ME GATE-08

MCQ 1.1 In the Taylor series expansion of e^x about $x = 2$, the coefficient of $(x-2)^4$ is

GATE ME 2008 ONE MARK

(A) $1/4\,!$ (B) $2^4/4!$

(C) $e^2/4!$ (D) $e^4/4!$

SOL 1.1 Option (C) is correct.

Taylor's series expansion of $f(x)$ is given by,

$$f(x) = f(a) + \frac{(x-a)}{\lfloor 1}f'(a) + \frac{(x-a)^2}{\lfloor 2}f''(a) + \frac{(x-a)^3}{\lfloor 3}f'''(a) + \ldots.$$

Then from this expansion the coefficient of $(x-a)^4$ is $\frac{f''''(a)}{\lfloor 4}$

Given $\quad a = 2$

& $\quad f(x) = e^x$

$$f'(x) = e^x$$
$$f''(x) = e^x$$
$$f'''(x) = e^x$$
$$f''''(x) = e^x$$

Hence, for $a = 2$ the coefficient of $(x-a)^4$ is $\frac{e^2}{\lfloor 4}$

MCQ 1.2 Given that $\ddot{x} + 3x = 0$, and $x(0) = 1$, $\dot{x}(0) = 0$, what is $x(1)$?

GATE ME 2008 ONE MARK

(A) -0.99 (B) -0.16

(C) 0.16 (D) 0.99

SOL 1.2 Option (D) is correct.

Given : $\quad \ddot{x} + 3x = 0$ & $x(0) = 1$

$$(D^2 + 3)x = 0 \qquad D = \frac{d}{dt}$$

The auxiliary Equation is written as

$$m^2 + 3 = 0$$
$$m = \pm\sqrt{3}\,i = 0 \pm \sqrt{3}\,i$$

Here the roots are imaginary

$$m_1 = 0 \;\&\; m_2 = \sqrt{3}$$

& Solution is given by

$$x = e^{m_1 t}(A\cos m_2 t + B\sin m_2 t)$$

$$x = e^0[A\cos\sqrt{3}\,t + B\sin\sqrt{3}\,t]$$

$$= [A\cos\sqrt{3}\,t + B\sin\sqrt{3}\,t] \quad \text{...(i)}$$

Given : $x(0) = 1$ at $t = 0,\ x = 1$

Substitute in equation (i),

$$1 = [A\cos\sqrt{3}(0) + B\sin\sqrt{3}(0)] = A + 0$$

$$A = 1$$

Differentiate equation (i) w.r.t. t,

$$\dot{x} = \sqrt{3}[-A\sin\sqrt{3}\,t + B\cos\sqrt{3}\,t] \quad \text{...(ii)}$$

Given $\dot{x}(0) = 0$ at $t = 0,\ \dot{x} = 0$

Substitute in equation (ii), we get

$$0 = \sqrt{3}[-A\sin 0 + B\cos 0]$$

$$B = 0$$

Put A & B in equation (i)

$$x = \cos\sqrt{3}\,t$$

$$x(1) = \cos\sqrt{3} = 0.99$$

MCQ 1.3 The value of $\lim_{x \to 8} \frac{x^{1/3} - 2}{(x - 8)}$

GATE ME 2008 ONE MARK

(A) $\frac{1}{16}$ (B) $\frac{1}{12}$

(C) $\frac{1}{8}$ (D) $\frac{1}{4}$

SOL 1.3 Option (B) is correct.

Let $f(x) = \lim_{x \to 8} \frac{x^{1/3} - 2}{(x - 8)}$ $\frac{0}{0}$ form

$$= \lim_{x \to 8} \frac{\frac{1}{3}x^{-2/3}}{1} \quad \text{Applying L-Hospital rule}$$

Substitute the limits, we get

$$= \frac{1}{3}(8)^{-2/3} = \frac{1}{3}(2^3)^{-2/3} = \frac{1}{4 \times 3} = \frac{1}{12}$$

MCQ 1.4 A coin is tossed 4 times. What is the probability of getting heads exactly 3 times ?

GATE ME 2008 ONE MARK

(A) $\frac{1}{4}$ (B) $\frac{3}{8}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

SOL 1.4 Option (A) is correct.

In a coin probability of getting Head

$$p = \frac{1}{2} = \frac{\text{No. of Possible cases}}{\text{No. of Total cases}}$$

Probability of getting tail

$$q = 1 - \frac{1}{2} = \frac{1}{2}$$

So the probability of getting Heads exactly three times, when coin is tossed 4 times is

$$P = {}^4C_3(p)^3(q)^1$$

$$= {}^4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 = 4 \times \frac{1}{8} \times \frac{1}{2} = \frac{1}{4}$$

MCQ 1.5

GATE ME 2008 ONE MARK

The matrix $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$ has one eigen value equal to 3. The sum of the other two eigen value is

(A) p

(B) $p-1$

(C) $p-2$

(D) $p-3$

SOL 1.5

Option (C) is correct.

Let, $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$

Let the eigen values of this matrix are λ_1, λ_2 & λ_3

Here one values is given so let $\lambda_1 = 3$

We know that

Sum of eigen values of matrix = Sum of the diagonal element of matrix A

$$\lambda_1 + \lambda_2 + \lambda_3 = 1 + 0 + p$$

$$\lambda_2 + \lambda_3 = 1 + p - \lambda_1 = 1 + p - 3$$

$$= p - 2$$

MCQ 1.6

GATE ME 2008 ONE MARK

The divergence of the vector field $(x-y)\boldsymbol{i} + (y-x)\boldsymbol{j} + (x+y+z)\boldsymbol{k}$ is

(A) 0

(B) 1

(C) 2

(D) 3

SOL 1.6

Option (D) is correct.

We know that the divergence is defined as $\nabla \cdot \boldsymbol{V}$

Let $\boldsymbol{V} = (x-y)\boldsymbol{i} + (y-x)\boldsymbol{j} + (x+y+z)\boldsymbol{k}$

And $\nabla = \left(\frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k}\right)$

So,

$$\nabla \cdot \boldsymbol{V} = \left(\frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k}\right) \cdot [(x-y)\boldsymbol{i} + (y-x)\boldsymbol{j} + (x+y+z)\boldsymbol{k}]$$

$$= \frac{\partial}{\partial x}(x-y) + \frac{\partial}{\partial y}(y-x) + \frac{\partial}{\partial z}(x+y+z)$$

$$= 1 + 1 + 1 = 3$$

GATE ME 2008 ONE MARK

The transverse shear stress acting in a beam of rectangular cross-section, subjected to a transverse shear load, is

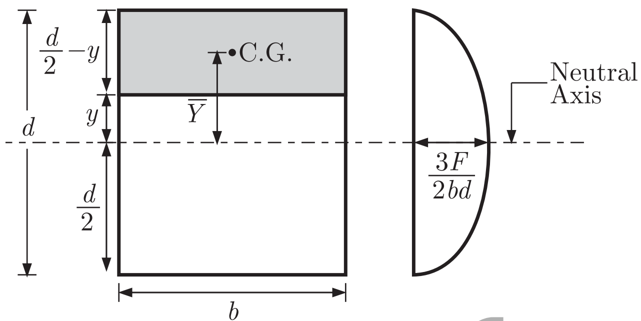
(A) variable with maximum at the bottom of the beam

(B) variable with maximum at the top of the beam

(C) uniform

(D) variable with maximum on the neutral axis

SOL 1.7 Option (D) is correct.



For a rectangle cross-section:

$$\tau_v = \frac{FA\overline{Y}}{Ib} = \frac{6F}{bd^3}\left(\frac{d^2}{4} - y^2\right) \qquad F = \text{Transverse shear load}$$

Maximum values of τ_v occurs at the neutral axis where, $y = 0$

$$\text{Maximum } \tau_v = \frac{6F}{bd^3} \times \frac{d^2}{4} = \frac{3F}{2bd}$$

$$= \frac{3}{2}\tau_{\text{mean}} \qquad \tau_{mean} = \frac{F}{bd}$$

So, transverse shear stress is variable with maximum on the neutral axis.

MCQ 1.8 GATE ME 2008 ONE MARK

A rod of length L and diameter D is subjected to a tensile load P. Which of the following is sufficient to calculate the resulting change in diameter ?

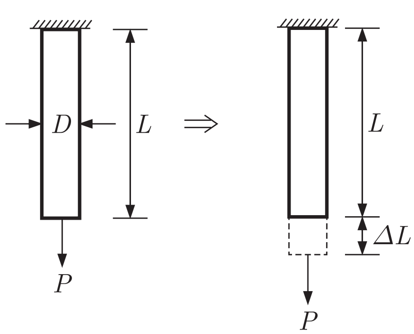
(A) Young's modulus

(B) Shear modulus

(C) Poisson's ratio

(D) Both Young's modulus and shear modulus

SOL 1.8 Option (D) is correct.



From the application of load P, the length of the rod increases by an amount of ΔL

$$\Delta L = \frac{PL}{AE} = \frac{PL}{\frac{\pi}{4}D^2 E} = \frac{4PL}{\pi D^2 E}$$

And increase in length due to applied load P in axial or longitudinal direction, the shear modulus is comes in action.

$$G = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{\tau_s}{\Delta L/L} = \frac{\tau_s L}{\Delta L}$$

So, for calculating the resulting change in diameter both young's modulus & shear modulus are used.

MCQ 1.9

GATE ME 2008 ONE MARK

A straight rod length $L(t)$, hinged at one end freely extensible at the other end, rotates through an angle $\theta(t)$ about the hinge. At time t, $L(t) = 1$ m, $\dot{L}(t) = 1$ m/s, $\theta(t) = \frac{\pi}{4}$ rad and $\dot{\theta}(t) = 1$ rad/s. The magnitude of the velocity at the other end of the rod is

(A) 1 m/s

(B) $\sqrt{2}$ m/s

(C) $\sqrt{3}$ m/s

(D) 2 m/s

SOL 1.9

Option (D) is correct.

Let : V_t =Tangential Velocity
V_r =Relative Velocity
V =Resultant Velocity

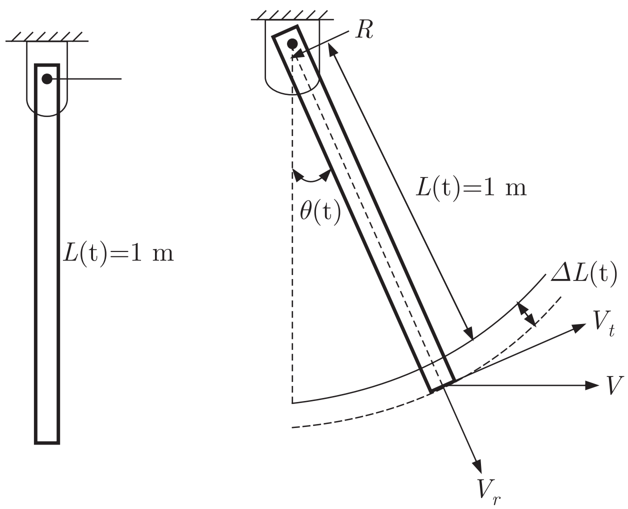
Let rod of length $L(t)$ increases by an amount $\Delta L(t)$.

Given $L(t) = 1$ m, $\dot{L}(t)$=1 m/sec, $\theta(t) = \frac{\pi}{4}$ rad, $\dot{\theta}(t)$=1 rad/sec

Time taken by the rod to turn $\frac{\pi}{4}$ rad is,

$$t = \frac{\text{dis tan ce}}{\text{velocity}} = \frac{\theta(t)}{\dot{\theta}(t)}$$

$$= \frac{\pi/4}{1} = \frac{\pi}{4} \text{ sec}$$



So, increase in length of the rod during this time will be

$$\Delta L(t) = L(t) \times t = \frac{\pi}{4} \times 1 = \frac{\pi}{4} \text{ meter}$$

Rod turn $\frac{\pi}{4}$ radian. So, increased length after $\frac{\pi}{4}$ sec, (New length)

$$= \left(1 + \frac{\pi}{4}\right) = 1.785 \text{ m}$$

Now, tangential velocity will be

$$V_t = R.\omega = 1.785 \times 1 = 1.785 \text{ m/sec} \qquad \omega = \dot{\theta}(t)$$

Radial velocity will be

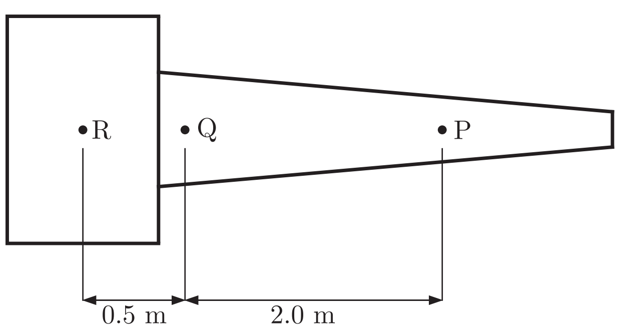
$$V_r = \dot{L}(t) = 1 \text{ m/sec}$$

Therefore, the resultant velocity will be

$$V_R = \sqrt{V_t^2 + V_r^2} = \sqrt{(1.785)^2 + (1)^2} = 2.04 \simeq 2 \text{ m/sec}$$

MCQ 1.10 GATE ME 2008 ONE MARK

A cantilever type gate hinged at Q is shown in the figure. P and R are the centers of gravity of the cantilever part and the counterweight respectively. The mass of the cantilever part is 75 kg. The mass of the counter weight, for static balance, is

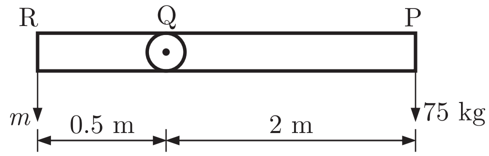


(A) 75 kg (B) 150 kg

(C) 225 kg (D) 300 kg

SOL 1.10 Option (D) is correct.

First of all we have to make the FBD of the given system.



Let mass of the counter weight $= m$.
Here point Q is the point of contraflexure or point of inflection or a virtual hinge.
So, $M_Q = 0$

$$m \times 0.5 = 75 \times 2 \Rightarrow m = 300 \text{ kg}$$

MCQ 1.11 GATE ME 2008 ONE MARK

A planar mechanism has 8 links and 10 rotary joints. The number of degrees of freedom of the mechanism, using Gruebler's criterion, is

(A) 0 (B) 1

(C) 2 (D) 3

SOL 1.11 Option (B) is correct.

From Gruebler's criterion, the equation for degree of freedom is given by,

$$n = 3(l-1) - 2j - h \quad \text{...(i)}$$

Given $l = 8$ and $j = 10,\ h = 0$

$$n = 3(8-1) - 2\times 10 = 1 \quad \text{from equation(i)}$$

MCQ 1.12 GATE ME 2008 ONE MARK

An axial residual compressive stress due to a manufacturing process is present on the outer surface of a rotating shaft subjected to bending. Under a given bending load, the fatigue life of the shaft in the presence of the residual compressive stress is

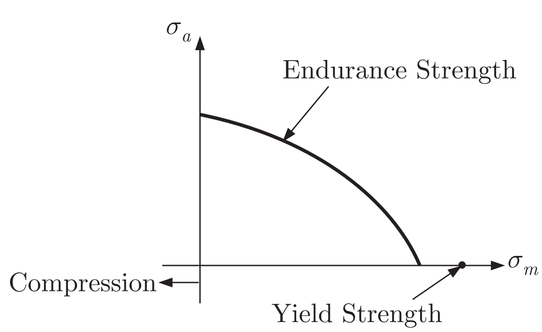
(A) decreased

(B) increased or decreased, depending on the external bending load

(C) neither decreased nor increased

(D) increased

SOL 1.12 Option (D) is correct.



(Gerber's Parabola)

The figure shown the Gerber's parabola. It is the characteristic curve of the fatigue

life of the shaft in the presence of the residual compressive stress.
The fatigue life of the material is effectively increased by the introduction of a compressive mean stress, whether applied or residual.

MCQ 1.13 GATE ME 2008 ONE MARK

2 moles of oxygen are mixed adiabatically with another 2 moles of oxygen in mixing chamber, so that the final total pressure and temperature of the mixture become same as those of the individual constituents at their initial states. The universal gas constant is given as R. The change in entropy due to mixing, per mole of oxygen, is given by

(A) $-R\ln 2$ (B) 0

(C) $R\ln 2$ (D) $R\ln 4$

SOL 1.13

Option (B) is correct.
Given : $T_1 = T_2,\ p_1 = p_2$
Universal Gas constant $= R$
Here given oxygen are mixed adiabatically

So, $dQ = 0$

We know, $ds = \frac{dQ}{T} = \frac{0}{T} = 0$

MCQ 1.14 GATE ME 2008 ONE MARK

For flow of fluid over a heated plate, the following fluid properties are known

Viscosity $= 0.001$Pa-s;
Specific heat at constant pressure $= 1$ kJ/kg.K;
Thermal conductivity $= 1$W/m - K

The hydrodynamic boundary layer thickness at a specified location on the plate is 1 mm. The thermal boundary layer thickness at the same location is

(A) 0.001 mm (B) 0.01 mm

(C) 1 mm (D) 1000 mm

SOL 1.14

Option (C) is correct.
Given : $\mu = 0.001\,\text{Pa - s},\ c_p = 1\,\text{kJ/kg K},\ k = 1\,\text{W/m K}$
The prandtl Number is given by,

$$\text{Pr} = \frac{\mu c_p}{k} = \frac{0.001 \times 1 \times 10^3}{1} = 1$$

And $$\frac{\delta}{\delta_t} = \frac{\text{hydrodynamic bondary layer thickness}}{\text{Thermal boundary layer thickness}} = (\text{Pr})^{1/3}$$

Given, $\delta = 1\,\text{m}$

$$\frac{\delta}{\delta_t} = (1)^{1/3} = 1$$

$$\delta = \delta_t = 1\,\text{mm}$$

Hence, thermal boundary layer thickness at same location is 1 mm.

