ab + ab

PLEASE ENSURE THAT THIS QUESTION BOOKLET CONTAINS 120 QUESTIONS SERIALLY NUMBERED FROM 1 TO 120.
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1. If $n(A) = 5$ and $n(B) = 7$, then the number of relations on $A \times B$ is

   (A) 2^{35} (B) 2^{49} (C) 2^{25} (D) 2^{70} (E) $2^{35\times35}$

2. Let $\phi(x) = \dfrac{b(x-a)}{b-a} + \dfrac{a(x-b)}{a-b}$, where $x \in R$ and a and b are fixed real numbers with $a \neq b$. Then $\phi(a+b)$ is equal to

   (A) $\phi(ab)$ (B) $\phi(-ab)$ (C) $\phi(a)+\phi(b)$

   (D) $\phi(a-b)$ (E) $\phi(0)$

$\dfrac{b((a+b)-a)}{b-a} + \dfrac{a((a+b)-b)}{a-b} = \dfrac{b^2}{b-a} + \dfrac{a^2}{a-b}$

$\dfrac{a^2-ab}{a-b} + \dfrac{b^2-ab}{b-a} = \dfrac{-(a^2-ab)+b^2-ab}{(b-a)+(b-a)}$

$= \dfrac{-a^2+ab+b^2-ab}{2b-2a}$

$= \dfrac{b^2-a^2}{2(b-a)}$

3. The range of the function $f(x) = \dfrac{x^2+8}{x^2+4}$, $x \in R$ is

   (A) $[-1, \frac{3}{2}]$ (B) $(1, 2]$ (C) $(1, 2)$ (D) $[1, 2]$ (E) $[\frac{3}{2}, 2]$

4. If $n(A) = 1000$, $n(B) = 500$ and if $n(A \cap B) \geq 1$ and $n(A \cup B) = p$, then

   (A) $500 \leq p \leq 1000$ (B) $1001 \leq p \leq 1498$

   (C) $1000 \leq p \leq 1498$ (D) $999 \leq p \leq 1499$

   (E) $1000 \leq p \leq 1499$

$-\cancel{b^2+a^2}$ $\dfrac{b^2-a^2}{(b-a)+(b-a)}$

$= \dfrac{b^2-a^2}{2b-a}$

5. The domain of the function $f(x) = \sin^{-1}\left(\dfrac{x+5}{2}\right)$ is

   (A) $[-1, 1]$ (B) $[2, 3]$ (C) $[3, 7]$ (D) $[-7, -3]$ (E) $(-\infty, \infty)$

Space for rough work

$A = \{1, 2, 3, 4, 5\}$

$B = \{1, 2, 3, 4, 5, 6, 7\}$

35

$A \times B = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7)$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (2,7)$
$(3,1)$ $(3,2)$ $(3,3)$ $(3,4)$ $(3,5)$ $(3,6)$ $(3,7)$
$(4,1)$ $(4,2)$ $(4,3)$ $(4,4)$ $(4,5)$ $(4,6)$ $(4,7)$
$(5,1)$ $(5,2)$ $(5,3)$ $(5,4)$ $(5,5)$ $(5,6)$ $(5,7)$

6. If $f(x) = x+1$ and $g(x) = 2x$, then $f\left(g(x)\right)$ is equal to

(A) $2(x+1)$ (B) $2x(x+1)$ (C) x (D) $2x+1$ (E) $2x^2+1$

7. If $z_k = e^{i\theta_k}$ for $k = 1, 2, 3, 4$, where $i^2 = -1$, and if $\left|\sum_{k=1}^{4} \frac{1}{z_k}\right| = 1$, then $\left|\sum_{k=1}^{4} z_k\right|$ is equal to

(A) 4 (B) 1 (C) 2 (D) 3 (E) $\frac{1}{4}$

8. If $z = \frac{-1}{2} + i\frac{\sqrt{3}}{2}$, then $8 + 10z + 7z^2$ is equal to

(A) $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2} + i\frac{\sqrt{3}}{2}$ (C) $\frac{-1}{2} + i\frac{3\sqrt{3}}{2}$

(D) $\frac{\sqrt{3}}{2}i$ (E) $-\frac{\sqrt{3}}{2}i$

9. Let $z \neq 1$ be a complex number and let $\omega = x + iy$, $y \neq 0$. If $\frac{\omega - \overline{\omega}z}{1-z}$ is purely real, then $|z|$ is equal to

(A) $|\omega|$ (B) $|\omega|^2$ (C) $\frac{1}{|\omega|^2}$ (D) $\frac{1}{|\omega|}$ (E) 1

Space for rough work

$8+10\left(\frac{-1}{2}+\frac{i\sqrt{3}}{2}\right)+$

$7\left(\frac{-1}{2}+\frac{i\sqrt{3}}{2}\right)^2$

$f(x)=x+1$
$g(x)=2x$

$f(g(x))$

$f(2x) = (2x+1)$

$2(x+1)$

$8+\left(\frac{-10}{2}+\frac{i10\sqrt{3}}{2}\right)+$

$7\left(\left(\frac{-1}{2}\right)^2+2\times\frac{-1}{2}\times\frac{i\sqrt{3}}{2}+\left(\frac{i\sqrt{3}}{2}\right)^2\right)$

$13-5i\sqrt{3}-7$

$=8+5-5i\sqrt{3}-\frac{7-7i\sqrt{3}}{2}$

$=\frac{16+10-10i\sqrt{3}-7-7i\sqrt{3}}{2} = \frac{19-17i\sqrt{3}}{2}$

$=\frac{16+10-7-10i\sqrt{3}-7i\sqrt{3}}{2}$

$=8+\left(\frac{-10+10i\sqrt{3}}{2}\right)+7\left(\frac{1}{4}-\frac{i\sqrt{3}}{2}\ldots\right)$

$=8-5(-1+i\sqrt{3})+7\left(\frac{-1-i\sqrt{3}}{2}\right)$

10. The locus of z such that $\left|\frac{1+iz}{z+i}\right| = 1$ is

(A) $y-x=0$ (B) $y+x=0$ (C) $y=0$ (D) $xy=1$ (E) $x=0$

11. The value of $\sum_{k=0}^{n}\left(i^k + i^{k+1}\right)$, where $i^2=-1$, is equal to

(A) $i-i^n$ (B) $-i+i^{n+1}$ (C) $i-i^{n+1}$ (D) $i-i^{n+2}$ (E) $-i-i^n$

12. Let $z_1 = \frac{2\sqrt{3}+i6\sqrt{7}}{6\sqrt{7}+i2\sqrt{3}}$ and $z_2 = \frac{\sqrt{11}+i3\sqrt{13}}{3\sqrt{13}-i\sqrt{11}}$. Then $\left|\frac{1}{z_1}+\frac{1}{z_2}\right|$ is equal to

(A) 47 (B) 264 (C) $|z_1 - z_2|$ (D) $|z_1 + z_2|$ (E) $|z_1 z_2|$

13. If the equation $ax^2+bx+c=0$, $a>0$, has two distinct real roots α and β such that $\alpha<-5$ and $\beta>5$, then

(A) $c>0$ (B) $c=0$ (C) $c=\frac{a+b}{2}$

(D) $c<0$ (E) $c=a+b$

Space for rough work

$\left|\frac{1}{z_1}\right| = \sqrt{}$

$\frac{1+i(x+iy)}{(x+iy)+i} = 1$

$2iy+y=1$

$y(2i)=1$

$2i=\frac{1}{y}$

$i=\frac{1}{2y}$

$1+ix\cancel{0}-y = x+2iy$

$x+2iy(-ix+y)=1$

$x+2xy+2iy^2=1$

$x(1+2y)+2iy^2=1$

14. If α and β are the distinct roots of $ax^2+bx+c=0$, where a, b and c are non-zero real numbers, then $\dfrac{a\alpha^2+b\alpha+6c}{a\beta^2+b\beta+9c}+\dfrac{a\beta^2+b\beta+19c}{a\alpha^2+b\alpha+13c}$ is equal to

