Assignments in Mathematics Class X (Term I)

1. REAL NUMBERS

IMPORTANT TERMS, DEFINITIONS AND RESULTS

- Given positive integers a and b, there exist unique integers q and r satisfying

  $a = bq + r$, $0 \leq r < b$. This result is known as **Euclid's division lemma.**

- An algorithm is a series of well defined steps which gives a procedure for solving a type of problem.

- A lemma is a proven statement used for proving another statement.

- HCF of two positive integers a and b is the largest positive integer d that divides both a and b.

- **Euclid's Division Algorithm :** To obtain the HCF of two positive integers, say c and d with $c > d$, we follow the steps below :

  Step 1. Apply Euclid's division lemma to find q and r where

  $c = dq + r$, $0 \leq r < d$.

  Step 2. If $r = 0$, then, d is the HCF of c and d. If $r \neq 0$, then apply Euclid's division lemma to d and r.

  Step 3. Continue this process till the remainder is zero. The divisor at this stage will be the required HCF.

- **The Fundamental Theorem of Arithmetic**: Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur. Or the prime factorisation of a natural number is unique, except for the order of its factors.

- Any number which cannot be expressed in the form $\frac{p}{q}$ where p, and q are integers and $q \neq 0$ is called an irrational number.

- Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

- The sum or difference of a rational and an irrational number is irrational.

- The product and quotient of a non-zero rational number and an irrational number is irrational.

- Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form $\frac{p}{q}$ where p and q are coprime and the prime factorisation of q is of the form $2^n 5^m$, where n and m are non negative integers.

- Let $x = \frac{p}{q}$ be a rational number such that the prime factorisation of q is of the form $2^n 5^m$, where n and m are non negative integers. Then, x has a decimal expansion which terminates.

- If $x = \frac{p}{q}$ is a rational number, such that the prime factorisation of q is of the form $2^m 5^n$, where m and n are whole numbers. If $m = n$, then the decimal expansion of x will terminate after m places of decimal. If $m > n$, then the decimal expansion of x will terminate after m places of decimal. If $n > m$, then the decimal expansion of x will terminate after n places of decimal.

- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form $2^n 5^m$, where n and m are non negative integers. Then x has a decimal expansion which is non terminating repeating (recurring).

- The decimal expansion of every rational number is either terminating or non-terminating repeating.

SUMMATIVE ASSESSMENT

MULTIPLE CHOICE QUESTIONS   [1 Mark]

A. Important Questions

1. Euclid's division algorithm can be applied to :
   (a) only positive integers
   (b) only negative integers
   (c) all integers
   (d) all integers except 0.

2. For some integer m, every even integer is of the form :
   (a) m (b) $m + 1$ (c) $2m$ (d) $2m + 1$

3. If the HCF of 65 and 117 is expressible in the form $65m - 117$, then the value of m is :
   (a) 1   (b) 2   (c) 3   (d) 4

4. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$, a; b being prime numbers, then LCM (p, q) is :
(a) ab (b) a^2b^2 (c) a^3b^2 (b) a^3b^3

5. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is :
(a) 10   (b) 100   (c) 504   (d) 2520

6. $7 \times 11 \times 13 \times 15 + 15$ is :
(a) composite number
(b) prime number
(c) neither composite nor prime
(d) none of these

7. $1.23\overline{48}$ is :
(a) an integer   (b) an irrational number
(c) a rational number   (d) none of these

8. $2.\overline{35}$ is :
(a) a terminating decimal
(b) a rational number
(c) an irrational number
(d) both (a) and (c)

9. 3.24636363... is :
(a) a terminating decimal number
(b) a non-terminating repeating decimal number
(c) a rational number
(d) both (b) and (c)

10. For some integer q, every odd integer is of the form :
(a) $2q$ (b) $2q + 1$ (c) q (d) $q + 1$

11. If the HCF of 85 and 153 is expressible in the form $85m - 153$, then the value of m is :
(a) 1   (b) 4   (c) 3   (d) 2

12. The decimal expansion of the rational number $\dfrac{47}{2^2.5}$ will terminate after :
(a) one decimal place   (b) three decimal places
(c) two decimal places
(d) more than 3 decimal places

13. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^2b$; a, b being prime numbers, then LCM (p, q) is :
(a) a^2b^2 (b) ab (c) ac^3b^3 (d) a^3b^2

14. Euclid's division lemma states that for two positive integers a and b, there exist unique integers q and r such that $a = bq + r$, where :
(a) $0 < r \le b$ (b) $1 < r < b$
(c) $0 < r < b$ (d) $0 \le r < b$

15. Following are the steps in finding the GCD of 21 and 333 :
$$333 = 21 \times m + 18$$
$$21 = 18 \times 1 + 3$$
$$n = 3 \times 6 + 0$$
The integers m and n are :
(a) $m = 15$, $n = 15$ (b) $m = 15$, $n = 18$
(c) $m = 15$, $n = 16$ (d) $m = 18$, $n = 15$

16. HCF and LCM of a and b are 19 and 152 respectively. If $a = 38$, then b is equal to :
(a) 152   (b) 19   (c) 38   (d) 76

17. $(n + 1)^2 - 1$ is divisible by 8, if n is :
(a) an odd integer   (b) an even integer
(c) a natural number   (d) an integer

18. The largest number which divides 71 and 126, leaving remainders 6 and 9 respectively is :
(a) 1750   (b) 13   (c) 65   (d) 875

