A

2009 – GP

Test Paper Code : GP

Time : 3 Hours Maximum Marks : 300

INSTRUCTIONS

1. The question-cum-answer booklet has **60** pages and has **66** questions. Please ensure that the copy of the question-cum-answer booklet you have received contains all the questions.
2. Write your **Roll Number, Name and the name of the Test Centre** in the appropriate space provided on the right side.
3. Write the answers to the objective questions against each Question No. in the **Answer Table for Objective Questions**, provided on Page No. **13**. Do not write anything else on this page.
4. Each objective question has **4 choices** for its answer : (A), (B), (C) and (D). Only **ONE** of them is the correct answer. There will be **negative marking** for wrong answers to objective questions. The following marking scheme for objective questions shall be used :
   (a) For each correct answer, you will be awarded **3 (Three)** marks.
   (b) For each wrong answer, you will be awarded **–1 (Negative one)** mark.
   (c) Multiple answers to a question will be treated as a wrong answer.
   (d) For each un-attempted question, you will be awarded **0 (Zero)** mark.
   (e) Negative marks for objective part(s) will be carried over to total marks.
5. Answer the subjective question only in the space provided after each question.
6. Do not write more than one answer for the same question. In case you attempt a subjective question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing last only will be evaluated.
7. All answers must be written in blue/ black/blue-black ink only. Sketch pen, pencil or ink of any other colour should not be used.
8. All rough work should be done in the space provided and scored out finally.
9. No supplementary sheets will be provided to the candidates.
10. **Clip board, log tables, slide rule, calculator, cellular phone, pager and electronic gadgets in any form are NOT allowed.**
11. The question-cum-answer booklet must be returned in its entirety to the Invigilator before leaving the examination hall. Do not remove any page from this booklet.

Sample

Sample

A

2009 – GP

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

ROLL NUMBER						

Name :

Test Centre :

Do not write your Roll Number or Name anywhere else in this question-cum-answer booklet.

I have read all the instructions and shall abide by them.

.....

Signature of the Candidate

I have verified the information filled by the Candidate above.

.....

Signature of the Invigilator

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IMPORTANT NOTE FOR CANDIDATES

- **Geology Section : Q. Nos. 1–15 (Objective Questions) and Q. Nos. 46–52 (Subjective Questions)**
- **Physics Section : Q. Nos. 16–30 (Objective Questions) and Q. Nos. 53–59 (Subjective Questions)**
- **Mathematics Section : Q. Nos. 31–45 (Objective Questions) and Q. Nos. 60–66 (Subjective Questions)**
- **Select any *TWO* Sections.**
- **Attempt objective and subjective questions of the selected *TWO* Sections.**
- **Questions 1-45 (objective questions) carry *three* marks each and questions 46-66 (subjective questions) carry *fifteen* marks each.**
- **Write the answers to the objective questions in the *Answer Table for Objective Questions* provided on page 13 only.**

GEOLOGY SECTION

Q.1 The drainage pattern in an area characterized by alternate hard and soft beds will be

(A) trellis (B) dendritic (C) radial (D) centripetal

Q.2 Consider the following statements :

I. Mohorovicic discontinuity marks the crust-mantle boundary

II. Gutenberg discontinuity lies at a depth of 2900 km below the Earth's surface

III. Lehman discontinuity lies at a depth of 5300 km below the Earth's surface

Choose the correct statement from the following :

(A) Only **I** is true (B) Only **I** and **III** are true

(C) Only **I** and **II** are true (D) Only **II** and **III** are true

Q.3 Parallel magnetic reversal patterns observed on the ocean floor near mid-oceanic ridges suggest the

(A) formation of new crust in the geologic past

(B) presence of mineral deposits in the oceanic crust

(C) origin of Earth's magnetic field in the inner core

(D) non-uniform movement of tectonic plates in the geologic past

Q.4 If the intercepts of a plane with crystallographic axes are $1a : 2b : \infty c$, its Miller indices will be

(A) 120 (B) 210 (C) 211 (D) 220

Q.5 Match the Mg-end members of minerals of **Group 1** with the corresponding Fe-end members of **Group 2**.

