a) Riswan's age = x , Nasrin's age = $2x$

b) Nasrin's age after four years = $x + 4$, Riswan's age after four years = $2x + 4$

$$(x + 4)(2x + 4) = 160$$

$$2x^2 + 12x + 16 = 160$$

$$2x^2 + 12x - 144 = 0$$

$$x^2 + 6x - 72 = 0$$

c) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 6 \times -72}}{2 \times 1}$$

$x = 6, -12$. Riswan's age = 6, Nasrin's age = 12.

1

Concepts

We are discussing the process of solving a second degree equation using the completing the square method. As the generalization of this method we can establish a formula to solve the second degree equation. It is not necessary to use the formula for solving the second degree equation.

In the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Worksheet 41

- 1) The speed of a boat in still water is 8 km in an hour. The boat travels 15 kilometre in upstream and 22 kilometre in downstream in 5 hours.
- If the speed of the stream is x what will be the speed attained by the boat in the downstream.
 - If the speed of the stream is x what will be the resulting speed in the upstream?
 - Form an equation using the given condition.
 - Calculate the speed of the stream.

Answers

a) $8 + x$

b) $8 - x$

c) Speed of boat in the downstream journey = $8 + x$. Time taken for the downward journey is $\frac{22}{8+x}$

Speed of boat in the upstream journey = $8 - x$. Time taken for the downstream journey $\frac{15}{8-x}$

$$\frac{15}{8-x} + \frac{22}{8+x} = 5$$

$$\frac{15(8+x) + 22(8-x)}{8^2 - x^2} = 5$$

$$5x^2 - 7x - 24 = 0.$$

d) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 5 \times -24}}{2 \times 5}$$

$$x = \frac{7 \pm \sqrt{529}}{10}$$

$$\frac{7+23}{10} = 3$$

speed of stream 3 km/h

- 2) A train travels with uniform speed in 300 km. If the speed of the train is increased by 5 km per hour, the journey would have taken 2 hours less.
- If the usual speed is x what will be the time taken for the journey.
 - If the speed is increased by 5 km per hour what will be the time taken for the journey?
 - Form an equation using the given condition.
 - Calculate the speed of the train.

Answers

$$\begin{aligned} \text{a) } & \frac{300}{x} \\ \text{b) } & \frac{300}{x+5} \\ \text{c) } & \frac{300}{x} - \frac{300}{x+5} = 2 \\ & \frac{1}{x} - \frac{1}{x+5} = \frac{2}{300} \\ & \frac{1}{x} - \frac{1}{x+5} = \frac{1}{150} \\ & x^2 + 5x - 750 = 0 \\ \text{d) } & x = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times -750}}{2 \times 1} \\ & x = 25, -30. \text{ Usual speed of train } 25 \text{ km/h.} \end{aligned}$$

- 3) There are 64 small squares in a chess board. The area of one small square is 6.25 sq.cm. There is a border of width 2cm around the chess board squares.
- If the length of the board is x what will be the total area of small squares?
 - Form a second degree equation using the given condition.
 - Calculate the length of the chess board.

Answers

$$\begin{aligned} \text{a) } & \text{On subtracting 2 from both sides area of the square formed by 64 small squares is } (x-4)^2 \\ \text{b) } & (x-4)^2 = 6.25 \times 64 \\ & x^2 - 8x + 16 = 400 \\ & x^2 - 8x - 384 = 0 \\ \text{c) } & x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ & x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 1 \times -384}}{2 \times 1} \\ & x = 24, -16. \\ & \text{Side of the chess board is } 24 \text{ cm} \end{aligned}$$

- 4) In a group of children each child gives a gift to every other child. If the total number of gifts is 132, then
- If the number of children is n then how many gifts each child give other children.
 - Form an equation using the given condition.
 - calculate the number of children in the group.

Answers

- a) $n - 1$ gifts should be given by a child
- b) Total number of gifts is $n(n - 1) = 132$
 $n^2 - n - 132 = 0$
- c) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times -132}}{2 \times 1}$
 $\frac{1 \pm \sqrt{529}}{2}$
 $x = 12, -11$. Number of children = 12

5) Teacher asked the children to draw a rectangle of area 5 sq.cm and perimeter 8 cm. Manju, a good student made a comment that it is possible to draw such a square after some algebraic calculations.

