ONLINE MATHS CLASS- X – 39 (17 / 09 / 2021)

4 . SECOND DEGREE EQUATIONS - CLASS - 6

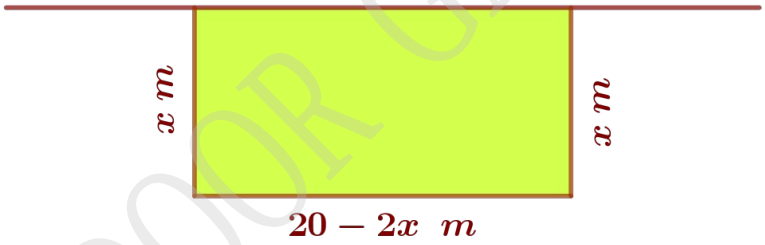
Important points

- Any second degree polynomial can be put in the form $p(x) = ax^2 + bx + c$
- To get $ax^2 + bx + c = 0$, we must take $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Activity 1

A rectangle is to be made on the ground using a 20 metre long rope , with a wall as one side . The area enclosed must be 50 square metres . What should be the length of the sides ?

Answer



Take , the length of the left and the right sides = x *metres* .

Length of the bottom side = $20 - 2x$ *metres* .

Area = 50 *sq . m .* ==> $x(20 - 2x) = 50$

$$20x - 2x^2 = 50$$

$$2x^2 - 20x = -50$$

$$\frac{2x^2 - 20x}{2} = -\frac{50}{2}$$

$$\frac{2x^2}{2} - \frac{20x}{2} = -\frac{50}{2}$$

$$x^2 - 10x = -25$$

$$x^2 - 10x + 5^2 = -25 + 5^2$$

$$(x - 5)^2 = -25 + 25 = 0$$

$$x - 5 = \sqrt{0} = 0$$

$$x = 0 + 5 = 5$$

Length of the shorter side of the rectangle $= x = 5\ m.$

Length of the longer side of the rectangle $= 20 - 2x = 20 - 2\times5 = 20 - 10$

$= 10\ m.$

Another method

$$2x^2 - 20x = -50 \implies 2x^2 - 20x + 50 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-20) \pm \sqrt{0}}{2 \times 2}$$

$$= \frac{20 \pm 0}{4}$$

$$x = \frac{20}{4} = 5$$

$a = 2\ ,\ b = -20\ ,\ c = 50$

$b^2 - 4ac = (-20)^2 - 4\times2\times50$

$= 400 - 400 = 0$

Length of the shorter side of the rectangle $= x = 5\ m.$

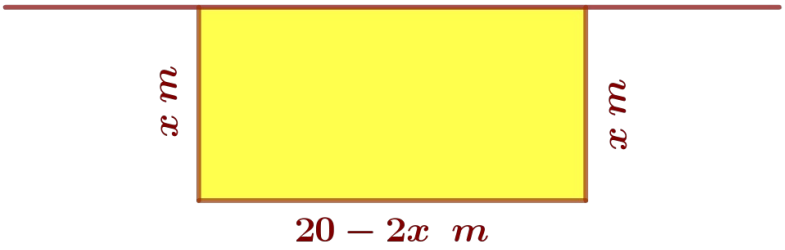
Length of the longer side of the rectangle $= 20 - 2x = 20 - 2\times5 = 20 - 10$

$= 10\ m.$

Activity 2

A rectangle is to be made on the ground using a 20 metre long rope , with a wall as one side . Can the area of the rectangle be 51 square metres ? Check .

Answer



Take , the length of the left and the right sides = x *metres*.

Length of the bottom side = $20-2x$ *metres*.

Area = 51 *sq.m.* ==> $x(20-2x) = 51$

$$20x - 2x^2 = 51$$

$$2x^2 - 20x = -51$$

$$2x^2 - 20x + 51 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-20) \pm \sqrt{-8}}{2 \times 2}$$

$$= \frac{20 \pm \sqrt{-8}}{4}$$

$a = 2$, $b = -20$, $c = 51$

$b^2 - 4ac = (-20)^2 - 4 \times 2 \times 51$

$= 400 - 408 = -8$

Since negative numbers don't have square roots , the equation $2x^2 - 20x + 51 = 0$ does not have a solution .

