3. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

IMPORTANT TERMS, DEFINITIONS AND RESULTS

- An equation which can be put in the form $ax + by + c = 0$, where a, b and c are real numbers and a and b are not both zero, is called a linear equation in two variables x and y.
- Every solution of the equation $ax + by + c = 0$ is a point on the line representing it. Or each solution (x, y), of a linear equation in two variables $ax + by + c = 0$, corresponds to a point on the line representing the equation and vice-versa.
- A linear equation in two variables has an infinite number of solutions.
- If we consider two equations of the form $a_1x + b_1y + c_1 = 0,\ a_2x + b_2y + c_2 = 0$, a pair of such equations is called a system of linear equations.
- We have three types of systems of two linear equations.
  - (i) **Independent System**, which has a unique solution. Such system is termed as a consistent system with unique solution.
  - (ii) **Inconsistent System**, which has no solution.
  - (iii) **Dependent System**, which represents a pair of equivalent equations and has an infinite number of solutions. Such system is also termed as a consistent system with infinite solutions.
- A pair of linear equations in two variables which has a common point, i.e., which has only one solution is called a consistent pair of linear equations.
- A pair of linear equations in two variables which has no solution, i.e., the lines are parallel to each other is called an inconsistent pair of linear equations.
- A pair of linear equations in two variables which are equivalent and has infinitely many solutions are called dependent pair of linear equations. Note that a dependent pair of linear equations is always consistent with infinite number of solutions.
- If a pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ represents
  - (i) intersecting lines, then $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$
  - (ii) parallel lines, then $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$
  - (iii) coincident lines, then $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

  Converse of the above statement is also true.
- **Graphical Method of Solving a Pair of Linear Equations**
  - (a) To solve a system of two linear equations graphically :
    - (i) Draw graph of the first equation.
    - (ii) On the same pair of axes, draw graph of the second equation.
  - (b) After representing a pair of linear equations graphically, only one of the following three possibilities can happen :
    - (i) The two lines will intersect at a point.
    - (ii) The two lines will be parallel.
    - (iii) The two lines will be coincident.
  - (c) (i) If the two lines intersect at a point, read the coordinates of the point of intersection to obtain the solution and verify your answer.
    - (ii) If the two lines are parallel, i.e., there is no point of intersection, write the system as inconsistent. Hence, no solution.
    - (iii) If the two lines have the same graph, then write the system as consistent with infinite number of solutions.
- **Algebraic Methods of Solving a Pair of Linear Equations**

(a) Substitution Method :

(i) Suppose we are given two linear equations in x and y. For solving these equations by the substitution method, we proceed according to the following steps :

Step 1. Express y in terms of x in one of the given equations.

Step 2. Substitute this value of y in terms of x in the other equation. This gives a linear equation in x.

Step 3. Solve the linear equation in x obtained in step 2.

Step 4. Substitute this value of x in the relation taken in step 1 to obtain a linear equation in y.

Step 5. Solve the above linear equation in y to get the value of y.

Note : We may interchange the role of x and y in the above method.

(ii) While solving a pair of linear equations, if we get statements with no variables, we conclude as below.

(a) If the statement is true, we say that the equations have infinitely many solutions.

(b) If the statement is false, we say that the equations have no solution.

(iii) When the two given equations in x and y are such that the coefficients of x and y in one equation are interchanged in the other, then we add and subtract the two equations to get a pair of very simple equations.

(b) Elimination Method :

In this method, we eliminate one of the variables and proceed using the following steps.

Step 1. Multiply the given equations by suitable numbers so as to make the coefficients of one of the variables equal.

Step 2. If the equal coefficients are opposite in sign, then add the new equations Otherwise, subtract them.

Step 3. The resulting equation is linear in one variable. Solve it to get the value of one of the unknown quantities.

Step 4. Substitute this value in any of the given equations.

Step 5. Solve it to get the value of the other variable.

(c) Cross Multiplication Method :

(i) The system of two linear equations

$a_1x+b_1y+c_1=0, a_2x+b_2y+c_2=0$, where

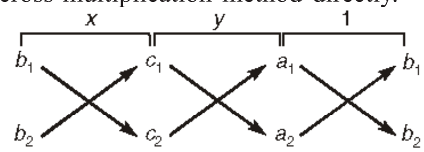
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ has a unique solution, given by

$$x=\frac{(b_1c_2-b_2c_1)}{(a_1b_2-a_2b_1)}, \quad y=\frac{(c_1a_2-c_2a_1)}{(a_1b_2-a_2b_1)}$$

We generally write it as

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

The following diagram will help to apply the cross-multiplication method directly.



The arrows between the numbers indicate that they are to be multiplied. The products with upward arrows are to be subtracted from the products with downward arrows.

(ii) The system of equations

$$a_1x+b_1y+c_1=0 \quad \text{...(i)}$$

$$a_2x+b_2y+c_2=0 \quad \text{...(ii)}$$

(a) is consistent with unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, i.e., lines represented by equations (i) and (ii) intersect at a point.

(b) is inconsistent, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, i.e., lines represented by equations (i) and (ii) are parallel and non coincident.

(c) is consistent with infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, i.e., lines represented by equations (i) and (ii) are coincident.

SUMMATIVE ASSESSMENT

MULTIPLE CHOICE QUESTIONS **[1 Mark]**

A. Important Questions

1. In the equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the equations will represent :

(a) coincident lines
(b) parallel lines
(c) intersecting lines
(d) none of the above

2. If a pair of linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$, represents parallel lines, then :

(a) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (b) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (d) none of these

3. If a pair of linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ represents coincident lines, then :

(a) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (d) none of these

4. The pair of equations $5x-15y=8$ and $3x-9y=\frac{24}{5}$ has :
(a) one solution
(b) two solutions
(c) infinitely many solutions
(d) no solution

5. Graphically, the pair of equations $6x-3y+10=0$ and $2x-y+9=0$ represents two lines which are:
(a) intersecting at exactly one point
(b) intersecting at exactly two points
(c) coincident
(d) parallel

6. A pair of linear equations which has a unique solution $x = 2, y = -3$ is :
(a) $x + y = -1, 2x - 3y = -5$
(b) $2x + 5y = -11, 4x + 10y = -22$
(c) $2x - y = 1, 3x + 2y = 0$
(d) $x - 4y - 14 = 0, 5x - y - 13 = 0$

7. The solution of the pair of equations $3x - y = 5$ and $x + 2y = 4$ is :
(a) $x = 1, y = 2$ (b) $x = 2, y = 1$
(c) $x = 2, y = 2$ (d) $x = 1, y = 1$

8. For what value of *k,* do the equations $3x - y + 8 = 0$ and $6x - ky = -16$, represent coincident lines ?
(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) –2

9. The pair of equations $x = 0$ and $x = 7$ has :
(a) two solutions (b) no solution
(c) infinitely many solutions
(d) one solution

10. If the lines given by $2x + 5y + a = 0$ and $3x + 2ky = b$ are coincident, then the value of *k* is :
(a) $\frac{15}{4}$ (b) $\frac{3}{2}$ (c) $\frac{-5}{4}$ (d) $\frac{2}{5}$

11. Solution of the system : $17x + 9y = 31$, $9x + 11y = 29$ is :
(a) $x = -2, y = -1$ (b) $x = -2, y = 1$
(c) $x = 2, y = 1$ (d) $x = 2, y = -1$

12. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages, in years, of the son and the father respectively are :
(a) 4 and 24 (b) 6 and 36
(c) 5 and 30 (d) 7 and 42

13. The pair of equations $x + 2y + 5 = 0$ and $3x - 6y + 1 = 0$ have :
(a) infinitely many solutions
(b) no solution
(c) a unique solution (d) exactly two solutions

14. Graphical representation of a system of linear equations $ax + by + c = 0$, $ex + fy = g$, is not intersecting lines. Also, $\frac{g}{c} \neq \frac{f}{b}$. What type of solution does the system have ?
(a) unique solution (b) infinite solution
(c) no solution
(d) solution cannot be determined

15. The pair of equations $x = a$ and $y = b$ graphically representing lines which are :
(a) coincident (b) intersecting at (a, b)
(c) parallel (d) intersecting at (b, a)

16. The value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 4$ will have infinitely many solutions is :
(a) –3 (b) 3 (c) –12 (d) 12

17. The sum of the digits of a two-digit number is 9. If 27 is added to it, digits of the number get reversed. The number is :
(a) 63 (c) 72 (c) 81 (d) 36

18. The value of *k* for which the system : $4x + 2y = 3$, $(k - 1)x - 6y = 9$ has no unique solution is :
(a) –13 (b) 9 (c) –11 (d) 13

19. For what value of *k*, do the equations $6x - ky = -16$ and $3x - y + 8 = 0$ represent coincident lines ?
(a) 2 (b) –2 (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

20. Arun has only Rs 2 and Re 1 coins with him. If the total number of coins that he has is 50 and the amount of money with him is Rs 80, then the number of Rs 2 and Re 1 coins respectively are,.
(a) 15 and 35 (b) 20 and 30
(c) 40 and 10 (d) 30 and 20

21. A pair of linear equations $a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$ is said to be inconsistent, if :

(a) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (b) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (d) $\frac{a_1}{a_2} \neq \frac{c_1}{c_2}$

22. The lines $2x - 3y = 1$ and $x + 3y = 5$ meet at :
(a) $x = 1, y = 2$ (b) $x = 2, y = -1$
(c) $x = 2, y = 1$ (d) $x = -2, y = 1$

B. Questions From CBSE Examination Papers

1. The number of solutions of the pair of linear equations $x + 2y - 8 = 0$ and $2x + 4y = 16$ is : **[2010 (T-I)]**

(a) 0 (b) 1
(c) infinitely many (d) none of these

2. The graphical representation of the pair of equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ represents : **[2010 (T-I)]**

(a) intersecting lines (b) parallel lines
(c) coincident lines (d) all of these

3. If a pair of linear equations is consistent, then the lines will be : **[2010 (T-I)]**

(a) parallel
(b) always coincident
(c) intersecting or coincident
(d) always intersecting

4. The condition so that the pair of linear equations $kx + 3y + 1 = 0$, $2x + y + 3 = 0$ has exactly one solution is : **[2010 (T-I)]**

(a) $k = 6$ (b) $k \neq 6$ (c) $k = 3$ (d) $k \neq 3$

5. The lines representing the linear equations $2x - y = 3$ and $4x - y = 5$: **[2010 (T-I)]**

(a) intersect at a point
(b) are parallel
(c) are coincident
(d) intersect at exactly two points