- a) If one side of the reactangle is x then what will be the other.
- b) Form a second degree equatio.
- c) Prove that it is not possible to construct such a rectangle.

Answers

- a) $2(l + b) = 8, l + b = 4$
ഒരു വശം x ആയാൽ മറ്റേ വശം $4 - x$
- b) $x(4 - x) = 5, -x^2 + 4x - 5 = 0, x^2 - 4x + 5 = 0$
- c) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $b^2 - 4ac = -4$. $\sqrt{b^2 - 4ac}$ is not a real number. Side is not real. Rectangle cannot be constructed.

Concepts

We are discussing the process of solving a second degree equation using the completing the square method. As the generalization of this method we can establish a formula to solve the second degree equation. It is not necessary to use the formula for solving the second degree equation.

In the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- 1) The participants of a meeting gave handshakes to each other. It is found that there are 190 handshakes in total.
- If there are n participants, what is the number of handshakes given by a participant to others?
 - Form a second degree equation using the number of participants and the number of handshakes.
 - Calculate the number of participants of the meeting.

Answers

- $n - 1$
- $\frac{n(n-1)}{2} = 190$
 $n^2 - n - 380 = 0$
- $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times -380}}{2 \times 1}$
 $x = \frac{1 \pm \sqrt{1521}}{2 \times 1}$
 $n = 20, -14$, Number of participants is 20

- 2) If the price of a book is reduced by 5 rupees, a person can buy 5 more books for 300 rupees.
- If the original price of the book is x , how many books can be purchased for 300 rupees?
 - If the price is decreased by 5 how many books can be purchased in 300 rupees
 - Form a second degree equation using the given condition.
 - Calculate the original price of the book

Answers

- a) $\frac{300}{x}$
- b) If the price is $x - 5$, the number of books = $\frac{300}{x-5}$
- c) $\frac{300}{x-5} - \frac{300}{x} = 5$
 $\frac{300x - 300x + 1500}{x(x-5)} = 5$
 $x^2 - 5x - 300 = 0$
- d) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-(-5) \pm \sqrt{(5)^2 - 4 \times 1 \times -300}}{2 \times 1}$
 $x = 20, -15$. The original price of the book is 20 rupees

3) The perimeter of a rectangle is 82cm, area 400 sq.cm

- a) If the length of one side is x then what is the length of other side ?
- b) Form a second degree equation
- c) Find the sides of the rectangle.

Answers

- a) $2(l + b) = 82, l + 2 = 41$. If one side is x the other side will be $41 - x$
- b) $x(41 - x) = 400, -x^2 + 41x - 400 = 0, x^2 - 41x + 400 = 0$
- c) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-(-41) \pm \sqrt{(-41)^2 - 4 \times 1 \times 400}}{2 \times 1}$
 $x = 25, 16$
Sides are 25cm and 16cm

4) The hypotenuse of a right triangle is 25cm, the difference between other two sides is 5cm

- a) If one of the perpendicular sides is x what is the length of other perpendicular side?
- b) Form a second degree equation .
- c) Calculate the length of its sides.
- d) Calculate the area of the triangle.

Answers

- a) $x + 5$
- b) $x^2 + (x + 5)^2 = 25^2$
 $x^2 + x^2 + 10x + 25 = 625$
 $2x^2 + 10x - 600 = 0$
 $x^2 + 5x - 300 = 0$
- c) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times -300}}{2 \times 1}$
 $x = 15, -20$. perpendicular sides are 15cm and 20cm
- d) Area = $\frac{1}{2} \times 15 \times 20 = 150$ sq.cm

5) The denominator of a fraction is 1 more than two times its numerator. The sum of the fraction and its reciprocal is $2\frac{16}{21}$.

- If the numerator is x what is its denominator.
- Write the fraction in x
- Form a second degree equation using the given condition.
- Find the fraction.

Answers

a) $2x + 1$

b) Fraction is $\frac{x}{2x+1}$.

c) $\frac{x}{2x+1} + \frac{2x+1}{x} = 2\frac{16}{21}$
 $\frac{x}{2x+1} + \frac{2x+1}{x} = \frac{58}{21}$
 $11x^2 - 26x - 21 = 0$

d) Solving, $x = 3$.

Fraction is $\frac{3}{2 \times 3 + 1} = \frac{3}{7}$

Concepts

We are discussing the process of solving a second degree equation using the completing the square method. As the generalization of this method we can establish a formula to solve the second degree equation. It is not necessary to use the formula for solving the second degree equation.

