ANSWER KEY II TERM 2019 DECEMBER
FIRST YEAR HIGHER SECONDARY EXAMINATION
MATHEMATICS (SCIENCE)

SCORE : 60

Each Carries 3 scores.

1. a) $A = \{1, 2, 3, 4\}$

   b) B can be, $\{1, 3\}$ or $\{1, 3, 5\}$, or $\{1, 3, 6\}$ or $\{1, 3, 6, 5\}$.

   c) Depending upon the above.

2. a) Answer: - 0

   Since
   $$(\sin x + \cos x)^2 = 1$$
   $$\sin^2 x + \cos^2 x + 2\sin x \cos x = 1$$
   $$1 + \sin 2x = 1$$
   $$\sin 2x = 0.$$

   b) Answer: - $\sqrt{2}$
   $$\sin x + \cos x = \sin x + \sin(90 - x)$$
   $$= 2\sin\left(\frac{x + 90 - x}{2}\right)\cos\left(\frac{x - 90 + x}{2}\right)$$
   $$= 2\sin 45 \cos\left(\frac{2x - 90}{2}\right)$$
   $$= 2\sin 45 \cos(x - 45)$$
   Maximum of $\cos x$ is 1.
   $$= 2 \times \frac{1}{\sqrt{2}} \times 1$$
   $$= \sqrt{2}$$

   c) Range of $\sin x$ is $[-1, 1]$

   ∴ Range of $2\sin x$ is $[-2, 2]$

3. $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
   $$= \frac{-1 \pm i\sqrt{7}}{4}$$

4. a) $(n+1)!$

   b) $$6! \times \frac{7}{6} = \frac{7!}{(7-r)!}$$
   $$\frac{7!}{6} = \frac{7!}{(7-r)!}$$
   $$(7-r)! = 6 \quad \therefore r = 4$$

5. $(1+x)^4$
   $$= {}^4C_0 \cdot 1^4 + {}^4C_1 \cdot 1^3 \cdot x + {}^4C_2 \cdot 1^2 \cdot x^2 + {}^4C_3 \cdot 1 \cdot x^3 + {}^4C_4 x^4$$
   $$= 1 + 4x + 6x^2 + 4x^3 + x^4$$
   $$\therefore (101)^4 = (1 + 100)^4$$
   $$= 1 + 400 + 60000 + 4000000 + 100000000$$
   $$= 104060401$$

6. a) $d = -3$

   b) $$a_n = 110 + (n-1)(-3)$$
   $$= 113 - 3n$$

   c) Yes.

   Since $a_n = 20$
   $$113 - 3n = 20$$
   $$3n = 93$$
   $$n = 31$$
   31^{st} term is 20.

7. a) (i) — (c)
   $$3x + 4y + 1 = 0$$
   $$4y = -3x - 1$$
   $$y = \frac{-3}{4}x - \frac{1}{4}$$
   $$\therefore \text{slope} = -\frac{3}{4}$$

   (ii) — (d)
   $$-3x + 4y + 5 = 0$$
   $$-3x + 4y = -5$$
   $$\frac{-3x}{-5} + \frac{4y}{-5} = 1$$
   $$\frac{x}{(5/3)} + \frac{y}{(-5/4)} = 1$$
   $$y\text{-intercept} = \frac{-5}{4}$$

   (iii) — (b)
   $$3x - 4y = 0$$
   $$3x = 4y$$
   If $x = 0$, then $y = 0$.

   ∴ It passes through $(0, 0)$.

(iv) — (a)

$3x = 5$

$x = \frac{5}{3}$

It is a line perpendicular to x-axis and passing through $(\frac{5}{3}, 0)$

b) Equation of a line having slope $\frac{1}{2}$ passes through (1,2) is,

$(y-2) = \frac{1}{2}(x-1)$

$2y - 4 = x - 1$

$2y - x - 3 = 0.$

put $x = 3$, $y = 3$

$\therefore$ (3,3) is a point on that line.

