Online Class – X – 22 **11 / 08 / 2021**

2 . Circles – Class 10

To view class

Cyclic quadrilaterals are those quadrilaterals with opposite angles supplementary.

Quadrilaterals which are always cyclic are
(i) Square
(i) Rectangle
(iii) Isosceles Trapezium

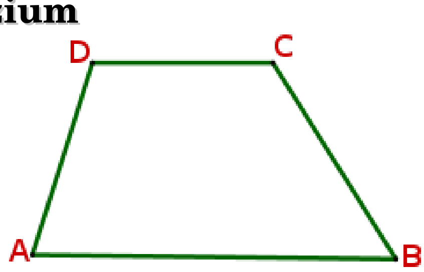
Assignment Answer

(Q) Prove that any non- isosceles trapezium is not cyclic ?

Ans)

Given ABCD is a non- isosceles trapezium

So $\angle A \neq \angle B$ . . . . (1)



Since ABCD is a trapezium, AB // CD.

So $\angle A + \angle D = 180°$ . . . . . (2) (Co-interior angles are supplementary)

From (1) & (2) we have $\angle B + \angle D \neq 180°$

Since the opposite angles are not supplementary ,
ABCD is not cyclic.
That is , any non- isosceles trapezium is not cyclic .

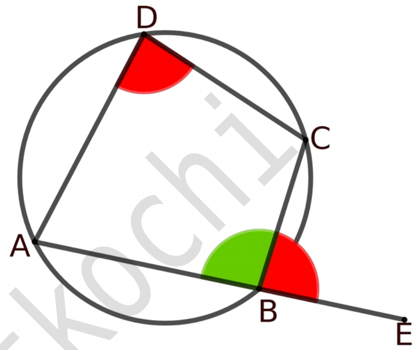
T .B page 59

Q2) Prove that any outer angle of a cyclic quadrilateral is equal to the inner angle at the opposite vertex.

Ans)

In the figure ∠CBE is an outer angle at a vertex of cyclic quadrilateral ABCD .

We have to prove ∠CBE =∠ADC



Since ABCD is cyclic

∠ABC + ∠ADC = 180° 1

Also

∠ABC +∠CBE = 180° 2

(Linear pair)

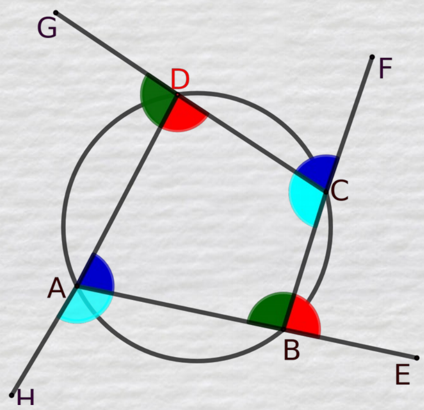
From 1 & 2 we have

∠ABC + ∠ADC = ∠ABC +∠CBE

∴ ∠ADC = ∠CBE

That is any outer angle of a cyclic quadrilateral is equal to the inner angle at the opposite vertex .

Any outer angle of a cyclic quadrilateral is equal to the inner angle at the opposite vertex.



∠ADC = ∠CBE

∠ABC = ∠GDA

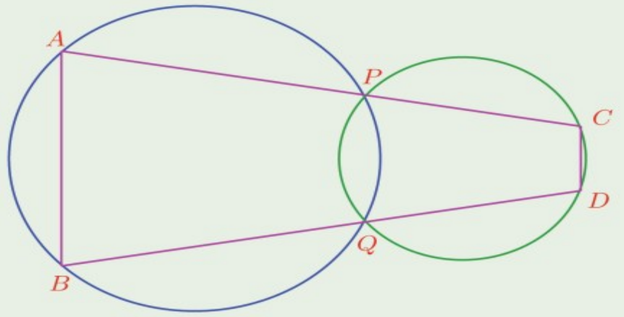
∠DAB = ∠DCF

∠HAB = ∠DCB

T .B page 60

Q6) i)

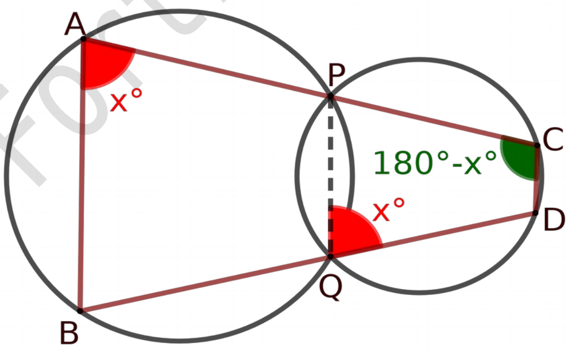
The two circles below intersect at P, Q and lines through these points meet the circles at A, B, C, D. The lines AC and BD are not parallel. Prove that if these lines are of equal length, then $ABDC$ is a cyclic quadrilateral.