Group 1	**Group 2**
P. Phlogopite	1. Grunerite
Q. Enstatite	2. Annite
R. Cummingtonite	3. Hedenbergite
	4. Ferrosilite

(A) P-3, Q-1, R-4
(B) P-3, Q-4, R-2
(C) P-2, Q-4, R-1
(D) P-4, Q-3, R-1

Q.6 Consider the following statements about igneous rocks :

I. Olivine and clinopyroxene are essential minerals in harzburgite
II. Olivine and orthopyroxene are essential minerals in wehrlite
III. Olivine, orthopyroxene and clinopyroxene are essential minerals in lherzolite
IV. Plagioclase and orthopyroxene are essential minerals in norite

Choose the correct option :

(A) Only **II** and **III** are true
(B) Only **III** and **IV** are true
(C) Only **I**, **II** and **III** are true
(D) Only **II**, **III** and **IV** are true

Q.7 Which of the following characteristics are true for current ripples?

(A) Gentle upstream and steep downstream sides
(B) Both upstream and downstream sides are gentle
(C) Both upstream and downstream sides are steep
(D) Steep upstream and gentle downstream sides

Q.8 Choose the correct mineral to complete the reaction :
Muscovite + Quartz = Sillimanite + _____ + H_2O

(A) Biotite (B) Corundum (C) Garnet (D) Orthoclase

Q.9 Which of the following statements is true for folds in sedimentary rocks?

(A) Older rocks occur towards the core of antiforms in an inverted sequence
(B) Younger rocks occur towards the core of antiforms in a normal sequence
(C) Older rocks occur towards the core of synforms in a normal sequence
(D) Younger rocks occur towards the core of antiforms in an inverted sequence

Q.10 The correct sequence of older to younger Groups in the Dharwar Supergroup is :

(A) Chitradurga Group, Bababudan Group, Ranibennur Group
(B) Bababudan Group, Chitradurga Group, Ranibennur Group
(C) Ranibennur Group , Chitradurga Group, Bababudan Group
(D) Bababudan Group, Ranibennur Group, Chitradurga Group

Q.11 The reservoir rock of petroleum in the Bombay High oil field is

(A) Oligocene sandstone
(B) Oligocene limestone
(C) Miocene sandstone
(D) Miocene limestone

Q.12 Which of the following is the largest lithospheric plate?

(A) Antarctic plate
(B) Eurasian plate
(C) Pacific plate
(D) African plate

Q.13 Match the stratigraphic units listed in **Group 1** with appropriate economic mineral deposits listed in **Group 2**.

Group 1	**Group 2**
P. Sausar Group	1. Iron
Q. Bailadila Group	2. Coal
R. Damuda Group	3. Copper
S. Aravalli Supergroup	4. Manganese
	5. Lead-Zinc

(A) P-4, Q-1, R-2, S-5
(B) P-5, Q-4, R-2, S-1
(C) P-3, Q-2, R-4, S-5
(D) P-1, Q-4, R-2, S-5

Q.14 On a fault plane dipping 40° towards N50°, the pitch of slickensides is 90°. The plunge of the slickensides will be

(A) 90° towards N50°
(B) 90° towards N40°
(C) 40° towards N50°
(D) 50° towards N40°

Q.15 Match the following plutonic rocks of **Group 1** with their equivalent volcanic rocks of **Group 2**.

Group 1	**Group 2**
P. Granite	1. Trachyte
Q. Diorite	2. Rhyolite
R. Syenite	3. Dacite
	4. Andesite

(A) P-4, Q-3, R-1
(B) P-2, Q-4, R-1
(C) P-3, Q-1, R-4
(D) P-1, Q-4, R-3

PHYSICS SECTION

Q.16 Under the influence of a force of 2 kN, a wire of diameter 2 mm gets elongated by 4 mm. What will be the elongation in a wire of same material and same length but of diameter 4 mm?