8. i) $A' = \{2,4,6,7,8,9\}$

$B' = \{1,3,7,9\}$

$A' \cap B' = \{7,9\}$

$A \cup B = \{1,2,3,4,5,6,8\}$

$(A \cup B)' = \{7,9\}$

$\therefore (A \cup B)' = A' \cap B'$

ii) $A' \cup B' = \{1,2,3,4,6,7,8,9\}.$

$A \cap B = \{5\}$

$(A \cap B)' = \{1,2,3,4,6,7,8,9\}$

$\therefore (A \cap B)' = A' \cup B'$

Each carries 4 scores

9. a) $|x|$

b) $x^2 - 8x + 12 \neq 0.$

If $x^2 - 8x + 12 = 0$

then $(x-2)(x-6) = 0$

$x = 2, 6.$

$\therefore$ Domain $= \mathbb{R} - \{2,6\}$

10. a) $P(n): \frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

For $n = 1$

$LHS = \frac{1}{1\times2} = \frac{1}{2}$

$RHS = \frac{1}{1+1} = \frac{1}{2}$

$LHS = RHS$

$\therefore P(1)$ is true.

b) Assume that $P(k)$ is true.

$\therefore P(k) = \frac{1}{1\times2} + \frac{1}{2\times3} + \dots + \frac{1}{k(k+1)} = \frac{k}{k+1}$

Now $P(k+1)$

$= \frac{1}{1\times2} + \frac{1}{2\times3} + \dots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)}$

$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$

$= \frac{1}{k+1}\left\{k + \frac{1}{k+2}\right\}$

$= \frac{k^2 + 2k + 1}{(k+1)(k+2)}$

$= \frac{(k+1)^2}{\cancel{(k+1)}(k+2)}$

$= \frac{k+1}{k+2}$

$\therefore P(k+1)$ is true.

$\therefore$ By principal of mathematical induction $P(n)$ is true.

11. a) $Z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$= 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= 1 + i\sqrt{3}$

b) $|z| = \sqrt{1+3} = 2.$

$\sqrt{a+ib} = \pm\sqrt{\frac{|z|+a}{2}} \pm i\sqrt{\frac{|z|-a}{2}}$

$= \pm\sqrt{\frac{2+1}{2}} \pm i\sqrt{\frac{2-1}{2}}$

$= \pm\sqrt{\frac{3}{2}} \pm i\sqrt{\frac{1}{2}}$

$\therefore$ roots are $\sqrt{\frac{3}{2}} + i\sqrt{\frac{1}{2}}$, $-\sqrt{\frac{3}{2}} - i\sqrt{\frac{1}{2}}$

12. a) ${}^{n}C_0 = 1$

b) i) ${}^{5}C_4 = \underline{5}$

ii) ${}^{3}C_2 \times {}^{5}C_2 + {}^{3}C_3 \times {}^{5}C_1$

$= \underline{35}$

13. a) 11

b) $n = 10$

middle term $= \left(\frac{n}{2}+1\right)^{th}$ term

$= 6^{th}$ term.

$T_{r+1} = {}^{n}C_r \, a^{n-r} \, b^{r}$

$T_{r+1} = {}^{10}C_r \left(\frac{x}{3}\right)^{n-r} (-9y)^{r}$

$\therefore T_{5+1} = {}^{10}C_5 \left(\frac{x}{3}\right)^{10-5} (-9y)^{5}$

$= \underline{61236\, x^5 y^5}$

14. a) $a_n = ar^{n-1}$

$\therefore a_3 = 4 \Rightarrow ar^2 = 4$ — ①

$a_6 = \frac{1}{2} \Rightarrow ar^5 = \frac{1}{2}$ — ②

$\frac{②}{①}$ $r^3 = \frac{1}{2\times 4}$

$r^3 = \frac{1}{8}$

$r^3 = \left(\frac{1}{2}\right)^3$

$\therefore r = \frac{1}{2}$

in ① $a \times \frac{1}{4} = 4$

$\therefore a = 16.$

$\therefore$ The G.P is, 16, 8, 4, 2, 1, . . .

b) $S_\infty = \frac{a}{1-r}$

$= \underline{32}$

15. a) L_1 passing through (1,0) and (3,6)

$\therefore$ slope $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{6-0}{3-1} = \underline{3}$

b) L_1 and L_2 are $\perp^r$ to each other

$\therefore m_1 m_2 = -1$

$m_2 = \frac{-1}{3}$

Equation of the line having slope $\frac{-1}{3}$ and passes through (3, 6) is,

$(y-6) = \frac{-1}{3}(x-3)$

$3y - 18 = -x + 3$

$3y + x - 21 = 0.$

c) L_2 have x-intercept and y-intercept.