NOTE :

Whether a number is positive or negative , its square is positive

Activity 3

The perimeter of a rectangle is 42 metres and its diagonal is 15 metres . What are the lengths of its sides ?

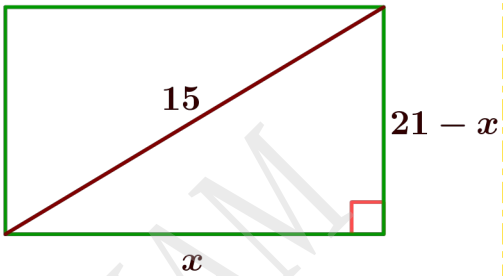
Answer

perimeter = 42 *metres*. ==> 2 length + 2 breadth = 42 *m*.

$$\Rightarrow \quad \text{length} + \text{breadth} = \frac{42}{2} = 21\ m.$$

Take , length = x *metres*. ,

then breadth = $21 - x$ *metres*.



Diagonal = 15 *metres*. $\Rightarrow x^2 + (21 - x)^2 = 15^2$

$$x^2 + 21^2 - 2 \times 21 \times x + x^2 = 15^2$$

$$x^2 + 441 - 42x + x^2 = 225$$

$$2x^2 - 42x + 441 = 225$$

$$2x^2 - 42x + 441 - 225 = 0 \Rightarrow 2x^2 - 42x + 216 = 0$$

$$\frac{2x^2 - 42x + 216}{2} = \frac{0}{2}$$

$$\frac{2x^2}{2} - \frac{42x}{2} + \frac{216}{2} = 0$$

$$x^2 - 21x + 108 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-21) \pm \sqrt{9}}{2 \times 1}$$

$$= \frac{21 \pm 3}{2}$$

$a = 1$, $b = -21$, $c = 108$

$b^2 - 4ac = (-21)^2 - 4 \times 1 \times 108$

$= 441 - 432 = 9$

$$x = \frac{21 + 3}{2} \quad \text{Or} \quad x = \frac{21 - 3}{2}$$

$$x = \frac{24}{2} \qquad \text{Or} \qquad x = \frac{18}{2}$$

$$x = 12 \qquad \text{Or} \qquad x = 9$$

Length = $x = 12\ m$. ==> breadth = $21 - x = 21 - 12 = 9\ m$.

Activity 4

In writing the equation to construct a rectangle of specified perimeter and area , the perimeter was wrongly written as 24 instead of 42 . The length of a side was found to be 10 .

a) What is the area in the problem ?

b) What are the lengths of the sides of the rectangle in the correct problem ?

Answer

Wrongly written perimeter = 24 ==> 2 length + 2 breadth = 24

$$\Rightarrow \text{length} + \text{breadth} = \frac{24}{2} = 12$$

Length of a sides = 10 , 2

a) Area = length × breadth = $10 \times 2 = 20$

Correct perimeter = 42 ==> 2 length + 2 breadth = 42

$$\Rightarrow \text{length} + \text{breadth} = \frac{42}{2} = 21$$

Take , length = x , then breadth = $21 - x$

Area = 20 ==> $x(21 - x) = 20$

$$21x - x^2 = 20$$

$$x^2 - 21x = -20$$

$$x^2 + 21x + 20 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-21) \pm \sqrt{361}}{2 \times 1}$$

$$= \frac{21 \pm 19}{2}$$

$a = 1 \ , \ b = -21 \ , \ c = 20$

$b^2 - 4ac = (-21)^2 - 4 \times 1 \times 20$

$= 441 - 80 = 361$

$x = \frac{21 + 19}{2}$ Or $x = \frac{21 - 19}{2}$

$x = \frac{40}{2}$ Or $x = \frac{2}{2}$

$x = 20$ Or $x = 1$

b) Length = $x = 20$ ==> Breadth = $21 - x = 21 - 20 = 1$