(A) 0.5 mm (B) 1.0 mm (C) 1.5 mm (D) 2.0 mm

Q.17 A plane harmonic wave traveling through a medium is represented (in SI system) by

$$E_x(y, t) = E_0 \sin 2\pi\left(\frac{y}{3\times10^{-7}} - 5\times10^{14}\, t\right).$$

The refractive index of the medium at this frequency will be

(A) 1.3 (B) 1.5 (C) 2.0 (D) 2.5

Q.18 In a diffraction pattern produced by N parallel slits of equal width and separation, the number of minima between the adjacent principal maxima is

(A) $N-2$ (B) $N-1$ (C) N (D) $N+1$

Q.19 A body P (temperature T_P) has twice the mass and twice the specific heat compared to that of the body Q (temperature T_Q). If the bodies are supplied equal amount of heat, the relationship between their resulting temperature-changes (ΔT_P and ΔT_Q) will be

(A) $\Delta T_P = 4\,\Delta T_Q$ (B) $\Delta T_P = 2\,\Delta T_Q$ (C) $\Delta T_Q = 2\,\Delta T_P$ (D) $\Delta T_Q = 4\,\Delta T_P$

Q.20 In an *npn* transistor, 95% of emitted electrons reach the collector. If the collector current is 19 mA, the base current will be

(A) 0.5 mA (B) 1.0 mA (C) 1.5 mA (D) 2.0 mA

Q.21 A particle is acted upon by a force $\vec{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$. Which of the following statements is true?

(A) $\vec{F}$ is not conservative

(B) $\vec{F}$ is conservative and there exists a potential V such that $\vec{F} = -\vec{\nabla}V$, $V = x^2y + y^2z + z^2x$

(C) $\vec{F}$ is conservative and there exists a potential V such that $\vec{F} = -\vec{\nabla}V$, $V = -xyz$

(D) $\vec{F}$ is not conservative and there exists a potential V such that $\vec{F} = -\vec{\nabla}V$, $V = xyz$

Q.22 If an ideal gas is subjected to an **isothermal** process, then

(A) no work is done by the system
(B) no heat is supplied to the system
(C) the heat supplied to the system equals the change in internal energy of the gas
(D) the heat supplied to the system equals the work done by the system

Q.23 With regard to entropy, which of the following statements is false?

(A) In a reversible process, the entropy change of the universe is zero
(B) For any process, the entropy of the universe never decreases
(C) In an irreversible process, the entropy of the universe increases
(D) When a system changes state, the resulting entropy change depends upon the process by which the change of state occurs

Q.24 Consider a parallel-plate vacuum-capacitor with capacitance C. A dielectric with relative permittivity ε_r is inserted in the capacitor such that it touches both the plates and fills up half the volume between the plates. The new capacitance is given by

(A) $\dfrac{1}{\dfrac{2}{C}+\dfrac{2}{C\varepsilon_r}}$ (B) $\dfrac{1}{\dfrac{2}{C}+\dfrac{2\varepsilon_r}{C}}$ (C) $\dfrac{C}{2}(1+\varepsilon_r)$ (D) $\dfrac{C\varepsilon_r}{2}$

Q.25 If μ_o is the permeability of free space, the correct relation (in SI System) between the three magnetic vectors $\vec{B}$, $\vec{H}$ and $\vec{M}$ is

(A) $\vec{B}=\mu_o\left(\vec{H}+\vec{M}\right)$ (B) $\vec{B}=\mu_o\vec{H}+\vec{M}$

(C) $\vec{B}=\vec{H}+\mu_o\vec{M}$ (D) $\vec{B}=\vec{H}+\vec{M}$

Q.26 If a piece of an **intrinsic** Silicon semiconductor carrying a constant current J is placed in a uniform magnetic field B transverse to J, then

(A) electrons and holes deflect in same direction, and the Hall-voltage will be non-zero
(B) electrons and holes deflect in same direction, and the Hall-voltage will be zero
(C) electrons and holes deflect in opposite directions, and the Hall-voltage will be non-zero
(D) electrons and holes deflect in opposite directions, and the Hall-voltage will be zero

Q.27 Unpolarized light is incident at Brewster's angle on the surface of a medium. Which of the following statements is false?

(A) The parallel component of the light is completely reflected
(B) The reflected light is completely polarized
(C) Some of the incident light is reflected and some is refracted
(D) The reflected light is perpendicular to the refracted light

Q.28 Assuming an adiabatic motion of an ideal fluid with entropy S, velocity $\vec{v}$ and density ρ which of the following is correct?