$3y + x = 21$

$\frac{3y}{21} + \frac{x}{21} = 1$

$\frac{y}{7} + \frac{x}{21} = 1.$

$\therefore$ x-intercept $= 21.$

Length of OC $= 21$

y-intercept $= 7.$

Length of OD $= 7.$

16. a) $4a = 16$

$\therefore a = 4$

i) $(0, -4)$

ii) Axis along y-axis

iii) Length of latus rectum

$= 4a$

$= \underline{16}$

b) Hyperbola, since $c > a$.

Each carries 6 scores

17. a) $\frac{\pi}{3}$

b) $\frac{\sin x + \sin 5x}{\cos x + \cos 5x} = \frac{2 \sin 3x \cos(-2x)}{2 \cos 3x \cos(-2x)}$

$= \frac{\sin 3x}{\cos 3x}$

$= \tan 3x$

c) $\sin x + \sin 3x + \sin 5x = 0$

$2 \sin 3x \cos 2x + \sin 3x = 0$

$\sin 3x (2 \cos 2x + 1) = 0$.

$\sin 3x = 0$ or $2 \cos 2x + 1 = 0$

$3x = n\pi$ $\qquad 2 \cos 2x = -1$

$x = \frac{n\pi}{3}$ $\qquad \cos 2x = -\frac{1}{2}$

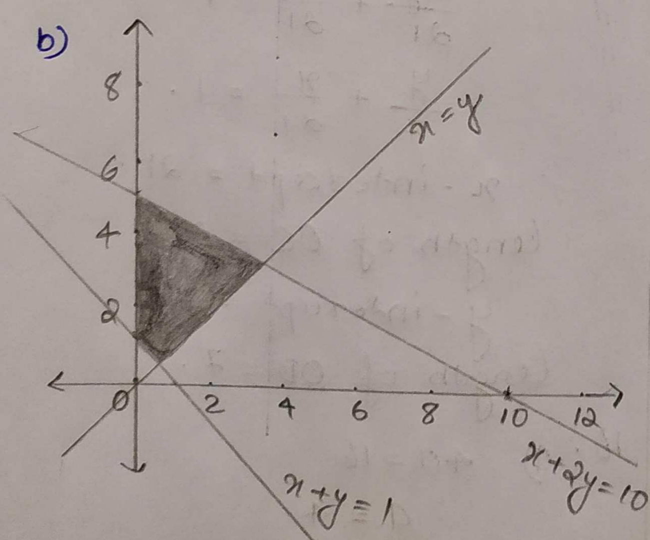
$2x = \frac{2\pi}{3}$

$x = \frac{\pi}{3}$

$x = n\pi \pm \frac{\pi}{3}$

18. a) ii

b)



c) Any point on the solution region.

19. a) Circle - general equation.

$(x-h)^2 + (y-k)^2 = r^2$

Circle passing through (0,0)

$h^2 + k^2 = r^2$ — (1)

Circle passing through (8,0)

$(8-h)^2 + k^2 = r^2$ — (2)

Circle passing through (0,6)

$h^2 + (6-k)^2 = r^2$ — (3).

Equating (1) and (2),

$(8-h)^2 + k^2 = h^2 + k^2$

$(8-h)^2 = h^2$

$h = 4$

Equating (1) and (3)

$h^2 + (6-k)^2 = h^2 + k^2$

$(6-k)^2 = k^2$

$k = 3$.

In (1) $r = 5$.

∴ Centre is (4,3) and radius 5.

b) $(x-4)^2 + (y-3)^2 = 5^r$

ie $x^2 + y^2 - 8x - 6y = 0$.

c) Equation of the line passing through (8,0) and (0,6)

$y - 0 = -\frac{3}{4}(x-8)$

$3x + 4y - 24 = 0$

d) tangent is $\perp^r$ to AB.

∴ slope $= \frac{4}{3}$.

Equation of the line having slope $\frac{4}{3}$ and passing through (8,0)

$3y - 4x + 32 = 0$.

20. $a = 3$, $b = 2$, $c = \sqrt{5}$

a) Length of major axis = 6

Length of minor axis = 4

b) foci $(0, \pm\sqrt{5})$

Vertices $(0, \pm 6)$

c) Length of latus rectum

$= \frac{8}{3}$

eccentricity $= \frac{\sqrt{5}}{3}$