MCQ 1.15 GATE ME 2008 ONE MARK

For the continuity equation given by $\nabla \cdot \boldsymbol{V} = 0$ to be valid, where $\boldsymbol{V}$ is the velocity vector, which one of the following is a necessary condition ?

(A) steady flow (B) irrotational flow
(C) inviscid flow (D) incompressible flow

SOL 1.15 Option (D) is correct.

The continuity equation in three dimension is given by,

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

For incompressible flow ρ=Constant

$$\rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right] = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\nabla \cdot \boldsymbol{V} = 0$$

So, the above equation represents the incompressible flow.

MCQ 1.16 GATE ME 2008 ONE MARK

Which one of the following is NOT a necessary assumption for the air-standard Otto cycle ?

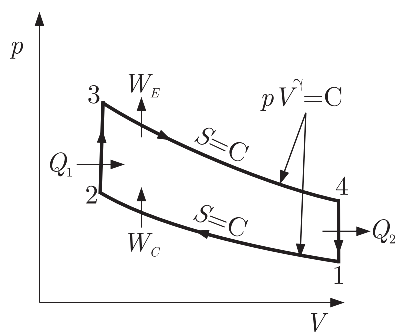
(A) All processes are both internally as well as externally reversible.

(B) Intake and exhaust processes are constant volume heat rejection processes.

(C) The combustion process is a constant volume heat addition process.

(D) The working fluid is an ideal gas with constant specific heats.

SOL 1.16 Option (B) is correct.



Assumptions of air standard otto cycle :-

(A) All processes are both internally as well as externally reversible.

(B) Air behaves as ideal gas

(C) Specific heats remains constant (c_p & c_v)

(D) Intake process is constant volume heat addition process & exhaust process is constant volume heat rejection process.

Intake process is a constant volume heat addition process, From the given options, option (2) is incorrect.

GATE ME 2008 ONE MARK

In an $M/M/1$ queuing system, the number of arrivals in an interval of length T is a

Poisson random variable (i.e. the probability of there being arrivals in an interval of length T is $\frac{e^{-\lambda T}(\lambda T)^n}{n!}$).

The probability density function $f(t)$ of the inter-arrival time is given by

(A) $\lambda^2(e^{-\lambda^2 t})$ (B) $\frac{e^{-\lambda^2 t}}{\lambda^2}$

(C) $\lambda e^{-\lambda t}$ (D) $\frac{e^{-\lambda t}}{\lambda}$

SOL 1.17 Option (C) is correct.

The most common distribution found in queuing problems is poisson distribution. This is used in single-channel queuing problems for random arrivals where the service time is exponentially distributed.

Probability of n arrivals in time t

$$P = \frac{(\lambda T)^n \cdot e^{-\lambda T}}{n!} \qquad \text{where } n = 0,1,2.......$$

So, Probability density function of inter arrival time (time interval between two consecutive arrivals)

$$f(t) = \lambda \cdot e^{-\lambda t}$$

MCQ 1.18 GATE ME 2008 ONE MARK

A set of 5 jobs is to be processed on a single machine. The processing time (in days) is given in the table below. The holding cost for each job is Rs. K per day.

Job	Processing time
P	5
Q	2
R	3
S	2
T	1

A schedule that minimizes the total inventory cost is

(A) T-S-Q-R-P (B) P-R-S-Q-T

(C) T-R-S-Q-P (D) P-Q-R-S-T

SOL 1.18 Option (A) is correct.

Total inventory cost will be minimum, when the holding cost is minimum. Now, from the Johnson's algorithm, holding cost will be minimum, when we process the least time consuming job first. From this next job can be started as soon as possible.

Now, arrange the jobs in the manner of least processing time.

T-S-Q-R-P or T-Q-S-R-P (because job Q and S have same processing time).

MCQ 1.19 GATE ME 2008 ONE MARK

For generating a Coon's surface we require

(A) a set of grid points on the surface

(B) a set of grid control points

(C) four bounding curves defining the surface

(D) two bounding curves and a set of grid control points

SOL 1.19 Option (C) is correct

Coon's surface is obtained by blending four boundary curves. The main advantage of Coon's surface is its ability to fit a smooth surface through digitized points in space such as those used in reverse engineering.

MCQ 1.20 Internal gear cutting operation can be performed by

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(A) milling

(B) shaping with rack cutter

(C) shaping with pinion cutter

(D) hobbing

SOL 1.20 Option (C) is correct.

Internal gear cutting operation can be performed by shaping with pinion cutter. In the case of 'rotating pinion type cutter', such an indexing is not required, therefore, this type is more productive and so common.

MCQ 1.21 Consider the shaded triangular region P shown in the figure. What is $\iint_P xy\,dx\,dy$?

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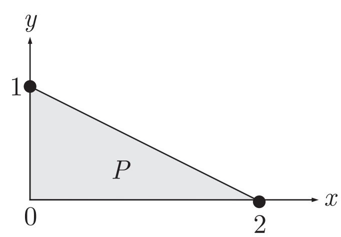


(A) $\frac{1}{6}$

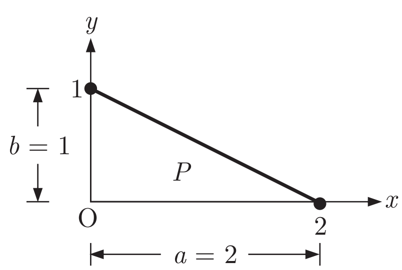
(B) $\frac{2}{9}$

(C) $\frac{7}{16}$

(D) 1

SOL 1.21 Option (A) is correct.

Given :



We know that the equation of line in intercept form is given by

$$\frac{x}{2}+\frac{y}{1}=1 \qquad \frac{x}{a}+\frac{y}{b}=1$$

$$x+2y=2 \Rightarrow x=2(1-y)$$

The limit of x is between 0 to $x=2(1-y)$ & y is 0 to 1,

Now
$$\iint_p xydxdy = \int_{y=0}^{y=1}\int_{x=0}^{2(1-y)} xydxdy$$
$$= \int_{y=0}^{y=1}\left[\frac{x^2}{2}\right]_0^{2(1-y)} ydy$$
$$= \int_{y=0}^{y=1} y\left[\frac{4(1-y)^2}{2}-0\right]dy$$
$$= \int_{y=0}^{y=1} 2y(1+y^2-2y)\,dy = \int_{y=0}^{y=1} 2(y+y^3-2y^2)\,dy$$

Again Integrating and substitute the limits, we get
$$= 2\left[\frac{y^2}{2}+\frac{y^4}{4}-\frac{2y^3}{3}\right]_0^1 = 2\left[\frac{1}{2}+\frac{1}{4}-\frac{2}{3}-0\right]$$
$$= 2\left[\frac{6+3-8}{12}\right] = \frac{2}{12} = \frac{1}{6}$$

MCQ 1.22 The directional derivative of the scalar function $f(x,y,z)=x^2+2y^2+z$ at the point $P=(1,1,2)$ in the direction of the vector $\boldsymbol{a}=3\boldsymbol{i}-4\boldsymbol{j}$ is

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(A) -4 (B) -2

(C) -1 (D) 1

SOL 1.22 Option (B) is correct.

We know that direction derivative of a function f along a vector $\boldsymbol{P}$ is given by
$$\boldsymbol{a} = \text{grad } f \cdot \frac{\boldsymbol{a}}{|\boldsymbol{a}|}$$

where
$$\text{grad } f = \left(\frac{\partial f}{\partial x}\boldsymbol{i}+\frac{\partial f}{\partial y}\boldsymbol{j}+\frac{\partial f}{\partial z}\boldsymbol{k}\right)$$

&
$$f(x,y,z) = x^2+2y^2+z,\quad \boldsymbol{a}=3\boldsymbol{i}-4\boldsymbol{j}$$
$$\boldsymbol{a} = \text{grad}(x^2+2y^2+z)\cdot\frac{3\boldsymbol{i}-4\boldsymbol{j}}{\sqrt{(3)^2+(-4)^2}}$$
$$= (2x\boldsymbol{i}+4y\boldsymbol{j}+\boldsymbol{k})\cdot\frac{(3\boldsymbol{i}-4\boldsymbol{j})}{\sqrt{25}} = \frac{6x-16y}{5}$$

At point $P(1,\,1,\,2)$ the direction derivative is
$$\boldsymbol{a} = \frac{6\times1-16\times1}{5} = -\frac{10}{5} = -2$$

MCQ 1.23 For what value of a, if any will the following system of equation in x,y and z have a solution ?

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$$2x+3y=4$$
$$x+y+z=4$$

$$3x + 2y - z = a$$

(A) Any real number (B) 0

(C) 1 (D) There is no such value

SOL 1.23 Option (B) is correct.

Given :
$$2x + 3y = 4$$
$$x + y + z = 4$$
$$x + 2y - z = a$$

It is a set of non-homogenous equation, so the augmented matrix of this system is

$$[A:B] = \begin{bmatrix} 2 & 3 & 0 & : & 4 \\ 1 & 1 & 1 & : & 4 \\ 1 & 2 & -1 & : & a \end{bmatrix}$$

Applying row operations,

$$R_3 \to R_3 + R_2,\; R_2 \to 2R_2 - R_1$$

$$[A:B] = \begin{bmatrix} 2 & 3 & 0 & : & 4 \\ 0 & -1 & 2 & : & 4 \\ 2 & 3 & 0 & : & 4+a \end{bmatrix}$$

Again applying row operation

$$R_3 \to R_3 - R_1$$

$$[A:B] = \begin{bmatrix} 2 & 3 & 0 & : & 4 \\ 0 & -1 & 2 & : & 4 \\ 0 & 0 & 0 & : & a \end{bmatrix}$$

So, for a unique solution of the system of equations, it must have the condition

$$\rho[A:B] = \rho[A]$$

So, when putting $a = 0$

We get $\rho[A:B] = \rho[A]$

MCQ 1.24 Which of the following integrals is unbounded ?

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(A) $\int_0^{\pi/4} \tan x dx$ (B) $\int_0^{\infty} \frac{1}{x^2+1} dx$

(C) $\int_0^{\infty} x e^{-x} dx$ (D) $\int_0^1 \frac{1}{1-x} dx$

SOL 1.24 Option (D) is correct.

Here we check all the four options for unbounded condition.

(A)
$$\int_0^{\pi/4} \tan x dx = [\log|\sec x|]_0^{\pi/4} = \left[\log\left|\sec\frac{\pi}{4}\right| - \log|\sec 0|\right]$$
$$= \log\sqrt{2} - \log 1 = \log\sqrt{2}$$

(B)
$$\int_0^{\infty} \frac{1}{x^2+1} dx = [\tan^{-1} x]_0^{\infty}$$
$$= \tan^{-1}\infty - \tan^{-1}(0) = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

(C) $\int_0^{\infty} xe^{-x}\,dx$

Let $I = \int_0^{\infty} xe^{-x}\,dx$

$$= x\int_0^{\infty} e^{-x}\,dx - \int_0^{\infty}\left[\frac{d}{dx}(x)\int e^{-x}\,dx\right]dx$$

$$= [-xe^{-x}]_0^{\infty} + \int_0^{\infty} e^{-x}\,dx$$

$$= [-xe^{-x} - e^{-x}]_0^{\infty} = [-e^{-x}(x+1)]_0^{\infty}$$

$$= -[0-1] = 1$$

(D) $\int_0^1 \frac{1}{1-x}\,dx = -\int_0^1 \frac{1}{x-1}\,dx = -[\log(x-1)]_0^1$

$$-[\log 0 - \log(-1)]$$

both log 0 & log (–1) undefined so it is unbounded.

MCQ 1.25 GATE ME 2008 TWO MARK

The integral $\oint f(z)\,dz$ evaluated around the unit circle on the complex plane for $f(z) = \frac{\cos z}{z}$ is

(A) $2\pi i$ (B) $4\pi i$

(C) $-2\pi i$ (D) 0

SOL 1.25 Option (A) is correct.

Let $I = \oint f(z)\,dz$ & $f(z) = \frac{\cos z}{z}$

Then $I = \oint \frac{\cos z}{z}\,dz = \oint \frac{\cos z}{|z-0|}\,dz$ ...(i)

Given that $|z| = 1$ for unit circle

From the Cauchy Integral formula

$$\oint \frac{f(z)}{z-a}\,dz = 2\pi i f(a) \quad ...(ii)$$

Compare equation (i) & (ii), we can say that,

$$a = 0 \;\&\; f(z) = \cos z$$

Or, $f(a) = f(0) = \cos 0 = 1$

Now from equation (ii) we get

$$\oint \frac{f(z)}{z-0}\,dz = 2\pi i \times 1 = 2\pi i \qquad a = 0$$

MCQ 1.26 GATE ME 2008 TWO MARK

The length of the curve $y = \frac{2}{3}x^{3/2}$ between $x = 0$ and $x = 1$ is

(A) 0.27 (B) 0.67

(C) 1 (D) 1.22

SOL 1.26 Option (D) is correct.

Given $y = \frac{2}{3}x^{3/2}$ ...(i)

We know that the length of curve is given by $\int_{x_1}^{x_2}\left\{\sqrt{\left(\frac{dy}{dx}\right)^2 + 1}\right\}dx$ ...(ii)

Differentiate equation(i) w.r.t. x

$$\frac{dy}{dx} = \frac{2}{3} \times \frac{3}{2}x^{\frac{3}{2}-1} = x^{1/2} = \sqrt{x}$$

Substitute the limit $x_1 = 0$ to $x_2 = 1$ & $\frac{dy}{dx}$ in equation (ii), we get

$$\mathcal{L} = \int_0^1\left(\sqrt{(\sqrt{x})^2 + 1}\right)dx = \int_0^1 \sqrt{x+1}\,dx$$

Integrating the equation and put the limits

$$= \left[\frac{2}{3}(x+1)^{3/2}\right]_0^1$$

$$\mathcal{L} = 1.22$$

MCQ 1.27 GATE ME 2008 TWO MARK

The eigen vector of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ are written in the form $\begin{bmatrix} 1 \\ a \end{bmatrix}$ and $\begin{bmatrix} 1 \\ b \end{bmatrix}$. What is $a + b$?

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

SOL 1.27 Option (B) is correct.

Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$

And λ_1 & λ_2 is the eigen values of the matrix.

We know for eigen values characteristic matrix is,

$$|A - \lambda I| = 0$$

$$\left|\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right| = 0$$

$$\begin{vmatrix} (1-\lambda) & 2 \\ 0 & (2-\lambda) \end{vmatrix} = 0 \quad \text{...(i)}$$

$$(1-\lambda)(2-\lambda) = 0$$

$$\lambda = 1 \,\&\, 2$$

So, Eigen vector corresponding to the $\lambda = 1$ is,

$$\begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ a \end{bmatrix} = 0$$

$$2a + a = 0 \Rightarrow \quad a = 0$$

Again for $\lambda = 2$

$$\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ b \end{bmatrix} = 0$$

$$-1+2b=0 \qquad b=\frac{1}{2}$$

Then sum of $a\,\&\,b \Rightarrow a+b=0+\frac{1}{2}=\frac{1}{2}$

MCQ 1.28 Let $f=y^x$. What is $\frac{\partial^2 f}{\partial x\partial y}$ at $x=2,\ y=1$?

GATE ME 2008 TWO MARK

(A) 0 (B) $\ln 2$

(C) 1 (D) $\frac{1}{\ln 2}$

SOL 1.28 Option (C) is correct.

Given $f(x,y)=y^x$

First partially differentiate the function w.r.t. y

$$\frac{\partial f}{\partial y}=xy^{x-1}$$

Again differentiate. it w.r.t. x

$$\frac{\partial^2 f}{\partial x\partial y}=y^{x-1}(1)+x(y^{x-1}\log y)=y^{x-1}(x\log y+1)$$

At : $x=2,\ y=1$

$$\frac{\partial^2 f}{\partial x\partial y}=(1)^{2-1}(2\log 1+1)=1(2\times 0+1)=1$$

MCQ 1.29 It is given that $y''+2y'+y=0,\ y(0)=0,\ y(1)=0$. What is $y(0.5)$?

GATE ME 2008 TWO MARK

(A) 0 (B) 0.37

(C) 0.62 (D) 1.13

SOL 1.29 Option (A) is correct.