(A) $18c$ (B) $27c$ (C) $\dfrac{36}{27}$ (D) $\dfrac{17}{8}$ (E) $\dfrac{19}{13}$

15. If the equations $x^2+ax+bc=0$ and $x^2+bx+ca=0$ have a common root and if a, b and c are non zero distinct real numbers, then their other roots satisfy the equation

(A) $x^2+x+abc=0$ (B) $x^2-(a+b)x+ab=0$
(C) $x^2+(a+b)x+ab=0$ (D) $x^2+x+ab=0$
(E) $x^2+abx+abc=0$

16. If $y=x+\dfrac{1}{x}$, $x\neq 0$, then the equation $(x^2-3x+1)(x^2-5x+1)=6x^2$ reduces to

(A) $y^2-8y+7=0$ (B) $y^2+8y+7=0$ (C) $y^2-8y-9=0$
(D) $y^2-8y+9=0$ (E) $y^2-7y+13=0$

17. If $\log_e 5$, $\log_e(5^x-1)$ and $\log_e(5^x-\frac{11}{5})$ are in A.P., then the values of x are

(A) $\log_5 4$ and $\log_5 3$ (B) $\log_3 4$ and $\log_4 3$ (C) $\log_3 4$ and $\log_3 5$
(D) $\log_5 6$ and $\log_5 7$ (E) $12, 6$

Space for rough work

14
28
27.5
70.5
9

$\dfrac{14}{15}$

$\dfrac{\frac{10}{9}}{\frac{4}{3}}$

$\dfrac{10}{12}\times\dfrac{3}{9}$ $\dfrac{5}{6}$

$S_n = \frac{n}{2}[2a+(n-1)d]$

18. The sum of first n terms of the series $\frac{4}{3}, \frac{10}{9}, \frac{28}{27}, \frac{82}{81}, \frac{244}{243}, \ldots$ is

(A) $n+\frac{1}{2}(1+3^{-n})$ (B) $n-\frac{1}{2}(1+3^{-n})$ (C) $n+\frac{1}{2}(2+3^{-n})$

(D) $n+\frac{1}{2}(2-3^{-n})$ (E) $n+\frac{1}{2}(1-3^{-n})$

19. If $\sum_{k=1}^{n} k(k+1)(k-1) = pn^4+qn^3+tn^2+sn$, where p, q, t and s are constants, then the value of s is equal to

(A) $-\frac{1}{4}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$ (E) $\frac{3}{4}$

20. In an A.P., the first term is 2 and the sum of first five terms is 5. Then the 31st term is

(A) 13 (B) 17 (C) -13 (D) $\frac{27}{2}$ (E) $-\frac{27}{2}$

21. If a, b, c, d are in G.P., then $(a+b+c+d)^2$ is equal to

(A) $(a+b)^2+(c+d)^2+2(b+c)^2$ (B) $(a+b)^2+(c+d)^2+2(a+c)^2$

(C) $(a+b)^2+(c+d)^2+2(b+d)^2$ (D) $(a+b)^2+(c+d)^2+(b+c)^2$

(E) $(a+b)^2+(c+d)^2+(b-c)^2$

$\frac{5}{2}[4+4d]=5$

Space for rough work

$a_1 = 2$

$S_5 = 5$

$a_{31} = ?$

$d = \frac{1}{2}$

~~2+15~~

$a_n = a+(n-1)d$

$= 2+30d$

$= 2+30\times\frac{1}{2}$

$= 17$

$\frac{b}{a}$

$\frac{5}{2}\times 2[1+d]=5$

$10[1+d]=5$

$10+10d=5$

$10d=5$

$d=\frac{5}{10}$

$d=\frac{1}{2}$

$((a+b)+(c+d))^2$

$=(a+b)^2+2(a+b)(c+d)+(c+d)^2$

$=(a+b)^2+(c+d)^2+2(ac+ab+bc+b^2)$

$ac+ac+c^2+c^2$

$2+\frac{1}{2}=\frac{5}{2}$

$=(a+b)^2+(c+d)^2+2(c(ab)+(ab)+b^2)$

$2ac+2c^2$

$a^2+2ac+c^2$

$2a^2+4ac+2c^2$

$2(ac+c^2)$

$2c(a+c)$

$(a+c)^2$

$S_n = \frac{n}{2}[2a+(n-1)d]$

22. The sum of first n terms of the series $1+(1+x)y+(1+x+x^2)y^2+(1+x+x^2+x^3)y^3+\cdots$ is

(A) $\left(\frac{1}{1-x}\right)\left[\frac{1-y^n}{1-y}-y\left(\frac{1-x^n y^n}{1-xy}\right)\right]$ (B) $\left(\frac{1}{1-x}\right)\left[\frac{1-y^n}{1-y^2}-x\left(\frac{1-x^n y^n}{1-xy}\right)\right]$

(C) $\left(\frac{1}{1-x}\right)\left[\frac{1-y^n}{1-y}-x^2\left(\frac{1-x^n y^n}{1-xy}\right)\right]$ (D) $\left(\frac{1}{1-x}\right)\left[\frac{1-y^n}{1-y}-2x\left(\frac{1-x^n y^n}{1-xy}\right)\right]$

(E) $\left(\frac{1}{1-x}\right)\left[\frac{1-y^n}{1-y}-x\left(\frac{1-x^n y^n}{1-xy}\right)\right]$

23. If 3rd, 7th and 12th terms of an A.P. are three consecutive terms of a G.P., then the common ratio of the G.P. is

(A) $\frac{5}{4}$ (B) $\frac{9}{4}$ (C) $\frac{2}{9}$ (D) $\frac{1}{2}$ (E) $\frac{12}{7}$

24. If n is any positive integer, then $\frac{1}{2^n}\left({}^{2n}P_n\right)$ is equal to

(A) $2\cdot4\cdot6\cdots\cdot(2n)$ (B) $1\cdot2\cdot3\cdots\cdot n$ (C) $1\cdot3\cdot5\cdots\cdot(2n-1)$

(D) $1\cdot2\cdot3\cdots\cdot(3n)$ (E) $2\cdot4\cdot6\cdots\cdot(2n+2)$

Space for rough work

25. Which one of the following is true?

(A) $\left(1+\frac{1}{n}\right)^n < n^2$, n is a positive integer

(B) $\left(1+\frac{1}{n}\right)^n < 2$, n is a positive integer

(C) $\left(1+\frac{1}{n}\right)^n < n^3$, n is a positive integer

(D) $\left(1+\frac{1}{n}\right)^n > 2$, n is a positive integer

(E) $\left(1+\frac{1}{n}\right)^n = n^2+n+3$, n is a positive integer

$\left(\frac{n+1}{n}\right)^n$

$\frac{(n+1)^n}{n^n} < n^2$

26. When 2^{1505} is divided by 9, the remainder is

(A) 8 (B) 7 (C) 5 (D) 6 (E) 1

45
54
63

27. The term independent of x in the expansion of $\left(x+\frac{1}{x^2}\right)^6$ is

(A) 20 (B) 15 (C) 6 (D) 1 (E) 0

Space for rough work

$16 \times 2 = 32$

$9\overline{)32}$ 3, 27, 5

$2 \times 2 \times 2 \times 2 \times 2 \times 2$

$9\overline{)64}$

$2 \times 16 \times 2 = 64$

$32 \times 2 = 64$

$9\overline{)2^{1505}}$

28. If $C_0, C_1, C_2, C_3, \cdots$ are binomial coefficients in the expansion of $(1+x)^n$, then $\frac{C_0}{3} - \frac{C_1}{4} + \frac{C_2}{5} - \frac{C_3}{6} + \cdots$ is equal to