6. The pair of linear equations $2x + 5y = -11$ and $5x + 15y = -44$ has : **[2010 (T-I)]**

(a) many solutions (b) no solution
(c) one solution (d) two solutions

7. The pair of equations $y = 0$ and $y = -7$ has : **[2010 (T-I)]**

(a) one solution
(b) two solutions
(c) infinitely many solutions
(d) no solution

8. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is : **[2010 (T-I)]**

(a) –5/4 (b) 2/5 (c) 15/4 (d) 3/2

9. The pair of linear equations $8x - 5y = 7$ and $5x - 8y = -7$ have : **[2010 (T-I)]**

(a) one solution (b) two solutions
(c) no solution (d) many solutions

10. The pair of linear equations $x - 2y = 0$ and $3x + 4y = 20$ have : **[2010 (T-I)]**

(a) one solution (b) two solutions
(c) many solutions (d) no solution

11. The pair of linear equations $kx + 2y = 5$ and $3x + y = 1$ has unique solution, if : **[2010 (T-I)]**

(a) $k = 6$ (b) $k \neq 6$
(c) $k = 0$ (d) k has any value

12. One equation of a pair of dependent linear equations is $-5x + 7y = 2$, the second equation can be : **[2010 (T-I)]**

(a) $10x + 14y + 4 = 0$
(b) $-10x = 14y + 4 - 0$
(c) $-10x + 14y + 4 = 0$
(d) $10x - 14y = -4$

13. The value of k for which the pair of equations : $kx - y = 2$ and $6x - 2y = 3$ has a unique solution is : **[2010 (T-I)]**

(a) $k = 3$ (b) $k \neq 3$ (c) $k \neq 0$ (d) $k = 0$

14. The value of k for which the pair of linear equations $4x + 6y - 1 = 0$ and $2x + ky - 7 = 0$ represents parallel lines is : **[2010 (T-I)]**

(a) $k = 3$ (b) $k = 2$ (c) $k = 4$ (d) $k = -2$

15. If $x = a$, $y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then the values of a and b respectively are : **[2010 (T-I)]**

(a) 3 and 5 (b) 5 and 3
(c) 3 and 1 (d) –1 and –3

16. The pair of linear equations $3x + 4y + 5 = 0$ and $12x + 16y + 15 = 0$ has :

(a) unique solution (b) many solutions
(c) no solution (d) exactly two solutions

17. Which of the following pairs of equations represent inconsistent system ? **[2010 (T-I)]**

(a) $3x - 2y = 8$, $2x + 3y = 1$
(b) $3x - y = -8$, $3x - y = 24$
(c) $lx - y = m$, $x + my = l$
(d) $5x - y = 10$, $10x - 2y = 20$

18. Which of the following is not a solution of the pair of equations $3x - 2y = 4$ and $6x - 4y = 8$? **[2010 (T-I)]**

(a) $x = 2$, $y = 1$ (b) $x = 4$, $y = 4$
(c) $x = 6$, $y = 7$ (d) $x = 5$, $y = 3$

19. The pair of linear equations $2x + 5y = 3$ and $6x + 15y = 12$ represent : **[2010 (T-I)]**

(a) intersecting lines (b) parallel lines
(c) coincident lines (d) none of these

SHORT ANSWER TYPE QUESTIONS [2 Marks]

A. Important Questions

1. Do the equations $x + 2y + 2 = 0$ and $\frac{x}{2} \neq \frac{y}{4} - 1 = 0$ represent a pair of intersecting lines? Justify your answer.

2. Is the pair of equations $3x + 6y - 9 = 0$ and $x + 2y - 3 = 0$ consistent? Justify your answer.

3. Do the equations $2x + 4y = 3$ and $12x + 6y = 6$ represent a pair of parallel lines? Justify your answer.

4. For the pair of equations $\lambda x + 3y = -7$, $2x + 6y = 14$, to have infinitely many solutions, the value of λ should be 1. Is this statement true? Give reasons.

5. Find the value of k for which the following system of equations have infinitely many solutions.
$2x - 3y = 7$, $(k + 2)x - (2k + 1)y = 3(2k - 1)$

Solve the following pairs of linear equations by substitution method.

6. $3x - 5y - 4 = 0$, $9x - 2y = 7$

7. $3x - 5y = 20$, $6x - 10y = 40$

8. $\frac{x}{2} + \frac{2y}{3} = -1, \quad x - \frac{y}{3} = 3$

9. $x - 3y - 3 = 0$, $3x - 9y - 2 = 0$

Solve the following pairs of linear equations by elimination method.

10. $\frac{3x}{2} - \frac{5y}{3} = -2, \quad \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

11. $3x - y = 3$, $9x - 3y = 9$

12. $0.2x + 0.3y = 1.3$, $0.4x + 0.5y = 2.3$

13. $x - 3y - 7 = 0$, $3x - 9y - 15 = 0$

Solve the following pairs of linear equations by cross multiplication method.

14. $\sqrt{2}x + \sqrt{3}y = 0, \sqrt{3}x - \sqrt{8}y = 0$

15. $4x - 2y = 10$, $2x - y = 5$

16. $x - y = 3$, $\frac{x}{3} + \frac{y}{2} = 6$

17. $3x + 2y = 8$, $6x + 4y = 10$

18. Determine the value of k for which the following pair of linear equations has unique solution : $(k+1)x + 5y = 4; \quad 3x + 5y = 7$.

19. Determine the value of k for which the following pair of linear equations is consistent : $2x + (k-1)y = 5; \quad 3x + 6y = 5$

20. Determine the value of k for which the following pair of linear equations has no solution : $(k+1)x - (2k+1)y = 4; \quad 6x - 10y = 7$

21. Determine the value of k for which the following pair of linear equations is inconsistent : $4x - (k+2)y = 5; \quad 2x - (k-1)y = 5.$

22. Determine the value of k for which the following pair of linear equations represents a pair of parallel lines on the graph. $(3k-1)x + (k-1)y = 5; \quad (k+1)x + y = 3.$

23. Determine the value of k for which the following pair of linear equations is consistent (dependent) : $2x - 3y = 1; \quad -4x + (k+2)y = -2.$

24. Determine the value of k for which the following pair of linear equations represents a pair of coincident lines on the graph : $3x + 4y = k + 1; \quad 6x + 8y = 10.$

25. For which value(s) of k, do the pair of linear equations : $kx + y = k^2$ and $x + ky = 1$ have a unique solution ?

26. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that geometrical representation of the pair so formed is intersecting lines.

27. Find the relation between a, b, c and d for which the equations $ax + by = c$ and $cx + dy = a$ have a unique solution.

B. Questions From CBSE Examination Papers

1. For which value of k will the following pair of linear equations have no solution? **[2010 (T-I)]**
$3x + y = 1; \ (2k-1)x + (k-1)y = 2k+1$

2. For what value of k, $2x + 3y = 4$ and $(k + 2)x + 6y = 3k + 2$ will have infinitely many solutions. **[2010 (T-I)]**

3. Solve : $47x + 31y = 63$, $31x + 47y = 15$. **[2010 (T-I)]**

4. Solve : $\frac{3}{x} - 5y + 1 = 0, \ \frac{2}{x} - y + 3 = 0.$ **[2010 (T-I)]**

5. Solve : $x + \frac{6}{y} = 6, \ 3x - \frac{8}{y} = 5.$

6. For which values of p does the pair of equations given below has unique solution ? **[2010 (T-I)]**
$4x + py + 8 = 0; \ 2x + 2y + 2 = 0$

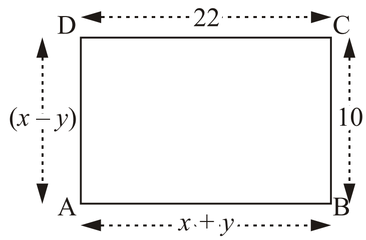
7. Determine a and b for which the following system of linear equations has infinite number of solutions
$2x - (a-4)y = 2b + 1; \ 4x - (a-1)y = 5b - 1.$
[2010 (T-I)]

8. For what value of p will the following system of equations have no solution
$(2p-1)x+(p-1)y = 2p+1;\ y+3x-1 = 0.$

[2010 (T-I)]

9. Solve: $148x+231y = 527,\ 231x+148y = 610$

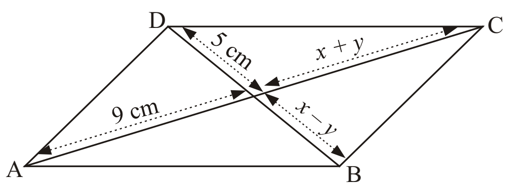
10. In the figure, ABCD is a rectangle. Find the values of x and y. **[2010 (T-I)]**



11. Determine a and b for which the system of linear equations has infinite number of solutions.
$(2a-1)x+3y-5 = 0;\ 3x+(b-1)y-2 = 0.$

[2010 (T-I)]

12. In the figure, ABCD is a parallelogram. Find the values of x and y. **[2010 (T-I)]**



13. Solve the following pair of linear equations :
$3x+4y = 10$ and $2x-2y = 2.$ **[2010 (T-I)]**

14. Is the system of linear equations $2x + 3y - 9 = 0$ and $4x + 6y - 18 = 0$ consistent ? Justify your answer. **[2010 (T-I)]**

15. Solve for x and y :

$$\frac{4}{x}+5y = 7;\ \frac{3}{x}+4y = 5$$

[2010 (T-I)]

16. Solve for x and y :

$$4x+\frac{6}{x} = 15;\ 3x-\frac{4}{x} = 7$$

[2010 (T-I)]

17. Find the value of m for which the pair of linear equations $2x + 3y - 7 = 0$ and $(m - 1)x + (m+1)y = (3m-1)$ has infinitely many solutions.

[2010 (T-I)]

18. Without drawing the graph, find out, whether the lines representing the following pair of linear equations intersect at a point, are parallel or coincident.

$$18x-7y = 24,\ \frac{9}{5}x-\frac{7}{10}y = \frac{9}{10}$$ **[2009]**

19. Find the value of k for which the pair of linear equations $kx + 3y = k - 2$ and $12x + ky = k$ has no solution. **[2010]**

SHORT ANSWER TYPE QUESTIONS [3 Marks]

A. Important Questions

1. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of m for which $y = mx + 3$.
2. The path of a Car I is given by the equation $3x + 4y - 12 = 0$ and the path of another Car II is given by the equation $6x + 8y - 48 = 0$. Represent this situation graphically.
3. Form the pair of linear equations in the following problems, and find their solutions graphically.
10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.
4. Show graphically that the system of equations $3x - y = 5$, $6x - 2y = 10$ has infinitely many solutions.
5. Determine algebraically, the vertices of the triangle formed by the lines.
$3x - y = 3$, $2x - 3y = 2$, $x + 2y = 8$
6. There are two numbers such that 3 times the greater is 18 times their difference and 4 times the smaller is 4 less than twice the sum of the two. What are the numbers?
7. If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?
8. Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?
9. The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.
10. Meena went to a bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all. Find how many notes of Rs 50 and Rs 100 she received?
11. Solve the following pair of linear equations
$\frac{2x-4}{4} = \frac{3y-2}{2};\ \frac{4x-3}{2}-\frac{6-y}{3} = 2\frac{5}{6}$

Represent the following pairs of linear equations graphically.