Ans)

Join PQ.

Now ABQP and PQDC becomes cyclic quadrilaterals.



Let ∠A = x° ,

Then ,

∠PQD = x° ***(any outer angle of a cyclic quadrilateral is equal to the inner angle at the opposite vertex)***

Since PQDC is cyclic ,∠C = 180°– x°

Now ,

∠A + ∠C = ~~x°~~ + 180°– ~~x°~~

So, ∠A + ∠C = 180°

Since ∠A + ∠C = 180° we can say AB parallel to CD

(***co-interior angles are supplementary*****)**

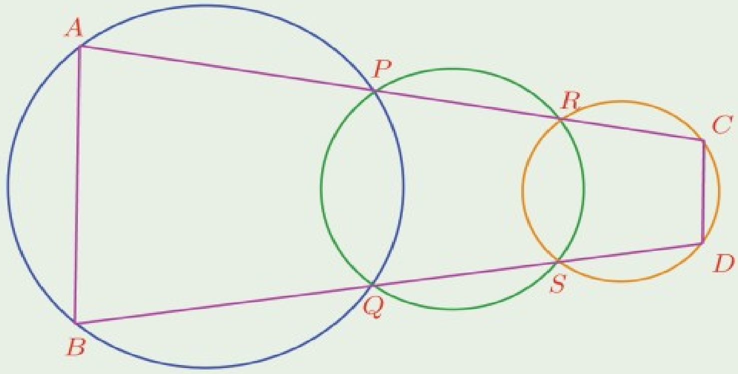
Since AB parallel to CD , and given AC = BD,

ABDC is an isosceles trapezium.

So ABDC is a cyclic quadrilateral.

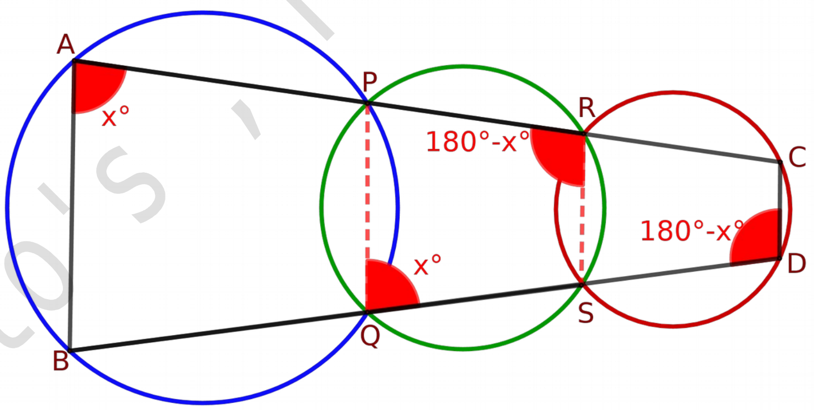
Q6) ii)

In the picture, the circles on the left and right intersect the middle circle at P, Q, R, S; the lines joining them meet the left and right circles at A, B, C, D. Prove that $ABDC$ is a cyclic quadrilateral.



Ans)

Draw PQ & RS



Let ∠BAP = x°, then ∠PQS = x° ***(any outer angle of a cyclic quadrilateral is equal to the inner angle at the opposite vertex)***

Since ∠PQS = x° , then ∠PRS =180°–x° ***(opposite angles of a cyclic quadrilateral are supplementary)***

Since ∠PRS =180° – x°, then ∠SDC = 180° – x° ***(any outer angle of a cyclic quadrilateral is equal to the inner angle at the opposite vertex)***

So ,

$$\angle BAP + \angle SDC = \cancel{x^\circ} + 180^\circ - \cancel{x^\circ}$$
$$= 180^\circ$$

So considering quadrilateral ABDC,

$\angle A + \angle D = 180^\circ$

$\angle B + \angle C = 360^\circ - 180^\circ = 180^\circ$

Since opposite angles of quadrilateral ABDC are supplementary,

ABDC is a cyclic quadrilateral.