Given : $y''+2y'+y=0$

$(D^2+2D+1)\,y=0$ where $D=d/dx$

The auxiliary equation is

$$m^2+2m+1=0$$

$$(m+1)^2=0,\ m=-1,-1$$

The roots of auxiliary equation are equal & hence the general solution of the given differential equation is,

$$y=(C_1+C_2x)\,e^{m_1x}=(C_1+C_2x)\,e^{-x} \qquad ..(i)$$

Given $y(0)=0$ at $x=0$, $\Rightarrow y=0$

Substitute in equation (i), we get

$$0=(C_1+C_2\times 0)\,e^{-0}$$

$$0=C_1\times 1 \Rightarrow C_1=0$$

Again $y(1)=0$, at $x=1 \Rightarrow y=0$

Substitute in equation (i), we get

$$0=[C_1+C_2\times(1)]\,e^{-1}=[C_1+C_2]\frac{1}{e}$$

$$C_1 + C_2 = 0 \quad \Rightarrow C_2 = 0$$

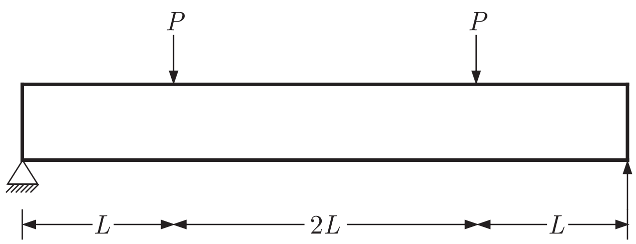
Substitute C_1 & C_2 in equation (i), we get

$$y = (0 + 0x)\, e^{-x} = 0$$

And $y(0.5) = 0$

MCQ 1.30 GATE ME 2008 TWO MARK

The strain energy stored in the beam with flexural rigidity EI and loaded as shown in the figure is



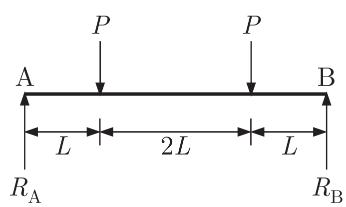
(A) $\frac{P^2L^3}{3EI}$

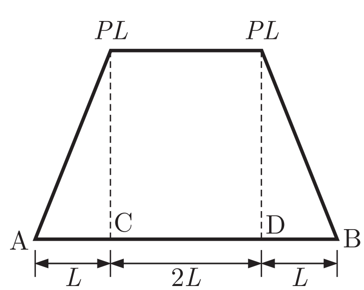
(B) $\frac{2P^2L^3}{3EI}$

(C) $\frac{4P^2L^3}{3EI}$

(D) $\frac{8P^2L^3}{3EI}$

SOL 1.30 Option (C) is correct.





B.M.D.

In equilibrium condition of forces,

$$R_A + R_B = 2P \qquad \text{...(i)}$$

Taking the moment about point A,

$$R_B \times 4L - P \times L - P \times 3L = 0$$

$$R_B \times 4L - 4PL = 0$$

$$R_B = \frac{4PL}{4L} = P$$

From equation (i),

$$R_A = 2P - P = P$$

With the help of R_A & R_B, we have to make the Bending moment diagram of the given beam. From this B.M.D, at section AC & BD Bending moment varying with distance but at section CD, it is constant

We know that,

$$\text{Strain energy } U = \int \frac{M^2}{2EI} dx$$

Where M is the bending moment of beam.

Total strain energy is given by

$$U = \underbrace{\int_0^L \frac{(Px)^2 dx}{2EI}}_{\{\text{for section } AC\}} + \underbrace{\frac{(PL)^2 2L}{2EI}}_{\{\text{for section } CD\}} + \underbrace{\int_0^L \frac{(Px)^2 dx}{2EI}}_{\{\text{for section } BD\}}$$

$$= 2\int_0^L \frac{(Px)^2 dx}{2EI} + \frac{P^2 L^3}{EI} = \frac{P^2}{EI}\int_0^L x^2 dx + \frac{P^2 L^3}{EI}$$

On integrating above equation, we get

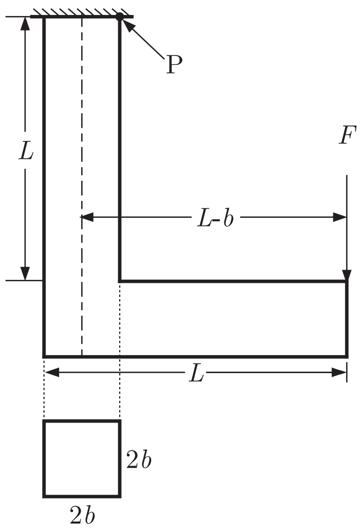
$$U = \frac{P^2}{EI}\left[\frac{x^3}{3}\right]_0^L + \frac{P^2 L^3}{EI}$$

On substituting the limits, we get

$$U = \frac{P^2 L^3}{3EI} + \frac{P^2 L^3}{EI} = \frac{4P^2 L^3}{3EI}$$

MCQ 1.31 GATE ME 2008 TWO MARK

For the component loaded with a force F as shown in the figure, the axial stress at the corner point P is



(A) $\frac{F(3L-b)}{4b^3}$

(B) $\frac{3(3L+b)}{4b^3}$

(C) $\frac{F(3L-4b)}{4b^3}$

(D) $\frac{F(3L-2b)}{4b^3}$

SOL 1.31 Option (D) is correct.

Here corner point P is fixed.

At point P double stresses are acting, one is due to bending & other stress is due to the direct Load.

So, bending stress, (From the bending equation)

$$\sigma_b = \frac{M}{I} y$$

Distance from the neutral axis to the external fibre $y = \frac{2b}{2} = b$,

$$\sigma_b = \frac{F(L-b)}{\frac{(2b)^4}{12}} \times b \qquad \text{For square section } I = \frac{b^4}{12}$$

$$\sigma_b = \frac{12F(L-b)}{16b^3} = \frac{3F(L-b)}{4b^3}$$

& Direct stress,

$$\sigma_d = \frac{F}{(2b)^2} = \frac{F}{4b^2} = \frac{F}{4b^2} \times \frac{b}{b} = \frac{Fb}{4b^3}$$

Total axial stress at the corner point P is,

$$\sigma = \sigma_b + \sigma_d = \frac{3F(L-b)}{4b^3} + \frac{Fb}{4b^3}$$

$$\sigma = \frac{F(3L-2b)}{4b^3}$$

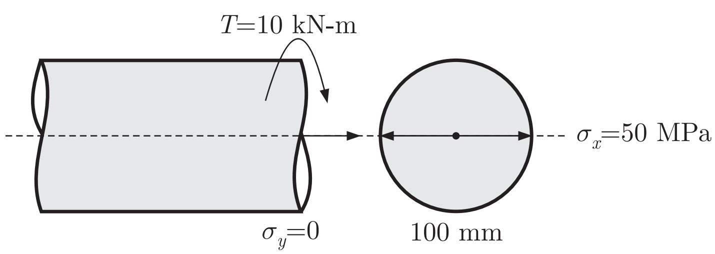
MCQ 1.32 GATE ME 2008 TWO MARK

A solid circular shaft of diameter 100 mm is subjected to an axial stress of 50 MPa. It is further subjected to a torque of 10 kNm. The maximum principal stress experienced on the shaft is closest to

(A) 41 MPa (B) 82 MPa

(C) 164 MPa (D) 204 MPa

SOL 1.32 Option (B) is correct.



The shaft is subjected to a torque of 10 kN-m and due to this shear stress is developed in the shaft,

$$\tau_{xy} = \frac{T}{J} \times r \qquad \text{From Torsional equation}$$

$$\tau_{xy} = \frac{10 \times 10^3}{\frac{\pi}{32} d^4} \times \frac{d}{2} = \frac{16 \times 10 \times 10^3}{\pi d^3}$$

$$\tau_{xy} = \frac{16 \times 10^4}{3.14 \times (10^{-1})^3} = \frac{160}{3.14} = 50.95 \text{ MPa}$$

Maximum principal stress,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

Substitute the values, we get

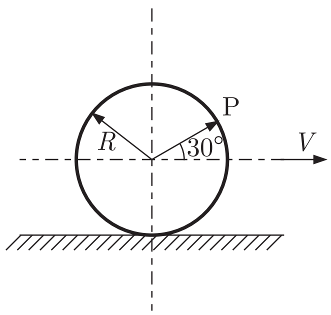
$$\sigma_1 = \frac{50}{2} + \frac{1}{2}\sqrt{(50)^2 + 4 \times (50.95)^2}$$

$$\sigma_1 = 25 + \frac{1}{2}\sqrt{12883.61} = 25 + \frac{113.50}{2}$$

$$\sigma_1 \; 25 + 56.75 = 81.75 \text{ MPa} \simeq 82 \text{ MPa}$$

MCQ 1.33 A circular disk of radius R rolls without slipping at a velocity V. The magnitude of the velocity at point P (see figure) is

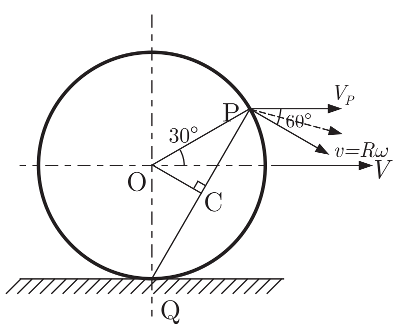
GATE ME 2008
TWO MARK



(A) $\sqrt{3}\, V$
(B) $\sqrt{3}\, V/2$
(C) $V/2$
(D) $2V/\sqrt{3}$

SOL 1.33 Option (A) is correct.

When disc rolling along a straight path, without slipping. The centre of the wheel O moves with some linear velocity and each particle on the wheel rotates with some angular velocity.



Thus, the motion of any particular on the periphery of the wheel is a combination of linear and angular velocity.

Let wheel rotates with angular velocity=ω rad/sec.

So, $$\omega = \frac{V}{R} \qquad \text{....(i)}$$

Velocity at point P is

$$V_P = \omega \times PQ \qquad \text{...(ii)}$$

From triangle OPQ

$$PQ \sqrt{(OQ)^2 + (OP)^2 - 2OQ \times OP \times \cos(\angle POQ)}$$

$$PQ = \sqrt{(R)^2 + (R)^2 - 2RR\cos 120^\circ}$$

$$PQ = \sqrt{(R)^2 + (R)^2 + (R)^2}$$

$$PQ = \sqrt{3}\,R \qquad \text{...(iii)}$$

From equation (i), (ii) and (iii)

$$V_P = \frac{V}{R} \times \sqrt{3}\,R$$

$$V_P = \sqrt{3}\,V$$

MCQ 1.34 Consider a truss PQR loaded at P with a force F as shown in the figure -

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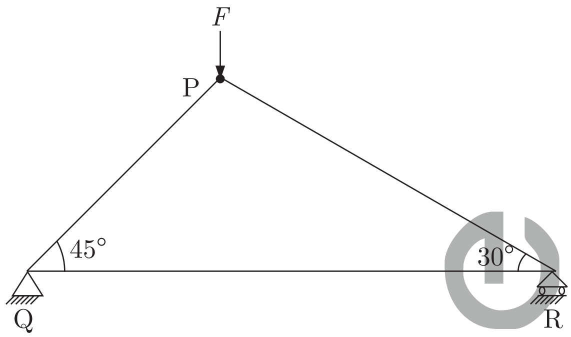
ONE MARK



The tension in the member QR is

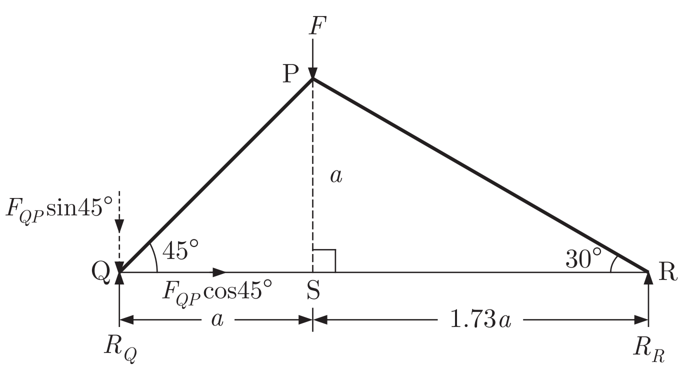
(A) 0.5 F (B) 0.63 F

(C) 0.73 F (D) 0.87 F

SOL 1.34 Option (B) is correct.

The forces which are acting on the truss PQR is shown in figure.

We draw a perpendicular from the point P, that intersects QR at point S.



Let $PS = QS = a$

R_Q & R_R are the reactions acting at point Q & R respectively.

Now from the triangle PRS

$$\tan 30^\circ = \frac{PS}{SR}$$

$$SR = \frac{PS}{\tan 30^\circ} = \frac{a}{\frac{1}{\sqrt{3}}} = \sqrt{3}\,a = 1.73a$$

Taking the moment about point R,

$$R_Q \times (a + 1.73a) = F \times 1.73a$$

$$R_Q = \frac{1.73a\,\text{F}}{2.73a} = \frac{1.73\,\text{F}}{2.73} = 0.634\,\text{F}$$

From equilibrium of the forces, we have

$$R_R + R_Q = F$$

$$R_R = F - R_Q = F - 0.634\,\text{F} = 0.366\,\text{F}$$

To find tension in QR we have to use the method of joint at point Q, and $\Sigma F_y = 0$

$$F_{QP} \sin 45^\circ = R_Q$$

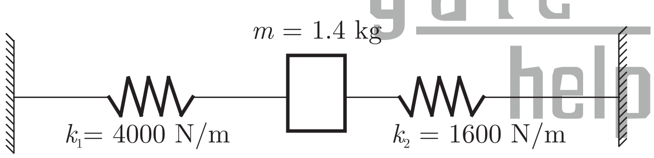
$$F_{QP} = \frac{0.634\,\text{F}}{\frac{1}{\sqrt{2}}} = 0.8966\,\text{F}$$

And, $\Sigma F_x = 0$

$$F_{QP} \cos 45^\circ = F_{QR} \Rightarrow F_{QR} = 0.8966\,\text{F} \times \frac{1}{\sqrt{2}} = 0.634\,\text{F} \simeq 0.63\,\text{F}$$

MCQ 1.35 The natural frequency of the spring mass system shown in the figure is closest to

GATE ME 2008
TWO MARK



(A) 8 Hz (B) 10 Hz

(C) 12 Hz (D) 14 Hz

SOL 1.35 Option (B) is correct.

Given $m = 1.4$ kg, $k_1 = 4000$ N/m, $k_2 = 1600$ N/m

In the given system k_1 & k_2 are in parallel combination

So, $k_{eq} = k_1 + k_2 = 4000 + 1600 = 5600$ N/m

Natural frequency of spring mass system is given by,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m}} = \frac{1}{2\pi}\sqrt{\frac{5600}{1.4}}$$

$$= \frac{1}{2\pi} \times 63.245 = 10.07 \simeq 10\,\text{Hz}$$

MCQ 1.36 The rod PQ of length L and with flexural rigidity EI is hinged at both ends. For what minimum force F is it expected to buckle ?

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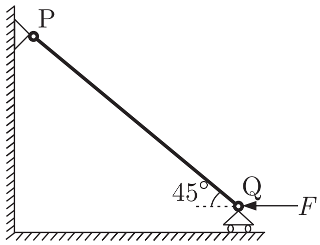
TWO MARK



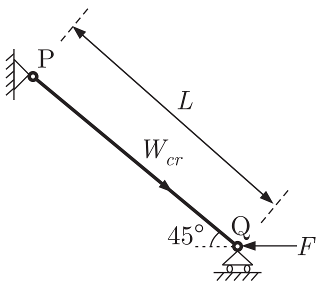
(A) $\frac{\pi^2 EI}{L^2}$

(B) $\frac{\sqrt{2}\,\pi^2 EI}{L^2}$

(C) $\frac{\pi^2 EI}{\sqrt{2}\,L^2}$

(D) $\frac{\pi^2 EI}{2L^2}$

SOL 1.36 Option (B) is correct.



We know that according to Euler's theory, the crippling or buckling load (W_{cr}) under various end conditions is represented by the general equation,

$$W_{cr} = \frac{C\pi^2 EI}{L^2} \qquad ...(i)$$

Where L = length of column

C = Constant, representing the end conditions of the column.

Here both ends are hinged,

$$C = 1$$

From equation (i),

$$W_{cr} = \frac{\pi^2 EI}{L^2}$$

Minimum force F required is

$$W_{cr} = F\cos 45^\circ$$

$$F = \frac{W_{cr}}{\cos 45^\circ} = \frac{\sqrt{2}\,\pi^2 EI}{L^2}$$

MCQ 1.37 GATE ME 2008 TWO MARK

In a cam design, the rise motion is given by a simple harmonic motion $(SHM)\ s = \frac{h}{2}\left(1 - \cos\frac{\pi\theta}{\beta}\right)$ where h is total rise, θ is camshaft angle, β is the total angle of the rise interval . The jerk is given by

(A) $\frac{h}{2}\left(1 - \cos\frac{\pi\theta}{\beta}\right)$

(B) $\frac{\pi}{\beta}\frac{h}{2}\sin\left(\frac{\pi\theta}{\beta}\right)$

(C) $\frac{\pi^2}{\beta^2}\frac{h}{2}\cos\left(\frac{\pi\theta}{\beta}\right)$ (D) $-\frac{\pi^3}{\beta^3}\frac{h}{2}\sin\left(\frac{\pi\theta}{\beta}\right)$

SOL 1.37 Option (D) is correct.