(A) $\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}$

(B) $\frac{1}{n+1} + \frac{2}{n+2} - \frac{1}{n+3}$

(C) $\frac{1}{n+2} - \frac{1}{n+1} + \frac{1}{n+3}$

(D) $\frac{2}{n+1} - \frac{1}{n+2} + \frac{2}{n+3}$

(E) $\frac{1}{n+2} - \frac{2}{n+1} + \frac{1}{n+3}$

29. There are 10 persons including 3 ladies. A committee of 4 persons including at least one lady is to be formed. The number of ways of forming such a committee is

(A) 160 (B) 170 (C) 180 (D) 175 (E) 155

30. The sum of coefficients in the expansion of $(1+3x-3x^2)^{1143}$ is equal to

(A) -1 (B) 0 (C) 1 (D) 2^{1143} (E) 2

31. The constant term in the expansion of $\left[1-(x-2)^2\right]^{10}$ is equal to

(A) 2^{10} (B) 6^{10} (C) 4^{10} (D) 5^{10} (E) 3^{10}

Space for rough work

$\sin^2\alpha \cdot \sin^2\alpha - \cos^2\alpha \cos^2\alpha = 0$

$1 -$

$\left[1-(x^2-4x+4)\right]^{10}$

$\left[-x^2+4x-3\right]^{10}$

$-x^2+1x+3x-3$

$x(-x+1)+3(-x+1)$

$(x-3)^{10}(-x+1)^{10}$

$6\overline{)180}$ 3

$\frac{\pi}{4}$ 45

$\frac{\pi}{6}$ 30

$\frac{\pi}{3}$ 60

32. If $e\begin{bmatrix} e^x & e^y \\ e^y & e^x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then the values of x and y are respectively

(A) $-1, -1$ (B) $1, 1$ (C) $0, 1$ (D) $1, 0$ (E) $0, 0$

33. If $A = \begin{bmatrix} \log x & -1 \\ -\log x & 2 \end{bmatrix}$ and if $\det(A) = 2$, then the value of x is equal to

(A) 2 (B) e^2 (C) -2 (D) e (E) $\log 2$

34. If $\begin{vmatrix} \sin^2\alpha & \cos^2\alpha \\ \cos^2\alpha & \sin^2\alpha \end{vmatrix} = 0$, $\alpha \in (0, \pi)$, then the values of α are

(A) $\frac{\pi}{2}$ and $\frac{\pi}{12}$ (B) $\frac{\pi}{2}$ and $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ and $\frac{3\pi}{4}$

(D) $\frac{\pi}{6}$ and $\frac{\pi}{3}$ (E) $\frac{\pi}{2}$ and $\frac{\pi}{3}$

35. If $A = \begin{bmatrix} a & x \\ y & a \end{bmatrix}$ and if $xy = 1$, then $\det(AA^T)$ is equal to

(A) $a^2 - 1$ (B) $(a^2+1)^2$ (C) $1 - a^2$ (D) $(a^2-1)^2$ (E) $(a-1)^2$

$(a^2+1)^2 = (a^2)^2 + 2a^2 + 1$

$= a^4$

1 2
2 1

$a^4 - 2a^2 + 1$

1 2
2 1

Space for rough work

$e\begin{bmatrix} e^{-1} & e^{-1} \\ e^{-1} & e^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$2\log x - \log x = 2$

$\log x = 2$

$\log e^2 = 2$

$A = \begin{bmatrix} a & x \\ y & a \end{bmatrix}$

$A^T = \begin{bmatrix} a & y \\ x & a \end{bmatrix}$

$e \times \frac{1}{e}$ $e\frac{1}{e}$

$(x+y)(x+y)$

$x^2 + xy + xy + y^2$

$\begin{bmatrix} 2 & -1 \\ -2 & 2 \end{bmatrix}$

$xy = 1$

$y = \frac{1}{x}$

$e \times \frac{1}{e}$ $e\frac{1}{e}$

$4 - 2 = 2$

$e^{\log 2}$

$\begin{bmatrix} a & x \\ y & a \end{bmatrix}\begin{bmatrix} a & y \\ x & a \end{bmatrix}$

$\begin{bmatrix} a^2+x^2 & ay+xa \\ ay+xa & y^2+a^2 \end{bmatrix}$

$(a^2+x^2)(y^2+a^2) - (ay+xa)(ay+xa)$

$(ay)^2 + 2ayxa + (xa)^2$

$a^2y^2 + a^2a^2 + x^2y^2 + a^2x^2 - \cancel{(ay)^2 + ayxa}$ $-2a^2xy - (xa)^2$

$a^2y^2 + a^4 + (xy)^2 + (ax)^2 - (ay)^2 - 2a^2 - (xa)^2$

$a^4 + 1 - 2a^2$

$a^2a^2 - 2a^2 + 1$

$a^2(a^2-2)+1$

36. If $f(x) = \begin{vmatrix} x & \lambda \\ 2\lambda & x \end{vmatrix}$, then $f(\lambda x) - f(x)$ is equal to

(A) $x(\lambda^2 - 1)$ (B) $2\lambda(x^2 - 1)$ (C) $\lambda^2(x^2 - 1)$ (D) $\lambda(x^2 - 1)$ (E) $x^2(\lambda^2 - 1)$

37. If $A=(a_{ij})_{m\times n}$, $B=(b_{ij})_{n\times p}$ and $C=(c_{ij})_{p\times q}$, then the product $(BC)A$ is possible only when

(A) $m = q$ (B) $n = q$ (C) $p = q$ (D) $m = p$ (E) $m = n$

38. If $\frac{2x+3}{5} < \frac{4x-1}{2}$, then x lies in the interval

(A) $\left[0, \frac{11}{16}\right)$ (B) $\left[\frac{11}{16}, \infty\right)$ (C) $\left(0, \frac{11}{16}\right)$ (D) $\left(-\infty, \frac{11}{16}\right)$ (E) $\left(\frac{11}{16}, \infty\right)$

39. If $7x - 2 < 4 - 3x$ and $3x - 1 < 2 + 5x$, then x lies in the interval

(A) $\left(\frac{3}{5}, \frac{3}{2}\right)$ (B) $\left(\frac{-3}{2}, \frac{3}{5}\right)$ (C) $\left[-\frac{3}{2}, \frac{3}{5}\right)$ (D) $\left[-\frac{3}{2}, \frac{3}{5}\right]$ (E) $\left(-\frac{3}{5}, \frac{3}{2}\right)$

40. The value of $\sqrt{2}\left(\cos 15^\circ - \sin 15^\circ\right)$ is equal to

(A) $\sqrt{3}$ (B) $\sqrt{2}$ (C) 1 (D) 2 (E) $2\sqrt{3}$

Space for rough work

$f(x) = \begin{vmatrix} x & \lambda \\ 2\lambda & x \end{vmatrix}$

$f(\lambda x) = \begin{vmatrix} \lambda x & \lambda \\ 2\lambda & \lambda x \end{vmatrix}$