19. If two integers a and b are written as $a = x^3y^2$ and $b = xy^4$; x, y are prime numbers, then H.C.F. (a, b) is :
(a) x^3y^3 (b) x^2y^2 (c) xy (d) xy^2

20. The decimal expansion of the rational number $\dfrac{14517}{1250}$ will terminate after :
(a) 4 decimal places
(b) 3 decimal places
(c) 2 decimal places
(d) 1 decimal place

B. Questions From CBSE Examination Papers

1. Euclid's division lemma states that if a and b are any two +ve integers, then there exist unique integers q and r such that : **[2010 (T-I)]**
(a) $a = bq + r$, $0 < r < b$
(b) $a = bq + r$, $0 \le r \le b$
(c) $a = bq + r$, $0 \le r < b$
(d) $a = bq + r$, $0 < b < r$

2. Which of the following rational numbers have a terminating decimal expansion ? **[2010 (T-I)]**

(a) $\dfrac{125}{441}$ (b) $\dfrac{77}{210}$

(c) $\dfrac{15}{1600}$ (d) $\dfrac{129}{2^2 \times 5^2 \times 7^2}$

3. According to Euclid's division algorithm, HCF of any two positive integers a and b with $a > b$ is obtained by applying Euclid's division lemma to a and b to find q and r such that $a = bq + r$, where r must satisfy : **[2010 (T-I)]**

(a) $1 < r < b$ (b) $0 < r < b$
(c) $0 \le r < b$ (d) $0 < r \le b$

4. The decimal expansion of $\dfrac{141}{120}$ will terminate after how many places of decimals ? **[2010 (T-I)]**
   (a) 1   (b) 2
   (c) 3   (d) will not terminate

5. Which of the following is not an irrational number ? **[2010 (T-I)]**
   (a) $5-\sqrt{3}$ (b) $\sqrt{5}+\sqrt{3}$ (c) $4+\sqrt{2}$ (d) $5+\sqrt{9}$

6. The value of x in the factor tree is : **[2010 (T-I)]**
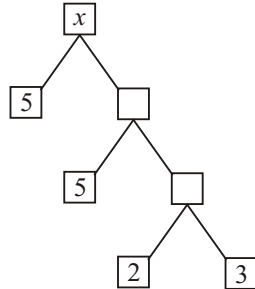
   (a) 30   (b) 150   (c) 100   (d) 50

7. Which of the following rational numbers have non terminating and repeating decimal expansion? **[2010 (T-I)]**
   (a) $\dfrac{15}{1600}$ (b) $\dfrac{17}{6}$ (c) $\dfrac{23}{8}$ (d) $\dfrac{35}{50}$

8. If two positive integers a and b are written as $a = x^2y^2$ and $b = xy^2$; x, y are prime number then HCF (a, b) is : **[2010 (T-I)]**
   (a) xy (b) xy^2 (c) x^2y^3 (d) x^2y^2

9. For the decimal number $0.\overline{6}$, the rational number is : **[2010 (T-I)]**
   (a) $\dfrac{33}{50}$ (b) $\dfrac{2}{3}$ (c) $\dfrac{111}{167}$ (d) $\dfrac{1}{3}$

10. A rational number can be expressed as a terminating decimal if the denominator has factors : **[2010 (T-I)]**
    (a) 2, 3 or 5   (b) 2 or 3
    (c) 3 or 5   (d) 2 or 5

11. Which of the following numbers has terminating decimal expansion ? **[2010 (T-I)]**
    (a) $\dfrac{37}{45}$ (b) $\dfrac{21}{2^3 5^6}$ (c) $\dfrac{17}{49}$ (d) $\dfrac{89}{2^2 3^2}$

12. $n^2 - 1$ is divisible by 8, if n is : **[2010 (T-I)]**
    (a) an integer   (b) a natural number
    (c) an odd integer   (d) an even integer

13. If p, q are two prime numbers, then LCM(p, q) is : **[2010 (T-I)]**

14. If p, q are two consecutive natural numbers, then HCF(p, q) is : **[2010 (T-I)]**
    (a) q (b) p (c) 1   (d) pq

15. $(\sqrt{2}-\sqrt{3})(\sqrt{3}+\sqrt{2})$ is : **[2010 (T-I)]**
    (a) A rational number
    (b) A whole number
    (c) An irrational number
    (d) A natural number

16. $(2+\sqrt{3})(2+\sqrt{5})$ is : **[2010 (T-I)]**
    (a) a rational number   (b) a whole number
    (c) an irrational number   (d) a natural number

17. Given that HCF (26, 91) = 13, then LCM of (26, 91) is : **[2010 (T-I)]**
    (a) 2366   (b) 182   (c) 91   (d) 364

18. Which of the following is a non-terminating repeating decimal ? **[2010 (T-I)]**
    (a) $\dfrac{35}{14}$ (b) $\dfrac{14}{35}$ (c) $\dfrac{1}{7}$ (d) $\dfrac{7}{8}$

19. If $x = 2^3 \times 3 \times 5^2, y = 2^2 \times 3^3$, then HCF (x, y) is : **[2010 (T-I)]**
    (a) 12   (b) 108   (c) 6   (d) 36

20. Given that HCF (253, 440) = 11 and LCM (253, 440) = 253 × R. The value of R is : **[2010 (T-I)]**
    (a) 400   (b) 40   (c) 440   (d) 253