12. $2x - y = 2;\ 4x - y = 8$

13. $2x + y = 6;\ \ x - 2y = -2$

14. $2x - 3y = 1;\ \ 4x - 6y = 2.$

Solve graphically

15. $x + y = 4;\ 2x + 3y = 11.$

16. $x + y = 5;\ 4x + 3y = 17.$

17. If $2x + y = 23$ and $4x - y = 19$, find the value of $5y - 2x.$

18. For which values of a and b, will be the following pair of linear equations have infinitely many solutions?

$x + 2y = 1, (a - b)x + (a + b)y = a + b - 2.$

19. Find the values of p and q for which the following pair of equations has infinitely many solutions : $2x + 3y = 7$ and $2px + py = -28 - qy.$

Solve the following pairs of equations after reducing them to linear equations :

20. $x - y = 0.8,\ 20/(x + y) = 2.$

21. $\frac{15}{x} + \frac{4}{y} = 7,\ \frac{9}{x} - \frac{16}{y} = -5.$

22. $\frac{1}{2x} - \frac{1}{y} = -1,\ \frac{1}{x} + \frac{1}{2y} = 8, x, y \neq 0.$

23. $2(x + 1) + 3(y + 1) = 15,\ 3(x + 1) - 2(y + 1) = 3.$

24. $3(x + 3y) = 11xy;\ 3(2x + y) = 7xy.$

25. $\frac{22}{3x + 2y} + \frac{7}{3x - 2y} = 3,\ \frac{33}{3x + 2y} - \frac{14}{3x - 2y} = 1.$

26. $\frac{2xy}{x + y} + \frac{3}{2},\ \frac{xy}{2x - y} = \frac{-3}{10}, x + y \neq 0, 2x - y \neq 0.$

27. Two audio cassettes and three video cassettes cost Rs. 425, whereas three audio cassettes and two video cassettes cost Rs. 350. What are the prices of an audio cassette and a video cassette?

28. A part of monthly expenses of a family is constant and the remaining varies with the price of wheat. When the rate of wheat is Rs. 250 a quintal, the total monthly expenses of the family are Rs. 1000 and when it is Rs. 240 a quintal, the total monthly expenses of the family are Rs. 980. Find the total monthly expenses of the family when the cost of wheat is Rs. 350 a quintal.

29. In a triangle ABC, ∠C = 3∠B = 2(∠A + ∠B). Find the three angles.

30. A lady has only 25 p and 50 p coins in her purse. If in all she has 40 coins totalling Rs 12.50, find the number of coins of each type in her purse.

B. Questions From CBSE Examination Papers

1. Solve for x and y : **[2010 (T-I)]**

$(a - b)x + (a + b)y = a^2 - 2ab - b^2$

$(a + b)(x + y) = a^2 + b^2$

2. The sum of the digits of a two digit number is 12. The number obtained by interchanging the two digits exceeds the given number by 18. Find the number. **[2010 (T-I)]**

3. The taxi charges in a city consists of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs. 105 and for journey of 15 km, the charge paid is Rs. 155. What are the fixed charges and the charges per km? **[2010 (T-I)]**

4. The monthly incomes of A and B are in the ratio of 5 : 4 and their monthly expenditures are in the ratio of 7 : 5. If each saves Rs. 3000 per month, find the montly income of each. **[2010 (T-I)]**

5. A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days, she has to pay Rs. 1000 as hostel charges whereas a student B, who takes food 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of the food per day. **[2010 (T-I)]**

6. The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. **[2010 (T-I)]**

7. Nine times a two-digit number is the same as twice the number obtained by interchanging the digits of the number. If one digit of the number exceeds the other number by 7, find the number. **[2010 (T-I)]**

8. The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save Rs. 2000 per month, find their monthly incomes. **[2010 (T-I)]**

9. Solve for x and y : **[2010 (T-I)]**

$$\frac{5}{x - 1} + \frac{1}{y - 2} = 2;\quad \frac{6}{x - 1} + \frac{3}{y - 2} = 1$$

10. Solve for x and $y : \frac{x}{a} + \frac{y}{b} = 2, ax - by = a^2 - b^2.$ **[2010 (T-I)]**

11. Solve for x and y : $mx - ny = m^2 + n^2$; $x - y = 2n.$ **[2010 (T-I)]**

12. Solve for u and v by changing into linear equations $2(3u - v) = 5uv$; $2(u + 3v) = 5uv$. **[2010 (T-I)]**

13. Solve the following system of linear equations by cross multiplication method : **[2010 (T-I)]**

$2(ax - by) + (a + 4b) = 0$

$2(bx + ay) + (b - 4a) = 0$

14. For what values of a and b does the following pair of linear equations have an infinite number of solutions : $2x + 3y = 7$; $a(x + y) - b(x - y) = 3a + b - 2$. **[2010 (T-I)]**

15. If 4 times the area of a smaller square is subtracted from the area of a larger square, the result is 144 m^2. The sum of the areas of the two squares is 464 m^2. Determine the sides of the two squares.

16. Half the perimeter of a rectangular garden, whose length is 4 m more than its breadth is 36 m. Find the dimensions of the garden. **[2010 (T-I)]**

17. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test? **[2010 (T-I)]**

18. A man travels 370 km partly by train and partly by car. If he covers 250 km by train and rest by car, it takes him 4 hours. But if he travels 130 km by train and rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car. **[2010 (T-I)]**

19. Six years hence a man's age will be three times his son's age and three years ago, he was nine times as old as his son. Find their present ages. **[2010 (T-I)]**

20. A boat goes 24 km upstream and 28 km downstream in 6 hours. It goes 30 km upstream and 21 km downstream in 6 hours 30 minutes. Find the speed of the boat in still water. **[2010 (T-I)]**

21. A person travels 600 km partly by train and partly by car. If he covers 400 km by train and the rest by car, it takes 6 hours 30 minutes. But if he travels 200 km by train and the rest by car, he takes half an hour longer. Find the speed of the car and that of the train. **[2010 (T-I)]**

22. The age of a father is equal to sum of the ages of his 6 children. After 15 years, twice the age of the father will be the sum of ages of his children. Find the age of the father. **[2010 (T-I)]**

23. The auto fare for the first kilometer is fixed and is different from the rate per km for the remaining distance. A man pays Rs. 57 for the distance of 16 km and Rs. 92 for a distance of 26 km. Find the auto fare for the first kilometer and for each successive kilometer. **[2010 (T-I)]**

24. A lending library has a fixed charge for first three days and an additional charge for each day there after. Bhavya paid Rs. 27 for a book kept for seven days, while Vrinda paid Rs. 21 for a book kept for five days. Find the fixed charge and the charge for each extra day. **[2010 (T-I)]**

25. Father's age is 3 times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of the two children. Find the age of father. **[2010 (T-I)]**

26. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars ? **[2010 (T-I)]**

27. Rekha's mother is five times as old as her daughter Rekha. Five years later, Rekha's mother will be three times as old as her daughter Rekha. Find the present age of Rekha and her mother's age. **[2010 (T-I)]**

28. Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers. **[2010 (T-I)]**

29. Solve : $\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2; x + y = 2ab$ **[2010 (T-I)]**

30. Solve : $ax + by - a + b = 0$; $bx - ay - a - b = 0$ **[2004 C]**

31. If $(x + 3)$ is a factor of $x^3 + ax^2 - bx + 6$ and $a + b = 7$, find the values of a and b. **[2004]**

32. If $(x + 1)$ is a factor of $2x^3 + ax^2 + 2bx + 1$, then find the values of a and b given that $2a + 3b = 4$. **[2004]**

33. Solve : $\frac{bx}{a} - \frac{ay}{b} + a + b = 0; bx - ay + 2ab = 0$ **[2006]**

34. Solve : $\frac{x}{a} + \frac{y}{b} + a - b; ax + by = a^3 + b^3$ **[2005]**

35. Find the values of a and b for which the following system of linear equations has infinite number of solutions: **[2003]**

$2x + 3y = 7$; $(a + b + 1)\, x + (a + 2b + 2)\, y = 4(a + b) + 1$

36. For what value of k, will the system of equations $x + 2y = 5$, $3x + ky + 15 = 0$ has (i) a unique solution, (ii) no solution. **[2001]**

37. The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction. **[2010]**

A. Important Questions

1. Solve for x and y : $\frac{x+y}{xy}=2, \frac{x-y}{xy}=6$

2. Solve for x and y : $\frac{1}{7x}+\frac{1}{6y}=3, \frac{1}{2x}-\frac{1}{3y}=5$

3. Solve : $\frac{5}{x+1}-\frac{2}{y-1}=\frac{1}{2}; \frac{10}{x+1}-\frac{2}{y-1}=\frac{5}{2}$, where $x \neq -1$ and $y \neq 1$.

4. Solve :

$$\frac{2}{3x+2y}+\frac{3}{3x-2y}=\frac{17}{5}; \frac{5}{3x+2y}+\frac{1}{3x-2y}=2$$

Solve the following pair of linear equations for *x* and *y*.

5. $x + y = a + b$, $ax + by = a^2 + b^2$.

6. $ax + by = a - b$; $bx - ay = a + b$.

7. $\frac{x}{a}+\frac{y}{b}=a+b, \frac{x}{a^2}+\frac{y}{b^2}=2, a,b \neq 0.$

8. $ax+by=1, bx+ay=\frac{2ab}{a^2+b^2}$

9. A two digit number is obtained by either multiplying the sum of the digits by 8 and adding 1, or by multiplying the difference of the digits by 13 and adding 2. Find the number. How many such numbers are there?

10. A man wished to give Rs 12 to each person and found that he fell short of Rs 6 when he wanted to give to all persons. He therefore, distributed Rs 9 to each person and found that Rs 9 were left over. How much money did he have and how many persons were there?

11. A person sells two articles together for Rs. 46, making a profit of 10% on one and 20% on the other. If he had sold each article at 15% profit, the result would have been the same. At what price does he sell each article ?

12. The area of a rectangle gets reduced by 9 square units if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

13. A railway half ticket costs half the full fare, but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket from station *A* to *B* cots Rs 2530. Also, one reserved first class ticket and one reserved first class half ticket from *A* to *B* cost Rs 3810. Find the full first class fare from station *A* to *B* and also the reservation charges for a ticket.

