Online Class – X – 20 **06/ 08 / 2021**

2 . Circles – Class 8

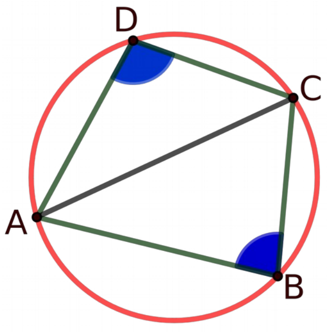
To view class

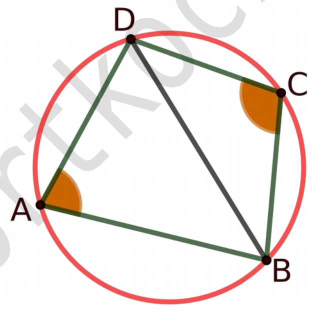
Circle and Quadrilateral

Finding the relation between the angles of a quadrilateral if the four vertices are on a circle

Consider quadrilateral ABCD.

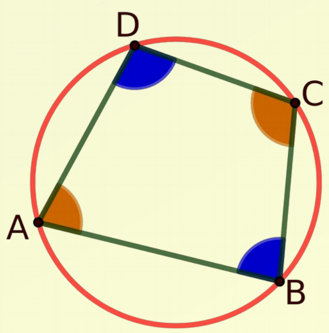
Draw diagonals AC & BD





Diagonals AC & BD are chords of the circle .
Since a pair of angles on an arc and its alternate arc are supplementary ,
we get , $\angle B + \angle D = 180°$ & $\angle A + \angle C = 180°$

Conclusion:



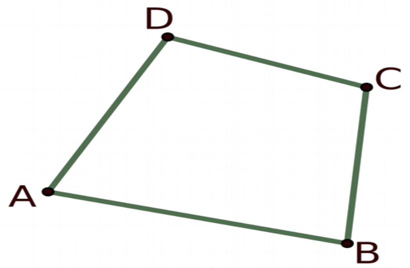
If all the four vertices of a quadrilateral are on a circle, then its **opposite angles are supplementary.**

$\angle B + \angle D = 180°$

$\angle A + \angle C = 180°$

To check ,if the opposite angles of a quadrilateral are supplementary, then all its vertices are on a circle

Consider quadrilateral ABCD.



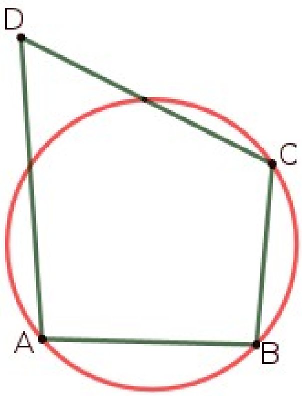
We can draw a circle passing through three of the vertices A, B, C .

Then fourth vertex D may be

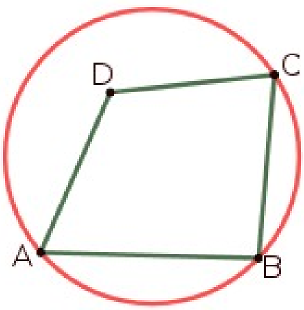
(i) Outside the circle

(ii) Inside the circle

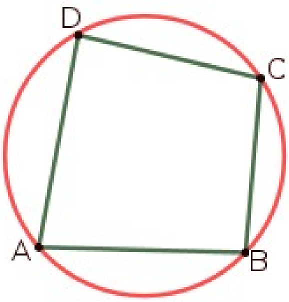
(iii) On the circle



(i)



(ii)



(iii)

Case (i) When the fourth vertex D is outside the circle

Let CD intersect the circle at E.

Join AE

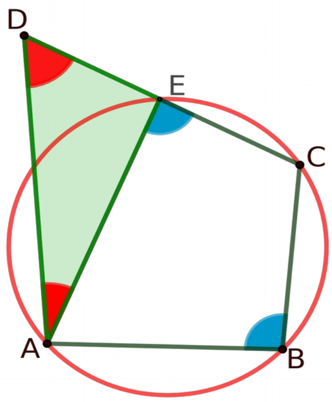
Consider quadrilateral ABCE

$\angle B + \angle AEC = 180°$. . . . . (1)

Consider $\triangle$ AED

$\angle AEC = \angle EAD + \angle D$

So, $\angle D < \angle AEC$ . . . . (2)



