

EC

Heading

Sol 1

In small signal model of the transistor

$$r_{\pi} = \frac{\beta V_T}{I_C}$$

$V_T \rightarrow$ Thermal voltage

$$= \frac{V_T}{I_C / \beta} = \frac{V_T}{I_B}$$

$$\frac{I_C}{\beta} = I_B$$

$$= \frac{V_T}{I_B}$$

So, $r_{\pi} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \Omega$

$$V_T = 25 \text{ mV}$$

$$I_B = 1 \text{ mA (DC)}$$

Hence (c) —

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The power spectral density consists of two components

Centered

- 1) A delta function at $\omega_0 = 10^4$ rad/s and amplitude of 400 whose inverse fourier transform is

$$400 \delta(\omega - 10^4) \xleftrightarrow{f^{-1}} \frac{400 e^{j10^4 t}}{\pi}$$

- 2) A triangular component of amplitude 6 and width $\tau = 10^3$ rad/s centered at $\omega_0 = 10^4$ rad/s

$$6 \text{ tri}\left(\frac{\omega - 10^4}{10^3}\right) \xleftrightarrow{f^{-1}} \frac{6 \times 10^3}{\pi} \text{sinc}^2\left(\frac{t\tau}{2\pi}\right) e^{j10^4 t}$$

Correlation function

$$R_x(\tau) = \frac{400}{\pi} e^{j10^4 \tau} + \frac{6 \times 10^3}{\pi} \text{sinc}^2\left(\frac{t\tau}{2\pi}\right) e^{j10^4 \tau}$$

$$E[x^2(t)] = R_x(0)$$

$$= \frac{400}{\pi} + \frac{6 \times 10^3}{\pi}$$

$$= \frac{6400}{\pi}$$

Heading

Sol

For raised cosine spectrum transmission bandwidth is given as

$$B_T = \text{BW } W(1+\alpha) \quad \alpha \rightarrow \text{Roll off factor}$$

$$B_T = \frac{R_b}{2} (1+\alpha) \quad R_b \rightarrow \text{Maximum signaling rate}$$

$$3500 = \frac{R_b}{2} (1+0.75)$$

$$R_b = \frac{3500 \times 2}{1.75} = 4000$$

Hence (C) —

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Incident wave

$$E_i = (8\hat{a}_x + 6\hat{a}_y - 5\hat{a}_z) e^{j(\omega t + 3x + 4y)}$$

$$E_r = \tau E_i$$

($\tau \rightarrow$ reflection Coefficient)

$$E_r = \tau \cos[\omega t - (-3x + 4y)]$$

$$E_{rx} = 8 e^{j[\omega t - (-3x + 4y)]} \hat{a}_x$$

$$E_{ry} = 6 e^{j[\omega t - (-3x + 4y)]} \hat{a}_y$$

For reflected wave ($\tau = -1$ for perfect conductor)

$$E_{rx} = -8 e^{j[\omega t + (-3x + 4y)]} \hat{a}_x$$

$$= -8 e^{j[\omega t - 3x + 4y]} \hat{a}_x$$

$$E_{ry} = -6 e^{j[\omega t + (-3x + 4y)]} \hat{a}_y$$

$$= -6 e^{j[\omega t - 3x + 4y]} \hat{a}_y$$

$$E_{rz} = -5 e^{j[\omega t + (-3x + 4y)]}$$

$$= -5 e^{j[\omega t - 3x + 4y]} \hat{a}_z$$

Hence $\odot \longrightarrow$

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(5)

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The field in circular polarization is found to be

$$E_s = E_0 (a_x \pm j a_y) e^{-j\beta z}$$

where plus sign is used for left circular polarization and minus sign for right circular polarization.

The given problem has left circular polarization.

$$\beta = 2.5 = \frac{\omega}{c}$$

$$2.5 = \frac{2\pi f}{c}$$

$$f = \frac{2.5 \times c}{2\pi} = \frac{2.5 \times 3 \times 10^8}{2 \times 3.14}$$

$$= 1.2 \text{ GHz}$$

Hence (A)

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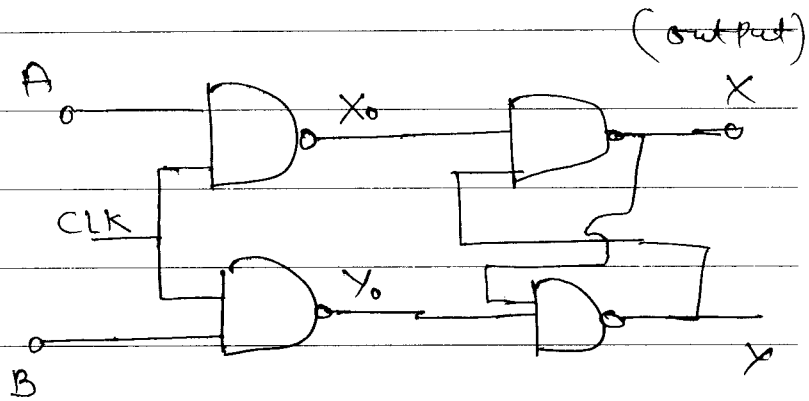
Race around condition occurs when output oscillates between 0 and 1 for a stable input.

Now, we consider all the given options

(B) CLK = 0

$X_0 = Y_0 = 1$ (always)

$Y = \bar{X}$
 $X = \bar{Y}$ } True



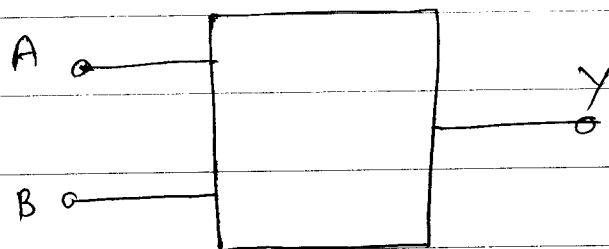
(C) CLK = 1 & A = B = 1

If CLK = 1, then $X_0 = Y_0 = 0$

So, $X = Y = 1$

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$$Y = 1, \text{ when } A > B$$

$$A = a_1 a_0 \quad B = b_1 b_0$$

a_1	a_0	b_1	b_0	Y
0	0	0	0	1
1	0	0	0	1
1	0	0	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1

Total combination = 6

Hence (B)

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Let $V > 0.7V$ and diode is in FW bias.

By applying kirchoff's voltage law

$$10 - i' \times 1k - V = 0$$

$$10 - \left[\frac{V - 0.7}{500} \right] (1000) - V = 0$$

$$10 - (V - 0.7) \times 2 - V = 0$$

$$10 - 3V + 1.4 = 0$$

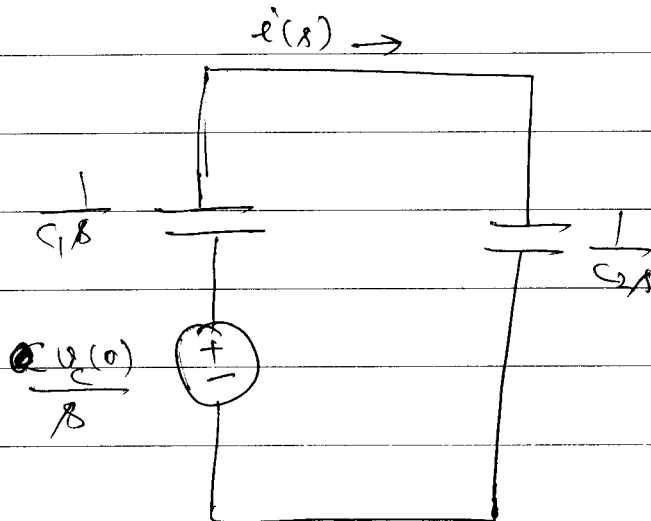
$$V = \frac{11.4}{3} = 3.8V \quad (\text{Assumption is true})$$

$$\text{So } i' = \frac{V - 0.7}{500} = \frac{3.8 - 0.7}{500} = 6.2 \text{ mA}$$

Hence (D) _____

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The s-domain equivalent circuit is shown as below



$$\underline{I}(s) = \frac{V_c(0)/s}{\frac{1}{C_1s} + \frac{1}{C_2s}} = \frac{V_c(0)}{\frac{1}{C_1} + \frac{1}{C_2}}$$

$$\underline{I}(s) = \left(\frac{C_1 C_2}{C_1 + C_2} \right) (12V)$$

for $\underline{I}(s) = 12 C_{eq}$

taking inverse Laplace transform

$$i(t) = 12 C_{eq} \delta(t) \quad (\text{Impulse})$$

~~Theoretically~~ Hence (D) ———.