Jerk is given by triple differentiation of s w.r.t. t,

$$\text{Jerk} = \frac{d^3 s}{dt^3}$$

Given
$$s = \frac{h}{2}\left(1 - \cos\frac{\pi\theta}{\beta}\right) = \frac{h}{2}\left[1 - \cos\frac{\pi(\omega t)}{\beta}\right] \qquad \theta = \omega t$$

Differentiating above equation w.r.t. t, we get

$$\frac{ds}{dt} = \frac{h}{2}\left[-\frac{\pi\omega}{\beta}\left\{-\sin\frac{\pi(\omega t)}{\beta}\right\}\right]$$

Again Differentiating w.r.t. t,

$$\frac{d^2 s}{dt^2} = \frac{h}{2}\frac{\pi^2\omega^2}{\beta^2}\left[\cos\frac{\pi(\omega t)}{\beta}\right]$$

Again Differentiating w.r.t. t,

$$\frac{d^3 s}{dt^3} = -\frac{h}{2}\frac{\pi^3\omega^3}{\beta^3}\sin\frac{\pi\theta}{\beta}$$

Let $\omega = 1\,\text{rad/sec}$

$$\frac{d^3 s}{dt^3} = -\frac{h}{2}\frac{\pi^3}{\beta^3}\sin\left(\frac{\pi\theta}{\beta}\right)$$

MCQ 1.38 A uniform rigid rod of mass $m = 1$ kg and length $L = 1$ m is hinged at its centre and laterally supported at one end by a spring of spring constant $k = 300\,\text{N/m}$. The natural frequency ω_n in rad/s is

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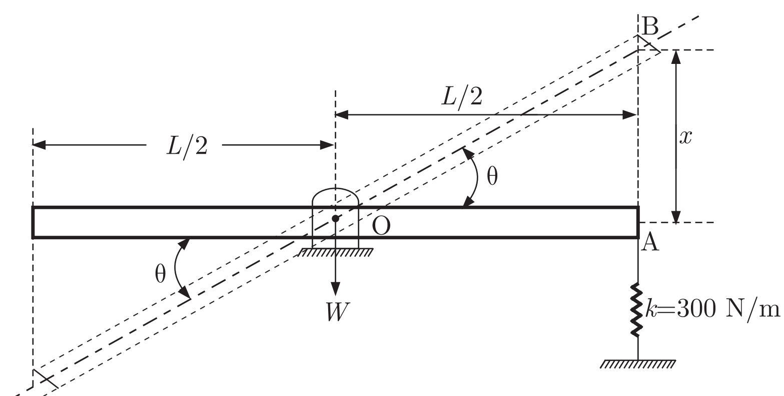
TWO MARK

(A) 10 (B) 20

(C) 30 (D) 40

SOL 1.38 Option (C) is correct.



Given $m = 1\,\text{kg},\ L = 1\,\text{m},\ k = 300\,\text{N/m}$

We have to turn the rigid rod at an angle θ about its hinged point, then rod moves upward at a distance x and also deflect in the opposite direction with the same amount. Let θ is very very small and take $\tan\theta \simeq \theta$

From ΔAOB, $\theta = \frac{x}{\frac{L}{2}} \Rightarrow x = \frac{L}{2}\theta$ ...(i)

& $\theta = \omega t \Rightarrow \dot{\theta} = \omega$ ...(ii)

By using the principal of energy conservation,

$$\frac{1}{2}I\omega^2 + \frac{1}{2}kx^2 = \text{Constant}$$

$$\frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}k\left(\frac{L}{2}\theta\right)^2 = c \qquad \text{From equation (i) and (ii)}$$

$$\frac{1}{2}I\dot{\theta}^2 + \frac{1}{8}L^2k\theta^2 = c$$

On differentiating w.r.t. t, we get

$$\frac{1}{2}I \times 2\dot{\theta}\ddot{\theta} + \frac{kL^2}{8} \times 2\theta\dot{\theta} = 0 \qquad \text{...(iii)}$$

For a rigid rod of length L & mass m, hinged at its centre, the moment of inertia,

$$I = \frac{mL^2}{12}$$

Substitute I in equation (iii), we get

$$\frac{1}{2} \times \frac{mL^2}{12} \times 2\dot{\theta}\ddot{\theta} + \frac{kL^2}{4}\theta\dot{\theta} = 0$$

$$\ddot{\theta} + \frac{3k}{m}\theta = 0 \qquad \text{...(iv)}$$

Compare equation (iv) with the general equation,

$$\ddot{\theta} + \omega_n^2\theta = 0$$

So, we have $\omega_n^2 = \frac{3k}{m}$

$$\omega_n = \sqrt{\frac{3k}{m}} = \sqrt{\frac{3 \times 300}{1}} = 30 \text{ rad/sec}$$

MCQ 1.39 GATE ME 2008 TWO MARK

A compression spring is made of music wire of 2 mm diameter having a shear strength and shear modulus of 800 MPa and 80 GPa respectively. The mean coil diameter is 20 mm, free length is 40 mm and the number of active coils is 10. If the mean coil diameter is reduced to 10 mm, the stiffness of the spring is approximately

(A) decreased by 8 times (B) decreased by 2 times

(C) increased by 2 times (D) increased by 8 times

SOL 1.39 Option (D) is correct.

We know that deflection in a compression spring is given by

$$\delta = \frac{64PR^3n}{d^4G} = \frac{8PD^3n}{d^4G}$$

Where n = number of active coils

D = Mean coil Diameter

d = Music wire Diameter

And $$k = \frac{P}{\delta} = \frac{d^4 G}{8D^3 n}$$

$$k \propto \frac{1}{D^3}$$

Given that mean coil diameter is reduced to 10 mm.

So, $D_1 = 20\text{ mm}$

$D_2 = 20 - 10 = 10\text{ mm}$

& $$\frac{k_2}{k_1} = \left(\frac{D_1}{D_2}\right)^3 = \left(\frac{20}{10}\right)^3 = 8$$

$$k_2 = 8k_1$$

So, stiffness is increased by 8 times.

MCQ 1.40 GATE ME 2008 TWO MARK

A journal bearing has a shaft diameter of 40 mm and a length of 40 mm. The shaft is rotating at 20 rad/s and the viscosity of the lubricant is 20 mPa-s. The clearance is 0.020 mm. The loss of torque due to the viscosity of the lubricant is approximately.

(A) 0.040 N-m (B) 0.252 N-m

(C) 0.400 N-m (D) 0.652 N-m

SOL 1.40

Option (A) is correct.

Given : $d = 40\text{ mm}$, $l = 40\text{ mm}$, $\omega = 20\text{ rad/sec}$

$Z(\mu) = 20\text{ mPa-s} = 20 \times 10^{-3}\text{ Pa-s}$, $c(y) = 0.020\text{ mm}$

Shear stress, $\tau = \mu \frac{u}{y}$ From the Newton's law of viscosity..(i)

$$u = r\omega = 0.020 \times 20 = 0.4\text{ m/sec}$$

$$\tau = \frac{20 \times 10^{-3} \times 0.4}{0.020 \times 10^{-3}} = 400\text{ N/m}^2$$

Shear force is generated due to this shear stress,

$$F = \tau A = \tau \times \pi dl \qquad A = \pi dl = \text{Area of shaft}$$

$$= 400 \times 3.14 \times 0.040 \times 0.040 = 2.0096\text{ N}$$

Loss of torque, $$T = F \times r = 2.0096 \times 0.020$$

$$= 0.040192\text{ N-m} \simeq 0.040\text{ N-m}$$

MCQ 1.41 GATE ME 2008 TWO MARK

A clutch has outer and inner diameters 100 mm and 40 mm respectively. Assuming a uniform pressure of 2 MPa and coefficient of friction of liner material is 0.4, the torque carrying capacity of the clutch is

(A) 148 N-m (B) 196 N-m

(C) 372 N-m (D) 490 N-m

SOL 1.41

Option (B) is correct.

Given : $d_1 = 100\text{ mm} \Rightarrow r_1 = 50\text{ mm}$, $d_2 = 40\text{ mm} \Rightarrow r_1 = 20\text{ mm}$

$p = 2\text{ MPa} = 2 \times 10^6\text{ Pa}$, $\mu = 0.4$

When the pressure is uniformly distributed over the entire area of the friction faces, then total frictional torque acting on the friction surface or on the clutch is given by,

$$T = 2\pi\mu p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times 3.14 \times 0.4 \times 2 \times 10^6[(50)^3 - (20)^3] \times 10^{-9}$$

$$= 195.39 \text{ N-m} \simeq 196 \text{ N-m}$$

MCQ 1.42 GATE ME 2008 TWO MARK

A spur gear has a module of 3 mm, number of teeth 16, a face width of 36 mm and a pressure angle of 20°. It is transmitting a power of 3 kW at 20 rev/s. Taking a velocity factor of 1.5 and a form factor of 0.3, the stress in the gear tooth is about.

(A) 32 MPa (B) 46 MPa

(C) 58 MPa (D) 70 MPa

SOL 1.42 Option (B) is correct.

Given : $m = 3\text{ mm},\ Z = 16,\ b = 36\text{ mm},\ \phi = 20^\circ,\ P = 3\text{ kW}$

$N = 20\text{ rev/sec} = 20 \times 60\text{ rpm} = 1200\text{ rpm},\ C_v = 1.5,\ y = 0.3$

Module, $$m = \frac{D}{Z}$$

$$D = m \times Z = 3 \times 16 = 48\text{ mm}$$

Power, $$P = \frac{2\pi NT}{60}$$

$$T = \frac{60P}{2\pi N} = \frac{60 \times 3 \times 10^3}{2 \times 3.14 \times 1200}$$

$$= 23.88\text{ N-m} = 23.88 \times 10^3\text{ N-mm}$$

Tangential load, $$W_T = \frac{T}{R} = \frac{2T}{D} = \frac{2 \times 23.88 \times 10^3}{48} = 995\text{ N}$$

From the lewis equation Bending stress (Beam strength of Gear teeth)

$$\sigma_b = \frac{W_T P_d}{by}$$

$$\sigma_b = \frac{W_T}{bym} \qquad \left[P_d = \frac{\pi}{P_C} = \frac{\pi}{\pi m} = \frac{1}{m}\right]$$

$$= \frac{995}{36 \times 10^{-3} \times 0.3 \times 3 \times 10^{-3}}$$

$$\sigma_b = \frac{995}{3.24 \times 10^{-5}} = 30.70 \times 10^6\text{ Pa} = 30.70\text{ MPa}$$

Permissible working stress

$$\sigma_W = \sigma_b \times C_v$$

$$= 30.70 \times 1.5 = 46.06\text{ MPa} \cong 46\text{ MPa}$$

MCQ 1.43 GATE ME 2008 TWO MARK

Match the type of gears with their most appropriate description.

Type of gear

P. Helical

Q. Spiral Bevel

C. Hypoid

S. Rack and pinion

Description

1. Axes non parallel and non intersecting
2. Axes parallel and teeth are inclined to the axis
3. Axes parallel and teeth are parallel to the axis
4. Axes are perpendicular and intersecting, and teeth are inclined to the axis.
5. Axes are perpendicular and used for large speed reduction
6. Axes parallel and one of the gears has infinite radius

(A) P-2, Q-4, R-1, S-6 (B) P-1, Q-4, R-5, S-6

(C) P-2, Q-6, R-4, S-2 (D) P-6, Q-3, R-1, S-5

SOL 1.43 Option (A) is correct.

	Types of Gear		Description
P.	Helical	**2.**	Axes parallel and teeth are inclined to the axis
Q.	Spiral Bevel	**4.**	Axes are perpendicular and intersecting, and teeth are inclined to the axis
R.	Hypoid	**1.**	Axes non parallel and non-intersecting
S.	Rack and pinion	**6.**	Axes are parallel and one of the gear has infinite radius

So, correct pairs are P-2, Q-4, R-1, S-6

MCQ 1.44 GATE ME 2008 TWO MARK

A gas expands in a frictionless piston-cylinder arrangement. The expansion process is very slow, and is resisted by an ambient pressure of 100 kPa. During the expansion process, the pressure of the system (gas) remains constant at 300 kPa. The change in volume of the gas is 0.01 m^3. The maximum amount of work that could be utilized from the above process is

(A) 0 kJ (B) 1 kJ

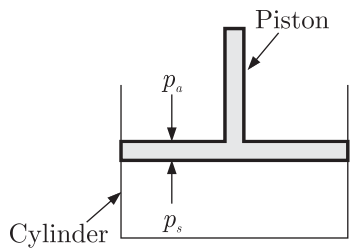
(C) 2 kJ (D) 3 kJ

SOL 1.44 Option (C) is correct.

Given : $p_a = 100\text{ kPa}$, $p_s = 300\text{ kPa}$, $\Delta\nu = 0.01\text{ m}^3$

Net pressure work on the system,

$$p = p_s - p_a = 300 - 100 = 200\text{ kPa}$$



For constant pressure process work done is given by

$$W = p\Delta\nu = 200 \times 0.01 = 2\,\text{kJ}$$

MCQ 1.45 GATE ME 2008 TWO MARK

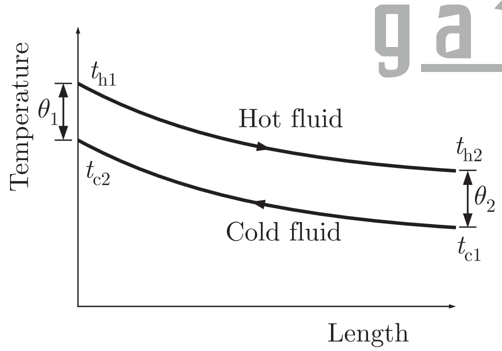
The logarithmic mean temperature difference (LMTD) of a counter flow heat exchanger is 20° C. The cold fluid enters at 20° C and the hot fluid enters at 100° C. Mass flow rate of the cold fluid is twice that of the hot fluid. Specific heat at constant pressure of the hot fluid is twice that of the cold fluid. The exit temperature of the cold fluid

(A) is 40° C
(B) is 60° C
(C) is 80° C
(D) cannot be determined

SOL 1.45

Option (C) is correct.

The $T-L$ curve shows the counter flow.



Given : $\theta_m = 20°\,C$, $t_{c1} = 20°\,C$, $t_{h1} = 100°\,C$

$$\dot{m}_c = 2\dot{m}_h \Rightarrow \frac{\dot{m}_c}{\dot{m}_h} = 2 \quad \text{...(i)}$$

$$c_{ph} = 2c_{pc} \Rightarrow \frac{c_{ph}}{c_{pc}} = 2 \quad \text{...(ii)}$$

Energy balance for counter flow is,

Heat lost by hot fluid = Heat gain by cold fluid

$$\dot{m}_h c_{ph}(t_{h1} - t_{h2}) = \dot{m}_c c_{pc}(t_{c2} - t_{c1})$$

$$\frac{c_{ph}}{c_{pc}}(t_{h1} - t_{h2}) = \frac{\dot{m}_c}{\dot{m}_h}(t_{c2} - t_{c1})$$

$$2(t_{h1} - t_{h2}) = 2(t_{c2} - t_{c1})$$

$$t_{h1} - t_{c2} = t_{h2} - t_{c1}$$

$$\theta_1 = \theta_2 \quad \text{...(iii)}$$

And $$\theta_m = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)} \quad ...(iv)$$

On substituting the equation (iii) in equation (iv), we get undetermined form.

Let $$\frac{\theta_1}{\theta_2} = x, \Rightarrow \theta_1 = \theta_2 x \quad ...(v)$$

Substitute θ_1 in equation(iv),

$$\theta_m = \lim_{x \to 1} \frac{\theta_2 x - \theta_2}{\ln\left(\frac{\theta_2 x}{\theta_2}\right)} = \lim_{x \to 1} \frac{\theta_2(x-1)}{\ln x} \quad ...(vi)$$

$\left[\frac{0}{0}\right]$ form, So we apply L-Hospital rule,

$$\theta_m = \lim_{x \to 1} \frac{\theta_2(1-0)}{\frac{1}{x}} = \lim_{x \to 1} x\theta_2$$

$$\theta_m = \theta_2 = \theta_1 \quad \text{From equation(iii)}$$

Now we have to find exit temperature of cold fluid (t_{c2}),

So, $$\theta_m = \theta_1 = t_{h1} - t_{c2}$$

$$t_{c2} = t_{h1} - \theta_m = 100 - 20 = 80^\circ \text{C}$$

MCQ 1.46 GATE ME 2008 TWO MARK

A two dimensional fluid element rotates like a rigid body. At a point within the element, the pressure is 1 unit. Radius of the Mohr's circle, characterizing the state of stress at that point, is

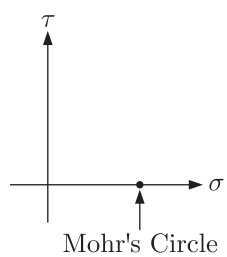
(A) 0.5 unit (B) 0 unit

(C) 1 unit (D) 2 unit

SOL 1.46 Option (B) is correct.