21. The decimal expansion of the rational number $\dfrac{11}{2^3 . 5^2}$ will terminate after : **[2010 (T-I)]**
    (a) one decimal place
    (b) two decimal places
    (c) three decimal places
    (d) more than 3 decimal places

22. The decimal expansion of $\dfrac{7}{125}$ will terminate after how many places of decimal : **[2010 (T-I)]**
    (a) 1   (b) 2   (c) 3   (d) 4

23. How many prime factors are there in prime factorisation of 5005 ? **[2010 (T-I)]**
    (a) 2   (b) 4   (c) 6   (d) 7

24. If least prime factor of a is 3 and least prime factor of b is 7, the least prime factor of $(a + b)$ is: **[2010 (T-I)]**
    (a) 2   (b) 3   (c) 5   (d) 11

25. If a, b are coprime, then a^2, b^2 are : **[2010 (T-I)]**
    (a) Coprime   (b) Not coprime
    (c) Odd numbers   (d) Even numbers

(a) 1   (b) P (c) q (d) pq

26. $119^2 - 111^2$ is : **[2010 (T-I)]**
    (a) a prime number
    (b) a composite number
    (c) an odd prime number
    (d) an odd composite number

27. If n is any natural number, then which of the following expressions ends with 0 ? **[2010 (T-I)]**
    (a) $(3 \times 2)^n$ (b) $(4 \times 3)^n$
    (c) $(2 \times 5)^n$ (d) $(6 \times 2)^n$

28. The decimal expansion of the rational number $\dfrac{43}{2^4 \times 5^3}$ will terminate after : **[2010 (T-I)]**
    (a) 3 places (b) 4 places (c) 5 places (d) 1 place

29. The product of the HCF and LCM of the smallest prime number and smallest composite number is:
    (a) 2   (b) 4   (c) 6   (d) 8

30. The decimal expansion of $\dfrac{6}{1250}$ will terminate after how many places of decimal ? **[2010 (T-I)]**
    (a) 1   (b) 2   (c) 3   (d) 4

SHORT ANSWER TYPE QUESTIONS   [2 Marks]

A. Important Questions

1. Show that every positive even integer is of the from $2m$, and that every positive odd integer is of the form $2m + 1$, where m is some integer.

2. Show that any positive odd integer is of form $4m + 1$ or $4m + 3$, where m is some integer.

3. Show that any positive odd integer is of the form $6m + 1$, or $6m + 3$, or $6m + 5$, where m is some integer.

4. Find the HCF of 1656 and 4025 by Euclid's method.

5. Factorise 34650 using factor tree.

6. Find the HCF of 255 and 867 by prime factorisation.

7. Find the largest number which can divide 3528 and 2835.

8. Find the LCM of 2520 and 2268 by prime factorisation.

9. Find the smallest number which is divisible by 85 and 119.

10. Show that $5 - \sqrt{3}$ is irrational.

11. Show that $3\sqrt{2}$ is irrational.

12. Show that $\dfrac{1}{\sqrt{2}}$ is irrational.

13. Write the denominator of the rational number $\dfrac{257}{5000}$ in the form $2^m \times 5^n$, where m, n are non-negative integers. Hence, write its decimal expansion, without actual division.

14. The values of the remainder r, when a positive integer a is divided by 3, are 0 and 1 only. Is it true? Justify your answer.

15. Two tankers contain 850 litres and 680 litres of petrol respectively. Find the maximum capacity of a container which can measure the petrol of either tanker in exact number of times.

16. Show that the sum and product of two irrational numbers $(5 + \sqrt{2})$ and $(5 - \sqrt{2})$ are rational numbers.

17. Without actually performing the long division, find if $\dfrac{987}{10500}$ will have terminating or non-terminating repeating decimal expansion. Give reason for your answer.

18. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

19. Show that any positive integer is of the form $3q$ or $3q + 1$ or $3q + 2$ for some integer q.

B. Questions From CBSE Examination Papers

1. Can the number 6^n, n being a natural number, end with the digit 5? Give reasons. **[2010 (T-I)]**

2. Use Euclid's division lemma to show that square of any positive integer is either of form $3m$ or $3m + 1$ for some integer m. **[2010 (T-I)]**

3. Find the L.C.M. of 120 and 70 by fundamental theorem of Arithmetic. **[2010 (T-I)]**

4. In the given factor tree, find the numbers m, n :

[2010 (T-I)]

4
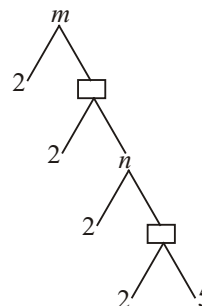
5. Without actually performing the long division, state whether the following number has a terminating decimal expansion or non terminating recurring decimal expansion $\dfrac{543}{225}$. **[2010 (T-I)]**

6. Use Euclid's division algorithm to find HCF of 870 and 225. **[2010 (T-I)]**

7. Check whether 6^n can end with the digit 0, for any natural number n. **[2010 (T-I)]**

8. Explain why $11 \times 13 \times 15 \times 17 + 17$ is a composite number. **[2010 (T-I)]**

9. Show that every positive even integer is of the form $2q$ and that every positive odd integer is of the form $2q + 1$, where q is some integer. **[2010 (T-I)]**

10. Check whether 15^n can end with digit zero for any natural number n. **[2010 (T-I)]**

11. Find the LCM of 336 and 54 by prime factorisation method. **[2010 (T-I)]**

12. Find the LCM and HCF of 120 and 144 by fundamental theorem of arithmetic. **[2010 (T-I)]**

13. Use Euclid's Lemma to show that square of any positive integer is of form $4m$ or $4m+1$ for some integer m. **[2010 (T-I)]**