$(\lambda x)^2 - 2\lambda^2 - (x^2 - 2\lambda^2)$

$\lambda^2 x^2 - \cancel{2\lambda^2} - x^2 + \cancel{2\lambda^2}$

$x^2(\lambda^2 - 1)$

$7x - 2 < 4 - 3x$

$7x + 3x < 4 + 2$

$10x < 6$

$x < \frac{6}{10}$

$x < \frac{3}{5}$

$3x - 1 < 2 + 5x$

$3x - 5x < 3$

$-2x < 3$

$x < \frac{3}{2}$

BC

$n \times p \times p \times q$

$n \times q$ $m \times n$

$2(2x+3) < 5(4x-1)$

$4x + 6 < 20x - 5$

$4x - 20x > -5 - 6$

$-16x > -11$

$x > \frac{11}{16}$

$p \wedge \sim q$

41. If p : It is snowing, q : I am cold, then the compound statement "It is snowing and it is not that I am cold" is given by

(A) $p \wedge (\sim q)$ (B) $p \wedge q$ (C) $(\sim p) \wedge q$

(D) $(\sim p) \wedge (\sim q)$ (E) $p \vee (\sim q)$

42. Which one of the following is not a statement?

(A) It is not that the sky is blue

(B) Is the sky blue?

(C) The sky is blue

(D) The sky is dark in the night

(E) The sky is not blue in the night

a and b

a or b

43. If p : The earth is round, q : $3 + 4 = 7$, then $(\sim p) \overset{\text{and}}{\vee} (\sim q)$ is

(A) It is not that the earth is round or 3+4 = 7

(B) The earth is round and 3+4 = 7

(C) It is not that the earth is round or it is not that 3+4 = 7

(D) The earth is round or 3+4 = 7

(E) The earth is round or it is not that 3+4 = 7

44. If $\cos x = -\frac{4}{5}$, where $x \in [0, \pi]$, then the value of $\cos\left(\frac{x}{2}\right)$ is equal to

(A) $\frac{1}{10}$ (B) $\frac{2}{5}$ (C) $\frac{1}{\sqrt{10}}$ (D) $-\frac{2}{5}$ (E) $-\frac{1}{\sqrt{10}}$

Space for rough work

$x = \cos$

45. The value of $\sec^2\theta + \text{cosec}^2\theta$ is equal to

(A) $\tan^2\theta + \cot^2\theta$ (B) $\sec^2\theta\ \text{cosec}^2\theta$ (C) $\sec\theta\ \text{cosec}\theta$

(D) $\sin^2\theta\ \cos^2\theta$ (E) $\sec^2\theta - \text{cosec}^2\theta$

46. If $x \in \left(\frac{\pi}{2}, \pi\right)$, then $\frac{\sec x - 1}{\sec x + 1}$ is equal to

(A) $(\text{cosec}\, x + \cot x)^2$ (B) $(\sin x - \cos x)^2$ (C) $(\text{cosec}\, x - \cot x)^2$

(D) $(\sec x + \tan x)^2$ (E) $(\sec x - \tan x)^2$

47. The value of $\sec\frac{2\pi}{3} + \text{cosec}\frac{5\pi}{6}$ is equal to

(A) 2 (B) -2 (C) 4 (D) -4 (E) 0

48. The value of $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right) + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is equal to

(A) $\tan^{-1}\left(\frac{5}{\sqrt{3}}\right)$ (B) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (C) $\tan^{-1}\left(\frac{1}{2}\right)$

(D) $\tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)$ (E) $\tan^{-1}\left(\frac{5}{2\sqrt{3}}\right)$

0 30 45

0 $\frac{1}{\sqrt{3}}$ 1 $\sqrt{3}$

Space for rough work

$\sec\left(\pi - \frac{\pi}{3}\right) + \text{cosec}\left(\pi - \frac{\pi}{6}\right)$

$\frac{1}{\cos\left(\pi - \frac{\pi}{3}\right)} + \frac{1}{\sin\left(\pi - \frac{\pi}{6}\right)}$

$\frac{1}{-\cos\frac{\pi}{3}} + \frac{1}{\sin\frac{\pi}{6}}$

$\frac{1}{-\frac{\sqrt{3}}{2}} + \frac{1}{\frac{\sqrt{3}}{2}}$

$-\frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}}$

$= \frac{0}{\sqrt{3}} = 0$

$\frac{1}{\cos^2\alpha} + \frac{1}{\sin^2\alpha}$

$\frac{\sin^2\alpha + \cos^2\alpha}{\sin^2\alpha \cdot \cos^2\alpha} = \frac{1}{\sin^2\alpha \cdot \cos^2\alpha}$

$= \text{cosec}^2\alpha\, \sec^2\alpha$

$\tan x = \frac{\sqrt{3}}{2}$ $\tan y = \frac{1}{\sqrt{3}}$

S A

$2\tan 60 = \sqrt{3}$

$\sqrt{3}\tan 45 = 1$

$2\tan 60 + \sqrt{3}\tan 45 =$

$\cos$

$\cos x =$ $\sin^{-1}\frac{2}{3} = \theta$

$\sin x = \frac{2}{3}$

49. The value of $\cos\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$ is equal to

(A) $\frac{\sqrt{3}}{5}$ (B) $\frac{5}{3}$ (C) $\frac{5}{\sqrt{3}}$ (D) $\sqrt{\frac{5}{3}}$ (E) $\frac{\sqrt{5}}{3}$

50. If $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{2}$, then the value of x is equal to

(A) $\frac{1}{\sqrt{6}}$ (B) $\frac{1}{6}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{3}$

51. If $\frac{\sin x}{\cos x} \times \frac{\sec x}{\operatorname{cosec} x} \times \frac{\tan x}{\cot x} = 9$, where $x \in \left(0, \frac{\pi}{2}\right)$, then the value of x is equal to

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) π (E) $\frac{\pi}{6}$

52. One of the principal solutions of $\sqrt{3}\sec x = -2$ is equal to

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{5\pi}{6}$ (D) $\frac{\pi}{3}$ (E) $\frac{\pi}{4}$

53. If the distance between the two points $(-1, a)$ and $(-1, -4a)$ is 10 units, then the values of a are

(A) ± 1 (B) ± 2 (C) ± 3 (D) ± 4 (E) ± 5

Space for rough work

$\cos(\sin^{-1}\frac{2}{3})$

$\cos$

$\tan^{-1} 2x = y$

$\tan^{-1} 3x$

$\tan y = 2x$

$\tan^{-1}(2x+3x)$

$\tan^{-1}(5x) = \frac{\pi}{2}$

$\tan\frac{\pi}{2} = 5x$

$0 = 5x$

$x = 0$

$x_2\, y_2$ $(-1, -4a)$

10 un

x_1, y_1 $(-1, a)$

$\sqrt{(-1-1)^2 + (-4a+a)^2} = 10$

$4 + 9a^2 = 10^2$

$4 + 9a^2 = 100$

$9a^2 = 96$

$a^2 = \frac{\cancel{96}\,32}{\cancel{9}\,3}$

$a^2 = \frac{4\sqrt{6}}{\sqrt{3}}$

$3|96$, $2|32$, $2|16$, $2|8$, $2|4$, 2

$\sqrt{96}$

$4\sqrt{6}$

$a = \frac{\sqrt{6}}{\sqrt{3}}$

54. If the slope of the line joining the points (3, 4) and (–2, a) is equal to $-\frac{2}{5}$, then the value of a is equal to