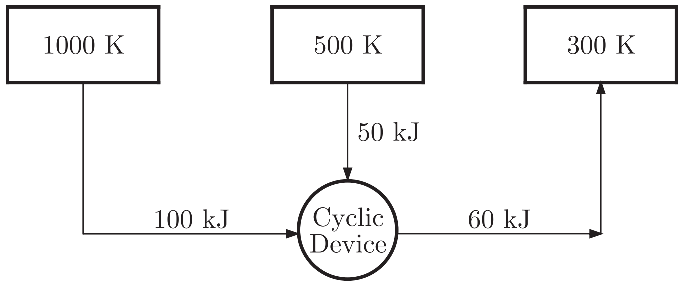
Pressure will remain uniform in all directions. So, hydrostatic load acts in all directions on the fluid element and Mohr's circle becomes a point on $\sigma - \tau$ axis and $\sigma_x = \sigma_y$ and $\tau_{xy} = 0$

So, $$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} = 0$$



MCQ 1.47 GATE ME 2008 TWO MARK

A cyclic device operates between three reservoirs, as shown in the figure. Heat is transferred to/from the cycle device. It is assumed that heat transfer between each thermal reservoir and the cyclic device takes place across negligible temperature difference. Interactions between the cyclic device and the respective thermal reservoirs that are shown in the figure are all in the form of heat transfer.



The cyclic device can be

(A) a reversible heat engine

(B) a reversible heat pump or a reversible refrigerator

(C) an irreversible heat engine

(D) an irreversible heat pump or an irreversible refrigerator

SOL 1.47 Option (A) is correct.

A heat engine cycle is a thermodynamic cycle in which there is a net Heat transfer from higher temperature to a lower temperature device. So it is a Heat Engine.

Applying Clausius theorem on the system for checking the reversibility of the cyclic device.

$$\oint_R \frac{dQ}{T} = 0$$

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} - \frac{Q_3}{T_3} = 0$$

$$\frac{100 \times 10^3}{1000} + \frac{50 \times 10^3}{500} - \frac{60 \times 10^3}{300} = 0$$

$$100 + 100 - 200 = 0$$

Here, the cyclic integral of dQ/T is zero. This implies, it is a reversible Heat engine.

MCQ 1.48 GATE ME 2008 TWO MARK

A balloon containing an ideal gas is initially kept in an evacuated and insulated room. The balloon ruptures and the gas fills up the entire room. Which one of the following statements is TRUE at the end of above process ?

(A) The internal energy of the gas decreases from its initial value, but the enthalpy remains constant

(B) The internal energy of the gas increases from its initial value, but the enthalpy remains constant

(C) Both internal energy and enthalpy of the gas remain constant

(D) Both internal energy and enthalpy of the gas increase

SOL 1.48 Option (C) is correct.

We know enthalpy,

$$h = U + p\nu \qquad \text{...(i)}$$

Where, U = Internal energy

$$p = \text{Pressure of the room}$$
$$\nu = \text{Volume of the room}$$

It is given that room is insulated, So there is no interaction of energy (Heat) between system (room) and surrounding (atmosphere).
It means Change in internal Energy $dU = 0$ & $U =$ Constant
And temperature is also remains constant.
Applying the perfect gas equation,

$$p\nu = nRT$$
$$p\nu = \text{Constant}$$

Therefore, from equation (i)

$$h = \text{Constant}$$

So this process is a constant internal energy & constant enthalpy process.

Alternate method

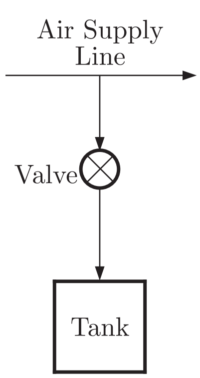
We know that enthalpy,

$$h = U + p\nu$$

Given that room is insulated, So there is no interaction of Energy (Heat) between system (room) and surrounding (atmosphere).
It means internal Energy $dU = 0$ & $U =$ constant.
Now flow work $p\nu$ must also remain constant thus we may conclude that during free expansion process $p\nu$ i.e. product of pressure and specific volume change in such a way that their product remains constant.
So, it is a constant internal energy & constant enthalpy process.

MCQ 1.49 GATE ME 2008 TWO MARK

A rigid, insulated tank is initially evacuated. The tank is connected with a supply line through which air (assumed to be ideal gas with constant specific heats) passes at 1 MPa, 350° C. A valve connected with the supply line is opened and the tank is charged with air until the final pressure inside the tank reaches 1 MPa. The final temperature inside the tank.



(A) is greater than 350° C

(B) is less than 350° C

(C) is equal to 350° C

(D) may be greater than, less than, or equal to, 350° C depending on the volume of the tank

SOL 1.49 Option (A) is correct.

Given : $p_1 = 1\,\text{MPa}$, $T_1 = 350^\circ\text{C} = (350 + 273)\,\text{K} = 623\,\text{K}$

For air $\gamma = 1.4$

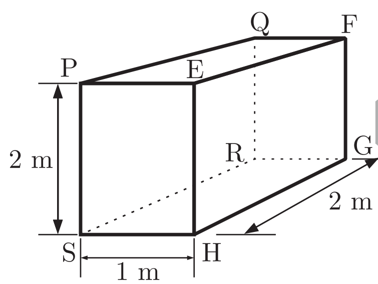
We know that final temperature (T_2) inside the tank is given by,

$$T_2 = \gamma T_1 = 1.4 \times 623 = 872.2\,\text{K} = 599.2^\circ\,\text{C}$$

T_2 is greater than 350° C.

MCQ 1.50 GATE ME 2008 TWO MARK

For the three-dimensional object shown in the figure below, five faces are insulated. The sixth face (PQRS), which is not insulated, interacts thermally with the ambient, with a convective heat transfer coefficient of $10\,\text{W/m}^2\,\text{K}$. The ambient temperature is 30° C. Heat is uniformly generated inside the object at the rate of $100\,\text{W/m}^3$. Assuming the face PQRS to be at uniform temperature, its steady state temperature is



(A) 10° C

(B) 20° C

(C) 30° C

(D) 40° C

SOL 1.50 Option (D) is correct.

Given : $h = 10\,\text{W/m}^2\,\text{K}$, $T_i = 30^\circ\,\text{C}$, $q_g = 100\,\text{W/m}^3$

Five faces of the object are insulated, So no heat transfer or heat generation by these five faces. Only sixth face (PQRS) interacts with the surrounding & generates heat.

Hence, Heat generated throughout the volume

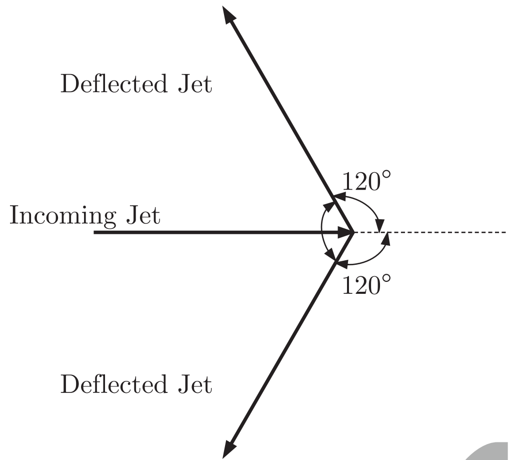
$$Q = \text{Rate of heat Generated} \times \text{Volume of object}$$
$$= 100 \times (1 \times 2 \times 2) = 400\,\text{W}$$

And heat transfer by convection is given by

$$Q = hA(T_f - T_i)$$
$$400 = 10 \times (2 \times 2)(T_f - 30)$$
$$T_f = 30 + 10 = 40^\circ\,\text{C}$$

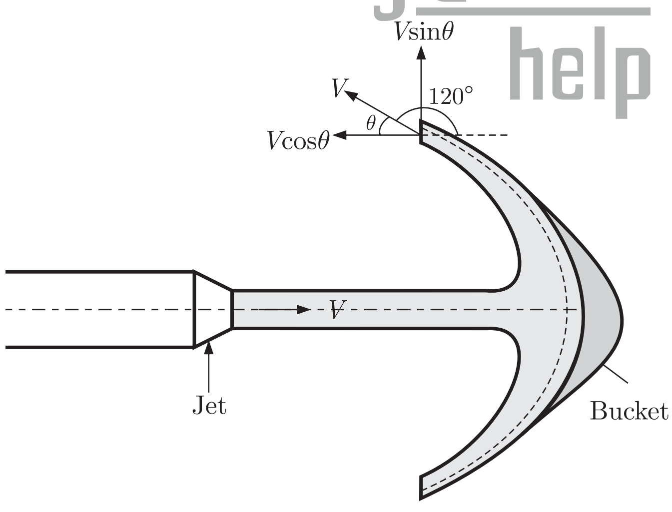
MCQ 1.51 GATE ME 2008 TWO MARK

Water, having a density of $1000\,\text{kg/m}^3$, issues from a nozzle with a velocity of $10\,\text{m/s}$ and the jet strikes a bucket mounted on a Pelton wheel. The wheel rotates at 10 rad/s. The mean diameter of the wheel is 1 m. The jet is split into two equal streams by the bucket, such that each stream is deflected by 120° as shown in the figure. Friction in the bucket may be neglected. Magnitude of the torque exerted by the water on the wheel, per unit mass flow rate of the incoming jet, is



(A) 0 (N-m)/(kg/s)
(B) 1.25 (N-m)/(kg/s)
(C) 2.5 (N-m)/(kg/s)
(D) 3.75 (N-m)/(kg/s)

SOL 1.51 None of these is correct.



Given : $\rho = 1000\,\text{kg/m}^3$, $V = 10$ m/sec, $\theta = 180 - 120 = 60^\circ$, $R = 0.5$ m
Initial velocity in the direction of jet $= V$
Final velocity in the direction of the jet $= -V\cos\theta$.
Force exerted on the bucket

$$F_x = \rho A V[V - (-V\cos\theta)] = \rho A V[1 + \cos\theta] V$$
$$= Q(1 + \cos\theta)\, V \qquad \text{Mass flow rate } Q = \rho A V$$

Torque, $T_x = F_x \times R = QV(1 + \cos\theta) R$

Torque per unit mass flow rate

$$\frac{T_x}{Q} = V(1+\cos\theta)R = 10(1+\cos 60°) \times 0.5 = 7.5 \text{ N-m/kg/sec}$$

And $$F_y = \rho A V(0 - V\sin\theta) = - QV\sin\theta$$

Torque in y-direction

$$T_y = F_y \times R = 0 \qquad R = 0$$

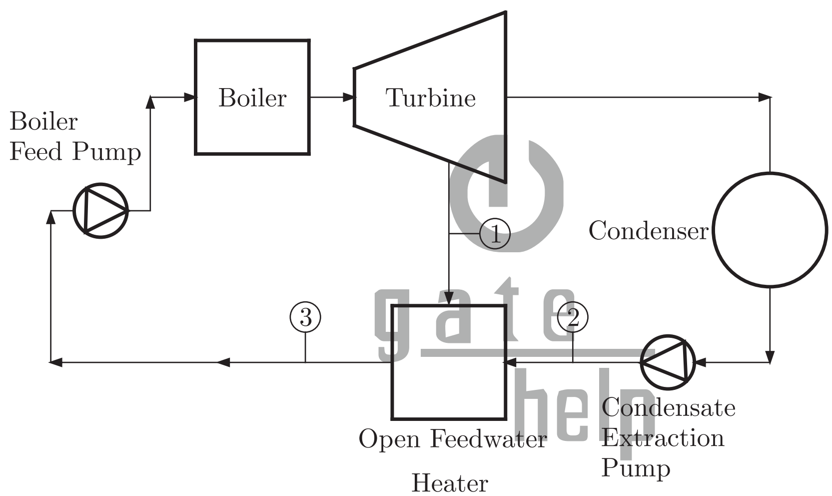
Total Torque will be

$$T = \sqrt{T_x^2 + T_y^2} = T_x = 7.5 \text{ N-m/kg/sec}$$

MCQ 1.52

GATE ME 2008
TWO MARK

A thermal power plant operates on a regenerative cycle with a single open feed water heater, as shown in the figure. For the state points shown, the specific enthalpies are: $h_1 = 2800$ kJ/kg and $h_2 = 200$ kJ/kg. The bleed to the feed water heater is 20% of the boiler steam generation rate. The specific enthalpy at state 3 is



(A) 720 kJ/kg (B) 2280 kJ/kg

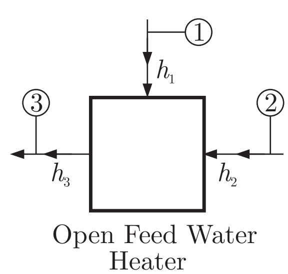
(C) 1500 kJ/kg (D) 3000 kJ/kg

SOL 1.52

Option (A) is correct.

Given : $h_1 = 2800$ kJ/kg, $h_2 = 200$ kJ/kg

From the given diagram of thermal power plant, point 1 is directed by the Boiler to the open feed water heater & point 2 is directed by the pump to the open feed water Heater. The bleed to the feed water heater is 20% of the boiler steam generation i.e. 20% of h_1



So, $$h_3 = 20\% \text{ of } h_1 + 80\% \text{ of } h_2$$

$$= 0.2 \times 2800 + 0.8 \times 200 = 720 \text{ kJ/kg}$$

MCQ 1.53 GATE ME 2008 TWO MARK

Moist air at a pressure of 100 kPa is compressed to 500 kPa and then cooled to 35° C in an aftercooler. The air at the entry to the aftercooler is unsaturated and becomes just saturated at the exit of the aftercooler. The saturation pressure of water at 35° C is 5.628 kPa. The partial pressure of water vapour (in kPa) in the moist air entering the compressor is closest to

(A) 0.57 (B) 1.13

(C) 2.26 (D) 4.52

SOL 1.53 Option (B) is correct.

Given : $p_1 = 100$ kPa, $p_2 = 500$ kPa, $p_{v1} = ?$

$p_{v2} = 5.628$ kPa (Saturated pressure at 35° C)

We know that,

Specific humidity $$W = 0.622\left(\frac{p_v}{p - p_v}\right)$$

For case II :

$$W = 0.622\left(\frac{5.628}{500 - 5.628}\right) = 7.08 \times 10^{-3} \text{ kg/kg of dry air}$$

For saturated air specific humidity remains same. So, for case (I) :

$$W = 0.622\left(\frac{p_{v1}}{p_1 - p_{v1}}\right)$$

On substituting the values, we get

$$7.08 \times 10^{-3} = 0.622\left(\frac{p_{v1}}{100 - p_{v1}}\right)$$

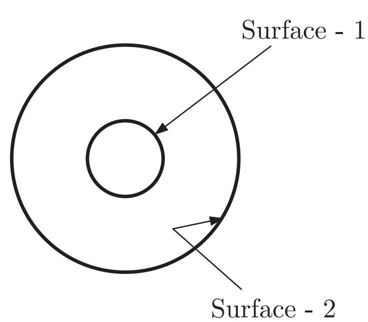
$$11.38 \times 10^{-3}(100 - p_{v1}) = p_{v1}$$

$$1.138 = 1.01138 p_{v1}$$

$$p_{v1} = 1.125 \text{ kPa} \simeq 1.13 \text{ kPa}$$

MCQ 1.54 GATE ME 2008 TWO MARK

A hollow enclosure is formed between two infinitely long concentric cylinders of radii 1 m and 2 m, respectively. Radiative heat exchange takes place between the inner surface of the larger cylinder (surface-2) and the outer surface of the smaller cylinder (surface-1). The radiating surfaces are diffuse and the medium in the enclosure is non-participating. The fraction of the thermal radiation leaving the larger surface and striking itself is



(A) 0.25 (B) 0.5

(C) 0.75 (D) 1

SOL 1.54 Option (B) is correct.

Given : $D_1 = 1\,\text{m},\ D_2 = 2\,\text{m}$

Hence, the small cylindrical surface (surface 1) cannot see itself and the radiation emitted by this surface strikes on the enclosing surface 2. From the conservation principal (summation rule).

For surface 1, $F_{12} + F_{11} = 1$ $\quad F_{11} = 0$

$$F_{12} = 1 \qquad ...(i)$$

From the reciprocity theorem

$$A_1 F_{12} = A_2 F_{21}$$

$$F_{21} = \frac{A_1}{A_2} = \frac{\pi D_1 L}{\pi D_2 L} = \frac{D_1}{D_2} = \frac{1}{2} = 0.5$$

and from the conservation principal, for surface 2, we have

$$F_{21} + F_{22} = 1$$

$$F_{22} = 1 - F_{21} = 1 - 0.5 = 0.5$$

So, the fraction of the thermal radiation leaves the larger surface & striking itself is $F_{22} = 0.5$.

MCQ 1.55 GATE ME 2008 TWO MARK

Air (at atmospheric pressure) at a dry bulb temperature of 40°C and wet bulb temperature of 20°C is humidified in an air washer operating with continuous water recirculation. The wet bulb depression (i.e. the difference between the dry and wet bulb temperature) at the exit is 25% of that at the inlet. The dry bulb temperature at the exit of the air washer is closest to

(A) 10°C (B) 20°C

(C) 25°C (D) 30°C

SOL 1.55 Option (C) is correct.