14. Using fundamental theorem of arithmetic, find the HCF of 26, 51 and 91. **[2010 (T-I)]**

SHORT ANSWER TYPE QUESTIONS [3 Marks]

A. Important Questions

1. Prove that one of every three consecutive positive integers is divisible by 3.

2. Prove that $n^2 - n$ is divisible by 2 for every positive integer n.

3. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

4. Prove that $\sqrt{p} + \sqrt{q}$ is irrational where, p, q are primes.

5. A baker has 444 sweet biscuits and 276 salty biscuits. He wants to stack them in such a way that each stack has the same number and same type of biscuits and they take up the least area of the tray. What is the number of biscuits that can be placed in each stack for this purpose?

6. On a morning walk, three boys step off together and their steps measure 45 cm, 40 cm and 42 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

7. If a is a non-zero rational and $\sqrt{b}$ is irrational, then show that $a\sqrt{b}$ is an irrational.

8. Find the largest number which can divide 1001, 1287 and 1573.

9. Find the LCM of 693, 495 and 297.

10. Find the smallest number divisible by 115, 138 and 161.

11. Find the smallest number which when divided by 161, 207 and 184 leaves remainder 21 in each case.

12. Find the largest number that divides 1251, 9377 and 15628 leaving remainders 1, 2 and 3 respectively.

B. Questions From CBSE Examination Papers

1. Show that any positive odd integer is of the form $6q+1$ or $6q+3$, where q is a positive integer. **[2010 (T-I)]**

2. Prove that $(5-\sqrt{3})$ is an irrational number. **[2010 (T-I)]**

3. Show that any positive odd integer is of the form $4q + 1$ or $4q + 3$, where q is a positive integer. **[2010 (T-I)]**

4. Prove that $\dfrac{5\sqrt{2}}{3}$ is an irrational number. **[2010 (T-I)]**

5. Prove that $\sqrt{2}$ is an irrational number. **[2010 (T-I)]**

6. Show that the square of any positive odd integer is of the form $8m + 1$, for some integer m. **[2010 (T-I)]**

7. Prove that $\sqrt{7}$ is an irrational number. **[2010 (T-I)]**

8. Use Euclid's division algorithm to find the HCF of 10224 and 9648. **[2010 (T-I)]**

9. Prove that $3\sqrt{5} - 2$ is an irrational number. **[2010 (T-I)]**

10. Prove that $2\sqrt{3} - 4$ is an irrational number.
[2010 (T-I)]

11. A merchant has 120 litres of oil of one kind, 180 litres of another kind and 240 litres of third kind. He wants to sells the oil by filling, the three kinds of oil in tins of equal capacity. What should be the greatest capacity of such a tin? **[2010 (T-I)]**

12. Prove that $\dfrac{1}{2+\sqrt{3}}$ is an irrational number.
[2010 (T-I)]

13. Show that 9^n can't end with 2 for any integer n.
[2010 (T-I)]

14. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march? **[2010 (T-I)]**

15. Find the HCF and LCM of 306 and 54. Verify that HCF × LCM = Product of the two numbers.
[2010 (T-I)]

16. Use Euclid division lemma to show that cube of any positive integer is either of the form 9*m,* 9*m* + 1, or 9*m* + 8. **[2010 (T-I)]**

17. Show that the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for any integer q. **[2010 (T-I)]**

18. Show that any positive even integer is of the form 6*m,* 6*m* + 2 or 6*m* + 4. where *m* is some integer. **[2010 (T-I)]**

19. Show that 5^n can't end with the digit 2 for any natural number *n*. **[2010 (T-I)]**

20. Prove that $\sqrt{5}$ is an irrational number.
[2010 (T-I)]

21. Show that any positive odd integer is of the form $6p+1$, $6p+3$ or $6p+5$, where p is some integer. **[2010 (T-I)]**

22. Prove that $\dfrac{1}{2+\sqrt{3}}$ is an irrational number.
[2010 (T-I)]

23. Find the LCM and HCF of 15, 18, 45 by the prime factorisation method. **[2010 (T-I)]**

24. Prove that $2+3\sqrt{2}$ is irrational. **[2010 (T-I)]**

25. Prove that $\dfrac{7}{5}\sqrt{2}$ is not a rational number.
[2010 (T-I)]

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FORMATIVE ASSESSMENT

INVESTIGATION -1

(i) Express $\dfrac{1}{9}$ as a decimal.

(ii) Use the result from (i) to express $0.\overline{7}$ as a fraction.

(iii) Try to find a rule for expressing any recurring decimal (in which only one digit repeats) as a fraction.

(iv) Express $\dfrac{1}{99}$ as a decimal.

(v) Use the result from (iv) to express $0.\overline{21}$ as a fraction.

(vi) Find a rule for expressing any recurring decimal (in which only two digits repeat) as a fraction.

(vii) Express $\dfrac{1}{999}$ as a decimal.

(viii) Use the result from (vii) to express $0.\overline{035}$ as a fraction.

(ix) Find a rule for expressing any recurring decimal (in which three digits repeat) as a fraction.

(x) Now, find a rule for expressing any recurring decimal as a fraction and verify your rule with recurring decimals in which four or five digits repeat.