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In phasor form

$$Z = 4 - j3$$

$$Z = 5 \angle -36.86^\circ \Omega$$

$$I = 5 \angle 100^\circ \text{ A}$$

Average power delivered

$$P_{\text{avg}} = \frac{1}{2} |I|^2 Z \cos \theta$$

$$= \frac{1}{2} 25 \times 5 \cos 36.86^\circ$$

$$= 50 \text{ W}$$

Hence (B) _____

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Using s-domain differentiation property of Laplace transform

$$\begin{aligned} \text{If } f(t) &\xleftrightarrow{\mathcal{L}} F(s) \\ tf(t) &\xleftrightarrow{\mathcal{L}} -\frac{dF(s)}{ds} \end{aligned}$$

$$\text{So, } \mathcal{L}[tf(t)] = -\frac{d}{ds} \left[\frac{1}{s^2 + s + 1} \right]$$

$$= \frac{2s + 1}{(s^2 + s + 1)^2}$$

Hence (B) —————.

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$$t \frac{dx}{dt} + x = t$$

$$\frac{dx}{dt} + \frac{x}{t} = 1$$

$$\frac{dx}{dt} + px = Q \quad (\text{General form})$$

Integrating factor, $IF = e^{\int p dt}$

$$= e^{\int \frac{1}{t} dt}$$

$$= e^{\ln t} = t$$

Solution has the form

$$x IF = \int Q IF dt + C$$

$$xt = \int (1)(t) dt + C$$

$$xt = \frac{t^2}{2} + C$$

$$x(1) = 0.5$$

$$0.5 = \frac{1}{2} + C$$

$$C = 0$$

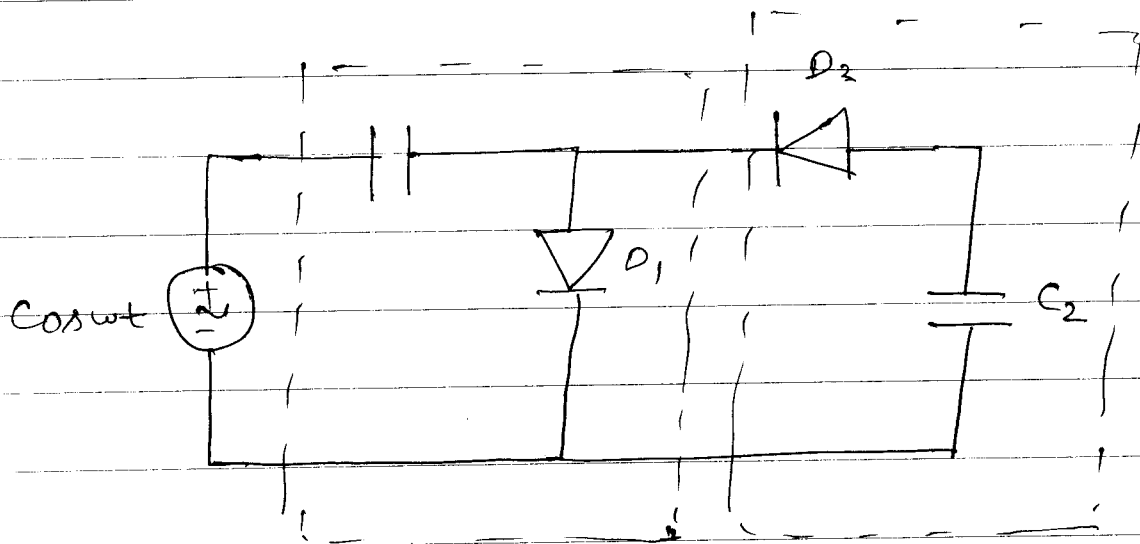
So $xt = \frac{t^2}{2} \Rightarrow x = \frac{t}{2}$

Hence (D) _____.

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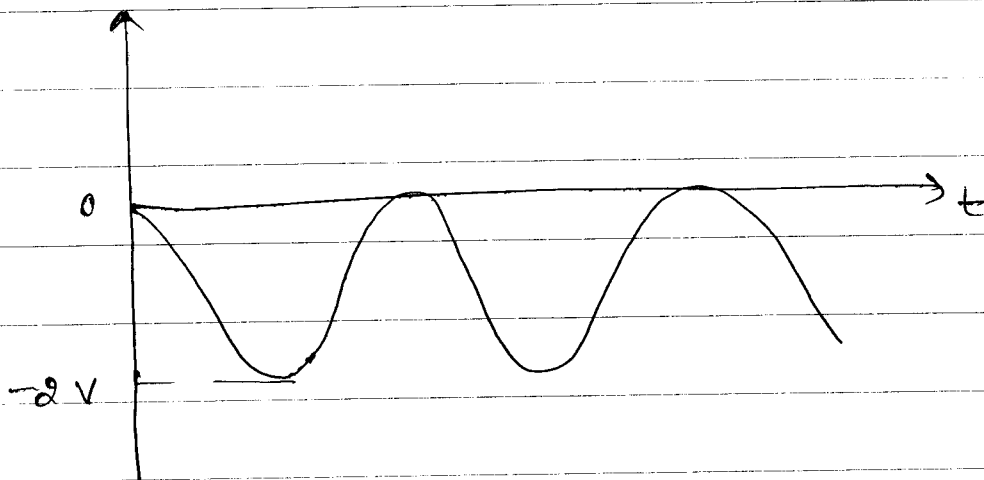
The circuit composed of a clamper and a peak rectifier as shown



Clamper

Peak rectifier

Clamper clamps the voltage to zero volt.



Heading

The peak rectifier adds +1 to peak voltage, so the overall peak voltage slows down by -1 volt.

$$V_0 = \cos \omega t - 2 + 1$$
$$= \cos \omega t - 1$$

Hence (A) — .

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Parallel connection of MOS \Rightarrow OR operation
Series connection of MOS \Rightarrow AND operation

The pull-up network acts as an inverter. From pull down network

$$Y = \overline{(A+B)C}$$

$$Y = \overline{(A+B)} + \overline{C}$$
$$= \overline{A} \cdot \overline{B} + \overline{C}$$

Hence (A) ———

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Heading

Entropy function of a discrete memory less system is given as

$$H = \sum_{k=0}^{N-1} P_k \log \left(\frac{1}{P_k} \right)$$

where P_k is probability of symbol s_k .

For first two symbols probability is same, so

$$H = P_1 \log \left(\frac{1}{P_1} \right) + P_2 \log \left(\frac{1}{P_2} \right) + \sum_{k=3}^{N-1} P_k \log \left(\frac{1}{P_k} \right)$$

$$= - \left(P_1 \log P_1 + P_2 \log P_2 + \sum_{k=3}^{N-1} P_k \log P_k \right)$$

$$= - \left(2P \log P + \sum_{k=3}^{N-1} P_k \log P_k \right) \quad (P_1 = P_2 = P)$$

Now, $P_1 = P + \epsilon$, $P_2 = P - \epsilon$

$$\text{So } H_0 = - \left[(P + \epsilon) \log (P + \epsilon) + (P - \epsilon) \log (P - \epsilon) + \sum_{k=1}^{N-1} P_k \log P_k \right]$$

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By comparing

$$H_0 < H$$

Entropy of source decreases.

Hence (D) _____ .

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Characteristic impedance

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right)$$

 $b \rightarrow$ Outer diameter $a \rightarrow$ inner diameter

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}} \ln\left(\frac{b}{a}\right)$$

$$= \sqrt{\frac{4\pi \times 10^{-7} \times 36\pi}{10^{-9} \times 10.89}} \ln\left(\frac{2.4}{1}\right)$$

$$= 100 \Omega$$

Hence (B)

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Heading

The directivity is defined as

$$D = \frac{F_{\max}}{F_{\text{ave}}}$$

$$F_{\max} = 1$$

$$F_{\text{ave}} = \frac{1}{4\pi} \int F(\theta, \phi) d\Omega$$

$$= \frac{1}{4\pi} \left[\int_0^{2\pi} \int_0^{2\pi} F(\theta, \phi) \sin\theta d\theta d\phi \right]$$

$$= \frac{1}{4\pi} \left[\int_0^{2\pi} \int_0^{\pi/2} \cos^4\theta \sin\theta d\theta d\phi \right]$$

$$= \frac{1}{4\pi} \left[\int_0^{\pi/2} \cos^4\theta \sin\theta d\theta d\phi \right]$$

$$= \frac{1}{4\pi} \left[2\pi \frac{\cos^5\theta}{5} \right]_0^{\pi/2}$$

$$= \frac{1}{4\pi} \times \frac{2\pi}{5} = \frac{1}{10}$$

$$D = \frac{1}{1/10} = 10$$

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$$\text{Or, } D(\text{in dB}) = 10 \log 10 = 10 \text{ dB}$$

Hence (A)

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$$x[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^n u[n]$$

$$x[n] = \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{3}\right)^{-n} u[-n-1] - \left(\frac{1}{2}\right)^n u[n]$$

Taking z-transform

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} u[n] + \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^{-n} z^{-n} u[-n-1]$$

$$- \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} u[n]$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} + \sum_{n=-\infty}^{-1} \left(\frac{1}{3}\right)^{-n} z^{-n} - \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} + \sum_{n=-\infty}^{-1} \left(\frac{1}{3}\right)^{-n} z^{-n} - \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3z}\right)^n + \sum_{m=1}^{\infty} \left(\frac{1}{3}\right)^m z^m - \sum_{n=0}^{\infty} \left(\frac{1}{2z}\right)^n$$

$$\underbrace{\hspace{10em}}_I$$

$$\underbrace{\hspace{10em}}_{II}$$

$$\underbrace{\hspace{10em}}_{III}$$

Series I converges if $\left|\frac{1}{3z}\right| < 1$ or $|z| > \frac{1}{3}$

Series II converges if $\left|\frac{1}{3}z\right| < 1$ or $|z| < 3$

Heading

Series (1) Converges if $|\frac{1}{az}| < 1$ or $|z| > \frac{1}{2}$

Region of convergence of $x(z)$ will be intersection of above three

So,

$$\text{Roc: } \frac{1}{2} < |z| < 3$$

Hence (c) —

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Heading

Prime implicants are the number of terms that we get by solving k-map

X \ yz	00	01	11	10
1	1	1		
0			1	1

$$F = \underbrace{x\bar{y} + \bar{x}y}_{\text{prime implicants}}$$

Hence (A) is correct option.