(A) $\frac{dS}{dt}=0,\ \frac{\partial(S\rho)}{\partial t}+\vec{\nabla}\cdot(S\rho\vec{v})=0$

(B) $\frac{\partial S}{\partial t}=0,\ \frac{\partial(S\rho)}{\partial t}+\vec{\nabla}\cdot(S\rho\vec{v})=0$

(C) $\frac{dS}{dt}=0,\ \frac{\partial(S\rho)}{\partial t}-\vec{\nabla}\cdot(S\rho\vec{v})=0$

(D) $\frac{\partial S}{\partial t}=0,\ \frac{\partial(S\rho)}{\partial t}-\vec{\nabla}\cdot(S\rho\vec{v})=0$

Q.29 The activity of a radioactive isotope decreases from 80000 to 10000 in 60 years. The half life of this isotope will be

(A) 10 years (B) 20 years (C) 30 years (D) 40 years

Q.30 If at $t = 0$, the charge density in a medium having conductivity σ and permittivity ε is $\rho_0(\vec{r})$, then the charge density $\rho(\vec{r}, t)$ at any later time t is given by

(A) $\rho_0(\vec{r})\frac{\sigma t}{\varepsilon}$

(B) $\rho_0(\vec{r})\exp\left(\frac{\sigma t}{\varepsilon}\right)$

(C) $\rho_0(\vec{r})$

(D) $\rho_0(\vec{r})\exp\left(-\frac{\sigma t}{\varepsilon}\right)$

MATHEMATICS SECTION

Q.31 If the integral of $y(x)$ from $x=1$ to $x=5$ by Simpson's one-third rule for the following data is 4 :

x :	1	2	3	4	5
$y(x)$:	3	1	2	0	α

then the value of α is

(A) -1 (B) 0 (C) 1 (D) 2

Q.32 If $\sin(x-y)=y^2\cos x$, then $\frac{dy}{dx}$ is

(A) $\frac{y\sin x+\cos(x-y)}{2y\sin x+\cos(x-y)}$

(B) $\frac{y^2\sin x+\cos(x-y)}{2y\cos x+\cos(x-y)}$

(C) $\frac{y^2\sin x+\cos(x+y)}{2y\cos x+\cos(x-y)}$

(D) $\frac{y^2\sin x+\cos(x-y)}{y\cos x+\cos(x-y)}$

Q.33 The absolute maximum and minimum values of the function $f(x)=2(\sin 2x+2\cos x)$ in the interval $\left[0, \frac{\pi}{3}\right]$ are

(A) $3\sqrt{3}$ and 4

(B) $3\sqrt{3}$ and $2\sqrt{3}$

(C) 4 and $2\sqrt{3}$

(D) $3\sqrt{3}$ and $2+\sqrt{3}$

Q.34 The series $\frac{1}{3\sqrt{1}}+\frac{x^2}{4\sqrt{2}}+\frac{x^4}{5\sqrt{3}}+\frac{x^6}{6\sqrt{4}}+\cdots$ is

(A) convergent for $|x|\leq 1$ and divergent for $|x|>1$

(B) convergent for $|x|<1$ and divergent for $|x|\geq 1$

(C) convergent for $|x|\leq 2$ and divergent for $|x|>2$

(D) convergent for $|x|<2$ and divergent for $|x|\geq 2$

Q.35 If $\lim_{x\to 0}\frac{e^{4x}-\alpha x-1}{x^2}=8$, then the value of α is

(A) 2 (B) 4 (C) 6 (D) 8

Q.36 If $f(x)=x^2$ and $g(x)=\frac{1}{x^2}$, then in $0<x<1$,

(A) both $f(x)$ and $g(x)$ are uniformly continuous
(B) neither $f(x)$ nor $g(x)$ is uniformly continuous
(C) $f(x)$ is uniformly continuous, while $g(x)$ is not
(D) $g(x)$ is uniformly continuous, while $f(x)$ is not