INVESTIGATION -2

Use the digits 2 and 8 to make two distinct 2-digit numbers. These numbers are 28 and 82. Now, find the difference of squares of these numbers.

$$
\begin{aligned}
82^2 - 28^2 &= 6724 - 784 = 5940 \\
&= 11 \times 540 \\
&= 99 \times 60 \\
&= 99 \times 10 \times 6 = 99 \times (8+2)\,(8-2) \\
&= 99 \times \text{sum of the digits} \\
&\qquad\qquad \times \text{difference of the digits.}
\end{aligned}
$$

Taking the digits as *x* and *y* prove that

$$(xy)^2 - (yx)^2 = 99(x+y)(x-y)$$

[**Note:** Here, *xy* is a 2-digit number, not the product of *x* and *y.*]

Try the above with other pairs of digits and verify that it always works.

INVESTIGATION -3

Multiply 123456789 by 4 and then multiply the result by 9. What do you observe?

Repeat the above step by multiplying first by a different number less than 9.

6

Do this twice.

Did you find a rule for predicting the answer when 123456789 is multiplied by one of the numbers 2,3,4,5,6,7,8 or 9 and the result is multiplied by 9. If yes, write the rule and check its validity.

TIME-DISTANCE INVESTIGATION

The distance between two places X and Y is 10 km. The distance between X and Z is x km.

Divya cycled from X to Y at 4 km/h.

Anya cycled from X to Y at 7 km/h. Divya left X at 8 am and Anya left X at 8:30 am. Anya crossed Divya at Z.

Express the time taken by Divya to reach Z in terms of x. Express the time taken by Anya to reach Z in terms of x.

Using the information, form an equation and find when and where the two girls meet.

FINDING ANYONE'S AGE

Mayank wanted to know his grandmother's age without asking her. He applied the following trick to know his grandmother's age. He asked his grandmother to do the following calculations;

Think of your age and add 10. Double your answer. Now subtract your age from the answer. Finally, tell the final answer.

His grandmother replied that her answer was 96. Mayank immediately replied that you must be 76 years old.

Investigate, how did Mayank work out his grandmother's age.

Form an algebraic equation which shows that the method will always work with anyone's age.

TWINS, TRIPLETS, QUADS

Find the number which when multiplied by 75 gives 7575.

Find the prime number whose product with any 2-digit number xy gives $xyxy$.

Find the number which when multiplied by 84 gives 848484.

Find four prime numbers whose product with any 2-digit number xy gives $xyxyxy$.

Find the number whose product with 21 gives 21212121.

Find three prime numbers whose product with any 2-digit number xy gives $xyxyxyxy$.

ABC OF NUMBERS

Find numbers a, b and c such that
$$a^b \times c^a = abca$$

A MATHEMATICAL GAME

This is a number game for two players. Player-1 calls a number from 1 to 10. Then player-2 increases this number by any number from 1 to 10. They take turns by always increasing the last number called, by any number from 1 to 10. Here the aim of the game is to reach 100. i.e., the player who gets 100 first will win the game. Investigate and device a winning strategy.

AMAZING NUMBERS

The number 59 has a special property that when it is divided by :

6, the remainder is 5
5, the remainder is 4
4, the remainder is 3
3, the remainder is 2
2, the remainder is 1

Can you find the next number having this property?

Find two numbers less than 10,000 with the property that on division by, 10, 9, 8, 7, 6, 5, 4, 3 and 2 the remainder is always 1 less than the divisor.

WHERE IS THE FALLACY?

Let $x = y$ be any non-zero number

$\Rightarrow x \times x = xy$ [Multiplying both sides by x]

$\Rightarrow x^2 - y^2 = xy - y^2$
                    [Subtracting y^2 from both sides]

$\Rightarrow (x + y)(x - y) = y(x - y)$

$\Rightarrow x + y = y$ [Dividing both sides by the
                                    common factors $(x - y)$]

$\Rightarrow 2x = x$ [As $x = y$]

$\Rightarrow 2 = 1$

Investigate and trace the fallacy in the argument.

PUZZLE

Three cartons contain stationery items, one has pens, one has pencil while the third has pens and pencils. These cartons are labelled as 'pens' 'pencils' and pens and pencils, but none of the labels is on the correct carton. You are allowed to select only one item from one carton and then tell which label should go on which carton.

Exercise 1.1

Question 1:

Use Euclid's division algorithm to find the HCF of:

(i)   135 and 225   (ii)   196 and 38220   (iii)   867 and 255

Answer:

(i) 135 and 225

Since 225 > 135, we apply the division lemma to 225 and 135 to obtain

$225 = 135 \times 1 + 90$

Since remainder 90 ≠ 0, we apply the division lemma to 135 and 90 to obtain

$135 = 90 \times 1 + 45$

We consider the new divisor 90 and new remainder 45, and apply the division lemma to obtain

$90 = 2 \times 45 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 45,

Therefore, the HCF of 135 and 225 is 45.

(ii)196 and 38220

Since 38220 > 196, we apply the division lemma to 38220 and 196 to obtain

$38220 = 196 \times 195 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

(iii)867 and 255

Since 867 > 255, we apply the division lemma to 867 and 255 to

obtain

867 = 255 × 3 + 102

Since remainder 102 ≠ 0, we apply the division lemma to 255 and 102

to obtain

255 = 102 × 2 + 51

We consider the new divisor 102 and new remainder 51, and apply the

division lemma to obtain

102 = 51 × 2 + 0

Since the remainder is zero, the process stops.

Since the divisor at this stage is 51,

Therefore, HCF of 867 and 255 is 51.