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Heading

$$G(s) = \frac{(s^2 + 9)(s + 2)}{(s + 1)(s + 3)(s + 4)}$$

$$G(j\omega) = \frac{(-\omega^2 + 9)(j\omega + 2)}{(j\omega + 1)(j\omega + 3)(j\omega + 4)}$$

The steady state output will be zero if

$$|G(j\omega)| = 0$$

$$-\omega^2 + 9 = 0$$

$$\omega = 3 \text{ rad/sec}$$

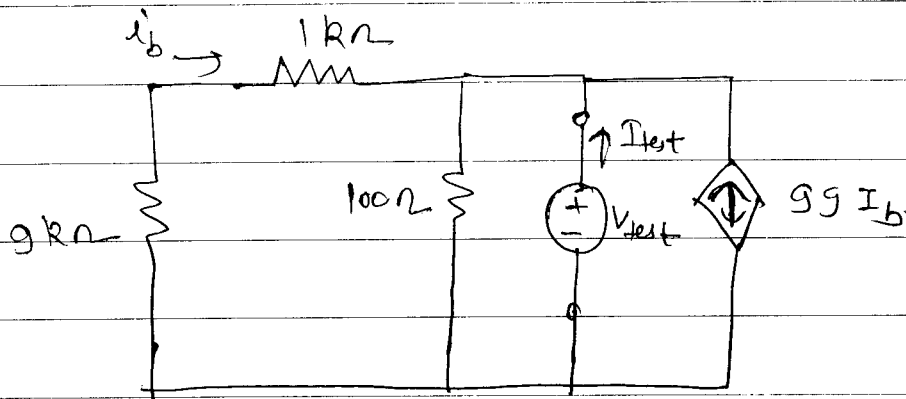
Hence (c) _____

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Soln:

We put a test source between terminals a-b to obtain equivalent impedance



$$Z_{Th} = \frac{V_{test}}{I_{test}}$$

By applying KCL at top right node

$$\frac{V_{test}}{9k + 1k} + \frac{V_{test}}{100} + 99I_b = I_{test}$$

$$\text{But } V_{test} \text{ and } I_b = \frac{-V_{test}}{9k + 1k}$$

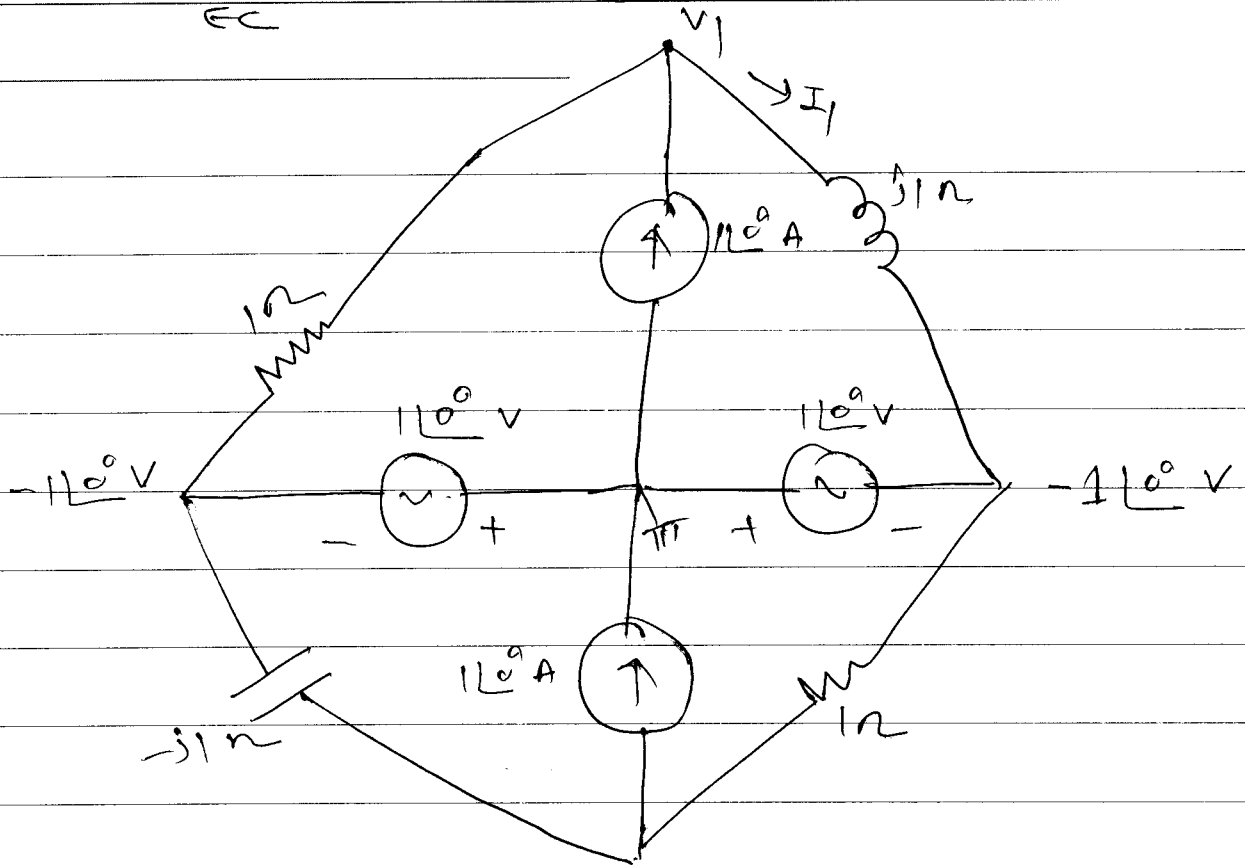
$$80 \frac{V_{test}}{9k + 1k} + \frac{V_{test}}{100} + \frac{99V_{test}}{9k + 1k} = I_{test}$$

$$\frac{100V_{test}}{10 \times 10^3} + \frac{V_{test}}{100} = I_{test}$$

$$\frac{2V_{test}}{100} = I_{test} \Rightarrow Z_{Th} = \frac{V_{test}}{I_{test}} = 50 \Omega$$

Hence (A)

Heading



Applying nodal analysis at top node

$$\frac{V_1 - (-110^\circ)}{1} + \dots + \dots$$

$$\frac{V_1 + 110^\circ}{1} + \frac{V_1 + 110^\circ}{j1} = 110^\circ$$

$$V_1(j1 + 1) + j110^\circ = 110^\circ$$

$$V_1 = \frac{-j110^\circ}{1 + j1}$$

$$\text{Current } I_1 = \frac{V_1 + 110^\circ}{j1} = \frac{-j110^\circ + 110^\circ}{j1}$$

$$= \frac{j110^\circ}{(1 + j)j}$$

$$= \frac{110^\circ}{1 + j} \text{ A}$$

Hence (B)

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$$f(z) = \frac{1}{z+1} - \frac{2}{z+3}$$

$\frac{1}{2\pi j} \oint_C f(z) dz =$ sum of the residues
 of the poles which lie
 inside the given closed
 region

$$C \Rightarrow |z+1| = 1$$

only pole $z = -1$ lies inside the circle, so
 residue at $z = -1$ is

$$f(z) = \frac{-z+1}{(z+1)(z+3)}$$

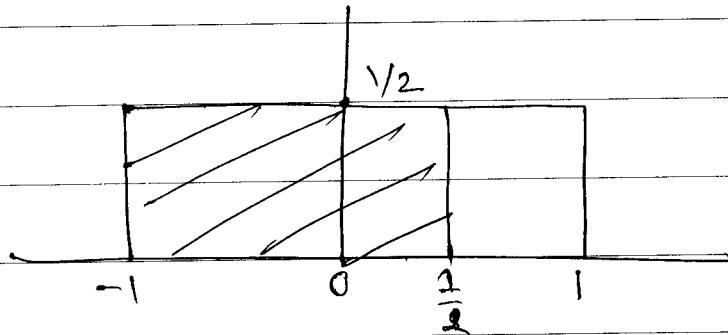
$$= \lim_{z \rightarrow -1} \frac{(z+1)(-z+1)}{(z+1)(z+3)} = \frac{2}{2} = 1$$

So $\frac{1}{2\pi j} \oint_C f(z) dz = 1$

Hence (C) _____.