14. It takes 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half the pool can be filled. How long would it take for each pipe to fill the pool separately?

15. Ages of two friends *A* and *B* differ by 3 years. *A'*s father *D* is twice as old as *A*, and *B* is twice as old as his sister *C*. Ages of *C* and *D* differ by 30 years. Find the ages of *A* and *B*.

16. In a rectangle, if the length is increased and breadth reduced each by 2 meters, the area is reduced by 28 sq. m. If the length is reduced by 1 m and the breadth is increased by 2 m, the area increases by 33 sq. m. Find the length and the breadth of the rectangle.

17. There are two classrooms *A* and *B* containing students. If 5 students are shifted from room *A* to room *B,* the resulting number of students in the two rooms become equal. If 5 students are shifted from room *B* to room *A,* the resulting number of students in room *A* becomes double the number of students left in room *B*. Find the original number of students in the two rooms.

18. Students of a class are made to stand in rows. If four students are extra in each row, there would be two rows less. If 4 students are less in each row, there would be 4 more rows. Find the number of students in the class.

19. A person invested some amount @ 12% simple interest and some other amount @ 10% simple interest. He received an yearly interest of Rs. 13000. But if he had interchanged the invested amounts, he would have received Rs. 400 more as interest. How much amount did he invest at different rates?

20. 2 men and 5 women can together finish a piece of work in 4 days, while 3 men and 6 women can finish it in 3 days. Find the time taken by 1 man alone to finish the work, and also that taken by 1 woman alone.

21. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the *x*-axis and shade the triangular region.

22. Draw the graphs of the equations $x = 3$, $y = 5$ and $2x - y - 4 = 0$. Also, find the area of the quadrilateral formed by these lines and the y-axis.

23. Draw the graphs of the lines $x = -2$ and $y = 3$. Write the vertices of the figure formed by these lines, the x-axis and the y-axis. Also, find the area of the figure.

24. Use a single graph paper to draw the graphs of $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$. Obtain the vertices of the triangle so obtained.

25. Use a single graph paper to draw the graphs of $x + y = 7$, $2x - 3y + 1 = 0$ and $3x - 2y - 1 = 0$. Obtain the vertices of the triangle so obtained.

B. Questions From CBSE Examination Papers

1. Draw the graphs of $2x + y = 6$ and $2x - y + 2 = 0$. Shade the region bounded by these lines and x-axis. Find the area of the shaded region. **[2010 (T-I)]**

2. Solve the following system of equations graphically and from the graph, find the points where these lines intersect the y-axis : **[2010 (T-I)]**

$$x - 2y = 2,\ 3x + 5y = 17.$$

3. Solve the following system of equations graphically and find the vertices of the triangle formed by these lines and the x-axis : **[2010 (T-I)]**

$$4x - 3y + 4 = 0,\ 4x + 3y - 20 = 0$$

4. Solve graphically the following system of equations : **[2010 (T-I)]**

$$x + 2y = 5,\ 2x - 3y = -4.$$

Also, find the points where the lines meet the x-axis.

5. Draw the graph of the pair of equations $2x + y = 4$ and $2x - y = 4$. Write the vertices of the triangle formed by these lines and the y-axis. Also shade this triangle. **[2010 (T-I)]**

6. Draw the graphs of the equations $4x - y - 8 = 0$ and $2x - 3y + 6 = 0$. Shade the region between two lines and x-axis. Also, find the co-ordinates of the vertices of the triangle formed by these lines and the x-axis. **[2010 (T-I)]**

7. Solve the following system of equations graphically and find the vertices of the triangle bounded by these lines and the x-axis. $2x - 3y - 4 = 0$, $x - y - 1 = 0$.

8. Solve the following system of equations graphically and from the graph, find the points where the lines intersect x-axis. **[2010 (T-I)]**

$$2x - y = 2,\ 4x - y = 8.$$

9. Solve graphically the pair of linear equations :

$$3x + y - 3 = 0;\quad 2x - y + 8 = 0$$

Write the co-ordinates of the vertices of the triangle formed by these lines with x-axis. **[2010 (T-I)]**

10. A sailor goes 8 km downstream in 40 minutes and returns in 1 hour. Find the speed of the sailor in still water and the speed of current. **[2010 (T-I)]**

11. Draw the graphs of equations $3x + 2y = 14$ and $4x - y = 4$. Shade the region between these lines and y-axis. Also, find the co-ordinates of the triangle formed by these lines with y-axis. **[2010 (T-I)]**

12. Check graphically whether the pair of linear equations $4x - y - 8 = 0$ and $2x - 3y + 6 = 0$ is consistent. Also, find the vertices of the triangle formed by these lines with the x-axis. **[2010 (T-I)]**

13. A boat goes 24 km upstream and 28 km downstream in 6 hours. It goes 30 km upstream and 21 km downstream in $6\frac{1}{2}$ hours. Find the speed of boat in still water and also the speed of the stream.

14. 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by one man alone and that by one boy alone to finish the work.

15. Solve graphically : $x + y + 1 = 0$, $3x + 2y = 12$

(i) Find the solution from the graph.

(ii) Shade the triangular region formed by the lines and the x-axis. **[2010 (T-I)]**

16. Solve graphically; $x - y = 1$, $2x + y = 8$. Shade the region bounded by these lines and y-axis. Also find its area. **[2010 (T-I)]**

17. Solve graphically the pair of linear equations : $x - y = -1$ and $2x + y - 10 = 0$. Also find the area of the region bounded by these lines and x-axis. **[2010 (T-I)]**

18. Solve graphically $4x - y = 4$ and $4x + y = 12$. Shade the triangular region formed by these lines and x-axis. Also, write the coordinate of the vertices of the triangle formed by these lines and x-axis. **[2010 (T-I)]**

19. Solve for x and y : $6(ax + by) = 3a + 2b$, $6(bx - ay) = 3b - 2a$. **[2004]**

20. Solve for x and y : $(a - b)x + (a + b)y = a^2 - 2ab - b^2$, $(a + b)(x + y) = a^2 + b^2$ **[2008]**

21. Solve : $\frac{44}{x+y}+\frac{30}{x-y}=10; \frac{55}{x+y}+\frac{40}{x-y}=13$

[2002 C]

22. Solve :

$$\frac{1}{2(2x+3y)}+\frac{12}{7(3x+2y)}=\frac{1}{2}; \frac{7}{(2x+3y)}+\frac{4}{3x+2y}=2,$$

where $(2x+3y) \neq 0$ and $(3x+2y) \neq 0$

[2004 C]

FORMATIVE ASSESSMENT

Activity

Objective : To obtain the conditions for consistency of given pairs of linear equations in two variables by graphical method.

Materials Required : Squared paper/graph paper, colour pencils, geometry box, etc.

Procedure :

Case 1. Let us consider the following pair of linear equations :

$2x - y - 1 = 0, \quad 3x + 2y - 12 = 0$

1. Obtain a table of ordered pairs which satisfy the given equations. Find at least three such pairs for each equation.

$2x - y - 1 = 0$

$\Rightarrow y = 2x - 1$

x	1	2	0
y	1	3	–1

Points are : (1, 1), (2, 3) and (0, –1).

$3x + 2y - 12 = 0$

$\Rightarrow y = \frac{12-3x}{2}$

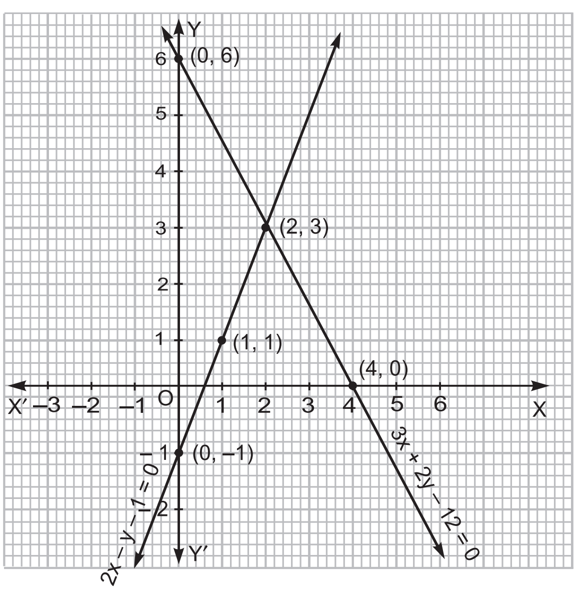


Figure 1

x	0	2	4
y	6	3	0

Points are : (0, 6), (2, 3) and (4, 0)

2. Take a graph paper and plot the above points to get the graphs of the two equations on the same pair of axes.

Case 2. Let us consider the following pair of linear equations :

$2x + y - 3 = 0, \quad 4x + 2y - 4 = 0$

1. Obtain a table of ordered pairs which satisfy the given equations. Find at least three such pairs for each equation.

$2x + y - 3 = 0$

$\Rightarrow y = 3 - 2x$

x	0	1	2
y	3	1	–1

Points are : (0, 3), (1, 1) and (2, – 1).

$4x + 2y - 4 = 0$

$\Rightarrow y = \frac{4-4x}{2}$

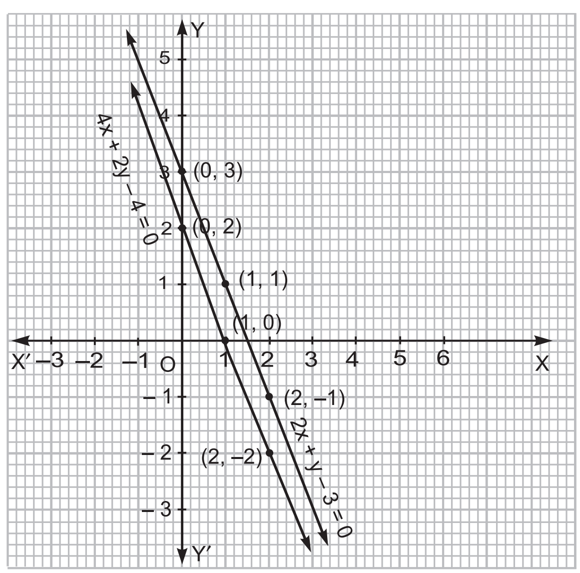


Figure 2

x	1	0	2
y	0	2	–2

Points are : (1, 0), (0, 2) and (2, –2).

2. Plot the above points on a graph paper to get the graphs of the two equations on the same pair of axes.

Case 3. Let us consider the following pair of linear equations :

$x - y = 4, \qquad -2x + 2y = -8$

1. Obtain a table of ordered pairs which satisfy the given equations. Find at least three such pairs for each equation.

$x - y = 4$

$\Rightarrow y = x - 4$

x	1	0	2
y	–3	–4	–2

Points are : (1, –3), (0, –4) and (2, –2).