Question 2:

Show that any positive odd integer is of the form $6q+1$, or $6q+3$, or

$6q+5$, where *q* is some integer.

Answer:

Let *a* be any positive integer and *b* = 6. Then, by Euclid's algorithm,

a = 6*q* + *r*for some integer *q* ≥ 0, and *r* = 0, 1, 2, 3, 4, 5 because 0 ≤

r < 6.

Therefore, *a* = 6*q* or 6*q* + 1 or 6*q* + 2 or 6*q* + 3 or 6*q* + 4 or 6*q* + 5

Also, 6*q* + 1 = 2 × 3*q* + 1 = $2k_1$ + 1, where k_1 is a positive integer

$6q + 3 = (6q + 2) + 1 = 2 (3q + 1) + 1 = 2k_2 + 1$, where k_2 is an integer

$6q + 5 = (6q + 4) + 1 = 2 (3q + 2) + 1 = 2k_3 + 1$, where k_3 is an integer

Clearly, $6q + 1, 6q + 3, 6q + 5$ are of the form $2k + 1$, where k is an integer.

Therefore, $6q + 1, 6q + 3, 6q + 5$ are not exactly divisible by 2.

Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$,

or $6q + 3$,

or $6q + 5$

Question 3:

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer:

HCF (616, 32) will give the maximum number of columns in which they can march.

We can use Euclid's algorithm to find the HCF.

$616 = 32 × 19 + 8$

$32 = 8 × 4 + 0$

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

Question 4:

Use Euclid's division lemma to show that the square of any positive integer is either of form 3m or 3m + 1 for some integer m.

[**Hint:** Let x be any positive integer then it is of the form 3q, 3q + 1 or 3q + 2. Now square each of these and show that they can be rewritten in the form 3m or 3m + 1.]

Answer:

Let a be any positive integer and b = 3.

Then a = 3q + r for some integer q ≥ 0

And r = 0, 1, 2 because 0 ≤ r < 3

Therefore, a = 3q or 3q + 1 or 3q + 2

Or,

$$a^2 = \left(3q\right)^2 \text{ or } \left(3q+1\right)^2 \text{ or } \left(3q+2\right)^2$$
$$a^2 = \left(9q^2\right) \text{ or } 9q^2 + 6q + 1 \text{ or } 9q^2 + 12q + 4$$
$$= 3\times\left(3q^2\right) \text{ or } 3\left(3q^2 + 2q\right)+1 \text{ or } 3\left(3q^2 + 4q + 1\right)+1$$
$$= 3k_1 \quad \text{or} \quad 3k_2 + 1 \quad \text{or} \quad 3k_3 + 1$$

Where k_1, k_2, and k_3 are some positive integers

Hence, it can be said that the square of any positive integer is either of the form 3m or 3m + 1.

Question 5:

Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Answer:

Let a be any positive integer and b = 3

$a = 3q + r$, where $q \geq 0$ and $0 \leq r < 3$

$\therefore a = 3q$ or $3q+1$ or $3q+2$

Therefore, every number can be represented as these three forms.

There are three cases.

Case 1: When $a = 3q$,

$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m,$

Where m is an integer such that $m = 3q^3$

Case 2: When $a = 3q + 1$,

$a^3 = (3q +1)^3$

$a^3 = 27q^3 + 27q^2 + 9q + 1$

$a^3 = 9(3q^3 + 3q^2 + q) + 1$

$a^3 = 9m + 1$

Where m is an integer such that $m = (3q^3 + 3q^2 + q)$

Case 3: When $a = 3q + 2$,

$a^3 = (3q +2)^3$

$a^3 = 27q^3 + 54q^2 + 36q + 8$

$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$

$a^3 = 9m + 8$

Where m is an integer such that $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.

Exercise 1.2

Question 1:

Express each number as product of its prime factors:

(i)   140   (ii)   156   (iii)   3825   (iv)   5005   (v)   7429

Answer:

(i) $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

(ii) $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

(iii) $3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$

(iv) $5005 = 5 \times 7 \times 11 \times 13$

(v) $7429 = 17 \times 19 \times 23$

Question 2:

Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i)   26 and 91   (ii)   510 and 92   (iii)   336 and 54

Answer:

(i)   26 and 91

$26 = 2 \times 13$

$91 = 7 \times 13$

$HCF = 13$

$LCM = 2 \times 7 \times 13 = 182$

Product of the two numbers $= 26 \times 91 = 2366$

$HCF \times LCM = 13 \times 182 = 2366$

Hence, product of two numbers = HCF × LCM

(ii)   510 and 92

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

$HCF = 2$

$LCM = 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

Product of the two numbers $= 510 \times 92 = 46920$

$HCF \times LCM = 2 \times 23460$

$\qquad\qquad = 46920$

Hence, product of two numbers = HCF × LCM

(iii)   336 and 54

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$336 = 2^4 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3$

$54 = 2 \times 3^3$

$HCF = 2 \times 3 = 6$

$LCM = 2^4 \times 3^3 \times 7 = 3024$

Product of the two numbers $= 336 \times 54 = 18144$

$HCF \times LCM = 6 \times 3024 = 18144$

Hence, product of two numbers = HCF × LCM

Question 3:

Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i)   12, 15 and 21   (ii)   17, 23 and 29   (iii)   8, 9 and 25

Answer:

(i)   12,15 and 21

$12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

$HCF = 3$

$LCM = 2^2 \times 3 \times 5 \times 7 = 420$

(ii)   17,23 and 29

$17 = 1 \times 17$

$23 = 1 \times 23$

$29 = 1 \times 29$

$HCF = 1$

$LCM = 17 \times 23 \times 29 = 11339$

(iii) 8,9 and 25

$8 = 2 \times 2 \times 2$

$9 = 3 \times 3$

$25 = 5 \times 5$

$HCF = 1$

$LCM = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

Question 4:

Given that HCF (306, 657) = 9, find LCM (306, 657).