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Probability density function of uniformly distributed variables x and y is shown as



$$P[\max(x, y)] < \frac{1}{2}$$

Since x and y are independent

$$\left\{ P[\max(x, y)] < \frac{1}{2} \right\} = P(x < \frac{1}{2}) \cdot P(y < \frac{1}{2})$$

$$P(x < \frac{1}{2}) = \text{shaded area} = \frac{3}{4}$$

$$\text{So } P\left\{ [\max(x, y)] < \frac{1}{2} \right\} = \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$$

Hence (B)

Heading

$$x = \sqrt{-1} = i = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$$

$$\text{So } x = e^{j\pi/2}$$

$$x^x = (e^{j\pi/2})^x = (e^{j\pi/2})^i = e^{-\pi/2}$$

Hence (A) ←

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Heading

For the semiconductor

$$n_0 p_0 = n_i^2$$

$$p_0 = \frac{n_i^2}{n_0} = \frac{10^{20}}{10^{19}} = 10 \text{ per cm}^3$$

Volume of given device

$$V = \text{Area} \times \text{depth}$$

$$= 1 \mu\text{m}^2 \times 1 \mu\text{m}$$

$$= 10^{-8} \text{ cm}^2 \times 1 \times 10^{-4} \text{ cm}$$

$$= 10^{-12} \text{ cm}^3$$

So total no. of holes is

$$P = p_0 \times V$$

$$= 10 \times 10^{-12}$$

$$= 10^{-11}$$

which is approximately equal to zero.

Hence (D) _____.

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In a coherent binary PSK system, the pair of signals $s_1(t)$ and $s_2(t)$ used to represent binary system 1 and 0 respectively

$$s_1(t) = \sqrt{\frac{2E}{T}} \sin \omega_c t$$

$$s_2(t) = -\sqrt{\frac{2E}{T}} \sin \omega_c t$$

where $0 \leq t \leq T$, E is the transmitted energy per bit.

General function of local oscillator

$$\phi_1(t) = \sqrt{\frac{2}{T}} \sin(\omega_c t), \quad 0 \leq t < T$$

But here local oscillator is ahead with 45° so,

$$\phi_1(t) = \sqrt{\frac{2}{T}} \sin(\omega_c t + 45^\circ)$$

Transmitted signal $s_1(t)$ and $s_2(t)$ in terms of $\phi_1(t)$ are

$$s_1(t) = \phi_1(t)$$

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Since $Z_0 = \sqrt{Z_1 Z_2}$

$$100 = \sqrt{50 \times 200}$$

This is quarter wave matching. The length would be odd multiple of $\lambda/4$

$$l = (2m+1) \frac{\lambda}{4}$$

$$f_1 = 429 \text{ MHz}, \quad l_1 = \frac{c}{f_1 \times 4} = \frac{3 \times 10^8}{429 \times 10^6 \times 4} = 0.174 \text{ m}$$

$$f_2 = 1 \text{ GHz}, \quad l_2 = \frac{c}{f_2 \times 4} = \frac{3 \times 10^8}{1 \times 10^9 \times 4} = 0.075 \text{ m}$$

Only option (c) is odd multiple of both l_1 and l_2 .

$$\frac{1.58}{l_1} = 9$$

$$(2m+1) = \frac{1.58}{l_2} \approx 21$$

Hence (c) ———.

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$$y(t) = \int_{-\infty}^t x(\tau) \cos(3\tau) d\tau$$

Time invariance:

$$\text{Let } x(t) = \delta(t)$$

$$y(t) = \int_{-\infty}^t \delta(\tau) \cos(3\tau) d\tau$$

$$= u(t) \cos(0)$$

$$= u(t)$$

For a delayed input $x(t-t_0)$ output is

$$y(t, t_0) = \int_{-\infty}^t \delta(\tau-t_0) \cos(3\tau) d\tau$$

$$= u(t) \cos(3t_0)$$

Delayed output

$$y(t-t_0) = u(t-t_0)$$

$$y(t, t_0) \neq y(t-t_0)$$

system is not time-invariant.

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Stability:

Consider a bounded input $x(t) = \cos 3t$

$$y(t) = \int_{-\infty}^t \cos^2 3t \, dt = \int_{-\infty}^t \frac{1 - \cos 6t}{2} \, dt$$

$$= \frac{1}{2} \int_{-\infty}^t 1 \cdot dt - \frac{1}{2} \int_{-\infty}^t \cos 6t \, dt$$

As $t \rightarrow \infty$, $y(t) \rightarrow \infty$ (unbounded)

System is not stable.

Hence (D) _____

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$$Y(s) = \frac{k(s+1)}{s^3 + as^2 + 2s + 1} [R(s) - Y(s)]$$

$$Y(s) \left[1 + \frac{k(s+1)}{s^3 + as^2 + 2s + 1} \right] = \frac{k(s+1)}{s^3 + as^2 + 2s + 1} R(s)$$

$$Y(s) [s^3 + as^2 + s(2+k) + (1+k)] = k(s+1) R(s)$$

Transfer Function

$$H(s) = \frac{Y(s)}{R(s)} = \frac{k(s+1)}{s^3 + as^2 + s(2+k) + (1+k)}$$

Routh table:

s^3	1	$2+k$
s^2	a	$1+k$
s^1	$\frac{a(2+k) - (1+k)}{a}$	0

For oscillation $\frac{a(2+k) - (1+k)}{a} = 0$

$$a = \frac{k+1}{k+2}$$

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Auxiliary equation

$$A(s) = as^2 + (R+1) = 0$$

$$s^2 = -\frac{R+1}{a}$$

$$s^2 = \frac{-(R+1)(R+2)}{(R+1)}$$

$$s^2 = -(R+2)$$

$$s = j\sqrt{R+2}$$

$$j\omega = j\sqrt{R+2}$$

$$\omega = \sqrt{R+2} = 2 \quad (\text{Oscillation frequency})$$

$$R = 2$$

$$\text{and } a = \frac{2+1}{2+2} = \frac{3}{4} = 0.75$$

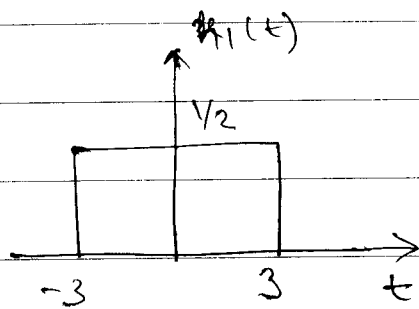
Hence (A) _____.

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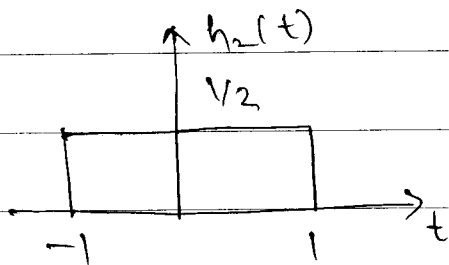
$$H(j\omega) = \frac{(2 \cos \omega)(\sin 2\omega)}{\omega}$$

$$= \cos \omega \cdot \frac{\sin 3\omega}{\omega} + \frac{\sin \omega}{\omega}$$

We know that inverse fourier transform of sinc function is a rectangular function.



$$\xleftrightarrow{f} \frac{\sin 3\omega}{\omega}$$



$$\xleftrightarrow{f} \frac{\sin \omega}{\omega}$$

So Inverse FT of $H(j\omega)$

$$h(t) = h_1(t) + h_2(t)$$

$$h(0) = h_1(0) + h_2(0)$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

Hence (c) —

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Heading

General form of state equations

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

For the given problem

$$A = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ a_2 \\ 0 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 0 & 0 & a_1 a_2 \\ a_2 a_3 & 0 & 0 \\ 0 & a_3 a_1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 a_2 \\ 0 \\ 0 \end{bmatrix}$$

For controllability it is necessary that the following matrix has a rank of $n=3$.