(A) 6 (B) 4 (C) 3 (D) 2 (E) 1

55. If the area of the triangle formed by (0, 0), (a, 0) and ($\frac{1}{2}$, a) is equal to $\frac{1}{2}$ square units, then the values of a are

(A) ± 2 (B) ± 3 (C) ± 1 (D) ± 4 (E) ± 5

56. The equation of the line perpendicular to the line $2x - 3y + 5 = 0$ and making an intercept 3 with y-axis is

(A) $3x + 2y - 6 = 0$ (B) $3x + 2y - 12 = 0$ (C) $3x - 2y - 6 = 0$

(D) $3x + 2y + 6 = 0$ (E) $3x + 2y - 5 = 0$

57. The perpendicular distance from the point (1, –1) to the line $x + 5y - 9 = 0$ is equal to

(A) $\sqrt{\frac{2}{13}}$ (B) $\sqrt{\frac{13}{2}}$ (C) $\frac{13}{2}$ (D) $\frac{2}{13}$ (E) $\frac{1}{\sqrt{13}}$

58. The angle between the lines $2x + 11y - 7 = 0$ and $x + 3y + 5 = 0$ is equal to

(A) $\tan^{-1}\frac{17}{31}$ (B) $\tan^{-1}\frac{11}{35}$ (C) $\tan^{-1}\frac{1}{7}$ (D) $\tan^{-1}\frac{33}{35}$ (E) $\tan^{-1}\frac{7}{33}$

Space for rough work

59. The distance between the parallel lines $5x-12y-14=0$ and $5x-12y+12=0$ is equal to

(A) $\frac{1}{13}$ (B) 2 (C) $\frac{2}{13}$ (D) 4 (E) $\frac{4}{13}$

60. Let C be a circle in the family of concentric circles $x^2+y^2=k^2$, where k is a parameter. If C passes through (1, 2), then the equation of C is

(A) $x^2+y^2=5$ (B) $x^2+y^2=25$ (C) $x^2+y^2=\sqrt{5}$

(D) $x^2+y^2=4$ (E) $x^2+y^2=2$

61. The equation of the circle with centre at (1, 1) and touching the line $3x+4y+3=0$ is

(A) $x^2+y^2-2x-2y+2=0$ (B) $x^2+y^2-2x-2y-2=0$

(C) $x^2+y^2+2x+2y+2=0$ (D) $x^2+y^2-2x-2y-4=0$

(E) $x^2+y^2+2x+2y+4=0$

62. If (4, 0) is a point on the circle $x^2+ax+y^2=0$, then the centre of the circle is at

(A) (– 2, 0) (B) (0, 2) (C) (2, 0) (D) (1, 0) (E) (3, 0)

Space for rough work

63. If (–2, 2) and (k, 0) are two diametrically opposite points of a circle of radius 1, then the equation of the circle is

(A) $x^2 + y^2 + 2x - 4y + 4 = 0$ (B) $x^2 + y^2 + 4x - 2y - 4 = 0$

(C) $x^2 + y^2 - 4x + 2y + 4 = 0$ (D) $x^2 + y^2 + 4x - 2y + 4 = 0$

(E) $x^2 + y^2 - 4x - 2y - 4 = 0$

64. If the ends of a focal chord of the parabola $y^2 = 8x$ are (x_1, y_1) and (x_2, y_2), then $x_1x_2 + y_1y_2$ is equal to

(A) 12 (B) 20 (C) 0 (D) – 12 (E) – 20

65. The eccentricity of the ellipse $12x^2 + 7y^2 = 84$ is equal to

(A) $\frac{\sqrt{5}}{7}$ (B) $\sqrt{\frac{5}{12}}$ (C) $\frac{\sqrt{5}}{12}$ (D) $\frac{5}{7}$ (E) $\frac{7}{12}$

66. If the eccentricity of a hyperbola is $\sqrt{2}$ and if the distance between the foci is 16, then its equation is

(A) $x^2 - y^2 = 4$ (B) $x^2 - y^2 = 8$ (C) $x^2 - y^2 = 24$

(D) $x^2 - y^2 = 32$ (E) $x^2 - y^2 = 64$

Space for rough work

67. If the equation $\dfrac{x^2}{9-c}+\dfrac{y^2}{5-c}=1$ represents an ellipse, then the foci are

(A) $(\pm 3,0)$ (B) $(\pm 2,3)$ (C) $(\pm 4,0)$ (D) $(\pm 2,1)$ (E) $(\pm 2,0)$

68. If the vectors $3\hat{i}-4\hat{j}-\hat{k}$ and $2\hat{i}+3\hat{j}-6\hat{k}$ represent the diagonals of a rhombus, then the length of the side of the rhombus is

(A) 15 (B) $15\sqrt{3}$ (C) $\dfrac{5\sqrt{3}}{2}$ (D) $\dfrac{15\sqrt{3}}{2}$ (E) $\dfrac{17\sqrt{3}}{2}$

69. If $\vec{a}=2\hat{i}+3\hat{j}+\alpha\hat{k}$ and $\vec{b}=3\hat{i}-\alpha\hat{j}+2\hat{k}$, then the angle between $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ is equal to

(A) 0 (B) $\dfrac{\pi}{6}$ (C) $\dfrac{\pi}{4}$ (D) $\dfrac{\pi}{3}$ (E) $\dfrac{\pi}{2}$

70. If $\vec{a}=2\hat{i}+2\hat{j}-\hat{k}$, $\vec{b}=\alpha\hat{i}+\beta\hat{j}+2\hat{k}$ and $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, then $\alpha+\beta$ is equal to

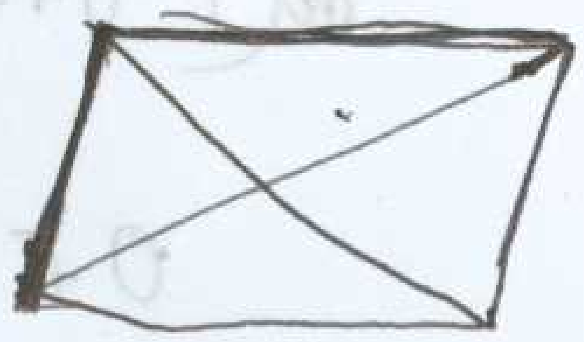
(A) 2 (B) 1 (C) 0 (D) –1 (E) –2

71. If the projection of $\vec{b}$ on $\vec{a}$ is twice the projection of $\vec{a}$ on $\vec{b}$, then $|\vec{b}|-|\vec{a}|$ is equal to

(A) $|\vec{a}-\vec{b}|$ (B) $|\vec{a}|+|\vec{b}|$ (C) $|\vec{b}|$ (D) $|\vec{a}|$ (E) 1

Space for rough work

$(2+\alpha)\hat{i}+(2+\beta)\hat{j}-1\hat{k} = (2-\alpha)\hat{i}+(2-\beta)\hat{j}+3\hat{k}$



$\sqrt{9+16+1}+\sqrt{4+9+36}$

$\sqrt{10+16}+\sqrt{49}$

~~$2(\vec{c}) = 2\hat{i}+3\hat{j}+$~~

$\sqrt{26}+7$

$2\,|\,26$
13

$6+12+16$

$26+1$

$\sqrt{26}+7$

$16+12 = 28+6 = 34$

$\sqrt{(2+\alpha)^2+(2+\beta)^2+(1)^2}=\sqrt{(2-\alpha)^2+(2-\beta)^2+9}$

$\cancel{4}+4\alpha+\cancel{\alpha^2}+\cancel{4}+4\beta+\cancel{\beta^2}+1 = \cancel{4}-4\alpha+\cancel{\alpha^2}+\cancel{4}-4\beta+\cancel{\beta^2}+9$