$-2x + 2y = -8$

$\Rightarrow y = \dfrac{-8+2x}{2}$

x	0	3	4
y	–4	–1	0

Points are : (0, –4), (3, –1) and (4, 0).

2. Plot the above points on a graph paper to get the graphs of the two equations on the same pair of axes.

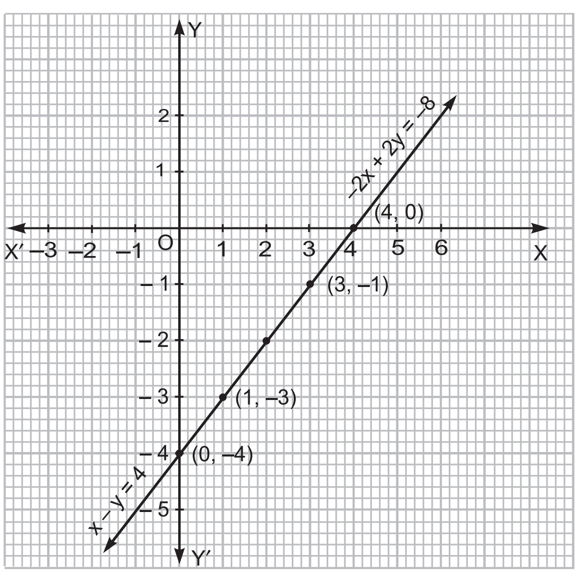


Figure 3

Observations :

1. The general form of a pair of linear equations is

   $a_1x + b_1y + c_1 = 0, \qquad a_2x + b_2y + c_2 = 0$

2. For case 1, we have

   $a_1 = 2, \quad b_1 = -1, \quad c_1 = -1$

   $a_2 = 3, \quad b_2 = 2, \quad c_2 = -12$

   $\therefore \dfrac{a_1}{a_2} = \dfrac{2}{3}, \ \dfrac{b_1}{b_2} = \dfrac{-1}{2}$

   $\Rightarrow \dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

3. Also, the graphs of these two equations represent a pair of intersecting lines [Figure 1]

4. So, from 2 and 3 above, we can say that for intersecting lines we must have $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$.

   Or, the given pair of equations is consistent with a unique solution if $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$.

5. For case 2, we have $a_1 = 2, \quad b_1 = 1, \quad c_1 = -3$

   $a_2 = 4, \quad b_2 = 2, \quad c_2 = -4$

   $\therefore \dfrac{a_1}{a_2} = \dfrac{2}{4} = \dfrac{1}{2}, \ \dfrac{b_1}{b_2} = \dfrac{1}{2}, \ \dfrac{c_1}{c_2} = \dfrac{3}{4}$

   $\Rightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

6. Also, the graphs of these two equations represent a pair of parallel lines [Figure 2]

7. So, from 5 and 6 above, we can say that for parallel lines we must have $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$.

   Or, the given pair of equations is inconsistent and has no solution if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$.

8. For case 3, we have

   $a_1 = 1, \quad b_1 = -1, \quad c_1 = -4$

   $a_2 = -2, \quad b_2 = 2, \quad c_2 = 8$

   $\therefore \dfrac{a_1}{a_2} = \dfrac{1}{-2}, \quad \dfrac{b_1}{b_2} = \dfrac{-1}{2}, \quad \dfrac{c_1}{c_2} = \dfrac{-1}{2}$

   $\Rightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

9. Also the graphs of these two equations represent a pair of coincident lines. [Figure 3]

10. So, from 8 and 9 above, we can say that for coincident lines we must have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Or the given pair of equations is consistent with infinite number of solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Conclusion :

From the above activity, we can say that the pair of linear equations in two variables $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

(i) is consistent with a unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

(ii) is inconsistent and has no solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

(iii) is consistent with infinite number of solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Do Yourself : Repeat the above activity taking the following pairs of linear equations :

(i) $x + y = 3, \quad 3x - 2y = 4$

(ii) $x + 2y = 5, \ 3x + 6y = 12$

(iii) $3x - y = 2, \ 12x - 4y = 8$

INVESTIGATION

Try this on a group of friends. Instruct your friend to do the following :

1. Think of a number.
2. Double it.
3. Add 4 to the answer.
4. Divide the result obtained by 2.
5. From the result so obtained subtract the number which you first thought of.

   Now, you tell him the answer, which is 2.

   Investigate what happens when you ask your friend to add 10, or 12 or 20 instead of 4 in step 3 above. Do you still get the final answer as 2?

   Try to use algebra to explain your answer.

Exercise 3.1

Question 1:

Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Answer:

Let the present age of Aftab be x.

And, present age of his daughter = y

Seven years ago,

Age of Aftab = $x - 7$

Age of his daughter = $y - 7$

According to the question,

$$(x-7)=7(y-7)$$
$$x-7 = 7y-49$$
$$x-7y=-42 \qquad (1)$$

Three years hence,

Age of Aftab = $x + 3$

Age of his daughter = $y + 3$

According to the question,

$$(x+3)=3(y+3)$$
$$x+3 = 3y+9$$
$$x-3y=6 \qquad (2)$$

Therefore, the algebraic representation is

$$x-7y=-42$$
$$x-3y=6$$

For $x-7y=-42$,

$$x=-42+7y$$

The solution table is

x	− 7	0	7
y	5	6	7

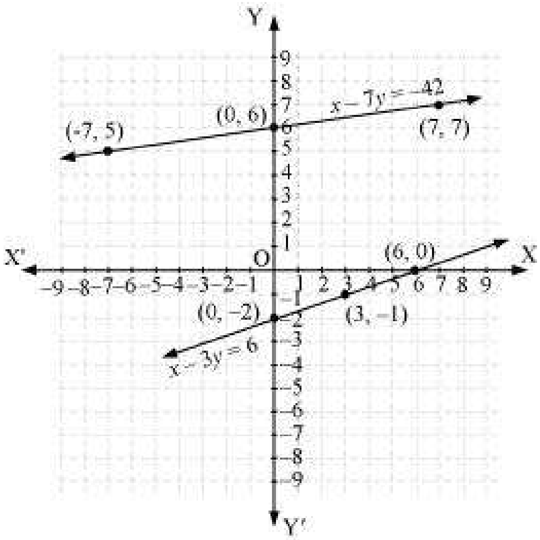
For $x-3y=6$,

$x=6+3y$

The solution table is

x	6	3	0
y	0	− 1	− 2

The graphical representation is as follows.



Question 2:

The coach of a cricket team buys 3 bats and 6 balls for Rs 3900. Later, she buys another bat and 2 more balls of the same kind for Rs 1300. Represent this situation algebraically and geometrically.

Answer:

Let the cost of a bat be Rs *x*.

And, cost of a ball = Rs *y*

According to the question, the algebraic representation is

$3x + 6y = 3900$

$x + 2y = 1300$

For $3x + 6y = 3900$,

$x = \frac{3900 - 6y}{3}$

The solution table is

x	300	100	− 100
y	500	600	700

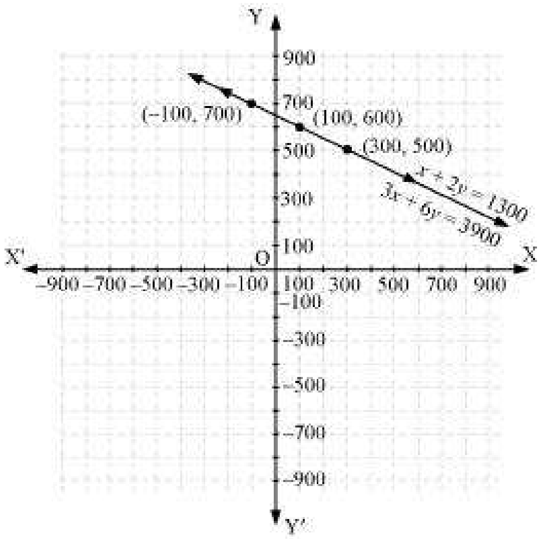
For $x + 2y = 1300$,

$x = 1300 - 2y$

The solution table is

x	300	100	− 100
y	500	600	700

The graphical representation is as follows.



Question 3:

The cost of 2 kg of apples and 1 kg of grapes on a day was found to be Rs 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs 300. Represent the situation algebraically and geometrically.

Answer:

Let the cost of 1 kg of apples be Rs *x*.

And, cost of 1 kg of grapes = Rs *y*

According to the question, the algebraic representation is

$$2x + y = 160$$
$$4x + 2y = 300$$

For $2x + y = 160$,

$$y = 160 - 2x$$

The solution table is

x	50	60	70
y	60	40	20

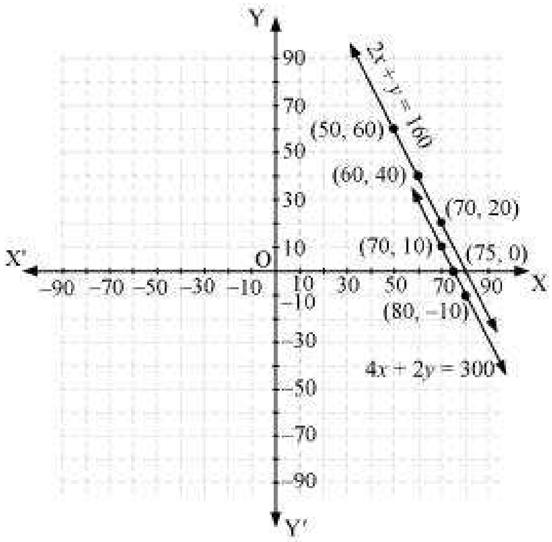
For $4x + 2y = 300$,

$$y = \frac{300 - 4x}{2}$$

The solution table is

x	70	80	75
y	10	−10	0

The graphical representation is as follows.



Exercise 3.2

Question 1:

Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and that of one pen.

Answer:

(i) Let the number of girls be x and the number of boys be y.