Q.37 If $\int_{y=0}^{1}\int_{x=0}^{y+4} dx\,dy = \int_{x=0}^{4}\int_{y=0}^{1} dy\,dx + \int_{x=4}^{5}\int_{y=g(x)}^{h(x)} dy\,dx$,

then the functions $g(x)$ and $h(x)$ are, respectively

(A) $(x-4)$ and 1
(B) $(x+4)$ and 1
(C) 1 and $(x-4)$
(D) 1 and $(x+4)$

Q.38 The volume of the portion of the cylinder $x^2+y^2=4$ in the first octant between the planes $z=0$ and $3x-z=0$ is

(A) 2
(B) 4
(C) 8
(D) 16

Q.39 The value of the integral $\oiint_S \vec{F}\cdot d\vec{S}$, where $\vec{F}=3x\,\hat{i}+2y\,\hat{j}+z\,\hat{k}$ and S is the closed surface given by the planes $x=0, x=1, y=0, y=2, z=0$ and $z=3$ is

(A) 6
(B) 18
(C) 24
(D) 36

Q.40 The values of the line integral $\int\left[\left(3x^2y+2xy\right)dx+\left(x^3+x^2\right)dy\right]$ from M(0, 0) to N(1, 1) along the paths $C_1: y=x$ and $C_2: y=x^2$ are, respectively

(A) 2 and -1
(B) 3 and 3
(C) -1 and 3
(D) 2 and 2

Q.41 The particular integral of the differential equation $y''+y'+3y=5\cos(2x+3)$ is

(A) $2\cos(2x+3)-\sin(2x+3)$
(B) $2\sin(2x+3)+\cos(2x+3)$
(C) $\sin(2x+3)-2\cos(2x+3)$
(D) $2\sin(2x+3)-\cos(2x+3)$

Q.42 The set $\{(1-k,\ k,\ 1-k), (0,\ 2-3k,\ 2), (1-k,\ -1,\ 0)\}$ forms a basis for $\mathbb{R}^3$ for k equal to

(A) 0 (B) 1 (C) 2 (D) 7/3

Q.43 If $f(x, y)=\begin{cases}\dfrac{(x+y)\sin(x+y)}{(x-y)}, & x\neq y\\ 0, & \text{otherwise,}\end{cases}$

then $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at (0, 0) are, respectively

(A) 0, −1 (B) 1, −1 (C) 1, 0 (D) 1, 1

Q.44 Let $f(z)=u(x, y)+iv(x, y)$ be analytic in a region **R**, in which the second order partial derivatives of $u(x, y)$ and $v(x, y)$ exist and are continuous. If $p(x, y)=\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}$ and $q(x, y)=\dfrac{\partial u}{\partial y}-\dfrac{\partial v}{\partial x}$, then which one of the following functions is analytic in **R**?

(A) $q(x, y)+i\,p(x, y)$
(B) $q(x, y)-i\,p(x, y)$
(C) $p(x, y)+iq(x, y)$
(D) $p(x, y)-iq(x, y)$

Q.45 If X and Y are two random variables having joint density function

$f(x, y)=\begin{cases}\dfrac{1}{8}(6-x-y), & 0<x<\alpha;\ 2<y<4\\ 0, & \text{elsewhere}\end{cases}$

then the value of α is

(A) 1 (B) 2 (C) 3 (D) 4

GEOLOGY SECTION

Q.46 (a) Answer the following :

(i) What are polymorphs? Name a gem variety of corundum along with its colour.

(ii) Write a note on the hardness of kyanite.

(iii) Give the compositions and crystal systems of barite and microcline. (9)

(b) Answer the following :

(i) Between olivine and orthopyroxene, which is richer in silicon per unit oxygen?

(ii) With the help of a labeled diagram, explain ophitic texture. (6)

Q.47 (a) With help of appropriate diagrams, explain how atolls develop progressively in oceans? (9)

(b) Briefly describe how stream terraces form. (6)

Q.48 (a) What is a seismograph? List at least three seismic phases (waves) that one observes on a seismic record after a large earthquake. Why does one feel two different kinds of motions after an earthquake (first push-pull type and, after a few seconds, swaying type)? (6)

(b) Describe an observation on the Earth's surface that can ascertain that the Earth's outer core is fluid. Draw suitable diagram(s). (9)

Q.49 (a) Answer the following :

(i) What is 'marker horizon' in stratigraphy?