Heading

$$U = [B : AB : A^2B]$$

$$= \begin{bmatrix} 0 & 0 & a_1 a_2 \\ 0 & a_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\text{so } a_2 \neq 0$$

$$a_1 a_2 \neq 0 \Rightarrow a_1 \neq 0$$

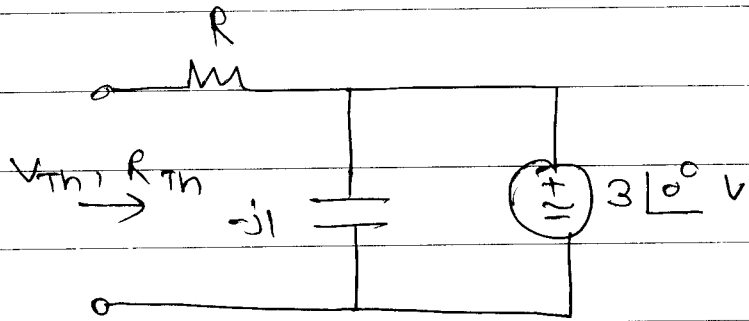
a_3 may be zero or not.

Hence (D) .

EC

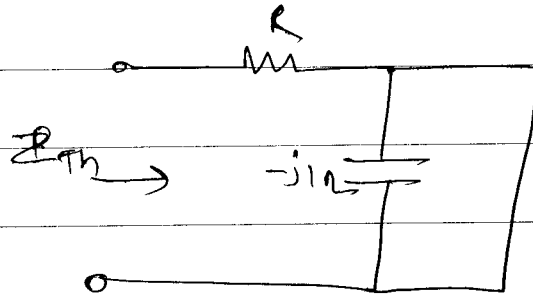
Heading

We obtain thevenin equivalent of circuit B



~~Resistance R_{Th}~~

Thevenin Impedance:

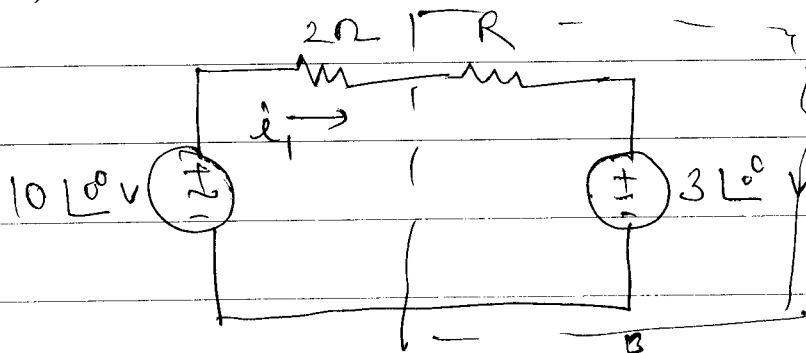


$$Z_{Th} = R$$

Thevenin Voltage:

$$V_{Th} = 3 \angle 0^\circ \text{ V}$$

Now, circuit becomes as



2012

Heading

Power transfer from circuit A to B

$$P = (i_1^2)^2 R + 3 i_1^2$$

$$P = \left[\frac{10-3}{2+R} \right]^2 R + 3 \left[\frac{10-3}{2+R} \right]^2$$

$$P = \frac{49R}{(2+R)^2} + \frac{21}{(2+R)}$$

$$P = \frac{49R + 21(2+R)}{(2+R)^2}$$

$$= \frac{42 + 70R}{(2+R)^2}$$

$$\frac{dP}{dR} = \frac{(2+R)^2 70 - (42+70R) 2(2+R)}{(2+R)^4} = 0$$

$$(2+R) [(2+R) 70 - (42+70R) 2] = 0$$

$$140 + 70R - 84 - 140R = 0$$

$$56 = 70R$$

$$R = 0.8 \Omega$$

Hence (A) —

2012

Heading

$$\frac{d^2y(t)}{dt^2} + 2 \frac{dy(t)}{dt} + y(t) = \delta(t)$$

By taking Laplace transform with initial conditions

$$\left[s^2 y(s) - s y(0) - \frac{dy}{dt} \Big|_{t=0} \right] + 2 \left[s y(s) - y(0) \right] + y(s) = 1$$

$$\left[s^2 y(s) + 2s - 0 \right] + 2 \left[s y(s) + 2 \right] + y(s) = 1$$

$$y(s) [s^2 + 2s + 1] = 1 - 2s - 4$$

$$y(s) = \frac{-2s - 3}{s^2 + 2s + 1}$$

we know that

$$\text{if } y(t) \xleftrightarrow{\mathcal{L}} Y(s)$$

$$\frac{dy(t)}{dt} \xleftrightarrow{\mathcal{L}} sY(s) - y(0)$$

$$\text{So, } sY(s) - y(0) = \frac{(-2s - 3)s + 2}{(s^2 + 2s + 1)}$$

$$= \frac{-2s^2 - 3s + 2s^2 + 4s + 2}{(s^2 + 2s + 1)}$$

Heading

$$\begin{aligned} sY(s) - y(0) &= \frac{s+2}{(s+1)^2} = \frac{s+1}{(s+1)^2} + \frac{1}{(s+1)^2} \\ &= \frac{1}{s+1} + \frac{1}{(s+1)^2} \end{aligned}$$

By taking inverse Laplace transform

$$\frac{dy(t)}{dt} = e^{-t} u(t) + t e^{-t} u(t)$$

at $t=0^+$

$$\left. \frac{dy}{dt} \right|_{t=0^+} = e^0 + 0 = 1$$

Hence (D) _____

2012

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Heading

Divergence of \vec{A} in spherical coordinates is given as

$$\nabla \cdot \vec{A} = \frac{1}{r^2} \frac{d}{dr} (r^2 A_r) \quad (\text{or, } \phi \text{ are zero})$$

$$= \frac{1}{r^2} \frac{d}{dr} (k r^{n+2})$$

$$= \frac{k}{r^2} (n+2) r^{n+1}$$

$$= k (n+2) r^{n-1} = 0 \quad (\text{given})$$

$$n+2 = 0$$

$$n = -2$$

✓

Hence (A) —

2012

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Heading

Probability of appearing a head is $\frac{1}{2}$.
If the number of required tosses is odd, we have following sequence of events

H, TTH, TTTTH, ---

Probability $P = \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \dots$

$$P = \frac{\frac{1}{2}}{1 - \frac{1}{4}} = \frac{2}{3}$$

Hence (C) —

2012

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Heading

General Equation of FM and PM

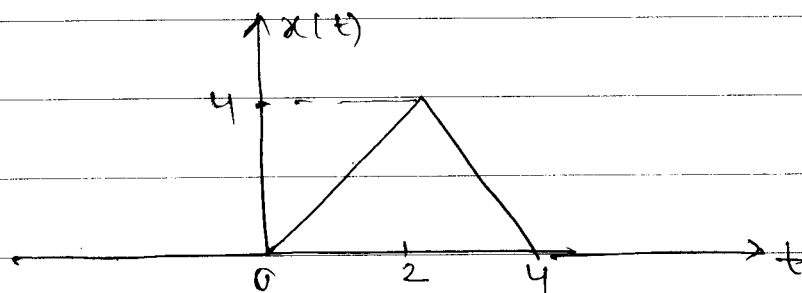
$$\phi_{FM}(t) = A_c \cos \left[\omega_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right]$$

$$\phi_{PM}(t) = A_c \cos [\omega_c t + k_p m(t)]$$

For same maximum phase deviation

$$k_p [m(t)]_{\max} = 2\pi k_f \left[\int_0^t m(\tau) d\tau \right]_{\max}$$

$$k_p(z) = 2\pi k_f \cdot [x(t)]_{\max}$$



$$x(t) = \int_0^t m(\tau) d\tau$$

$$[x(t)]_{\max} = 4$$

So, $k_p(z) = 2\pi k_f (4)$

Heading

$$\frac{k_p}{k_f} = 4\pi$$

Hence (B) _____

2012

Heading

$$H_z = 3 \cos(2.094 \times 10^2 x) \cos(2.618 \times 10^2 y) \cos(6.283 \times 10^{10} t - \beta z)$$

$$\beta_x = 2.094 \times 10^2$$

$$\beta_y = ~~6.283~~ 2.618 \times 10^2$$

$$\omega = 6.283 \times 10^{10} \text{ rad/s}$$

For the wave propagation

$$\beta = \sqrt{\frac{\omega^2}{c^2} - (\beta_x^2 + \beta_y^2)}$$

Substituting above values

$$\beta = \sqrt{\left(\frac{6.283 \times 10^{10}}{3 \times 10^8}\right)^2 - (2.094^2 + 2.618^2) \times 10^4}$$

$$\approx j261$$

β is imaginary so mode of operation is non-propagating

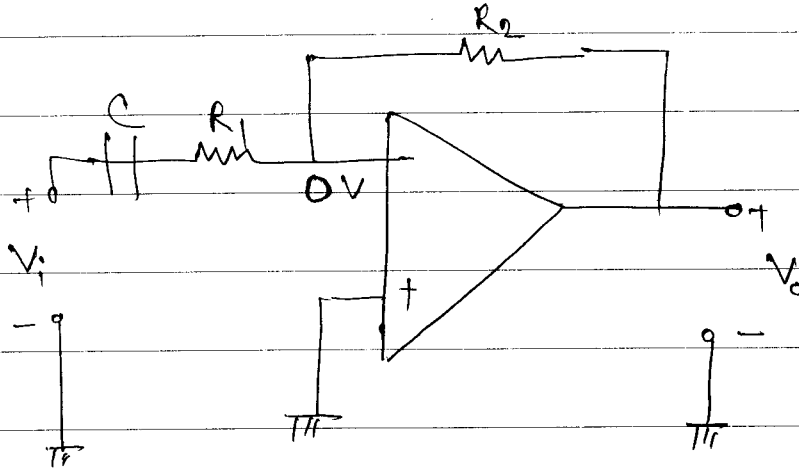
$$v_p = 0$$

Hence (D)