$4\alpha+4\beta+1=-4\alpha-4\beta+9$

$8\alpha+8\beta=8$ $\quad 8(\alpha+\beta)=8$

$\alpha+\beta=8/8$

$\alpha+\beta=1$

$-1+$

72. If $\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$, then $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2$ is equal to

(A) $\sqrt{2}$ (B) 2 (C) $\sqrt{6}$ (D) 4 (E) 6

73. If $|\vec{a}| = 1$, $|\vec{b}| = 3$ and $|\vec{a} - \vec{b}| = \sqrt{7}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(A) 0 (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$ (E) $\frac{\pi}{2}$

74. A vector of magnitude 7 units, parallel to the resultant of the vectors $\vec{a} = 2\hat{i} - 3\hat{j} - 2\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, is

(A) $\frac{7}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$ (B) $7(\hat{i} - \hat{j} - \hat{k})$ (C) $\frac{7}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

(D) $\frac{7}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$ (E) $7(\hat{i} + \hat{j} - \hat{k})$

75. The point which divides the line joining the points $(1, 3, 4)$ and $(4, 3, 1)$ internally in the ratio 2 : 1, is

(A) (2, –3, 3) (B) (2, 3, 3) (C) $\left(\frac{5}{2}, 3, \frac{5}{2}\right)$ (D) (–3, 3, 2) (E) (3, 3, 2)

Space for rough work

$$\begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = i(-1) - j(1) + k(1)$$

$= -i - j + k$

$= \left(\sqrt{1+1+1}\right)^2$

$= 3$

$(i - j + 0k)(0i + j + k)$

$0 - 1 + 0$

$7 = \cancel{(2\hat{i} - 3\hat{j}}$

$$\begin{vmatrix} i & j & k \\ 2 & -3 & -2 \\ -1 & 2 & 1 \end{vmatrix} = i(-3+4) - j(2-3) + k(4-3)$$

$= i + j + k$

76. The angle between the lines $\frac{x-7}{1}=\frac{y+3}{-5}=\frac{z}{3}$ and $\frac{2-x}{-7}=\frac{y}{2}=\frac{z+5}{1}$ is equal to

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$ (E) 0

77. The equation of the plane which is equidistant from the two parallel planes $2x-2y+z+3=0$ and $4x-4y+2z+9=0$ is

(A) $8x-8y+4z+15=0$ (B) $8x-8y+4z-15=0$

(C) $8x-8y+4z+3=0$ (D) $8x-8y+4z-3=0$

(E) $8x-8y+4z+4=0$

78. The angle between the planes $3x+4y+5z=3$ and $4x-3y+5z=9$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$ (E) $\frac{2\pi}{3}$

79. The vector equation of the plane through the point $(2,1,-1)$ and parallel to the plane $\vec{r}\cdot\left(\hat{i}+3\hat{j}-\hat{k}\right)=0$ is

(A) $\vec{r}\cdot\left(\hat{i}+9\hat{j}+11\hat{k}\right)=6$ (B) $\vec{r}\cdot\left(\hat{i}-9\hat{j}+11\hat{k}\right)=4$

(C) $\vec{r}\cdot\left(\hat{i}+3\hat{j}-\hat{k}\right)=6$ (D) $\vec{r}\cdot\left(\hat{i}+3\hat{j}-\hat{k}\right)=4$

(E) $\vec{r}\cdot\left(\hat{i}-3\hat{j}+\hat{k}\right)=6$

Space for rough work

80. If the foot of the perpendicular drawn from the point $(5,1,-3)$ to a plane is $(1,-1,3)$, then the equation of the plane is

(A) $2x+y-3z+8=0$
(B) $2x+y+3z+8=0$
(C) $2x-y-3z+8=0$
(D) $2x-y+3z+8=0$
(E) $2x+y-3z+6=0$

81. The equation of the plane through the line of intersection of the planes $x-y+z+3=0$ and $x+y+2z+1=0$ and parallel to x-axis is

(A) $2y-z=2$ (B) $2y+z=2$ (C) $4y+z=4$ (D) $4y-2z=3$ (E) $4y-z=4$

82. The equation of the straight line making angles 60°, 60° and 45° with positive direction of the coordinate axes and passing through the point $(2,1,-1)$ is

(A) $\sqrt{2}(x-2)=\sqrt{2}(y-1)=(z+1)$
(B) $(x-2)=\sqrt{2}(y-1)=(z+1)$
(C) $\sqrt{2}(x-2)=(y-1)=(z+1)$
(D) $(x-2)=\sqrt{2}(y-1)=\sqrt{2}(z+1)$
(E) $\sqrt{2}(x-2)=(y-1)=\sqrt{2}(z+1)$

83. If two numbers p and q are chosen randomly from the set $\{1,2,3,4\}$ with replacement, then the probability that $p^2 \geq 4q$ is equal to

(A) $\frac{1}{4}$ (B) $\frac{3}{16}$ (C) $\frac{1}{2}$ (D) $\frac{9}{16}$ (E) $\frac{7}{16}$

Space for rough work

84. A die is rolled three times. The probability that the sum of three numbers obtained is 15, is equal to

(A) $\frac{5}{108}$ (B) $\frac{5}{216}$ (C) $\frac{11}{216}$ (D) $\frac{7}{108}$ (E) $\frac{13}{216}$

85. The mean of five numbers is 0 and their variance is 2. If three of those numbers are -1, 1 and 2, then the other two numbers are

(A) -5 and 3 (B) -4 and 2 (C) -3 and 1 (D) -2 and 0 (E) -1 and -1

86. A batsman in his 16^{th} innings makes a score of 70 runs, and thereby increases his average by 2 runs. If he had never been 'not out', then his average after 16^{th} innings is

(A) 36 (B) 38 (C) 40 (D) 42 (E) 44

$a_{16} = 70$

87. The value of $\lim\limits_{x\to 0} \dfrac{\log(1+2x)}{x}$ is equal to

(A) 1 (B) 2 (C) 3 (D) $\frac{3}{2}$ (E) $\frac{1}{2}$

88. $\lim\limits_{x\to 0-} \dfrac{1}{3-2^{\frac{1}{x}}}$ is equal to

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{1}{3}$ (E) $-\infty$

Space for rough work

1, 2, 3, 4, 5, 6

~~6+3+5~~ =

6+5+4

~~5+3+5~~

5+5+5
6+5+4
4+6+5
5+6+4
5+4+6
6+4+5
4+5+6

89. When $x \geq 2$, the function $f(x) = 2|x-2| - |x+1| + x$ is reduced to

(A) $f(x) = -2x+3$ (B) $f(x) = 2x-5$ (C) $f(x) = 5$
(D) $f(x) = -1$ (E) $f(x) = -2x-5$

90. If the function $f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ cx+k, & \text{if } 1 < x < 4 \\ -2x, & \text{if } x \geq 4 \end{cases}$

is continuous everywhere, then the values of c and k are respectively

(A) $-3, -5$ (B) $-3, 5$ (C) $-3, -4$ (D) $-3, 4$ (E) $-3, 3$

91. If $y = 5^{\tan x}$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is equal to

(A) 5 log 5 (B) 10 log 5 (C) 0 (D) $(\log 5)^2$ (E) log 5

92. If $y = \sin^{-1}x$ and $z = \cos^{-1}\left(\sqrt{1-x^2}\right)$, then $\frac{dy}{dz}$ is equal to

(A) $\frac{x}{\sqrt{1-x^2}}$ (B) $\frac{1}{2}$ (C) $\frac{-x}{\sqrt{1-x^2}}$ (D) 1 (E) $\frac{-x}{2\sqrt{1-x^2}}$