(ii) List the Groups of Upper Vindhyans from older to younger.

(iii) Which stratigraphic unit records Cretaceous marine transgression near Jabalpur? (9)

(b) Answer the following :

(i) What is an inlier?

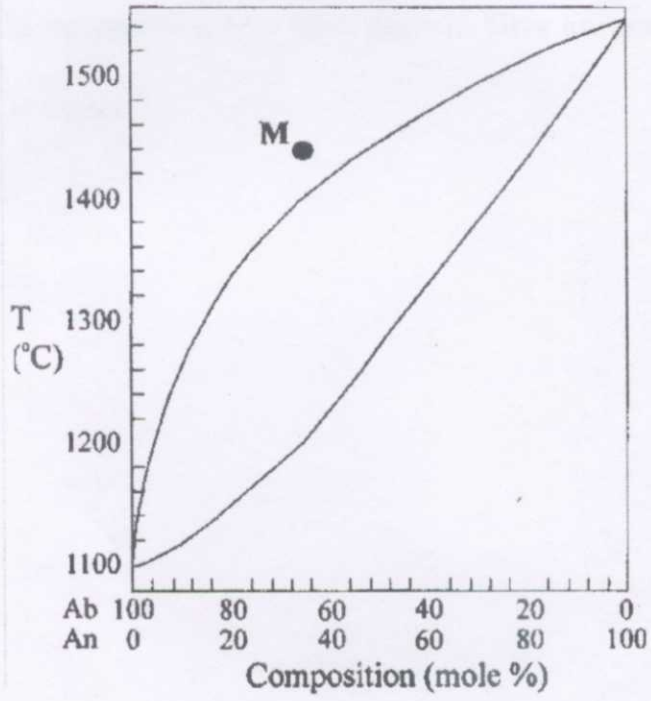
(ii) Name the stratigraphic Group in the Assam-Arakan region that contains thick coal seams. (6)

Q.50 (a) Discuss the host rock, ore mineralogy and mode of occurrence of gold in the Hutti schist belt. (9)

(b) (i) Explain magmatic segregation deposit. Give an example of such deposits in India.

(ii) What is gossan? (6)

Q.51 (a)



In the above figure, a plagioclase melt represented by **M** starts cooling. Answer the following :

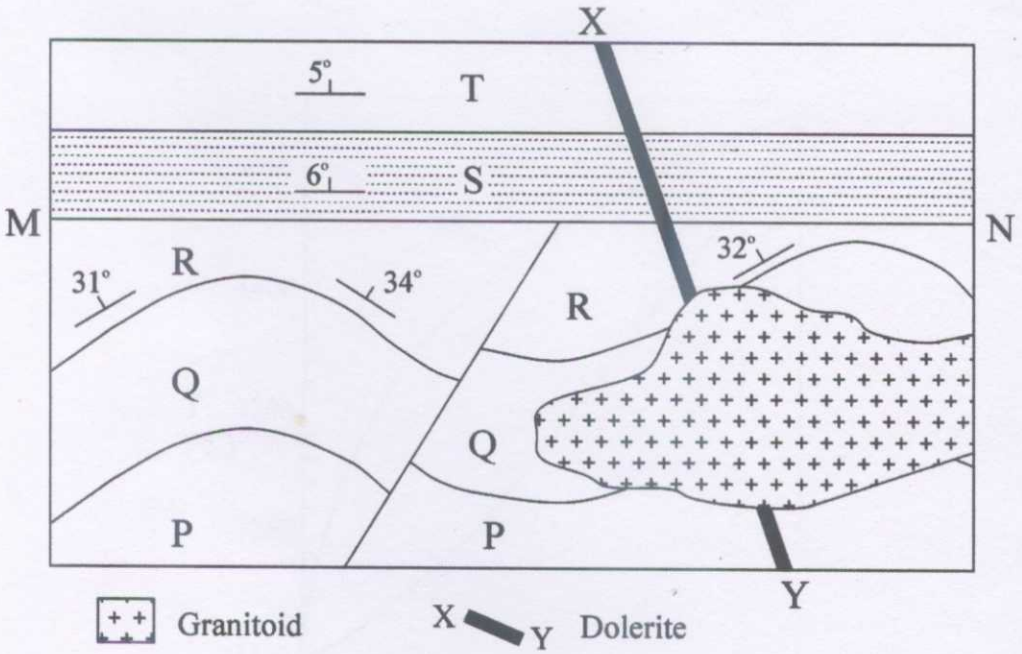
(i) At what temperature does crystallization start? What is the composition of the first crystal of plagioclase?