2012

Heading

First we obtain the transfer function



$$\frac{0 - V_i}{\frac{1}{j\omega C} + R_1} + \frac{0 - V_o}{R_2} = 0$$

$$\frac{V_o}{R_2} = \frac{-V_i}{\frac{1}{j\omega C} + R_1}$$

$$V_o = \frac{-V_i}{\frac{R_1}{R_2} + \frac{1}{j\omega C R_2}} = \frac{-V_i R_2}{R_1 - j \frac{1}{\omega C}}$$

At $\omega \rightarrow 0$ (Low frequencies)

$$\frac{1}{\omega C} \rightarrow \infty, \text{ so } V_o = 0$$

At $\omega \rightarrow \infty$ (higher frequencies)

$$\frac{1}{\omega C} \rightarrow 0, \text{ so } V_o = \frac{-V_i R_2}{R_1}$$

Heading

The filter passes high frequencies & 0
it is a high pass filter.

$$H(j\omega) = \frac{V_o}{V_i} = \frac{-R_2}{R_1 - j \frac{1}{\omega C}}$$

$$|H(\infty)| = \left| \frac{-R_2}{R_1} \right| = \frac{R_2}{R_1}$$

At 3 dB frequency

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}} |H(\infty)|$$

$$\frac{R_2}{\sqrt{R_1^2 + \frac{1}{\omega^2 C^2}}} = \frac{1}{\sqrt{2}} \cdot \frac{R_2}{R_1}$$

$$2R_1^2 = R_1^2 + \frac{1}{\omega^2 C^2}$$

$$R_1^2 = \frac{1}{\omega^2 C^2}$$

$$\omega = \frac{1}{R_1 C}$$

Hence (B) _____.

Heading

Convolution sum is defined as

$$y[n] = h[n] * g[n] = \sum_{k=-\infty}^{\infty} h[n] g[n-k]$$

For causal sequence

$$y[n] = \sum_{k=0}^{\infty} h[n] g[n-k]$$

$$y[n] = h[n] g[n] + h[n] g[n-1] + h[n] g[n-2] + \dots$$

for $n=0$

$$y[0] = h[0] g[0] + h[1] g[-1] + \dots$$

$$y[0] = h[0] g[0] \quad \text{--- (i)} \quad g[-1] = g[-2] = \dots = 0$$

for $n=1$

$$y[1] = h[1] g[1] + h[2] g[0] + h[1] g[-1] + \dots$$

$$y[1] = h[1] g[1] + h[1] g[0]$$

$$\frac{1}{2} = \frac{1}{2} g[1] + \frac{1}{2} g[0] \quad h[1] = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$$

$$1 = g[1] + g[0]$$

$$g[1] = 1 - g[0]$$

From eq (i)

$$g[0] = \frac{y[0]}{h[0]} = \frac{1}{1} = 1$$

$$\text{So } g[1] = 1 - 1 = 0$$

Hence (A) ———

Heading

Let Q_{n+1} is next state and Q_n is the present state. From the given figure

$$D = Y = \bar{A}x_0 + Ax_1$$

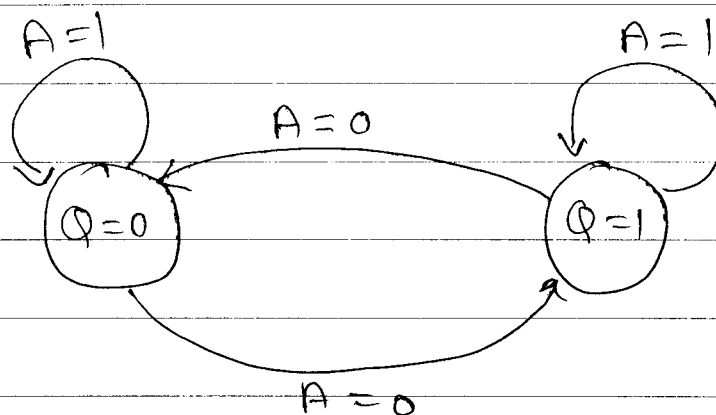
$$Q_{n+1} = D = \bar{A}x_0 + Ax_1$$

$$Q_{n+1} = \bar{A}\bar{Q}_n + AQ_n \quad x_0 = \bar{Q}, \quad x_1 = Q$$

If $A=0$, $Q_{n+1} = \bar{Q}_n$ (toggle of previous state)

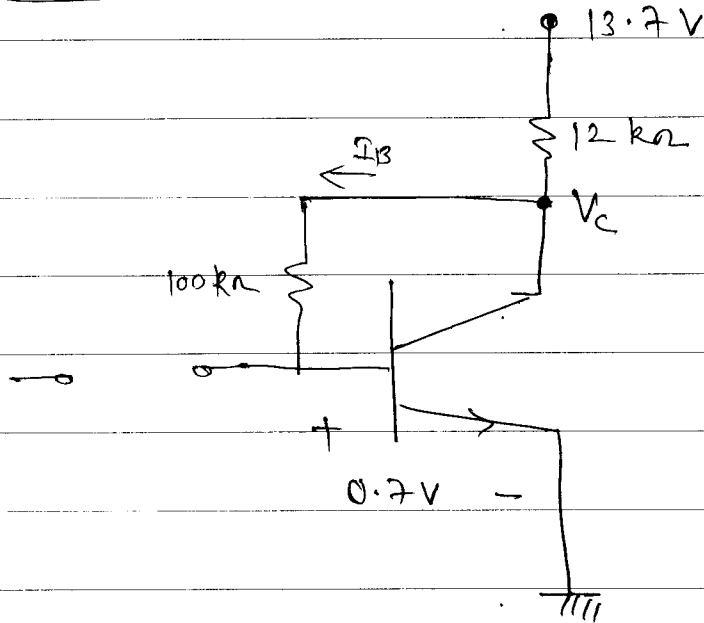
If $A=1$, $Q_{n+1} = Q_n$

So state diagram is



Hence (D) _____

Heading

DC Analysis

using KVL in input loop

$$V_C - 100 I_B - 0.7 = 0$$

$$V_C = 100 I_B + 0.7 \quad \text{--- (i)}$$

$$I_C \approx I_E = \frac{13.7 - V_C}{12k} = (\beta + 1) I_B$$

$$\frac{13.7 - V_C}{12 \times 10^3} = 100 I_B \quad \text{--- (ii)}$$

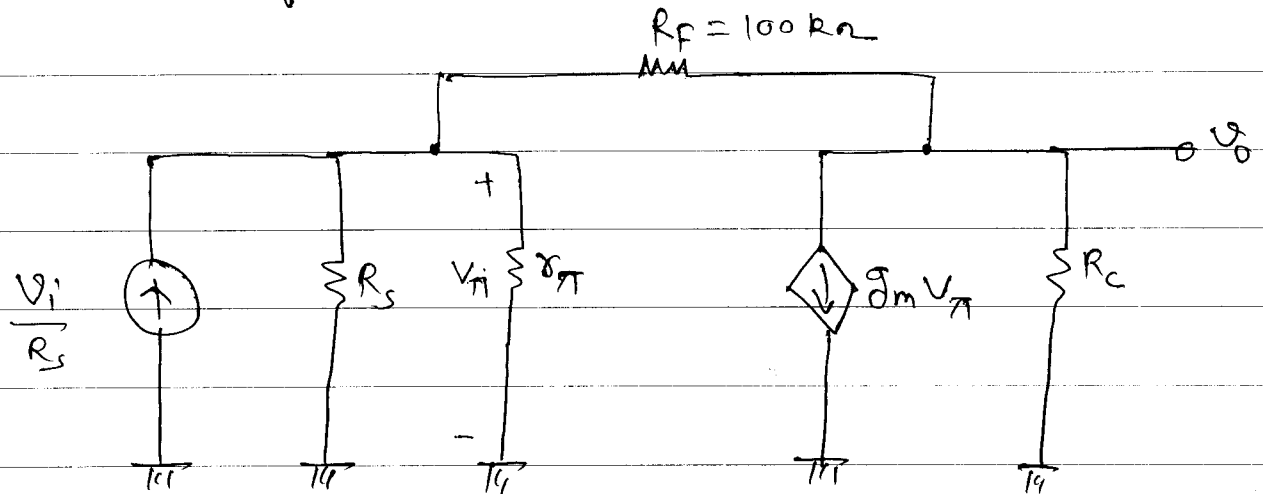
Solving (i) & (ii)

$$I_B = 0.01 \text{ mA}$$

Heading

Small Signal Analysis :-

Transforming given input voltage source into equivalent current source



This is a shunt-shunt feedback amplifier.