According to the question, the algebraic representation is

$x + y = 10$

$x - y = 4$

For $x + y = 10$,

$x = 10 - y$

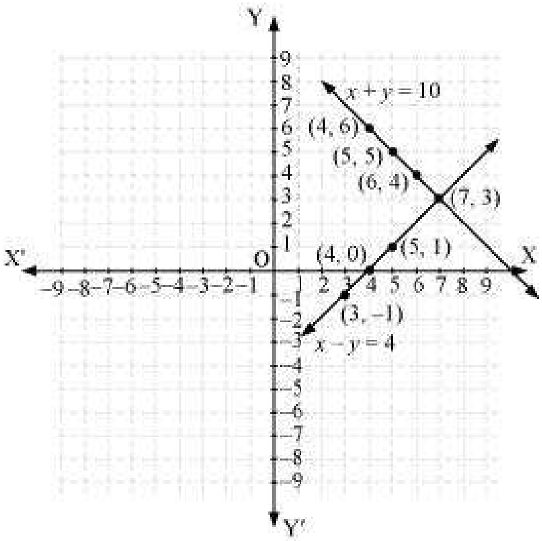
x	5	4	6
y	5	6	4

For $x - y = 4$,

$x = 4 + y$

x	5	4	3
y	1	0	−1

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines intersect each other at point (7, 3).

Therefore, the number of girls and boys in the class are 7 and 3 respectively.

(ii) Let the cost of 1 pencil be Rs x and the cost of 1 pen be Rs y.

According to the question, the algebraic representation is

$5x + 7y = 50$

$7x + 5y = 46$

For $5x + 7y = 50$,

$$x = \frac{50 - 7y}{5}$$

x	3	10	− 4
y	5	0	10

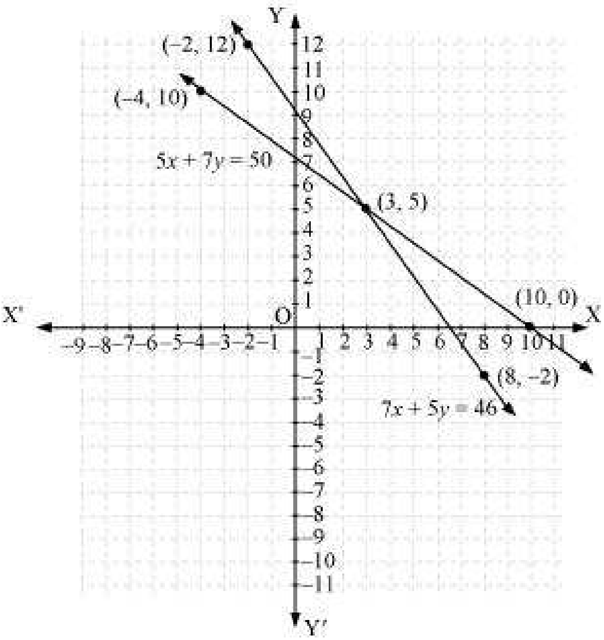
$7x + 5y = 46$

$$x = \frac{46 - 5y}{7}$$

x	8	3	− 2

y	− 2	5	12

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines intersect each other at point (3, 5).

Therefore, the cost of a pencil and a pen are Rs 3 and Rs 5 respectively.

Question 2:

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations at a point, are parallel or coincident:

$$\text{(i) } 5x - 4y + 8 = 0 \quad \text{(ii) } 9x + 3y + 12 = 0 \quad \text{(iii) } 6x - 3y + 10 = 0$$
$$7x + 6y - 9 = 0 \quad 18x + 6y + 24 = 0 \quad 2x - y + 9 = 0$$

Answer:

(i) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Comparing these equations with $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$, we obtain

$a_1 = 5, \quad b_1 = -4, \quad c_1 = 8$

$a_2 = 7, \quad b_2 = 6, \quad c_2 = -9$

$\frac{a_1}{a_2} = \frac{5}{7}$

$\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

Hence, the lines representing the given pair of equations have a unique solution and the pair of lines intersects at exactly one point.

(ii) 9*x* + 3*y* + 12 = 0

18*x* + 6*y* + 24 = 0

Comparing these equations with $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$, we obtain

$a_1 = 9, \quad b_1 = 3, \quad c_1 = 12$

$a_2 = 18, \quad b_2 = 6, \quad c_2 = 24$

$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, Hence, the lines representing the given pair of equations are coincident and there are infinite possible solutions for the given pair of equations.

(iii)$6x - 3y + 10 = 0$

$2x - y + 9 = 0$

Comparing these equations with $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$, we obtain

$a_1 = 6, \quad b_1 = -3, \quad c_1 = 10$

$a_2 = 2, \quad b_2 = -1, \quad c_2 = 9$

$\frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1}$

$\frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1}$

$\frac{c_1}{c_2} = \frac{10}{9}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$,

Hence, the lines representing the given pair of equations are parallel to each other and hence, these lines will never intersect each other at any point or there is no possible solution for the given pair of equations.

Question 3:

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5; \quad 2x - 3y = 7$ (ii) $2x - 3y = 8; \quad 4x - 6y = 9$

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7; \quad 9x - 10y = 14$ (iv) $5x - 3y = 11; -10x + 6y = -22$

(v) $\frac{4}{3}x + 2y = 8; \quad 2x + 3y = 12$

Answer:

(i) $3x + 2y = 5$

$2x - 3y = 7$

$\frac{a_1}{a_2} = \frac{3}{2}, \quad \frac{b_1}{b_2} = \frac{-2}{3}, \quad \frac{c_1}{c_2} = \frac{5}{7}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

These linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

(ii)$2x - 3y = 8$

$4x - 6y = 9$

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{8}{9}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$,

Therefore, these linear equations are parallel to each other and thus have no possible solution. Hence, the pair of linear equations is inconsistent.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$

$9x - 10y = 14$

$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{1}{6}, \quad \frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = \frac{-1}{6}, \quad \frac{c_1}{c_2} = \frac{7}{14} = \frac{1}{2}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

Therefore, these linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

(iv)$5x - 3y = 11$

$-10x + 6y = -22$

$\frac{a_1}{a_2} = \frac{5}{-10} = \frac{-1}{2}, \quad \frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}, \quad \frac{c_1}{c_2} = \frac{11}{-22} = \frac{-1}{2}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$,

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.

(v) $\frac{4}{3}x + 2y = 8$

$2x + 3y = 12$

$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}, \quad \frac{b_1}{b_2} = \frac{2}{3}, \quad \frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$,

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.

Question 4:

Which of the following pairs of linear equations are consistent/ inconsistent? If consistent, obtain the solution graphically:

(i) $x + y = 5, \quad 2x + 2y = 10$

(ii) $x - y = 8, \quad 3x - 3y = 16$

(iii) $2x + y - 6 = 0, \quad 4x - 2y - 4 = 0$

(iv) $2x - 2y - 2 = 0, 4x - 4y - 5 = 0$

Answer:

(i) $x + y = 5$

$2x + 2y = 10$

$\frac{a_1}{a_2} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{5}{10} = \frac{1}{2}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$,

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.

$x + y = 5$

$x = 5 - y$

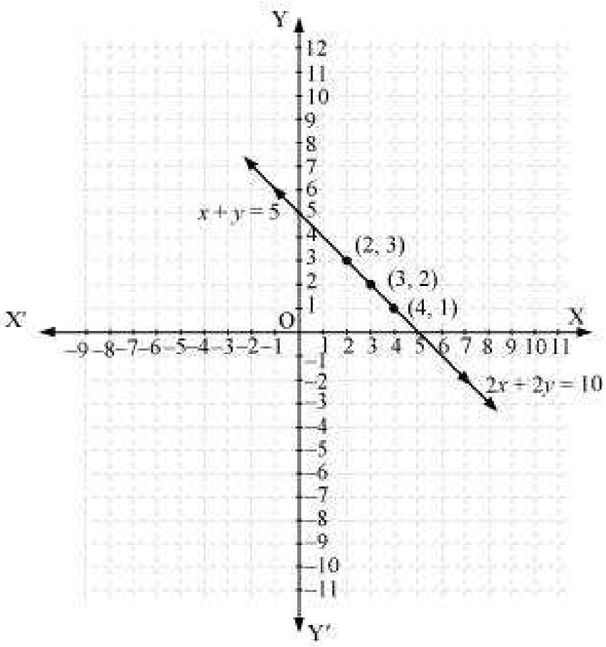
x	4	3	2
y	1	2	3

And, $2x + 2y = 10$

$$x = \frac{10 - 2y}{2}$$

x	4	3	2
y	1	2	3

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are overlapping each other. Therefore, infinite solutions are possible for the given pair of equations.

(ii) $x - y = 8$

$3x - 3y = 16$

$$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$,

Therefore, these linear equations are parallel to each other and thus have no possible solution. Hence, the pair of linear equations is inconsistent.

(iii) $2x + y - 6 = 0$

$4x - 2y - 4 = 0$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-1}{2}, \quad \frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

Therefore, these linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

$2x + y - 6 = 0$

$y = 6 - 2x$

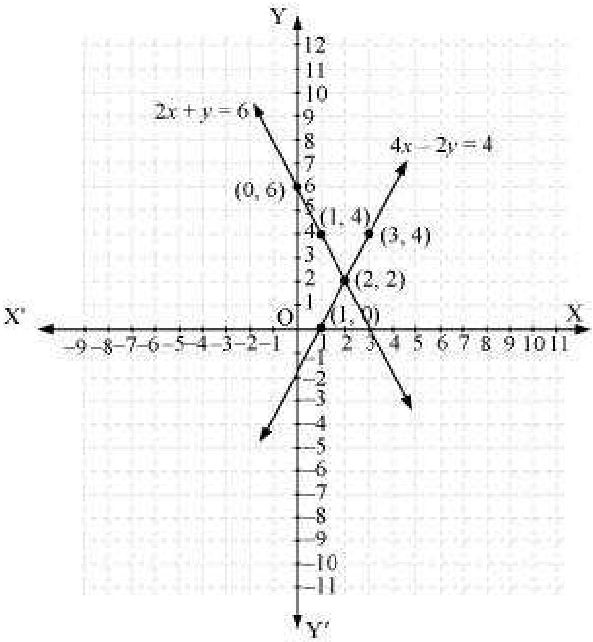
x	0	1	2
y	6	4	2

And $4x - 2y - 4 = 0$

$$y = \frac{4x - 4}{2}$$

x	1	2	3
y	0	2	4

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are intersecting each other at the only point i.e., (2, 2) and it is the solution for the given pair of equations.

(iv) $2x - 2y - 2 = 0$

$4x - 4y - 5 = 0$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{2}{5}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$,

Therefore, these linear equations are parallel to each other and thus have no possible solution. Hence, the pair of linear equations is inconsistent.

Question 5:

Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Answer:

Let the width of the garden be x and length be y.