(ii) At what temperature does crystallization stop? What is the composition of the final melt? Assume equilibrium crystallization.

(iii) What difference would you expect between the plagioclase crystal formed by equilibrium crystallization and that formed by disequilibrium crystallization. (9)

(b) What do you understand by sorting of sediments? Name two sedimentary environments and give the types of sorting found in them. (6)

Q.52



In the figure given above, P, Q, R, S and T represent different sedimentary beds exposed in a flat terrane. Discuss the geologic history of this area including the development of various lithologies, structures and events in proper sequence. What does the line M-N represent? (15)

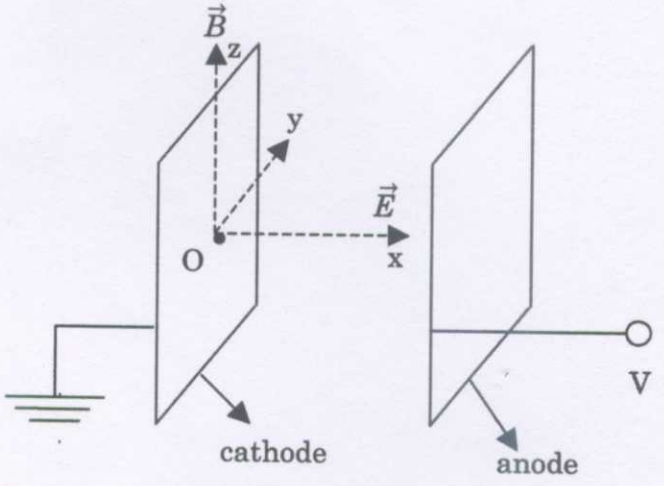
PHYSICS SECTION

Q.53 A 2 m long wire having a linear mass density of 0.0025 kg/m is stretched between two fixed supports such that two adjacent harmonic frequencies are 252 Hz and 336 Hz.

(a) Calculate the fundamental frequency of the wire. (9)

(b) Determine the tension in the wire. (6)

Q.54 An electron initially at rest at point O (lying on the cathode) is acted upon by a magnetic field $\vec{B}=B_o\hat{z}$ and electric field $\vec{E}=E_o\hat{x}$. This electric field results due to the potential difference V between the cathode and anode separated by distance d (see figure below). For what value of B_o, the electron does not reach the anode? (15)



Q.55 An *fcc* lattice is formed by atoms having radius r.

(a) Obtain the expression, in terms of r, for the areal density of atoms (atoms/m^2) for the (111) plane. (9)

(b) Obtain the relationship between r and the maximum radius R of a sphere that can just occupy the centre of the unit cell. (6)

Q.56 On fission, ^{235}U yields ^{133}Sb at a constant rate R_{Sb}. Being unstable, ^{133}Sb undergoes following decay-sequence

$$^{133}Sb \rightarrow {}^{133}Te \rightarrow {}^{133}I \rightarrow {}^{133}Cs$$

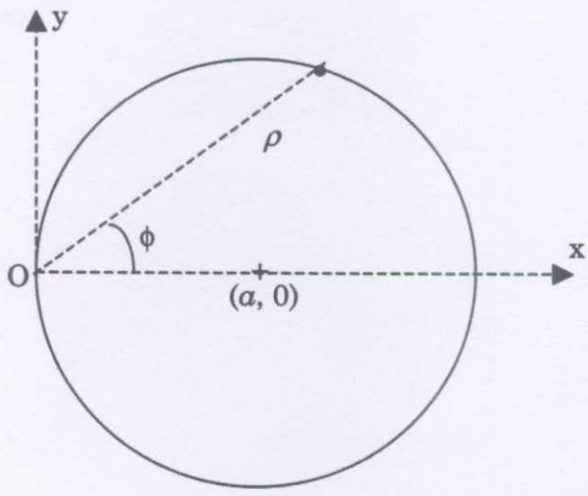
If the decay constants of ^{133}Sb and ^{133}Te are λ_{Sb} and λ_{Te} respectively, obtain the expressions for number of ^{133}Sb and ^{133}Te atoms as functions of time (i.e., $N_{Sb}(t)$ and $N_{Te}(t)$). (15)