In the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) First term of an arithmetic sequence is 7 and common difference 3.

- What is its algebraic form?
- Find the sum of the first n terms
- How many terms of this sequence beginning from the first term makes the sum 710?

Answers

a) $x_n = dn + (f - d) = 3n + (7 - 3) = 3n + 4$

b) $S_n = (x_1 + x_n) \times \frac{n}{2}$
 $S_n = (7 + 3n + 4) \times \frac{n}{2}$
 $(11 + 3n) \times \frac{n}{2} = \frac{11n}{2} + \frac{3n^2}{2}$

c) $\frac{3n^2}{2} + \frac{11n}{2} = 710$
 $\frac{3n^2 + 11n}{2} = 710. 3n^2 + 11n - 1420 = 0$
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-11 \pm \sqrt{11^2 - 4 \times 3 \times -1420}}{2 \times 3}$
 $x = \frac{-11 \pm \sqrt{17161}}{6}, n = \frac{120}{6}, \frac{-142}{6}$
 $n = 20.$ Twenty terms make the sum 720

2) On joining two vertices of a polygon we get either a side or a diagonal. Consider a polygon of n sides.

- How many diagonals can be drawn from a vertex?
- How many diagonals are there in a polygon of n sides ?
- Find the number of sides of a polygon having 35 diagonals.
- Name the polygon having number of sides and diagonals equal.

Answers

a) $n - 3$

(Joining a vertex to other vertices on either side of it forms a side of the polygon. That is, $n - 3$ diagonals can be drawn from a vertex)

b) $\frac{n(n-3)}{2}$

(Diagonal from the vertex A to B and the diagonal from B to A are treated as a single diagonal. So we should take half of $n(n - 3)$)

c) $\frac{n(n-3)}{2} = 35$

$$n(n - 3) = 70, n^2 - 3n = 70, n^2 - 3n - 70 = 0.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times -70}}{2 \times -3}$$

$$\frac{3 \pm \sqrt{289}}{2}, n = 10, -7. \text{ The number of sides is } 10$$

d) Pentagon.

3) The points $A_1, A_2, A_3 \dots A_n$ are marked in a circle. On joining two points we get a chord .

a) How many chords can be drawn from a given point to other points?

b) What is the total number of chords?

c) How many points should be marked on the circle to get 120 chords.

Answers

a) $n - 1$

b) $\frac{n(n-1)}{2}$

c) $\frac{n(n-1)}{2} = 120$

$$n(n - 1) = 240, n^2 - n - 240 = 0,$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times -240}}{2 \times 1}$$

$$n = \frac{1 \pm \sqrt{961}}{2}, n = 16, -15. \text{ The number of points marked on the circle is } 16$$

4) Consider the sequence of numbers which gives the remainder 3 on dividing by 4.

a) Write the algebraic form of this sequence ?

b) What is the sum of first n terms of this sequence ?

c) How many terms from the beginning make the sum 820?

d) Can the sum of any 25 terms of this sequence 2020?

Answers

a) The sequence is 3, 7, 11 ...

$$x_n = dn + (f - d) = 4n - 1$$

b) $S_n = (x_1 + x_n) \times \frac{n}{2} = (3 + 4n - 1) \times \frac{n}{2}$
 $n + 2n^2$

c) $2n^2 + n = 820, 2n^2 + n - 820 = 0$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-1 \pm \sqrt{1^2 - 4 \times 2 \times -820}}{2 \times 2}$$

$$n = \frac{-1 \pm \sqrt{6561}}{4}$$

$$n = \frac{-1 \pm 81}{4}, n = 20.$$

The sum of 20 terms is 820

d) All terms are odd numbers. The sum of 25 odd numbers cannot be an even number.

5) The sum of a number and its positive square root is $\frac{6}{25}$.

a) If x is the number, write an equation using the given conditions.

b) Write the equation in the form $ax^2 + bx + c = 0$

c) Find the number.

Answers

a) $x + \sqrt{x} = \frac{6}{25}$

b) $\sqrt{x} = \frac{6}{25} - x$
 $\sqrt{x} = \frac{6 - 25x}{25}$

Squaring on both sides and solving we get in the form $ax^2 + bx + c = 0$. It is $625x^2 - 925x + 36 = 0$

c) Solving we get $x = \frac{1}{25}$