According to the question,

$y - x = 4$ (1)

$y + x = 36$ (2)

$y - x = 4$

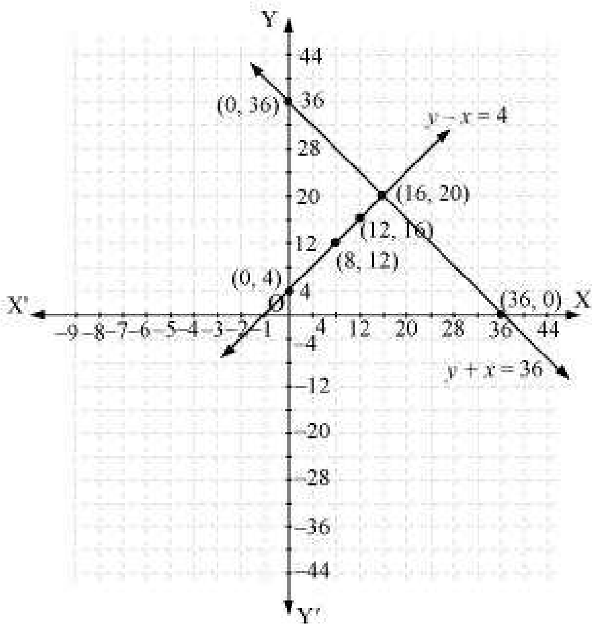
$y = x + 4$

x	0	8	12
y	4	12	16

$y + x = 36$

x	0	36	16
y	36	0	20

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are intersecting each other at only point i.e., (16, 20). Therefore, the length and width of the given garden is 20 m and 16 m respectively.

Question 6:

Given the linear equation 2*x* + 3*y* − 8 = 0, write another linear equations in two variables such that the geometrical representation of the pair so formed is:

(i) intersecting lines (ii) parallel lines

(iii) coincident lines

Answer:

(i)Intersecting lines:

For this condition,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

The second line such that it is intersecting the given line is $2x+4y-6=0$ as $\frac{a_1}{a_2}=\frac{2}{2}=1,\ \frac{b_1}{b_2}=\frac{3}{4}$ and $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

(ii) Parallel lines:

For this condition,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the second line can be

4*x* + 6y − 8 = 0

as $\frac{a_1}{a_2}=\frac{2}{4}=\frac{1}{2},\ \frac{b_1}{b_2}=\frac{3}{6}=\frac{1}{2},\ \frac{c_1}{c_2}=\frac{-8}{-8}=1$

And clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(iii)Coincident lines:

For coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, the second line can be

6*x* + 9*y* − 24 = 0

as $\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}$, $\frac{c_1}{c_2} = \frac{-8}{-24} = \frac{1}{3}$

And clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Question 7:

Draw the graphs of the equations *x* − *y* + 1 = 0 and 3*x* + 2*y* − 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the *x*-axis, and shade the triangular region.

Answer:

x − *y* + 1 = 0

x = *y* − 1

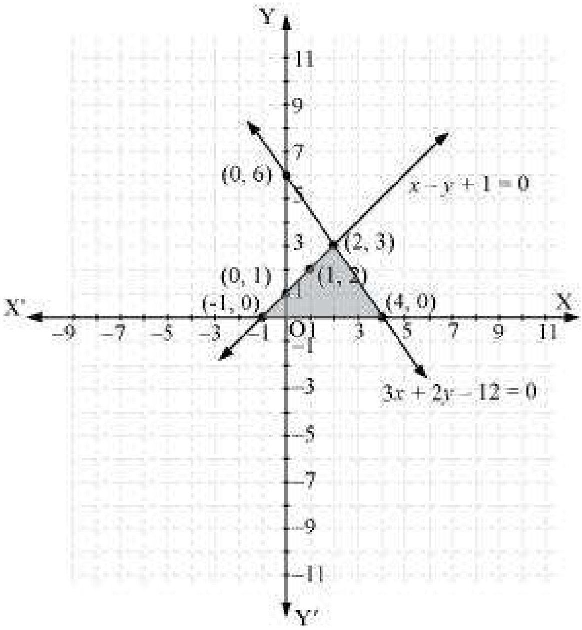
x	0	1	2
y	1	2	3

3*x* + 2*y* − 12 = 0

$$x = \frac{12 - 2y}{3}$$

x	4	2	0
y	0	3	6

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are intersecting each other at point (2, 3) and *x*-axis at (−1, 0) and (4, 0). Therefore, the vertices of the triangle are (2, 3), (−1, 0), and (4, 0).

Exercise 3.3

Question 1:

Solve the following pair of linear equations by the substitution method.

(i) $x + y = 14$

$x - y = 4$

(ii) $s - t = 3$

$\frac{s}{3} + \frac{t}{2} = 6$

(iii) $3x - y = 3$

$9x - 3y = 9$

(iv) $0.2x + 0.3y = 1.3$

$0.4x + 0.5y = 2.3$

(v) $\sqrt{2}x + \sqrt{3}y = 0$

$\sqrt{3}x - \sqrt{8}y = 0$

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2$

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Answer:

(i) $x + y = 14$ (1)

$x - y = 4$ (2)

From (1), we obtain

$x = 14 - y$ (3)

Substituting this value in equation (2), we obtain

$(14 - y) - y = 4$

$14 - 2y = 4$

$10 = 2y$

$y = 5 \qquad (4)$

Substituting this in equation (3), we obtain

$x = 9$

$\therefore x = 9, y = 5$

(ii) $s - t = 3 \qquad (1)$

$\frac{s}{3} + \frac{t}{2} = 6 \qquad (2)$

From (1), we obtain

$s = t + 3 \qquad (3)$

Substituting this value in equation (2), we obtain

$$\frac{t+3}{3} + \frac{t}{2} = 6$$

$$2t + 6 + 3t = 36$$

$$5t = 30$$

$$t = 6 \qquad (4)$$

Substituting in equation (3), we obtain

s = 9

∴ *s* = 9, *t* = 6

(iii)$3x - y = 3$ (1)

$9x - 3y = 9$ (2)

From (1), we obtain

$y = 3x - 3$ (3)

Substituting this value in equation (2), we obtain

$$9x - 3(3x - 3) = 9$$

$$9x - 9x + 9 = 9$$

9 = 9

This is always true.

Hence, the given pair of equations has infinite possible solutions and the relation between these variables can be given by

$y = 3x - 3$

Therefore, one of its possible solutions is $x = 1$, $y = 0$.

(iv) $0.2x + 0.3y = 1.3 \qquad (1)$

$0.4x + 0.5y = 2.3 \qquad (2)$

From equation (1), we obtain

$$x = \frac{1.3 - 0.3y}{0.2} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$0.4\left(\frac{1.3-0.3y}{0.2}\right)+0.5y=2.3$$

$$2.6-0.6y+0.5y=2.3$$

$$2.6-2.3=0.1y$$

$$0.3=0.1y$$

$$y=3 \qquad (4)$$

Substituting this value in equation (3), we obtain

$$x=\frac{1.3-0.3\times 3}{0.2}$$

$$=\frac{1.3-0.9}{0.2}=\frac{0.4}{0.2}=2$$

$$\therefore x=2,\ y=3$$

(v) $\sqrt{2}x+\sqrt{3}y=0 \qquad (1)$

$$\sqrt{3}x-\sqrt{8}y=0 \qquad (2)$$

From equation (1), we obtain

$$x=\frac{-\sqrt{3}y}{\sqrt{2}} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$\sqrt{3}\left(-\frac{\sqrt{3}y}{\sqrt{2}}\right)-\sqrt{8}y=0$$

$$-\frac{3y}{\sqrt{2}}-2\sqrt{2}y=0$$

$$y\left(-\frac{3}{\sqrt{2}}-2\sqrt{2}\right)=0$$

$$y=0 \qquad (4)$$

Substituting this value in equation (3), we obtain

$x = 0$

$\therefore x = 0, y = 0$

(vi) $\frac{3}{2}x - \frac{5}{3}y = -2 \qquad (1)$

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6} \qquad (2)$

From equation (1), we obtain

$9x - 10y = -12$

$x = \frac{-12 + 10y}{9} \qquad (3)$

Substituting this value in equation (2), we obtain

$$\frac{\frac{-12+10y}{9}}{3} + \frac{y}{2} = \frac{13}{6}$$

$$\frac{-12+10y}{27} + \frac{y}{2} = \frac{13}{6}$$

$$\frac{-24+20y+27y}{54} = \frac{13}{6}$$

$47y = 117 + 24$

$47y = 141$

$y = 3 \qquad (4)$

Substituting this value in equation (3), we obtain

$$x = \frac{-12 + 10 \times 3}{9} = \frac{18}{9} = 2$$

Hence, $x = 2$, $y = 3$

Question 2:

Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

Answer:

$$2x+3y=11 \qquad (1)$$
$$2x-4y=-24 \qquad (2)$$

From equation (1), we obtain

$$x=\frac{11-3y}{2} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$2\left(\frac{11-3y}{2}\right)-4y=-24$$
$$11-3y-4y=-24$$
$$-7y=-35$$
$$y=5 \qquad (4)$$

Putting this value in equation (3), we obtain

$$x=\frac{11-3\times5}{2}=-\frac{4}{2}=-2$$

Hence, $x = -2$, $y = 5$

Also,

$$y=mx+3$$
$$5=-2m+3$$
$$-2m=2$$
$$m=-1$$

Question 3:

Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, she buys 3 bats and 5 balls for Rs 1750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs 105 and for a journey of 15 km, the charge paid is Rs 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km.

(v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Answer:

(i) Let the first number be *x* and the other number be *y* such that *y* > *x*.

According to the given information,

$$y = 3x \qquad (1)$$

$$y - x = 26 \qquad (2)$$

On substituting the value of *y* from equation (1) into equation (2), we obtain

$$3x - x = 26$$

$$x = 13 \qquad (3)$$

Substituting this in equation (1), we obtain

y = 39

Hence, the numbers are 13 and 39.

(ii) Let the larger angle be *x* and smaller angle be *y*.

We know that the sum of the measures of angles of a supplementary pair is always 180º.

According to the given information,

$x + y = 180^\circ \quad (1)$

$x - y = 18^\circ \quad (2)$

From (1), we obtain

$x = 180^\circ - y$ (3)

Substituting this in equation (2), we obtain

$180^\circ - y - y = 18^\circ$

$162^\circ = 2y$

$81^\circ = y \quad (4)$

Putting this in equation (3), we obtain

$x = 180^\circ - 81^\circ$

$= 99^\circ$

Hence, the angles are 99º and 81º.

(iii) Let the cost of a bat and a ball be x and y respectively.