Useful Information : The solution of $\frac{dy(x)}{dx}+T(x)\,y(x)=Q(x)$ is $y(x)=\frac{1}{PI(x)}\left[\int PI(x)Q(x)dx+C\right]$, where $PI(x)=e^{\int T(x)dx}$

Q.57 For an **isentropic** fluid motion having velocity $\vec{v}$, show that the Euler's equation can be rewritten in terms of specific enthalpy $h\,(=u+PV,\ u \equiv$ specific internal energy, $V \equiv$ specific volume) as,

$$\frac{\partial \vec{v}}{\partial t} - \vec{v}\times(\vec{\nabla}\times\vec{v}) = -\vec{\nabla}\left(h+\frac{1}{2}v^2\right) \qquad (15)$$

Q.58 A particle of mass m is moving in a circular path (radius a) passing through the origin O as shown in the figure below.



The particle is acted upon by a force $\vec{F}$ pointing towards the origin. If l is the magnitude of the conserved angular momentum of the particle, obtain the expression of $\vec{F}(\rho)$. (15)

Q.59 For a reversible process, obtain the change in entropy as a function of the independent thermodynamic variables – temperature and volume, for a gas consisting of N number of particles obeying Van der Waals equation of state. Assume the specific heat C_V to be constant. (15)

MATHEMATICS SECTION

Q.60 Solve the following differential equations :

(a) $\sec^2 y\frac{dy}{dx}+2x\tan y=x$, given that $y(1)=\pi/4$. (6)

(b) $4\frac{d^2y}{dx^2}+4\frac{dy}{dx}+y=x\left(x+e^{-x/2}\right)$. (9)

Q.61 Find the eigenvalues and the corresponding eigenvectors of the matrix

$$\begin{bmatrix} 2 & 4 & 2 \\ 1 & 5 & 2 \\ 2 & 8 & 5 \end{bmatrix}$$

(15)

Q.62 Verify Stokes' theorem for

$\vec{F}=(2x-3y)\hat{i}+y^2z^3\,\hat{j}+y^3z^2\,\hat{k}$,

$S:\ x^2+y^2+z=1,\ z\geq 0,$

$C:$ the bounding curve of S. (15)

Q.63 Evaluate the integral $\int_{-\infty}^{\infty} \frac{(x-1)^2}{(x^2+4)\ (x^2+9)} dx$ using the method of residues. (15)

Q.64 (a) Find the Laurent series expansion of the function $f(z)=\frac{1}{1-z}$ about the point $z = i$ valid in the region $|z-i|>2$. (6)

(b) Find the value of α for which $f(x) = e^{-\alpha|x|}$, $-\infty<x<\infty$, is the probability density function of a continuous random variable. Also, find the mean and the variance of the distribution. (9)

Q.65 (a) Let the real valued functions $f(x)$ and $g(x)$ be continuous on $[a, b]$ and differentiable on (a, b). Also, let $f(a)=g(a)$ and $f'(x)<g'(x)$ for $a<x<b$. Then, using the mean value theorem, prove that $f(b)<g(b)$. (6)

(b) Nine individuals are chosen at random from a population and their heights (in cm.) are found to be 158, 160, 162, 165, 167, 171, 172, 173, and 175. Discuss the suggestion that the mean height of the population is 163 cm., given that for eight degrees of freedom, the value of Student's t at 5% level of significance is 2.31. Perform all calculations correct up to 2 decimal places. (9)

Q.66 (a) Find the missing values (*) in the following data using backward differences : (6)

x :	10	15	20	25	30
y :	1	*	*	–1	1

(b) Find the value of y at $x = 1.75$ using the Newton's forward interpolation formula from the following data : (9)

x :	1.6	1.7	1.8	1.9
y :	0.945	0.955	0.964	0.971