Answer:

$$\text{HCF}(306, 657) = 9$$

We know that, $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$

$$\therefore \text{LCM} \times \text{HCF} = 306 \times 657$$

$$\text{LCM} = \frac{306 \times 657}{\text{HCF}} = \frac{306 \times 657}{9}$$

$$\text{LCM} = 22338$$

Question 5:

Check whether 6^n can end with the digit 0 for any natural number n.

Answer:

If any number ends with the digit 0, it should be divisible by 10 or in other words, it will also be divisible by 2 and 5 as $10 = 2 \times 5$

Prime factorisation of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorisation of 6^n.

Hence, for any value of n, 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n.

Question 6:

Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Answer:

Numbers are of two types - prime and composite. Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1) = 13 \times (77 + 1)$

$= 13 \times 78$

$= 13 \times 13 \times 6$

The given expression has 6 and 13 as its factors. Therefore, it is a composite number.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$= 5 \times (1008 + 1)$

$= 5 \times 1009$

1009 cannot be factorised further. Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

Question 7:

There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Answer:

It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.

18 = 2 ✖ 3 ✖ 3

And, 12 = 2 ✖ 2 ✖ 3

LCM of 12 and 18 = 2 × 2 × 3 × 3 = 36

Therefore, Ravi and Sonia will meet together at the starting pointafter

36 minutes.

Exercise 1.3

Question 1:

Prove that $\sqrt{5}$ is irrational.

Answer:

Let $\sqrt{5}$ is a rational number.

Therefore, we can find two integers a, b ($b \neq 0$) such that $\sqrt{5} = \dfrac{a}{b}$

Let a and b have a common factor other than 1. Then we can divide them by the common factor, and assume that a and b are co-prime.

$a = \sqrt{5}b$

$a^2 = 5b^2$

Therefore, a^2 is divisible by 5 and it can be said that a is divisible by 5.

Let $a = 5k$, where k is an integer

$(5k)^2 = 5b^2$

$b^2 = 5k^2$ This means that b^2 is divisible by 5 and hence, b is divisible by 5.

This implies that a and b have 5 as a common factor.

And this is a contradiction to the fact that a and b are co-prime.

Hence, $\sqrt{5}$ cannot be expressed as $\dfrac{p}{q}$ or it can be said that $\sqrt{5}$ is irrational.

Question 2:

Prove that $3 + 2\sqrt{5}$ is irrational.

Answer:

Let $3+2\sqrt{5}$ is rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$3+2\sqrt{5} = \frac{a}{b}$$

$$2\sqrt{5} = \frac{a}{b} - 3$$

$$\sqrt{5} = \frac{1}{2}\left(\frac{a}{b} - 3\right)$$

Since a and b are integers, $\frac{1}{2}\left(\frac{a}{b} - 3\right)$ will also be rational and

therefore, $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption

that $3+2\sqrt{5}$ is rational is false. Therefore, $3+2\sqrt{5}$ is irrational.

Question 3:

Prove that the following are irrationals:

(i) $\dfrac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6+\sqrt{2}$

Answer:

(i) $\dfrac{1}{\sqrt{2}}$

Let $\dfrac{1}{\sqrt{2}}$ is rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$

$$\sqrt{2} = \frac{b}{a}$$

$\frac{b}{a}$ is rational as a and b are integers.

Therefore, $\sqrt{2}$ is rational which contradicts to the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is false and $\frac{1}{\sqrt{2}}$ is irrational.

(ii)　　$7\sqrt{5}$

Let $7\sqrt{5}$ is rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$7\sqrt{5} = \dfrac{a}{b}$ for some integers a and b

$\therefore \sqrt{5} = \dfrac{a}{7b}$

$\frac{a}{7b}$ is rational as a and b are integers.

Therefore, $\sqrt{5}$ should be rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Therefore, our assumption

that $7\sqrt{5}$ is rational is false. Hence, $7\sqrt{5}$ is irrational.

(iii)　　$6 + \sqrt{2}$

Let $6 + \sqrt{2}$ be rational.

Therefore, we can find two integers *a*, *b* (*b* ≠ 0) such that

$$6 + \sqrt{2} = \frac{a}{b}$$

$$\sqrt{2} = \frac{a}{b} - 6$$

Since *a* and *b* are integers, $\frac{a}{b} - 6$ is also rational and hence, $\sqrt{2}$ should be rational. This contradicts the fact that $\sqrt{2}$ is irrational. Therefore, our assumption is false and hence, $6 + \sqrt{2}$ is irrational.

Exercise 1.4

Question 1:

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\dfrac{13}{3125}$ (ii) $\dfrac{17}{8}$ (iii) $\dfrac{64}{455}$ (iv) $\dfrac{15}{1600}$

(v) $\dfrac{29}{343}$ (vi) $\dfrac{23}{2^3 5^2}$ (vii) $\dfrac{129}{2^2 5^7 7^5}$ (viii) $\dfrac{6}{15}$

(ix) $\dfrac{35}{50}$ (x) $\dfrac{77}{210}$

Answer:

(i) $\dfrac{13}{3125}$

$3125 = 5^5$

The denominator is of the form 5^m.