From (1) & (2) **$\angle B + \angle D < 180^\circ$**

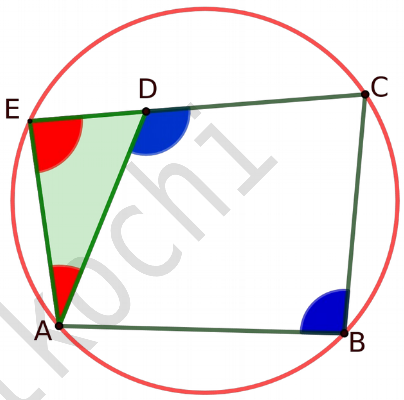
Case (ii) When the fourth vertex D is inside the circle

Extend CD to meet the circle at E

Join AE

Consider quadrilateral ABCE

$\angle B + \angle E = 180^\circ$ .... (1)



Consider $\triangle$ AED

$\angle ADC = \angle E + \angle EAD$

So, $\angle ADC > \angle E$ .... (2)

From (1) & (2) $\angle B + \angle ADC > 180^\circ$

i.e, **$\angle B + \angle D > 180^\circ$**

Case (iii)

From case(i) & case(ii) we have seen,

When the **fourth vertex** D is **outside** the circle then

$\angle B + \angle D < 180^\circ$

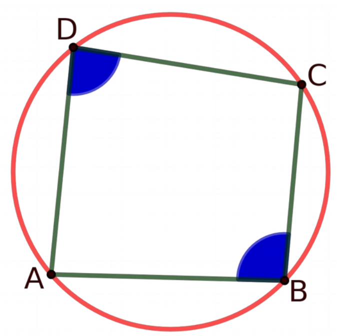
When the **fourth vertex** D is **inside** the circle then

$\angle B + \angle D > 180^\circ$

So,

If $\angle B + \angle D = 180^\circ$

then , **$\angle$D must be on the circle**

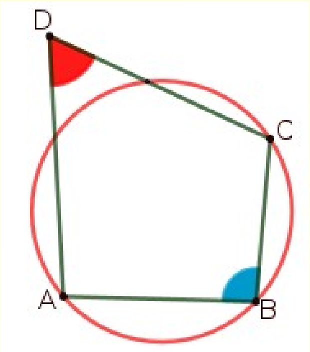


If the opposite angles of a quadrilateral are supplementary, we can draw a circle passing through all four of its vertices.

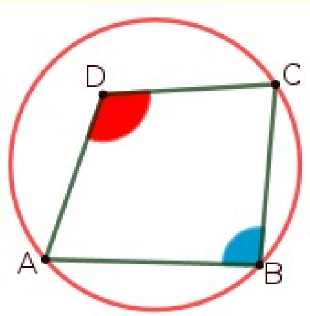
Conclusion

If vertex D of quadrilateral ABCD is ,

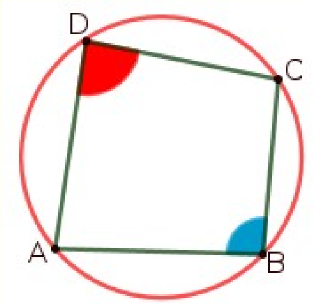
(i) **Outside** the circle drawn through the other three veritces, then $\angle B + \angle D < 180°$



(ii) **Inside** the circle drawn through the other three veritces, then $\angle B + \angle D > 180°$



(iii) **On** the circle drawn through the other three veritces, then $\angle B + \angle D = 180°$

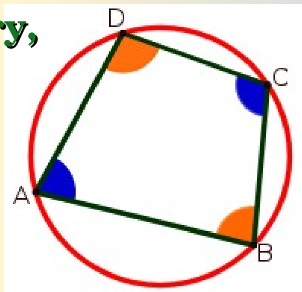


If all four vertices of a quadrilateral are on a circle,

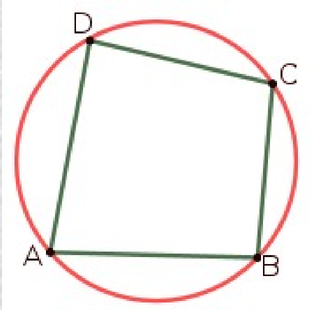
➜ **then its opposite angles are supplementary,**

If the opposite angles of a quadrilateral are supplementary,

➜ **then all its vertices are on a circle .**



If the opposite angles of a quadrilateral are supplementary, we can draw a circle passing through all four of its vertices.



This quadrilateral can be called as a

Cyclic Quadrilateral

Cyclic quadrilaterals are those quadrilaterals with opposite angles supplementary.

*** All rectangles are cyclic quadrilaterals**

Assignment

Q) ABCD is an isosceles trapezium. Check whether it is a cyclic quadrilateral.