Given parameters

$$r_{\pi} = \frac{V_T}{I_B} = \frac{25\text{ mV}}{0.01\text{ mA}} = 2.5\text{ k}\Omega$$

$$g_m = \frac{\beta}{r_{\pi}} = \frac{100}{2.5 \times 1000} = 0.04\text{ S}$$

writing KCL at output node

$$\frac{V_o}{R_c} + g_m V_{\pi} + \frac{V_o - V_{\pi}}{R_F} = 0$$

Heading

$$V_o \left[\frac{1}{R_c} + \frac{1}{R_F} \right] + V_{\pi} \left[g_m - \frac{1}{R_F} \right] = 0$$

Substituting $R_c = 12 \text{ k}\Omega$, $R_F = 100 \text{ k}\Omega$, $g_m = 0.04 \text{ S}$,

$$V_o (9.33 \times 10^{-5}) + V_{\pi} (0.04) = 0$$

$$V_o = -428.72 V_{\pi} \quad \text{--- (1)}$$

Writing KCL at input node

$$\frac{V_i}{R_s} = \frac{V_{\pi}}{R_s} + \frac{V_{\pi}}{r_{\pi}} + \frac{V_{\pi} - V_o}{R_F}$$

substituting all values

$$\frac{V_i}{R_s} = V_{\pi} \left[\frac{1}{R_s} + \frac{1}{r_{\pi}} + \frac{1}{R_F} \right] - \frac{V_o}{R_F}$$

$$\frac{V_i}{R_s} = V_{\pi} (5.1 \times 10^{-4}) - \frac{V_o}{R_F}$$

substituting V_{π} from equation (1)

$$\frac{V_i}{R_s} = \frac{-5.1 \times 10^{-4}}{428.72} V_o - \frac{V_o}{R_F}$$

Heading

$$\frac{V_o}{10 \times 10^3} = -1.16 \times 10^{-6} V_o - 1 \times 10^{-5} V_o$$

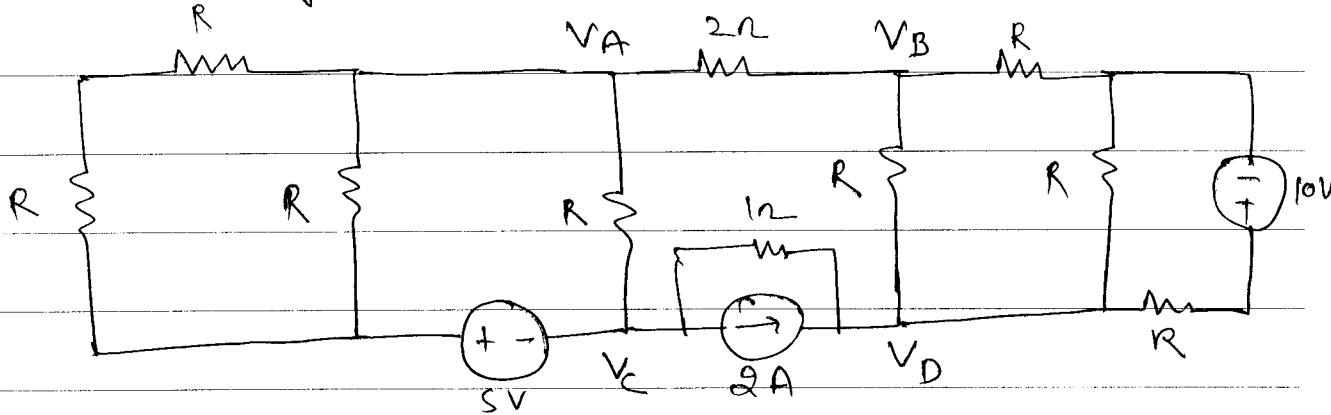
$$\frac{V_o}{10 \times 10^3} = -1.16 \times 10^{-5} V_o - 1 \times 10^{-5} V_o$$

$$|A_v| = \left| \frac{V_o}{V_i} \right| = \frac{1}{10 \times 10^3 \times 1.16 \times 10^{-5}} \approx 8.56$$

Hence (D)