Hence, the decimal expansion of $\dfrac{13}{3125}$ is terminating.

(ii) $\dfrac{17}{8}$

$8 = 2^3$

The denominator is of the form 2^m.

Hence, the decimal expansion of $\dfrac{17}{8}$ is terminating.

(iii) $\dfrac{64}{455}$

455 = 5 × 7 × 13

Since the denominator is not in the form $2^m × 5^n$, and it also contains

7 and 13 as its factors, its decimal expansion will be non-terminating

repeating.

(iv) $\dfrac{15}{1600}$

$1600 = 2^6 × 5^2$

The denominator is of the form $2^m × 5^n$.

Hence, the decimal expansion of $\dfrac{15}{1600}$ is terminating.

(v) $\dfrac{29}{343}$

$343 = 7^3$

Since the denominator is not in the form $2^m × 5^n$, and it has 7 as its

factor, the decimal expansion of $\dfrac{29}{343}$ is non-terminating repeating.

(vi) $\dfrac{23}{2^3 × 5^2}$

The denominator is of the form $2^m × 5^n$.

Hence, the decimal expansion of $\dfrac{23}{2^3 × 5^2}$ is terminating.

(vii) $\dfrac{129}{2^2 × 5^7 × 7^5}$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 7 as

its factor, the decimal expansion of $\dfrac{129}{2^2 \times 5^7 \times 7^5}$ is non-terminating

repeating.

(viii) $\dfrac{6}{15} = \dfrac{2 \times 3}{3 \times 5} = \dfrac{2}{5}$

The denominator is of the form 5^n.

Hence, the decimal expansion of $\dfrac{6}{15}$ is terminating.

(ix) $\dfrac{35}{50} = \dfrac{7 \times 5}{10 \times 5} = \dfrac{7}{10}$

$10 = 2 \times 5$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\dfrac{35}{50}$ is terminating.

(x) $\dfrac{77}{210} = \dfrac{11 \times 7}{30 \times 7} = \dfrac{11}{30}$

$30 = 2 \times 3 \times 5$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 3 as

its factors, the decimal expansion of $\dfrac{77}{210}$ is non-terminating repeating.

Question 2:

Write down the decimal expansions of those rational numbers in

Question 1 above which have terminating decimal expansions.

Answer:

(i) $\dfrac{13}{3125} = 0.00416$

$$\begin{array}{r}
0.00416 \\
3125{\overline{\smash{\big)}\,13.00000}} \\
\underline{0} \\
130 \\
\underline{0} \\
1300 \\
\underline{0} \\
13000 \\
\underline{12500} \\
5000 \\
3125 \\
\underline{} \\
18750 \\
\underline{18750} \\
\times
\end{array}$$

(ii) $\dfrac{17}{8} = 2.125$

$$\begin{array}{r}
2.125 \\
8{\overline{\smash{\big)}\,17}} \\
\underline{16} \\
10 \\
\underline{8} \\
20 \\
\underline{16} \\
40 \\
\underline{40} \\
\times
\end{array}$$

(iv) $\dfrac{15}{1600} = 0.009375$

$$\begin{array}{r} 0.009375 \\ 1600\overline{)15.000000} \\ 0 \\ \hline 150 \\ 0 \\ \hline 1500 \\ 0 \\ \hline 15000 \\ 14400 \\ \hline 6000 \\ 4800 \\ \hline 12000 \\ 11200 \\ \hline 8000 \\ 8000 \\ \hline \times \end{array}$$

(vi) $\dfrac{23}{2^3 \times 5^2} = \dfrac{23}{200} = 0.115$

$$\begin{array}{r} 0.115 \\ 200\overline{)23.000} \\ 0 \\ \hline 230 \\ 200 \\ \hline 300 \\ 200 \\ \hline 1000 \\ 1000 \\ \hline \times \end{array}$$

$$\begin{array}{r} 0.4 \\ 5\overline{)2.0} \\ 0 \\ \hline 20 \\ 20 \\ \hline \times \end{array}$$

(viii) $\dfrac{6}{15} = \dfrac{2 \times 3}{3 \times 5} = \dfrac{2}{5} = 0.4$

(ix) $\dfrac{35}{50} = 0.7$

$$\begin{array}{r} 0.7 \\ 50\overline{)35.0} \\ 0 \\ \hline 350 \\ 350 \\ \hline \times \end{array}$$

Question 3:

The following real numbers have decimal expansions as given below.

In each case, decide whether they are rational or not. If they are

rational, and of the form $\dfrac{p}{q}$, what can you say about the prime factor

of q?

(i) 43.123456789 (ii) 0.120120012000120000… (iii) $43.\overline{123456789}$

Answer:

 (i) 43.123456789

Since this number has a terminating decimal expansion, it is a rational

number of the form $\dfrac{p}{q}$ and q is of the form $2^m \times 5^n$

i.e., the prime factors of q will be either 2 or 5 or both.

(ii) 0.120120012000120000 …

The decimal expansion is neither terminating nor recurring. Therefore, the given number is an irrational number.

(iii) $43.\overline{123456789}$

Since the decimal expansion is non-terminating recurring, the given

number is a rational number of the form $\dfrac{p}{q}$ and q is not of the form

$2^m \times 5^n$ i.e., the prime factors of q will also have a factor other than 2 or 5.