Space for rough work

$f(x) = 2|1| - |4| + 3$
$= 2 - 4 + 3$
$= 5 - 4$
$= 1$

$\frac{d}{dx} 5^{\tan x}$

$\frac{dy}{dx} = 5 \log \tan x$

$\frac{dy}{dx} \tan$

$2^x = x \log 2$

$\frac{dy}{dx} = 5 \log 1$

$\tan x \log 5$

93. If $u = 2(t - \sin t)$ and $v = 2(1 - \cos t)$, then $\dfrac{dv}{du}$ at $t = \dfrac{2\pi}{3}$ is equal to

(A) $\sqrt{3}$ (B) $-\sqrt{3}$ (C) $2\sqrt{3}$ (D) $\dfrac{2}{\sqrt{3}}$ (E) $\dfrac{1}{\sqrt{3}}$

94. If $f(x) = \log\left[e^x \left(\dfrac{3-x}{3+x}\right)^{1/3}\right]$, then $f'(1)$ is equal to

(A) $\dfrac{3}{4}$ (B) $\dfrac{2}{3}$ (C) $\dfrac{1}{3}$ (D) $\dfrac{1}{2}$ (E) $\dfrac{1}{4}$

95. If $y = (\log x)^2$, then $\dfrac{dy}{dx}$ at $x = e$ is equal to

(A) 2 (B) $\dfrac{e}{2}$ (C) e (D) $\dfrac{2}{e}$ (E) $2e$

96. If $y^x = 2^x$, then $\dfrac{dy}{dx}$ is equal to

(A) $\dfrac{y}{x}\log\left(\dfrac{2}{y}\right)$ (B) $\dfrac{x}{y}\log\left(\dfrac{2}{y}\right)$ (C) $\dfrac{y}{x}\log\left(\dfrac{y}{2}\right)$

(D) $\dfrac{x}{y}\log\left(\dfrac{y}{2}\right)$ (E) $\dfrac{y}{x}\log(2y)$

Space for rough work

$\dfrac{d(\log x)^2}{dx}$ $\quad 2\log x \times \dfrac{1}{x}$

$\dfrac{dy}{dx}\ x=e$

$2\log e \times \dfrac{1}{e}$

y

$\dfrac{2}{e}$

97. If $x^2+2xy+2y^2=1$, then $\frac{dy}{dx}$ at the point where $y=1$ is equal to

(A) 1 (B) 2 (C) –1 (D) –2 (E) 0

98. The points on the graph $y=x^3-3x$ at which the tangent is parallel to x-axis are

(A) (2, 2) and (1, –2)
(B) (–1, 2) and (–2, –2)
(C) (2, 2) and (–1, 2)
(D) (–2, –2) and (2, 2)
(E) (1, –2) and (–1, 2)

99. The slope of the normal to the curve $y^3-xy-8=0$ at the point (0, 2) is equal to

(A) –3 (B) –6 (C) 3 (D) 6 (E) 8

100. If the straight line $y-2x+1=0$ is the tangent to the curve $xy+ax+by=0$ at $x=1$, then the values of a and b are respectively

(A) 1 and 2 (B) 1 and –1 (C) –1 and 2 (D) –1 and –2 (E) 1 and –2

101. If the angle between the curves $y=2^x$ and $y=3^x$ is α, then the value of $\tan\alpha$ is equal to

(A) $\frac{\log(3/2)}{1+(\log 2)(\log 3)}$ (B) $\frac{6}{7}$ (C) $\frac{1}{7}$

(D) $\frac{\log(6)}{1+(\log 2)(\log 3)}$ (E) 0

Space for rough work

$\cancel{x^2+2x+2=1}$

$x^2+2xy-1=-2y^2$

102. The function $f(x) = 3x^3 - 36x + 99$ is increasing for

(A) $-\infty < x < 2$ (B) $-2 < x < \infty$ (C) $-2 < x < 2$

(D) $x < -2$ or $x > 2$ (E) $-\infty < x < \infty$

103. Let $f(x) = x^3 - x + p \quad (0 \le x \le 2)$, where p is a constant. The value c of mean value theorem is

(A) $\dfrac{\sqrt{3}}{2}$ (B) $\dfrac{\sqrt{6}}{3}$ (C) $\dfrac{\sqrt{3}}{3}$ (D) $\dfrac{\sqrt{2}}{3}$ (E) $\dfrac{2\sqrt{3}}{3}$

104. The minimum value of the function $f(x) = \dfrac{1}{\sin x + \cos x}$ in the interval $\left[0, \dfrac{\pi}{2}\right]$ is

(A) $\dfrac{\sqrt{2}}{2}$ (B) $-\dfrac{\sqrt{2}}{2}$ (C) $\dfrac{2}{\sqrt{3}+1}$ (D) $-\dfrac{2}{\sqrt{3}+1}$ (E) 1

105. $\displaystyle\int \frac{5x\,dx}{(1-x)^3}$ is equal to

(A) $\dfrac{5}{2(x-1)^2} - \dfrac{5}{(x-1)} + C$ (B) $\dfrac{5}{2(x-1)^2} + \dfrac{5}{(x-1)} + C$

(C) $\dfrac{5}{3(x-1)^2} + \dfrac{5}{2(x-1)} + C$ (D) $\dfrac{5}{3(x-1)^2} - \dfrac{5}{2(x-1)} + C$

(E) $\dfrac{-5}{2(x-1)^2} + \dfrac{5}{(x-1)} + C$

Space for rough work

$\displaystyle\int \frac{5}{(1-x)^3}\,dx =$

106. $\int \frac{dx}{x-\sqrt{x}}$ is equal to

(A) $2\log|\sqrt{x}-1|+C$ (B) $2\log|\sqrt{x}+1|+C$ (C) $\log|\sqrt{x}-1|+C$

(D) $\frac{1}{2}\log|\sqrt{x}+1|+C$ (E) $\frac{1}{2}\log|\sqrt{x}-1|+C$

107. $\int \frac{dx}{4\sin^2 x+3\cos^2 x}$ is equal to

(A) $\frac{\sqrt{3}}{4}\tan^{-1}\left(\frac{2\tan x}{\sqrt{3}}\right)+C$ (B) $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{\tan x}{\sqrt{3}}\right)+C$

(C) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{3}}\right)+C$ (D) $\frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{\tan x}{\sqrt{3}}\right)+C$

(E) $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{3}}\right)+C$

108. $\int \frac{\sec x\, dx}{\sqrt{\cos 2x}}$ is equal to

(A) $2\sin^{-1}(\tan x)+C$ (B) $\tan^{-1}\left(\frac{\tan x}{2}\right)+C$

(C) $\sin^{-1}(\tan x)+C$ (D) $\frac{1}{2}\sin^{-1}(\tan x)+C$

(E) $\frac{1}{2}\tan^{-1}(2\tan x)+C$

Space for rough work

109. $\int \frac{e^x}{x}\left(x \log x + 1\right) dx$ is equal to

(A) $\frac{e^x}{x} + C$ (B) $xe^x \log|x| + C$ (C) $e^x \log|x| + C$

(D) $x\left(e^x + \log|x|\right) + C$ (E) $xe^x + \log|x| + C$

110. $\int \frac{1+\log x}{(1+x\log x)^2}\, dx$ is equal to

(A) $\frac{1}{1+x\log|x|} + C$ (B) $\frac{1}{1+\log|x|} + C$ (C) $\frac{-1}{1+x\log|x|} + C$