Heading

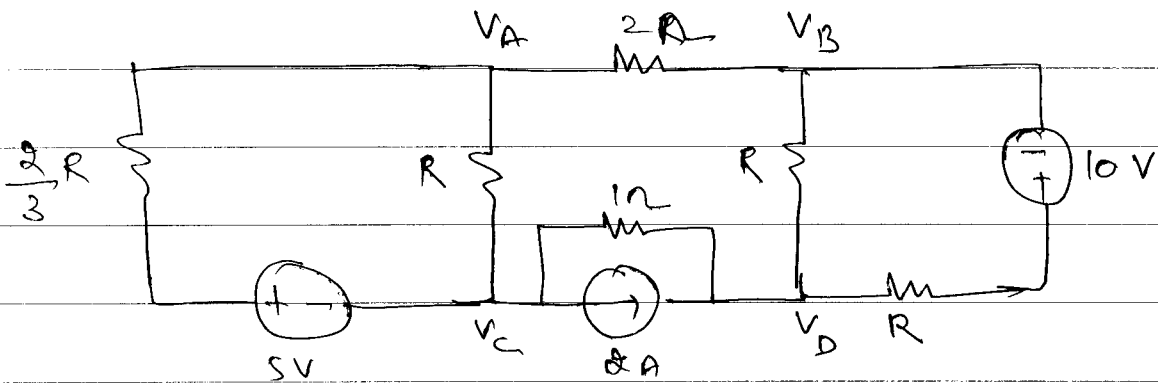
Simplifying the circuit



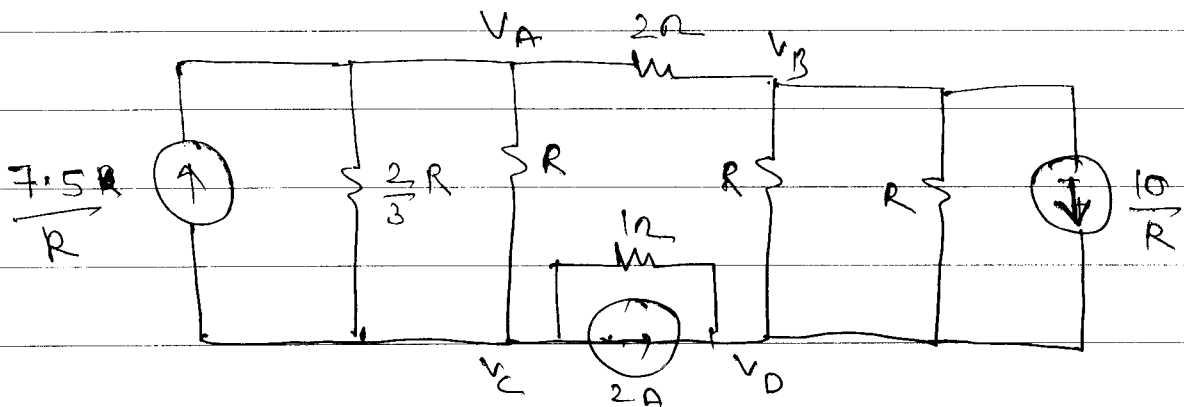
Combining resistances

$$R + R = 2R$$

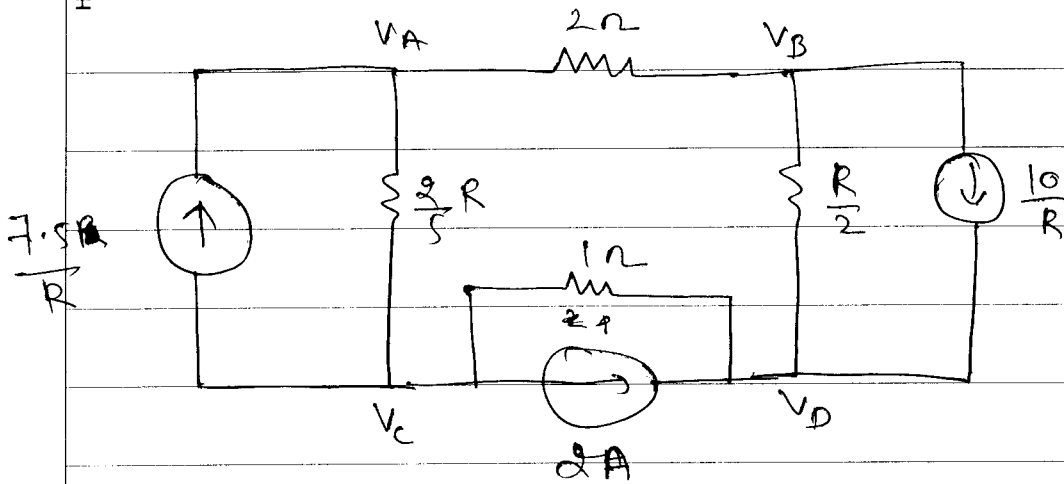
$$2R \parallel R = \frac{2}{3}R$$



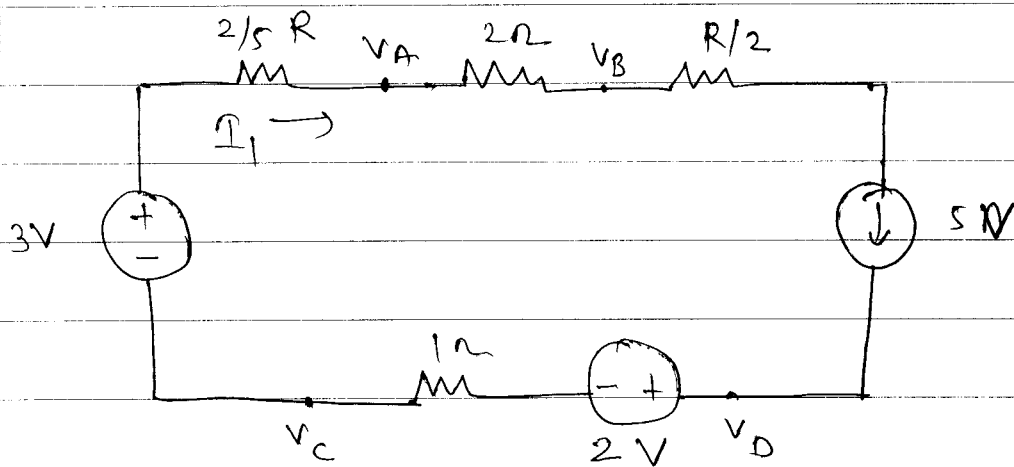
using source transformation



Heading



Again using source transformation of all the current sources



$$I_1 = \frac{V_A - V_B}{2} = \frac{6}{2} = 3 \text{ A}$$

$$V_C - V_D = \cancel{3} - I_1 - 2 \quad (\text{Carefully})$$

$$= - (3) - 2$$

$$= -5 \text{ V}$$

Hence (A) \longrightarrow

Heading

$$f(x) = x^3 - 9x^2 + 24x + 5$$

$$\frac{d f(x)}{d x} = 3x^2 - 18x + 24 = 0$$

$$x^2 - 6x + 8 = 0$$

$$x = 6, x = 2$$

$$\frac{d^2 f(x)}{d x^2} = 6x - 18$$

$$\text{for } x = 2, \frac{d^2 f(x)}{d x^2} = 12 - 18 = -6 < 0$$

so at $x = 2$, $f(x)$ will be maximum

$$f(x) \Big|_{\max} = (2)^3 - 9(2)^2 + 24(2) + 5$$

$$= 8 - 36 + 48 + 5$$

$$= 25$$

Hence (B) _____

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Heading

Characteristic equation

$$|A - dI| = 0$$

$$\begin{vmatrix} -s-d & -3 \\ * & -d \end{vmatrix} = 0$$

$$sd + d^2 + 6 = 0$$

$$d^2 + sd + 6 = 0$$

Since every characteristic equation satisfies its own matrix, so

$$A^2 + 5A + 6 = 0 \Rightarrow A^2 = -5A - 6I$$

Multiplying with A

$$A^3 + 5A^2 + 6A = 0$$

$$A^3 + 5(-5A - 6I) + 6A = 0$$

$$A^3 = 19A + 30I$$

Hence (B) —

Gate EC
2012

(42)

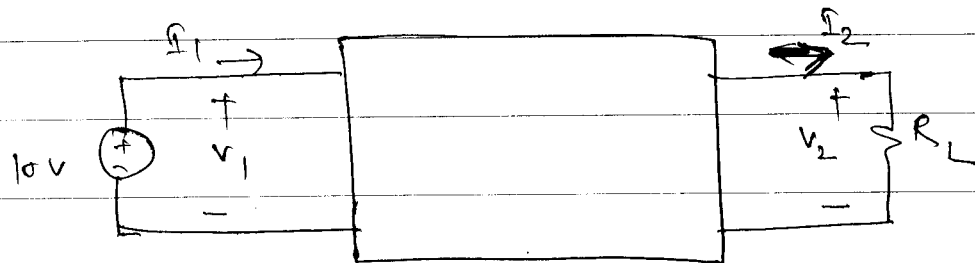
Heading

Let y -parameter matrix of the given two-port network

$$y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$$I_1 = y_{11} V_1 + y_{12} V_2$$

$$I_2 = y_{21} V_1 + y_{22} V_2$$



(i) $R_L = 1\Omega$, $I_2 = 3\text{ A}$

$$3 = y_{21}(10) + y_{22}(3)(1) \quad (V_2 = I_2 R_L)$$

(ii) $R_L = 0.5\Omega$, $I_2 = 2\text{ A}$

$$2 = y_{21}(10) + y_{22}(2)(0.5)$$

from above two equation

$$1 = y_{22}(-2)$$

2012

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Heading

$$y_{22} = -0.5 \text{ S}$$

$$y_{21} = 0.45 \text{ S}$$

So, when $R_L = 7 \Omega$

$$i_2 = 0.45 (10) - 0.5 (7) I_2$$

$$4.5 I_2 = 4.5$$

$$I_2 = 1 \text{ A}$$

Hence (C) —————

Heading

Gate source overlap capacitance

$$C_0 = \frac{\delta W \epsilon_{ox} \epsilon_0}{t_{ox}} \quad (\text{medium SiO}_2)$$

$$= \frac{20 \times 10^{-9} \times 1 \times 10^{-6} \times 3.9 \times 8.85 \times 10^{-12}}{1 \times 10^{-9}}$$

$$= 0.69 \times 10^{-15} \text{ F}$$

Hence (B) ———

Heading

Source body junction capacitance

$$C_s = \frac{A \epsilon_r \epsilon_0}{d}$$

$$A = (0.2 \mu\text{m} + 0.2 \mu\text{m} + 0.2 \mu\text{m}) \times 1 \mu\text{m} \\ + 2(0.2 \mu\text{m} \times 0.2 \mu\text{m})$$

$$= 0.68 \mu\text{m}^2$$

$d = 10 \text{ nm}$ (depletion width of all junction)

$$\text{A} \quad C_s = \frac{0.68 \times 10^{-12} \times 11.7 \times 8.9 \times 10^{-12}}{10 \times 10^{-9}}$$

$$= 7 \times 10^{-15} \text{ F}$$

Hence (B) —

Heading

For $r > a$

$$I_{\text{enclosed}} = (\pi a^2) J$$

$$\oint H \cdot d\ell = I_{\text{enclosed}}$$

$$H \times 2\pi r = (\pi a^2) J$$

$$H = \frac{I_0}{2\pi r}, \quad I_0 = (\pi a^2) J$$

$$H \propto \frac{1}{r} \quad \text{for } r > a$$

For $r < a$

$$I_{\text{enclosed}} = \frac{J(\pi r^2)}{\pi a^2} = \frac{J r^2}{a^2}$$

$$\text{So, } \oint H \cdot d\ell = I_{\text{enclosed}}$$

$$H \times 2\pi r = \frac{J r^2}{a^2}$$

$$H = \frac{J r}{2\pi a^2}$$

$$J \propto r \quad \text{for } r < a$$

Hence (C) —————.

2012

Heading

$$G_c(s) = \frac{s+a}{s+b} = \frac{j\omega + a}{j\omega + b}$$

Phase lead angle

$$\phi = \tan^{-1}\left(\frac{\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{b}\right)$$

$$\phi = \tan^{-1}\left(\frac{\frac{\omega}{a} - \frac{\omega}{b}}{1 + \frac{\omega^2}{ab}}\right)$$

$$= \tan^{-1}\left(\frac{\omega(b-a)}{ab + \omega^2}\right)$$

For phase-lead compensation $\phi > 0$

$$b - a > 0$$

$$b > a$$

Hence (A) —

Note: For phase lead compensator zero is nearer to the origin as compared to origin so option (c) can not be true.

2012

Heading

$$\phi = \tan^{-1}\left(\frac{\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{b}\right)$$

$$\frac{d\phi}{d\omega} = \frac{1/a}{1+(\frac{\omega}{a})^2} - \frac{1/b}{1+(\frac{\omega}{b})^2} = 0$$

$$\frac{1}{a} + \frac{1}{a} \frac{\omega^2}{b^2} = \frac{1}{b} + \frac{1}{b} \frac{\omega^2}{a^2}$$

$$\frac{1}{a} - \frac{1}{b} = \frac{\omega^2}{ab} \left(\frac{1}{a} - \frac{1}{b} \right)$$

$$\omega = \sqrt{ab}$$

$$\omega = \sqrt{1 \times 2} = \sqrt{2} \text{ rad/sec}$$

Hence (A) —