Given : At inlet $t_{DBT} = 40^\circ\ \text{C}$, $t_{WBT} = 20^\circ\ \text{C}$

We know that, wet bulb depression $= t_{DBT} - t_{WBT} = 40 - 20 = 20^\circ\ \text{C}$

And given wet bulb depression at the exit = 25% of wet bulb depression at inlet

This process becomes adiabatic saturation and for this process,

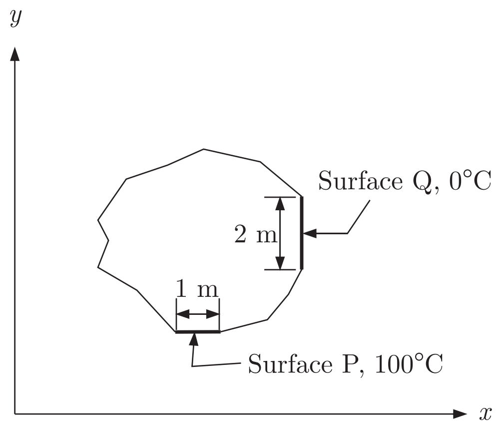
$$t_{WBT(\text{inlet})} = t_{WBT(\text{outlet})}$$

So,
$$t_{DBT(\text{exit})} - 20 = 0.25 \times 20$$

$$t_{DBT(\text{exit})} = 20 + 5 = 25^\circ\ \text{C}$$

MCQ 1.56 GATE ME 2008 TWO MARK

Steady two-dimensional heat conduction takes place in the body shown in the figure below. The normal temperature gradients over surfaces P and Q can be considered to be uniform. The temperature gradient $\partial T/\partial x$ at surface Q is equal to $10\,\text{K/m}$. Surfaces P and Q are maintained at constant temperature as shown in the figure, while the remaining part of the boundary is insulated. The body has a constant thermal conductivity of $0.1\,\text{W/mK}$. The values of $\frac{\partial T}{\partial x}$ and $\frac{\partial T}{\partial y}$ at surface P are



(A) $\frac{\partial T}{\partial x} = 20\,\text{K/m}, \quad \frac{\partial T}{\partial y} = 0\,\text{K/m}$
(B) $\frac{\partial T}{\partial x} = 0\,\text{K/m}, \quad \frac{\partial T}{\partial y} = 10\,\text{K/m}$

(C) $\frac{\partial T}{\partial x} = 10\,\text{K/m}, \quad \frac{\partial T}{\partial y} = 10\,\text{K/m}$
(D) $\frac{\partial T}{\partial x} = 0\,\text{K/m}, \quad \frac{\partial T}{\partial y} = 20\,\text{K/m}$

SOL 1.56 Option (D) is correct.

Given : $\left(\frac{\partial T}{\partial x}\right)_Q = 10\,\text{K/m}$, $(T)_P = (T)_Q$, $(k)_P = (k)_Q = 0.1\,\text{W/mK}$

Direction of heat flow is always normal to surface of constant temperature.
So, for surface P,

$$\frac{\partial T}{\partial x} = 0$$

Because, $Q = -kA\frac{\partial T}{\partial x}$ and ∂T is the temperature difference for a short perpendicular distance dx. Let width of both the bodies are unity.

From the law of energy conservation,

$$\text{Heat rate at } P = \text{Heat rate at } Q$$

$$-0.1 \times 1 \times \left(\frac{\partial T}{\partial y}\right)_P = -0.1 \times 2 \times \left(\frac{\partial T}{\partial x}\right)_Q$$

Because for P heat flow in y direction & for Q heat flow in x direction

$$\left(\frac{\partial T}{\partial y}\right)_P = \frac{0.1 \times 2 \times 10}{0.1} = 20\,\text{K/m}$$

MCQ 1.57 GATE ME 2008 TWO MARK

In a steady state flow process taking place in a device with a single inlet and a single outlet, the work done per unit mass flow rate is given by $W = -\int_{\text{inlet}}^{\text{outlet}} \nu dp$, where ν is the specific volume and p is the pressure.

The expression for W given above

(A) is valid only if the process is both reversible and adiabatic

(B) is valid only if the process is both reversible and isothermal

(C) is valid for any reversible process

(D) is incorrect; it must be $W = \int_{\text{inlet}}^{\text{outlet}} p d\nu$

SOL 1.57 Option (C) is correct.

From the first law of thermodynamic,

$$dQ = dU + dW$$

$$dW = dQ - dU \qquad \text{...(i)}$$

If the process is complete at the constant pressure & no work is done other than the $pd\nu$ work. So

$$dQ = dU + pd\nu$$

At constant pressure

$$pd\nu = d(p\nu)$$

$$(dQ) = dU + d(p\nu) = d(U + p\nu) = (dh) \qquad h = U + p\nu$$

From equation (i)

$$dW = -dh + dQ = -dh + Tds \qquad ds = dQ/T \qquad \text{...(ii)}$$

For an reversible process,

$$Tds = dh - \nu dp$$

$$-\nu dp = -dh + Tds \qquad \text{...(iii)}$$

From equation (ii) & (iii)

$$dW = -\nu dp$$

On integrating both sides, we get

$$W = -\int \nu dp$$

It is valid for reversible process.

MCQ 1.58 For the standard transportation linear programme with m source and n destinations and total supply equaling total demand, an optimal solution (lowest cost) with the smallest number of non-zero x_{ij} values (amounts from source i to destination j) is desired. The best upper bound for this number is

GATE ME 2008 TWO MARK

(A) mn

(B) $2(m + n)$

(C) $m + n$

(D) $m + n - 1$

SOL 1.58 Option (D) is correct.

In a transportation problem with m origins and n destinations, if a basic feasible solution has less than $m + n - 1$ allocations (occupied cells), the problem is said to be a degenerate transportation problem.

So, the basic condition for the solution to be optimal without degeneracy is.

Number of allocations $= m + n - 1$

MCQ 1.59 A moving average system is used for forecasting weekly demand $F_1(t)$ and $F_2(t)$ are sequences of forecasts with parameters m_1 and m_2, respectively, where m_1 and $m_2 (m_1 > m_2)$ denote the numbers of weeks over which the moving averages are taken. The actual demand shows a step increase from d_1 to d_2 at a certain time. Subsequently,

GATE ME 2008 TWO MARK

(A) neither $F_1(t)$ nor $F_2(t)$ will catch up with the value d_2

(B) both sequences $F_1(t)$ and $F_2(t)$ will reach d_2 in the same period

(C) $F_1(t)$ will attain the value d_2 before $F_2(t)$

(D) $F_2(t)$ will attain the value d_2 before $F_1(t)$

SOL 1.59 Option (D) is correct.

Here $F_1(t)$ & $F_2(t)$ = Forecastings

m_1 & m_2 = Number of weeks

A higher value of m results in better smoothing. Since here $m_1 > m_2$ the weightage of the latest demand would be more in $F_2(t)$.

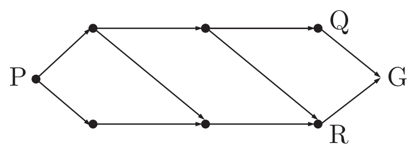
Hence, $F_2(t)$ will attain the value of d_2 before $F_1(t)$.

MCQ 1.60 GATE ME 2008 TWO MARK

For the network below, the objective is to find the length of the shortest path from node P to node G.

Let d_{ij} be the length of directed arc from node i to node j.

Let S_j be the length of the shortest path from P to node j. Which of the following equations can be used to find S_G ?



(A) $S_G = \text{Min}\{S_Q, S_R\}$

(B) $S_G = \text{Min}\{S_Q - d_{QG}, S_R - d_{RG}\}$

(C) $S_G = \text{Min}\{S_Q + d_{QG}, S_R + d_{RG}\}$

(D) $S_G = \text{Min}\{d_{QG}, d_{RG}\}$

SOL 1.60 Option (C) is correct.

There are two paths to reach from node P to node G.

(i) Path P-Q-G (ii) Path P-R-G

For Path P-Q-G,

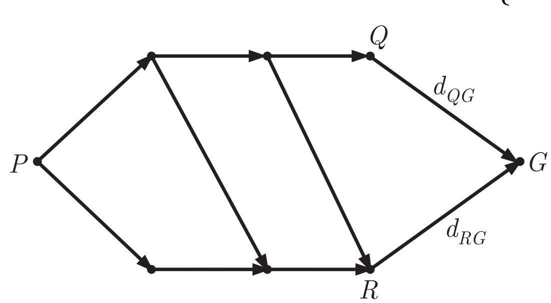
Length of the path $S_G = S_Q + d_{QG}$

For path P-R-G,

Length of the path $S_G = S_R + d_{RG}$

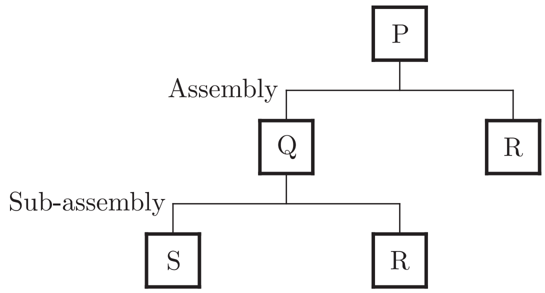
So, shortest path

$$S_G = \text{Min}\{S_Q + d_{QG},\ S_R + d_{RG}\}$$



GATE ME 2008 TWO MARK

The product structure of an assembly P is shown in the figure.



Estimated demand for end product P is as follows

Week	1	2	3	4	5	6
Demand	1000	1000	1000	1000	1200	1200

ignore lead times for assembly and sub-assembly. Production capacity (per week) for component R is the bottleneck operation. Starting with zero inventory, the smallest capacity that will ensure a feasible production plan up to week 6 is

(A) 1000 (B) 1200

(C) 2200 (D) 2400

SOL 1.61 Option (C) is correct.

From the product structure we see that 2 piece of R is required in production of 1 piece P.

So, demand of R is double of P

Week	**Demand** (P)	**Demand** (R)	**Inventory level** $I =$ Production $-$ Demand
1	1000	2000	$R - 2000$
2	1000	2000	$2R - 4000$
3	1000	2000	$3R - 6000$
4	1000	2000	$4R - 8000$
5	1200	2400	$5R - 10400$
6	1200	2400	$6R - 12800$

We know that for a production system with bottleneck the inventory level should be more than zero.

So,

$$6R - 12800 \geq 0$$

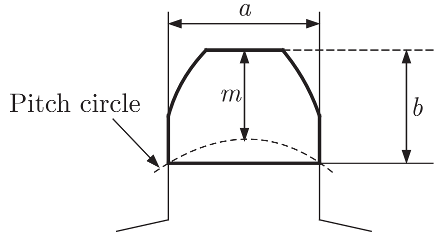
For minimum inventory

$$6R - 12800 = 0$$
$$6R = 12800$$
$$R = 2133$$
$$\simeq 2200$$

Hence, the smallest capacity that will ensure a feasible production plan up to week 6 is 2200.

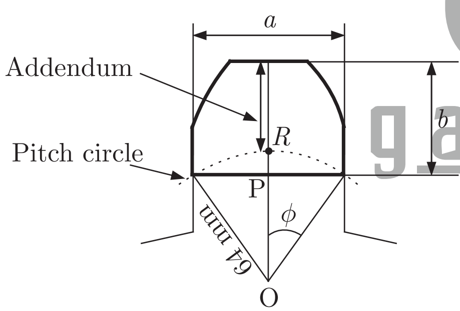
MCQ 1.62
GATE ME 2008
TWO MARK

One tooth of a gear having 4 module and 32 teeth is shown in the figure. Assume that the gear tooth and the corresponding tooth space make equal intercepts on the pitch circumference. The dimensions 'a' and 'b', respectively, are closest to



(A) 6.08 mm, 4 mm
(B) 6.48 mm, 4.2 mm
(C) 6.28 mm, 4.3 mm
(D) 6.28 mm, 4.1 mm

SOL 1.62 Option (D) is correct.



Given : $m = 4$, $Z = 32$, Tooth space = Tooth thickness = a

We know that, $m = \dfrac{D}{Z}$

Pitch circle diameter, $D = mZ = 4 \times 32 = 128$ mm

And for circular pitch, $P_c = \pi m = 3.14 \times 4 = 12.56$ mm

We also know that circular pitch,

$$P_c = \text{Tooth space} + \text{Tooth thickness}$$
$$P_c = a + a = 2a$$
$$a = \frac{P_c}{2} = \frac{12.56}{2} = 6.28 \text{ mm}$$

From the figure, $b = \text{addendum} + PR$

or $$\sin\phi = \frac{PQ}{OQ} = \frac{a/2}{64} = \frac{3.14}{64}$$

$$\phi = \sin^{-1}(0.049) = 2.81^\circ$$
$$OP = 64\cos 2.81^\circ = 63.9 \text{ mm}$$
$$PR = OR - OP = 64 - 63.9 = 0.1 \text{ mm} \qquad OR = \text{Pitch circle radius}$$

And $$b = m + PR = 4 + 0.1 = 4.1 \text{ mm}$$

Therefore, $a = 6.28$ mm and $b = 4.1$ mm

MCQ 1.63 GATE ME 2008 TWO MARK

While cooling, a cubical casting of side 40 mm undergoes 3%, 4% and 5% volume shrinkage during the liquid state, phase transition and solid state, respectively. The volume of metal compensated from the riser is

(A) 2% (B) 7%

(C) 8% (D) 9%

SOL 1.63 Option (B) is correct.

Since metal shrinks on solidification and contracts further on cooling to room temperature, linear dimensions of patterns are increased in respect of those of the finished casting to be obtained. This is called the "Shrinkage allowance".

The riser can compensate for volume shrinkage only in the liquid or transition stage and not in the solid state.

So, Volume of metal that compensated from the riser $= 3\% + 4\% = 7\%$

MCQ 1.64 GATE ME 2008 ONE MARK

In a single point turning tool, the side rake angle and orthogonal rake angle are equal. φ is the principal cutting edge angle and its range is $0^\circ \le \varphi \le 90^\circ$. The chip flows in the orthogonal plane. The value of φ is closest to

(A) 0° (B) 45°

(C) 60° (D) 90°

SOL 1.64 Option (D) is correct.

Interconversion between ASA (American Standards Association) system and ORS (Orthogonal Rake System)

$$\tan\alpha_s = \sin\phi\tan\alpha - \cos\phi\tan i$$

where α_s = Side rake angle

α = orthogonal rake angle

ϕ = principle cutting edge angle $= 0 \le \phi \le 90^\circ$

i = inclination angle ($i = 0$ for ORS)

$$\alpha_s = \alpha \text{ (Given)}$$

$$\tan\alpha_s = \sin\phi\tan\alpha - \cos\phi\tan(0^\circ)$$

$$\tan\alpha_s = \sin\phi\tan\alpha$$

$$\frac{\tan\alpha_s}{\tan\alpha} = \sin\phi$$

$$1 = \sin\phi$$

$$\phi = \sin^{-1}(1) = 90^\circ$$

MCQ 1.65 GATE ME 2008 TWO MARK

A researcher conducts electrochemical machining (ECM) on a binary alloy (density 6000 kg/m^3) of iron (atomic weight 56, valency 2) and metal (atomic weight 24, valency 4). Faraday's constant $= 96500$ coulomb/mole. Volumetric material removal rate of the alloy is 50 mm^3/s at a current of 2000 A. The percentage of the metal P in the alloy is closest to

(A) 40 (B) 25

(C) 15 (D) 79

SOL 1.65 Option (B) is correct.

Given : $\rho = 6000\ \text{kg/m}^3 = 6\ \text{gm/cm}^3,\ F = 96500\ \text{coulomb/mole}$

$MRR = 50\ \text{mm}^3/\text{s} = 50\times10^{-3}\ \text{cm}^3/\text{s},\ I = 2000\ \text{A}$

For Iron : Atomic weight $= 56$

Valency $= 2$

For Metal P: Atomic weight $= 24$

Valency $= 4$

The metal Removal rate

$$MRR = \frac{eI}{F\rho}$$

$$50\times10^{-3} = \frac{e\times2000}{96500\times6}$$

$$e = \frac{50\times10^{-3}\times96500\times6}{2000} = 14.475$$

Let the percentage of the metal P in the alloy is x.

So,
$$\frac{1}{e} = \frac{100-x}{100}\times\frac{V_{Fe}}{A_{t_{Fe}}}+\frac{x}{100}\times\frac{V_P}{A_{tP}}$$

$$\frac{1}{14.475} = \frac{100-x}{100}\times\frac{2}{56}+\frac{x}{100}\times\frac{4}{24}$$

$$\frac{1}{14.475} = \left(1-\frac{x}{100}\right)\frac{1}{28}+\frac{x}{100}\times\frac{1}{6}$$

$$\frac{1}{14.475} = x\left[\frac{1}{600}-\frac{1}{2800}\right]+\frac{1}{28}$$

$$\frac{1}{14.475}-\frac{1}{28} = x\times\frac{11}{8400}$$

$$\frac{541}{16212} = \frac{11x}{8400}$$

$$x = \frac{541\times8400}{16212\times11}\simeq 25$$

MCQ 1.66 GATE ME 2008 TWO MARK

In a single pass rolling operation, a 20 mm thick plate with plate width of 100 mm , is reduced to 18 mm. The roller radius is 250 mm and rotational speed is 10 rpm. The average flow stress for the plate material is 300 MPa. The power required for the rolling operation in kW is closest to

(A) 15.2 (B) 18.2

(C) 30.4 (D) 45.6

SOL 1.66 Option None of these.