(D) $\log\left|\frac{1}{1+\log|x|}\right| + C$ (E) $\log\left|1+x\log|x|\right| + C$

111. $\int\left(1-\tan^2 x\right) dx$ is equal to

(A) $\tan x + C$ (B) $\sec x + C$ (C) $2x - \sec x + C$

(D) $x - \tan x + C$ (E) $2x - \tan x + C$

112. The value of $\int_0^6 |x-3|\, dx$ is equal to

(A) 6 (B) 0 (C) 12 (D) 18 (E) 9

Space for rough work

$\int dx - \int \tan^2 x$

x

$1 - \tan^2 x = \sec^2 x$

$\int \sec^2 x = \tan x + C$

113. If $f(x) = \int_{2x}^{\sin x} \cos(t^3)dt$, then $f'(x)$ is equal to

(A) $\cos(\sin^3 x)\cos x - 2\cos(8x^3)$ (B) $\sin(\sin^3 x)\sin x - 2\sin(8x^3)$

(C) $\cos(\cos^3 x)\cos x - 2\cos(x^3)$ (D) $\cos(\sin^3 x) - \cos(8x^3)$

(E) $\sin(\sin^3 x)\cos x - 2\sin(8x^3)$

114. $\int_0^{10} \frac{x^{10}}{(10-x)^{10} + x^{10}} dx$ is equal to

(A) 10 (B) 5 (C) 2 (D) $\frac{1}{2}$ (E) 0

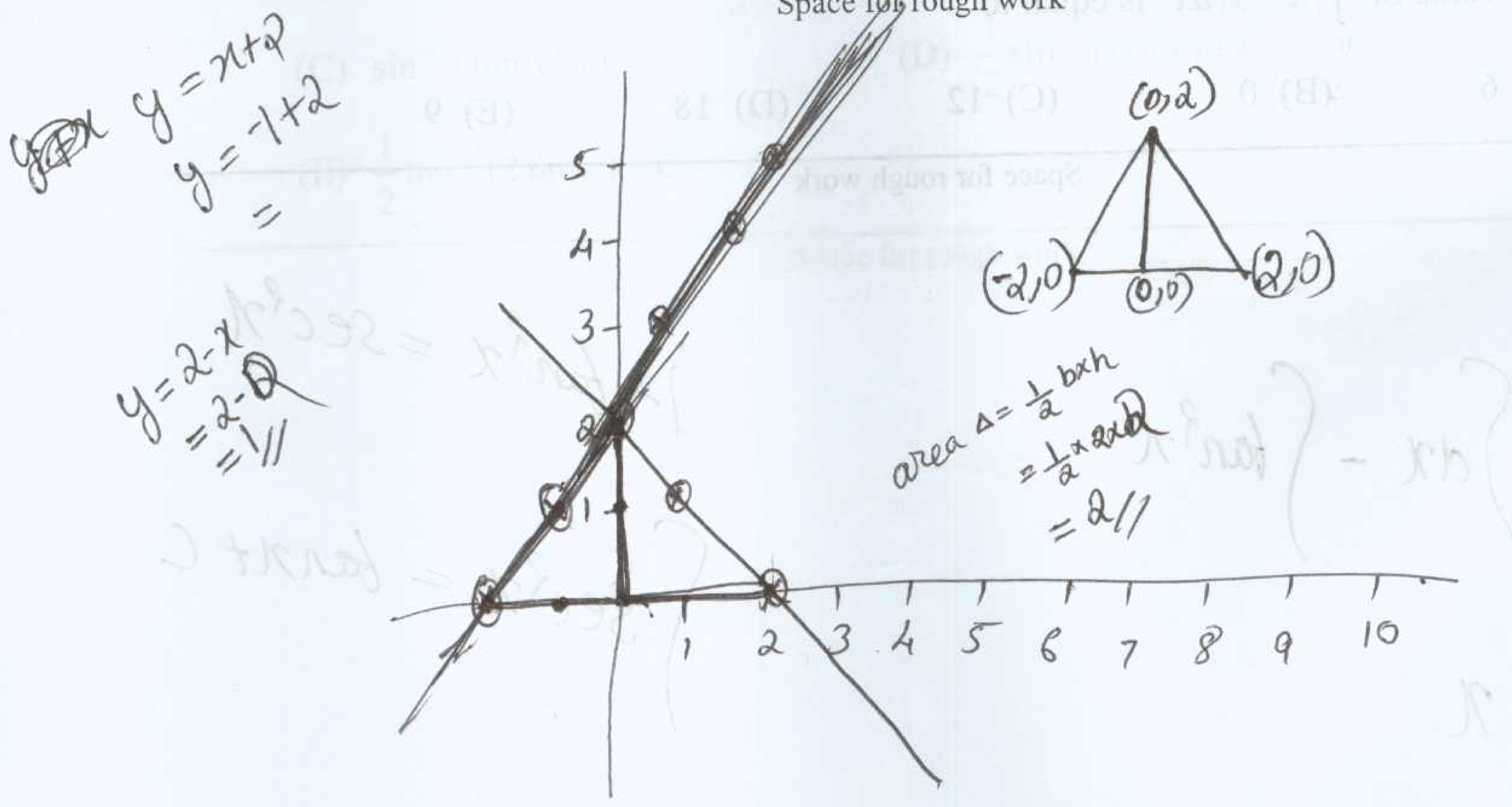
115. The area bounded by $y = x + 2$, $y = 2 - x$ and the x-axis is (in square units)

(A) 1 (B) 2 (C) 4 (D) 6 (E) 8

116. The value of $\int_0^1 \sqrt{x}\, e^{\sqrt{x}}\, dx$ is equal to

(A) $\frac{(e-2)}{2}$ (B) $2(e-2)$ (C) $2e-1$ (D) $2(e-1)$ (E) $\frac{e-1}{2}$

Space for rough work



117. The order and degree of the differential equation $\left(\dfrac{d^3y}{dx^3}\right)^{\frac{1}{3}} = 2\dfrac{d^2y}{dx^2} + \sqrt[3]{\cos^2 x}$ are, respectively

(A) 3 and 1 (B) 3 and 3 (C) 1 and 3 (D) 3 and 2 (E) 2 and 2

118. The differential equation representing the family of curves given by $y = ae^{-3x} + b$ where a and b are arbitrary constants, is

(A) $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} - 2y = 0$ (B) $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} = 0$ (C) $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} - 2y = 0$

(D) $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = 0$ (E) $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} = 0$

119. An integrating factor of the differential equation $xdy - ydx + x^2e^x dx = 0$ is

(A) $\dfrac{1}{x}$ (B) $\log\sqrt{1+x^2}$ (C) $\sqrt{1+x^2}$

(D) x (E) $\dfrac{1}{1+x^2}$

120. The solution of the differential equation $x\dfrac{dy}{dx} = \dfrac{y}{1+\log x}$ is

(A) $y = \log x + C$ (B) $y = \dfrac{C}{1+\log x}$ (C) $y = C(x + \log x)$

(D) $y = x + \log(Cx)$ (E) $y = C(1+\log x)$

Space for rough work

$\frac{dy}{dx} = \frac{y}{x x + x\log x}$

$\frac{dy}{dx} = \frac{y}{x(1+\log x)}$

$\frac{1}{y}dy = \frac{1}{x(1+\log x)}dx$