Given : $t_i = 20\ \text{mm},\ t_f = 18\ \text{mm},\ b = 100\ \text{mm},$

$R = 250\ \text{mm},\ N = 10\ \text{rpm},\ \sigma_0 = 300\ \text{MPa}$

We know, Roll strip contact length is given by,

$$L = \theta \times R = \sqrt{\frac{t_i - t_f}{R}} \times R$$
$$= \sqrt{R(t_i - t_f)}$$

So, $$L = \sqrt{250 \times 10^{-3}(20 - 18)10^{-3}}$$
$$= 22.36 \times 10^{-3}$$

Rolling load, $$F = Lb\sigma_0 = 22.36 \times 10^{-3} \times 100 \times 10^{-3} \times 300 \times 10^6$$
$$= 670.8\text{ kN}$$

Power $$P = F \times v = 670.8 \times \left(\frac{\pi DN}{60}\right)$$
$$= 670.8 \times \left(\frac{3.14 \times 0.5 \times 10}{60}\right) = 175.5\text{ kW}$$

MCQ 1.67 GATE ME 2008 TWO MARK

In arc welding of a butt joint, the welding speed is to be selected such that highest cooling rate is achieved. Melting efficiency and heat transfer efficiency are 0.5 and 0.7, respectively. The area of the weld cross section is 5 mm^2 and the unit energy required to melt the metal is 10 J/mm^3. If the welding power is 2 kW, the welding speed in mm/s is closest to

(A) 4 (B) 14

(C) 24 (D) 34

SOL 1.67 Option (B) is correct.

Given : $\eta_m = 0.5$, $\eta_h = 0.7$, $A = 5\text{ mm}^2$

$E_u = 10\text{ J/mm}^3$, $P = 2\text{ kW}$, V (mm/s) = ?

Total energy required to melt,

$$E = E_u \times A \times V = 10 \times 5 \times V = 50V\text{ J/sec}$$

Power supplied for welding,

$$P_s = P \times \eta_h \times \eta_m = 2 \times 10^3 \times 0.5 \times 0.7 = 700\text{ W}$$

From energy balance,

Energy required to melt = Power supplied for welding

$$50V = 700 \quad \Rightarrow \quad V = 14\text{ mm/sec}$$

MCQ 1.68 GATE ME 2008 TWO MARK

In the deep drawing of cups, blanks show a tendency to wrinkle up around the periphery (flange). The most likely cause and remedy of the phenomenon are, respectively,

(A) Buckling due to circumferential compression; Increase blank holder pressure

(B) High blank holder pressure and high friction; Reduce blank holder pressure and apply lubricant

(C) High temperature causing increase in circumferential length; Apply coolant to blank

(D) Buckling due to circumferential compression; decrease blank holder pressure

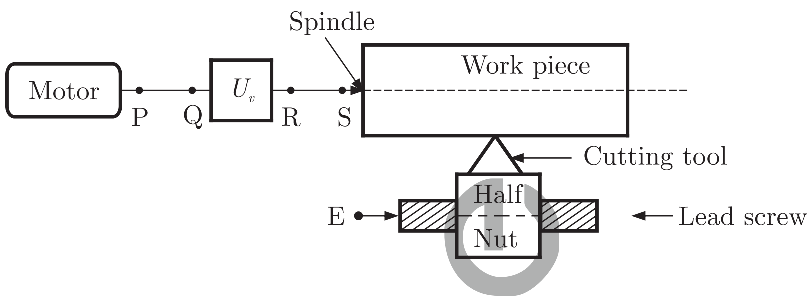
SOL 1.68 Option (A) is correct.

Seamless cylinders and tubes can be made by hot drawing or cupping.

The thickness of the cup is reduced and its length increased by drawing it through a series of dies having reduced clearance between the die and the punch. Due to reduction in its thickness, blanks shows a tendency to wrinkle up around the periphery because of buckling due to circumferential compression an due to this compression blank holder pressure increases.

MCQ 1.69
GATE ME 2008
TWO MARK

The figure shows an incomplete schematic of a conventional lathe to be used for cutting threads with different pitches. The speed gear box U_v is shown and the feed gear box U_s is to be placed. P, Q, R and S denote locations and have no other significance. Changes in U_v should NOT affect the pitch of the thread being cut and changes in U_s should NOT affect the cutting speed.



The correct connections and the correct placement of U_s are given by

(A) Q and E are connected. U_s is placed between P and Q.

(B) S and E are connected. U_s is placed between R and S

(C) Q and E are connected. U_s is placed between Q and E

(D) S and E are connected. U_s is placed between S and E

SOL 1.69

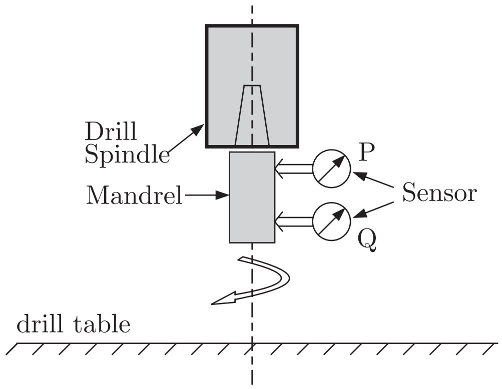
Option (C) is correct.

The feed drive serves to transmit power from the spindle to the second operative unit of the lathe, that is, the carriage. It, thereby converts the rotary motion of the spindle into linear motion of the carriage.

So, Q and E are connected & U_s is placed between Q and E.

MCQ 1.70
GATE ME 2008
TWO MARK

A displacement sensor (a dial indicator) measure the lateral displacement of a mandrel mounted on the taper hole inside a drill spindle. The mandrel axis is an extension of the drill spindle taper hole axis and the protruding portion of the mandrel surface is perfectly cylindrical measurements are taken with the sensor placed at two positions P and Q as shown in the figure. The reading are recorded as R_x = maximum deflection minus minimum deflection, corresponding to sensor position at X, over one rotation.



If $R_P = R_Q > 0$, which one of the following would be consistent with the observation ?

(A) The drill spindle rotational axis is coincident with the drill spindle taper hole axis

(B) The drill spindle rotational axis intersects the drill spindle taper hole axis at point P

(C) The drill spindle rotational axis is parallel to the drill spindle taper hole axis

(D) The drill spindle rotational axis intersects the drill spindle taper hole axis at point Q

SOL 1.70 Option (C) is correct.

A dial indicator (gauge) or clock indicator is a very versatile and sensitive instrument. It is used for :

(i) determining errors in geometrical form, for example, ovality, out-of roundness, taper etc.

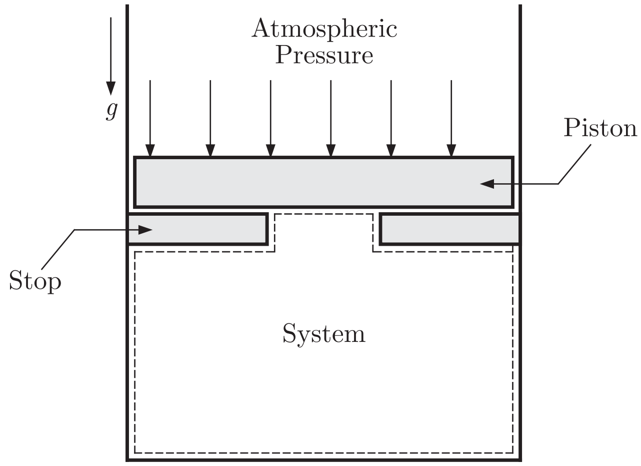
(ii) determining positional errors of surface

(iii) taking accurate measurements of deformation.

Here equal deflections are shown in both the sensor P and sensor Q. So drill spindle rotational axis is parallel to the drill spindle taper hole axis.

Common Data for Questions 71, 72 & 73 :

In the figure shown, the system is a pure substance kept in a piston-cylinder arrangement. The system is initially a two-phase mixture containing 1 kg of liquid and 0.03 kg of vapour at a pressure of 100 kPa. Initially, the piston rests on a set of stops, as shown in the figure. A pressure of 200 kPa is required to exactly balance the weight of the piston and the outside atmospheric pressure. Heat transfer takes place into the system until its volume increases by 50%. Heat transfer to the system occurs in such a manner that the piston, when allowed to move, does so in a very slow (quasi-static/quasi-equilibrium) process. The thermal reservoir from which heat is transferred to the system has a temperature of 400° C. Average temperature of the system boundary can be taken as 175° C. The heat transfer to the system is 1 kJ, during which its entropy increases by 10 J/K.



Specific volume of liquid (ν_f) and vapour (ν_g) phases, as well as values of saturation temperatures, are given in the table below.

Pressure (kPa)	**Saturation temperature,** T_{sat} (°C)	ν_f (m^3/kg)	ν_g (m^3/kg)
100	100	0.001	0.1
200	200	0.0015	0.002

MCQ 1.71 At the end of the process, which one of the following situations will be true ?

GATE ME 2008
TWO MARK

(A) superheated vapour will be left in the system

(B) no vapour will be left in the system

(C) a liquid + vapour mixture will be left in the system

(D) the mixture will exist at a dry saturated vapour state

SOL 1.71 Option (A) is correct.

When the vapour is at a temperature greater than the saturation temperature, it is said to exist as super heated vapour. The pressure & Temperature of superheated vapour are independent properties, since the temperature may increase while the pressure remains constant. Here vapour is at 400° C & saturation temperature is 200° C.

So, at 200 kPa pressure superheated vapour will be left in the system.

MCQ 1.72 The work done by the system during the process is

GATE ME 2008
TWO MARK

(A) 0.1 kJ (B) 0.2 kJ

(C) 0.3 kJ (D) 0.4 kJ

SOL 1.72 Option (D) is correct.

Given : $p_1 = 100\,\text{kPa}$, $p_2 = 200\,\text{kPa}$

Let, $\nu_1 = \nu$

Now, given that Heat transfer takes place into the system until its volume increases by 50%

So, $\nu_2 = \nu + 50\% \text{ of } \nu$

Now, for work done by the system, we must take pressure is $p_2 = 200\,\text{kPa}$, because work done by the system is against the pressure p_2 and it is a positive work done.

From first law of thermodynamics,

$$dQ = dU + dW \quad \text{...(i)}$$

But for a quasi-static process,

$$T = \text{Constant}$$

Therefore, change in internal energy is

$$dU = 0$$

From equation (i)

$$dQ = dW = pd\nu \qquad dW = pd\nu$$
$$= p[\nu_2 - \nu_1]$$

For initial condition at $100\,\text{kPa}$, volume

$$\nu_1 = m_{liquid} \times \frac{1}{\rho_f} + m_{vapour} \times \frac{1}{\rho_g}$$

Here $\frac{1}{\rho_f} = \nu_f = 0.001,\ \frac{1}{\rho_g} = \nu_g = 0.1$

$$m_{liquid} = 1\,\text{kg},\ m_{vapour} = 0.03\,\text{kg}$$

So

$$\nu_1 = 1 \times 0.001 + 0.03 \times 0.1$$
$$= 4 \times 10^{-3}\,\text{m}^3$$
$$\nu_2 = \frac{3}{2}\nu_1 = \frac{3}{2} \times 4 \times 10^{-3} = 6 \times 10^{-3}\,\text{m}^3$$
$$= 200 \times 10^3\left[\frac{3\nu}{2} - \nu\right]$$
$$= 200 \times [6 \times 10^{-3} - 4 \times 10^{-3}]$$
$$= 200 \times 2 \times 10^{-3} = 0.4\,\text{kJ}$$

MCQ 1.73 The net entropy generation (considering the system and the thermal reservoir together) during the process is closest to

GATE ME 2008
TWO MARK

(A) 7.5 J/K (B) 7.7 J/K

(C) 8.5 J/K (D) 10 J/K

SOL 1.73 Option (C) is correct.

$$\Delta s_{net} = (\Delta s)_{system} + (\Delta s)_{surrounding} \quad \text{...(i)}$$

And it is given that,

$$(\Delta s)_{system} = 10\,\text{kJ}$$

Also, $(\Delta s)_{surrounding} = \left(\frac{Q}{T}\right)_{surrounding}$

Heat transferred to the system by thermal reservoir,

$$T = 400^\circ \text{C} = (400+273)\,\text{K} = 673\,\text{K}$$
$$Q = 1\,\text{kJ}$$
$$(\Delta s)_{surrounding} = \frac{1000}{673} = 1.485\,\text{J/K}$$

From equation (i)

$$(\Delta s)_{net} = 10 - 1.485 = 8.515\,\text{J/K}$$

(Take Negative sign, because the entropy of surrounding decrease due to heat transfer to the system.)

Common Data for Questions 74 and 75 :

Consider the Linear Programme (LP)

Max $4x + 6y$

Subject to

$$3x + 2y \le 6$$
$$2x + 3y \le 6$$
$$x, y \ge 0$$

MCQ 1.74 GATE ME 2008 TWO MARK

After introducing slack variables s and t, the initial basic feasible solution is represented by the table below (basic variables are $s = 6$ and $t = 6$, and the objective function value is 0)

	-4	-6	0	0	0
s	3	2	1	0	6
t	2	3	0	1	6
	x	y	s	t	RHS

After some simplex iterations, the following table is obtained

	0	0	0	2	12
s	5/3	0	1	$-1/3$	2
y	2/3	1	0	1/3	2
	x	y	s	t	RHS

From this, one can conclude that

(A) the LP has a unique optimal solution

(B) the LP has an optimal solution that is not unique

(C) the LP is infeasible

(D) the LP is unbounded

SOL 1.74 Option (B) is correct.

The LP has an optimal solution that is not unique, because zero has appeared in the non-basic variable (x and y) column, in optimal solution.

MCQ 1.75 The dual for the LP in Q. 29 is

GATE ME 2008 TWO MARK

(A) Min $6u + 6v$
subject to
$3u + 2v \geq 4$
$2u + 3v \geq 6$
$u, v \geq 0$

(B) Max $6u + 6v$
subject to
$3u + 2v \leq 4$
$2u + 3v \leq 6$
$u, v \geq 0$

(C) Max $4u + 6v$
subject to
$3u + 2v \geq 6$
$2u + 3v \geq 6$
$u, v \geq 0$

(D) Min $4u + 6v$
subject to
$3u + 2v \leq 6$
$2u + 3v \leq 6$
$u, v \geq 0$

SOL 1.75 Option (A) is correct.

The general form of LP is

$$\text{Max } Z = CX$$

Subject to $AX \leq B$

And dual of above LP is represented by

$$\text{Min } Z = B^T Y$$

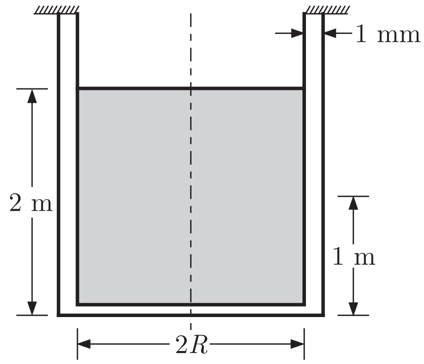
Subject to $A^T Y \geq C^T$

So, the dual is Min $6u + 6v$

Subject to
$$3u + 2v \geq 4$$
$$2u + 3v \geq 6$$
$$u, v \geq 0$$

Statement for Linked Answer Questions 76 and 77 :

A cylindrical container of radius $R = 1$ m, wall thickness 1 mm is filled with water up to a depth of 2 m and suspended along its upper rim. The density of water is $1000\,\text{kg/m}^3$ and acceleration due to gravity is $10\,\text{m/s}^2$. The self-weight of the cylinder is negligible. The formula for hoop stress in a thin-walled cylinder can be used at all points along the height of the cylindrical container.



MCQ 1.76 GATE ME 2008 TWO MARK

The axial and circumference stress (σ_d, σ_c) experienced by the cylinder wall at mid-depth (1 m as shown) are

(A) (10, 10)MPa (B) (5, 10)MPa

(C) (10, 5)MPa (D) (5, 5)MPa

SOL 1.76

Option (B) is correct.

Given : $R = 1\,\text{m}, \quad t = 1\,\text{mm} = 10^{-3}\,\text{m}$

We know that axial or longitudinal stress for a thin cylinder is,

$$\sigma_x = \sigma_a = \frac{p \times D}{4t} = \frac{p \times 2R}{4t} \quad \text{...(i)}$$

Here, p = Pressure of the fluid inside the shell

So, pressure at 1 m depth is,

$$p = \rho g h = 1000 \times 10 \times 1 = 10^4\,\text{N/m}^2$$

From equation (i),

$$\sigma_a = \frac{10^4 \times 2 \times 1}{4 \times 10^{-3}} = 5 \times 10^6\,\text{N/m}^2 = 5\,\text{MPa}$$

And hoop or circumferential stress,

$$\sigma_y = \sigma_c = \frac{p \times D}{2t}$$

$$\sigma_c = \frac{10^4 \times 2}{2 \times 10^{-3}} = 10 \times 10^6\,\text{N/m}^2 = 10\,\text{MPa}$$

MCQ 1.77 GATE ME 2008 TWO MARK

If the Young's modulus and Poisson's ratio of the container material are 100 GPa and 0.3, respectively, the axial strain in the cylinder wall at mid-depth is

(A) 2×10^{-5} (B) 6×10^{-5}

(C) 7×10^{-5} (D) 1.2×10^{-4}

SOL 1.77

Option (A) is correct.

Given : υ or $\frac{1}{m} = 0.3, \ E = 100\,\text{GPa} = 100 \times 10^9\,\text{Pa}$

Axial strain or longitudinal strain at mid – depth is,

$$\sigma_a = \sigma_x = \frac{pD}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

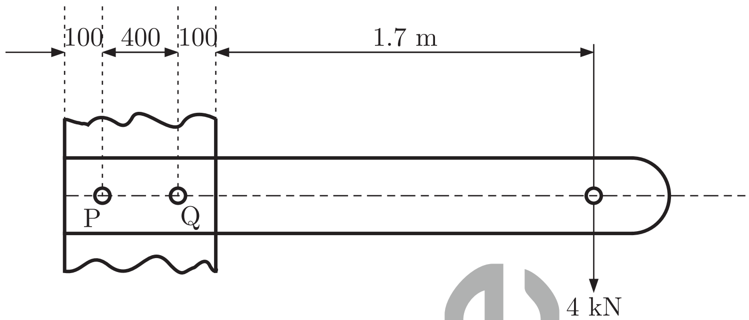
Substitute the values, we get

$$\sigma_a = \frac{10^4 \times 2 \times 1}{2 \times 10^{-3} \times 100 \times 10^9}\left(\frac{1}{2} - 0.3\right)$$

$$\sigma_a = \frac{10^4}{10^8}\left(\frac{1}{2} - 0.3\right) = 10^{-4} \times 0.2 = 2 \times 10^{-5}$$

Statement for Linked Answer Questions 78 and 79 :

A steel bar of 10 × 50 mm is cantilevered with two M 12 bolts (P and Q) to support a static load of 4 kN as shown in the figure.



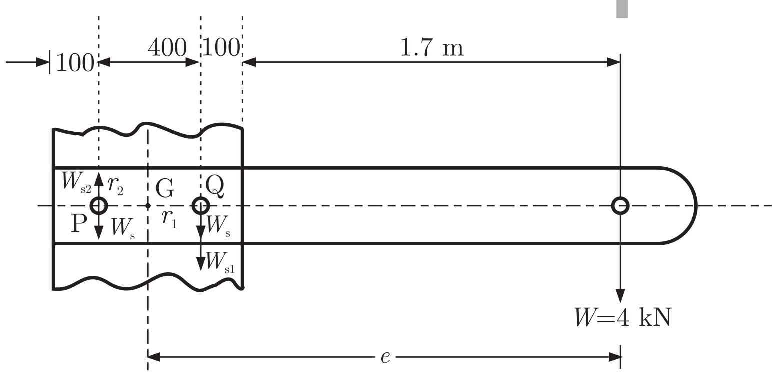
MCQ 1.78 GATE ME 2008 TWO MARK

The primary and secondary shear loads on bolt P, respectively, are

(A) 2 kN, 20 kN (B) 20 kN, 2 kN

(C) 20 kN, 0 kN (D) 0 kN, 20 kN

SOL 1.78 Option (A) is correct.



In this figure W_S represent the primary shear load whereas W_{S1} and W_{S2} represent the secondary shear loads.

Given : $A = 10 \times 50\,\text{mm}^2$, $n = 2$, $W = 4\,\text{kN} = 4 \times 10^3\,\text{N}$

We know that primary shear load on each bolt acting vertically downwards,

$$W_s = \frac{W}{n} = \frac{4\,\text{kN}}{2} = 2\,\text{kN}$$

Since both the bolts are at equal distances from the centre of gravity G of the two bolts, therefore the secondary shear load on each bolt is same.

For secondary shear load, taking the moment about point G,

$$W_{s1} \times r_1 + W_{s2} \times r_2 = W \times e$$

$r_1 = r_2$ and $W_{s1} = W_{s2}$

So, $$2r_1 W_{s1} = 4 \times 10^3 \times (1.7 + 0.2 + 0.1)$$

$$2 \times 0.2 \times W_{s1} = 4 \times 10^3 \times 2$$

$$W_{s1} = \frac{8 \times 10^3}{2 \times 0.2} = 20 \times 10^3 = 20 \text{ kN}$$

MCQ 1.79 The resultant shear stress on bolt P is closest to

GATE ME 2008 TWO MARK

(A) 132 MPa (B) 159 MPa

(C) 178 MPa (D) 195 MPa

SOL 1.79 Option (B) is correct.

From the figure, resultant Force on bolt P is

$$F = W_{s2} - W_s = 20 - 2 = 18 \text{ kN}$$

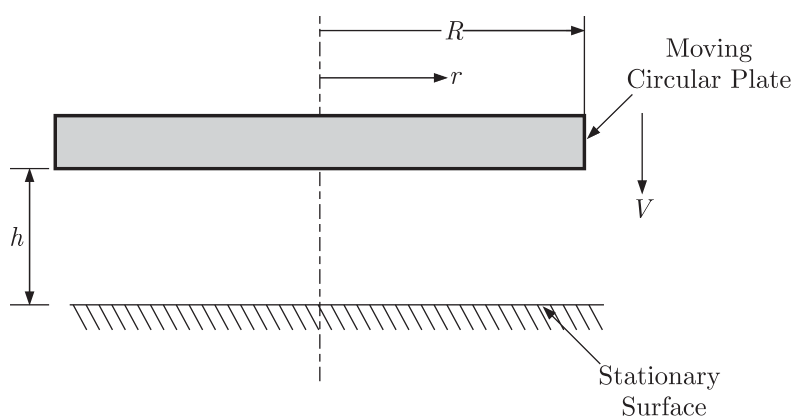
Shear stress on bolt P is,

$$\tau = \frac{F}{\text{Area}}$$ $M12$ Means bolts have 12 mm diameter

$$= \frac{18 \times 10^3}{\frac{\pi}{4} \times (12 \times 10^{-3})^2} = 159.23 \text{ MPa} \simeq 159 \text{ MPa}$$

Statement for linked Answer questions 80 & 81:

The gap between a moving circular plate and a stationary surface is being continuously reduced, as the circular plate comes down at a uniform speed V towards the stationary bottom surface, as shown in the figure. In the process, the fluid contained between the two plates flows out radially. The fluid is assumed to be incompressible and inviscid.



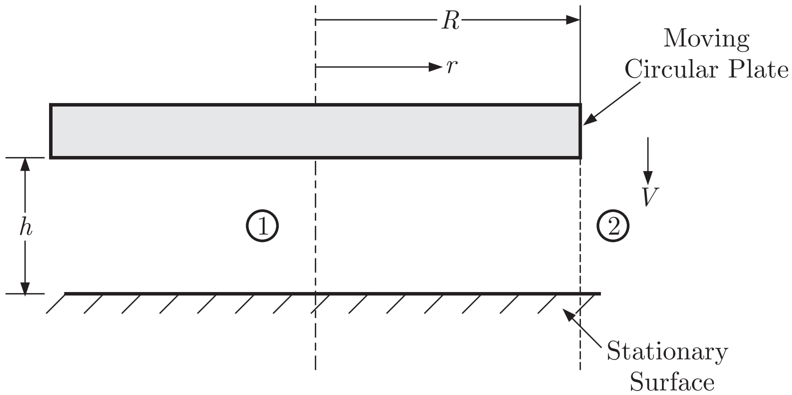
MCQ 1.80 The radial velocity V_r at any radius r, when the gap width is h, is

GATE ME 2008 TWO MARK

(A) $V_r = \frac{Vr}{2h}$ (B) $V_r = \frac{Vr}{h}$

(C) $V_r = \frac{2Vh}{r}$ (D) $V_r = \frac{Vh}{r}$

SOL 1.80 Option (A) is correct.



Here Gap between moving & stationary plates are continuously reduced, so we can say that

Volume of fluid moving out radially

$= $ Volume of fluid displaced by moving plate within radius r

Volume displaced by the moving plate

$= $ Velocity of moving plate $\times$ Area$= V \times \pi r^2$ ...(i)

Volume of fluid which flows out at radius r

$= V_r \times 2\pi r \times h$ ...(ii)

Equating equation (i) & (ii),

$$V \times \pi r^2 = V_r \times 2\pi rh$$

$$Vr = 2V_r h \Rightarrow V_r = \frac{Vr}{2h}$$

Alternate Method :

Apply continuity equation at point (i) & (ii),

$$A_1 V_1 = A_2 V_2$$

$$V \times \pi r^2 = V_r \times 2\pi rh$$

$$V_r = \frac{Vr}{2h}$$

MCQ 1.81 The radial component of the fluid acceleration at $r = R$ is

GATE ME 2008 TWO MARK

(A) $\frac{3V^2 R}{4h^2}$

(B) $\frac{V^2 R}{4h^2}$

(C) $\frac{V^2 R}{2h^2}$

(D) $\frac{V^2 h}{2R^2}$

SOL 1.81 Option (B) is correct.

From previous part of question,

$$V_r = \frac{Vr}{2h}$$

Acceleration at radius r is given by

$$a_r = V_r \times \frac{dV_r}{dr} = V_r \times \frac{d}{dr}\left[\frac{Vr}{2h}\right] = V_r \times \frac{V}{2h} \quad ...(i)$$

At $r = R$

$$a_r = \frac{VR}{2h} \times \frac{V}{2h} = \frac{V^2R}{4h^2}$$

Statement for Linked Answer Questions 82 and 83 :

Orthogonal turning is performed on a cylindrical workpiece with the shear strength of 250 MPa. The following conditions are used: cutting velocity is 180 m/ min, feed is 0.20 mm/rev, depth of cut is 3 mm, chip thickness ratio = 0.5. The orthogonal rake angle is 7°. Apply Merchant's theory for analysis.

MCQ 1.82 The shear plane angle (in degree) and the shear force respectively are

GATE ME 2008
TWO MARK

(A) 52, 320 N (B) 52, 400 N

(C) 28, 400 N (D) 28, 320 N

SOL 1.82 Option (D) is correct.

Given : $\tau_s = 250$ MPa, $V = 180$ m/ min, $f = 0.20$ mm/rev.

$d = 3$ mm, $r = 0.5$, $\alpha = 7°$

We know from merchant's theory,

Shear plane angle $\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha}$

$$\tan\phi = \frac{0.5\cos 7°}{1 - 0.7\sin 7°} = \frac{0.496}{0.915} = 0.54$$

$$\phi = \tan^{-1}(0.54) = 28.36 \simeq 28°$$

Average stress on the shear plane area are

$$\tau_s = \frac{F_s}{A_s} \Rightarrow F_s = \tau_s \times A_s$$

where, A_s is the shear plane area $= \frac{bt}{\sin\phi}$

for orthogonal operation $b \cdot t = d \cdot f$

So, $F_s = \frac{\tau_s \times d \times f}{\sin\phi}$

$$F_s = \frac{250 \times 3 \times 0.20}{\sin 28°} = 319.50 \simeq 320 \text{ N}$$

MCQ 1.83 The cutting and frictional forces, respectively, are

GATE ME 2008
TWO MARK

(A) 568 N, 387 N (B) 565 N, 381 N

(C) 440 N, 342 N (D) 480 N, 356 N

SOL 1.83 Option (B) is correct.

Now we have to find cutting force (F_c) and frictional force (F_t).

From merchant's theory,

$$2\phi + \beta - \alpha = 90°$$

$$\beta = 90° + \alpha - 2\phi$$

$$= 90° + 7 - 2 \times 28 = 41°$$

We know that

$$\frac{F_c}{F_s} = \frac{\cos(\beta - \alpha)}{\cos(\phi + \beta - \alpha)} \qquad F_s = \text{Share force}$$

$$F_c = 320 \times \frac{\cos(41^\circ - 7^\circ)}{\cos(28^\circ + 41^\circ - 7^\circ)}$$

$$= 320 \times 1.766 \simeq 565\,\text{N}$$

And $$F_s = F_c \cos\phi - F_t \sin\phi$$

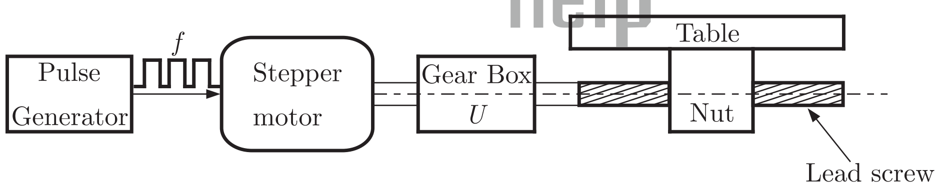
So, $$F_t = \frac{F_c \cos\phi - F_s}{\sin\phi}$$

$$F_t = \frac{565 \times \cos 28^\circ - 320}{\sin 28^\circ} = \frac{178.865}{0.47}$$

$$= 381.56\,\text{N} \simeq 381\,\text{N}$$

Statement for linked Answer Questions 84 and 85 :

In the feed drive of a Point-to-Point open loop CNC drive, a stepper motor rotating at 200 steps/rev drives a table through a gear box and lead screw-nut mechanism (pitch=4 mm, number of starts=1). The gear ratio $= \left(\frac{\text{Output rotational speed}}{\text{Input rotational speed}}\right)$ is given by $U = \frac{1}{4}$. The stepper motor (driven by voltage pulses from a pulse generator) executes 1 step/pulse of the pulse generator. The frequency of the pulse train from the pulse generator is $f = 10{,}000$ pulses per minute.



MCQ 1.84 GATE ME 2008 TWO MARK

The basic Length Unit (BLU), i.e, the table movement corresponding to 1 pulse of the pulse generator, is

(A) 0.5 microns (B) 5 microns

(C) 50 microns (D) 500 microns

SOL 1.84 Option (B) is correct.

Given : $N = 200$ step/rev., $p = 4$ mm, $U = \frac{1}{4}$, $f = 10000$ Pulse/min.

In a CNC machine basic length unit (BLU) represents the smallest distance.

$$\text{Revolution of motor in one step} = \frac{1}{200}\,\text{rev./step}$$

$$\text{Movement of lead screw} = \frac{1}{200} \times \frac{1}{4} = \frac{1}{800}\,\text{rev. of load screw}$$

Movement from lead screw is transferred to table.

$$\text{i.e. Movement of table} = \frac{1}{800} \times \text{Pitch} = \frac{1}{800} \times 4 = \frac{1}{200}$$

$$= 0.005 = 5 \text{ microns.}$$

MCQ 1.85 GATE ME 2008 TWO MARK

A customer insists on a modification to change the BLU of the CNC drive to 10 microns without changing the table speed. The modification can be accomplished by

(A) changing U to $\frac{1}{2}$ and reducing f to $\frac{f}{2}$

(B) changing U to $\frac{1}{8}$ and increasing f to $2f$

(C) changing U to $\frac{1}{2}$ and keeping f unchanged

(D) keeping U unchanged and increasing f to $2f$

SOL 1.85 Option (C) is correct.

If we change the gear ratio by a factor $\frac{1}{2}$ and f remains unchanged, then

$$\text{BLU} = \text{Revolution of motor} \times \text{Gear ratio} \times \text{pitch}$$
$$= \frac{1}{200} \times \frac{1}{2} \times 4 = \frac{1}{100} = 10 \text{ micros}$$

We see that f is unchanged and value of Gear ratio is changed by $\frac{1}{2}$.

Answer Sheet

1.	(C)	18.	(A)	35.	(B)	52.	(A)	69.	(C)
2.	(D)	19.	(C)	36.	(B)	53.	(B)	70.	(C)
3.	(B)	20.	(C)	37.	(D)	54.	(B)	71.	(A)
4.	(A)	21.	(A)	38.	(C)	55.	(C)	72.	(D)
5.	(C)	22.	(B)	39.	(D)	56.	(D)	73.	(C)
6.	(D)	23.	(B)	40.	(A)	57.	(C)	74.	(B)
7.	(D)	24.	(D)	41.	(B)	58.	(D)	75.	(A)
8.	(D)	25.	(A)	42.	(B)	59.	(D)	76.	(B)
9.	(D)	26.	(D)	43.	(A)	60.	(C)	77.	(A)
10.	(D)	27.	(B)	44.	(C)	61.	(C)	78.	(A)
11.	(B)	28.	(C)	45.	(C)	62.	(D)	79.	(B)
12.	(D)	29.	(A)	46.	(B)	63.	(B)	80.	(A)
13.	(B)	30.	(C)	47.	(A)	64.	(D)	81.	(B)
14.	(C)	31.	(D)	48.	(C)	65.	(B)	82.	(D)
15.	(D)	32.	(B)	49.	(A)	66.	(*)	83.	(B)
16.	(B)	33.	(A)	50.	(D)	67.	(B)	84.	(B)
17.	(C)	34.	(B)	51.	(*)	68.	(A)	85.	(C)

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