According to the given information,

$7x + 6y = 3800 \quad (1)$

$3x + 5y = 1750 \quad (2)$

From (1), we obtain

$$y = \frac{3800 - 7x}{6} \quad (3)$$

Substituting this value in equation (2), we obtain

$$3x + 5\left(\frac{3800 - 7x}{6}\right) = 1750$$

$$3x + \frac{9500}{3} - \frac{35x}{6} = 1750$$

$$3x - \frac{35x}{6} = 1750 - \frac{9500}{3}$$

$$\frac{18x-35x}{6}=\frac{5250-9500}{3}$$

$$-\frac{17x}{6}=\frac{-4250}{3}$$

$$-17x=-8500$$

$$x=500 \qquad (4)$$

Substituting this in equation (3), we obtain

$$y=\frac{3800-7\times 500}{6}$$

$$=\frac{300}{6}=50$$

Hence, the cost of a bat is Rs 500 and that of a ball is Rs 50.

(iv)Let the fixed charge be Rs *x* and per km charge be Rs *y*.

According to the given information,

$$x+10y=105 \qquad (1)$$

$$x+15y=155 \qquad (2)$$

From (3), we obtain

$$x=105-10y \qquad (3)$$

Substituting this in equation (2), we obtain

$$105-10y+15y=155$$

$$5y=50$$

$$y=10 \qquad (4)$$

Putting this in equation (3), we obtain

$$x=105-10\times 10$$

$$x=5$$

Hence, fixed charge = Rs 5

And per km charge = Rs 10

Charge for 25 km = *x* + 25*y*

= 5 + 250 = Rs 255

(v) Let the fraction be $\frac{x}{y}$.

According to the given information,

$$\frac{x+2}{y+2}=\frac{9}{11}$$

$$11x+22=9y+18$$

$$11x-9y=-4 \qquad (1)$$

$$\frac{x+3}{y+3}=\frac{5}{6}$$

$$6x+18=5y+15$$

$$6x-5y=-3 \qquad (2)$$

From equation (1), we obtain $x=\frac{-4+9y}{11} \qquad (3)$

Substituting this in equation (2), we obtain

$$6\left(\frac{-4+9y}{11}\right)-5y=-3$$

$$-24+54y-55y=-33$$

$$-y=-9$$

$$y=9 \qquad (4)$$

Substituting this in equation (3), we obtain

$$x=\frac{-4+81}{11}=7$$

Hence, the fraction is $\frac{7}{9}$.

(vi) Let the age of Jacob be *x* and the age of his son be *y*.

According to the given information,

$(x+5)=3(y+5)$

$x-3y=10 \qquad (1)$

$(x-5)=7(y-5)$

$x-7y=-30 \qquad (2)$

From (1), we obtain

$x=3y+10 \qquad (3)$

Substituting this value in equation (2), we obtain

$3y+10-7y=-30$

$-4y=-40$

$y=10 \qquad (4)$

Substituting this value in equation (3), we obtain

$x=3\times10+10$

$=40$

Hence, the present age of Jacob is 40 years whereas the present age of his son is 10 years.

Exercise 3.4

Question 1:

Solve the following pair of linear equations by the elimination method and the substitution method:

(i) $x+y=5$ and $2x-3y=4$ (ii) $3x+4y=10$ and $2x-2y=2$

(iii) $3x-5y-4=0$ and $9x=2y+7$ (iv) $\frac{x}{2}+\frac{2y}{3}=-1$ and $x-\frac{y}{3}=3$

Answer:

(i) ***By elimination method***

$x+y=5 \quad (1)$

$2x-3y=4 \quad (2)$

Multiplying equation (1) by 2, we obtain

$2x+2y=10 \quad (3)$

Subtracting equation (2) from equation (3), we obtain

$5y=6$

$y=\frac{6}{5} \quad (4)$

Substituting the value in equation (1), we obtain

$x=5-\frac{6}{5}=\frac{19}{5}$

$\therefore x=\frac{19}{5}, y=\frac{6}{5}$

By substitution method

From equation (1), we obtain

$x=5-y$ (5)

Putting this value in equation (2), we obtain

$2(5-y)-3y=4$

$-5y = -6$

$y = \frac{6}{5}$

Substituting the value in equation (5), we obtain

$x = 5 - \frac{6}{5} = \frac{19}{5}$

$\therefore x = \frac{19}{5}, y = \frac{6}{5}$

(ii) ***By elimination method***

$3x + 4y = 10 \quad (1)$

$2x - 2y = 2 \quad (2)$

Multiplying equation (2) by 2, we obtain

$4x - 4y = 4 \quad (3)$

Adding equation (1) and (3), we obtain

$7x = 14$

$x = 2 \quad (4)$

Substituting in equation (1), we obtain

$6 + 4y = 10$

$4y = 4$

$y = 1$

Hence, $x = 2$, $y = 1$

By substitution method

From equation (2), we obtain

$x = 1 + y$ (5)

Putting this value in equation (1), we obtain

$3(1 + y) + 4y = 10$

$7y = 7$

$y = 1$

Substituting the value in equation (5), we obtain

$x = 1 + 1 = 2$

$\therefore x = 2, y = 1$

(iii) ***By elimination method***

$3x - 5y - 4 = 0 \qquad (1)$

$9x = 2y + 7$

$9x - 2y - 7 = 0 \qquad (2)$

Multiplying equation (1) by 3, we obtain

$9x - 15y - 12 = 0 \qquad (3)$

Subtracting equation (3) from equation (2), we obtain

$13y = -5$

$y = \frac{-5}{13} \qquad (4)$

Substituting in equation (1), we obtain

$3x + \frac{25}{13} - 4 = 0$

$3x = \frac{27}{13}$

$x = \frac{9}{13}$

$\therefore x = \frac{9}{13}, y = \frac{-5}{13}$

By substitution method

From equation (1), we obtain

$x = \frac{5y + 4}{3} \qquad (5)$

Putting this value in equation (2), we obtain

$$9\left(\frac{5y+4}{3}\right)-2y-7=0$$

$$13y=-5$$

$$y=-\frac{5}{13}$$

Substituting the value in equation (5), we obtain

$$x=\frac{5\left(\frac{-5}{13}\right)+4}{3}$$

$$x=\frac{9}{13}$$

$$\therefore \quad x=\frac{9}{13}, y=\frac{-5}{13}$$

(iv)***By elimination method***

$$\frac{x}{2}+\frac{2y}{3}=-1$$

$$3x+4y=-6 \quad (1)$$

$$x-\frac{y}{3}=3$$

$$3x-y=9 \qquad (2)$$

Subtracting equation (2) from equation (1), we obtain

$$5y=-15$$

$$y=-3 \qquad (3)$$

Substituting this value in equation (1), we obtain

$$3x-12=-6$$

$$3x=6$$

$$x=2$$

Hence, x = 2, y = −3

By substitution method

From equation (2), we obtain

$$x = \frac{y+9}{3} \quad (5)$$

Putting this value in equation (1), we obtain

$$3\left(\frac{y+9}{3}\right) + 4y = -6$$

$5y = -15$

$y = -3$

Substituting the value in equation (5), we obtain

$$x = \frac{-3+9}{3} = 2$$

$\therefore x = 2, y = -3$

Question 2:

Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method:

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all. Find how many notes of Rs 50 and Rs 100 she received.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Answer:

(i)Let the fraction be $\frac{x}{y}$.

According to the given information,

$\frac{x+1}{y-1}=1 \qquad \Rightarrow x-y=-2 \qquad (1)$

$\frac{x}{y+1}=\frac{1}{2} \qquad \Rightarrow 2x-y=1 \qquad (2)$

Subtracting equation (1) from equation (2), we obtain

x = 3 (3)

Substituting this value in equation (1), we obtain

$3-y=-2$

$-y=-5$

$y=5$

Hence, the fraction is $\frac{3}{5}$.

(ii)Let present age of Nuri = x

and present age of Sonu = y

According to the given information,

$(x-5)=3(y-5)$

$x-3y=-10 \qquad (1)$

$(x+10)=2(y+10)$

$x-2y=10 \qquad (2)$

Subtracting equation (1) from equation (2), we obtain

y = 20 (3)

Substituting it in equation (1), we obtain

$x-60=-10$

$x=50$

Hence, age of Nuri = 50 years

And, age of Sonu = 20 years

(iii)Let the unit digit and tens digits of the number be x and y respectively. Then, number = $10y + x$

Number after reversing the digits = $10x + y$

According to the given information,

$x + y = 9$ (1)

$9(10y + x) = 2(10x + y)$

$88y - 11x = 0$

$-x + 8y = 0$ (2)

Adding equation (1) and (2), we obtain

$9y = 9$

$y = 1$ (3)

Substituting the value in equation (1), we obtain

$x = 8$

Hence, the number is $10y + x = 10 \times 1 + 8 = 18$

(iv)Let the number of Rs 50 notes and Rs 100 notes be x and y respectively.

According to the given information,

$x+y=25 \qquad (1)$

$50x+100y=2000 \qquad (2)$

Multiplying equation (1) by 50, we obtain

$50x+50y=1250 \qquad (3)$

Subtracting equation (3) from equation (2), we obtain

$50y=750$

$y=15$

Substituting in equation (1), we have $x = 10$

Hence, Meena has 10 notes of Rs 50 and 15 notes of Rs 100.

(v)Let the fixed charge for first three days and each day charge thereafter be Rs *x* and Rs *y* respectively.

According to the given information,

$x+4y=27 \quad (1)$

$x+2y=21 \quad (2)$

Subtracting equation (2) from equation (1), we obtain

$2y=6$

$y=3 \quad (3)$

Substituting in equation (1), we obtain

$x+12=27$

$x=15$

Hence, fixed charge = Rs 15

And Charge per day = Rs 3

Exercise 3.5

Question 1:

Which of the following pairs of linear equations has unique solution, no solution or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

(i) $x-3y-3=0$
$3x-9y-2=0$

(ii) $2x+y=5$
$3x+2y=8$

(iii) $3x-5y=20$
$6x-10y=40$

(iv) $x-3y-7=0$
$3x-3y-15=0$

Answer:

(i)
$$x-3y-3=0$$
$$3x-9y-2=0$$
$$\frac{a_1}{a_2}=\frac{1}{3},\quad \frac{b_1}{b_2}=\frac{-3}{-9}=\frac{1}{3},\quad \frac{c_1}{c_2}=\frac{-3}{-2}=\frac{3}{2}$$
$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Therefore, the given sets of lines are parallel to each other. Therefore, they will not intersect each other and thus, there will not be any solution for these equations.

(ii)
$$2x+y=5$$
$$3x+2y=8$$
$$\frac{a_1}{a_2}=\frac{2}{3},\quad \frac{b_1}{b_2}=\frac{1}{2},\quad \frac{c_1}{c_2}=\frac{-5}{-8}$$
$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication method,

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\frac{x}{-8-(-10)}=\frac{y}{-15+16}=\frac{1}{4-3}$$

$$\frac{x}{2}=\frac{y}{1}=1$$

$$\frac{x}{2}=1,\quad \frac{y}{1}=1$$

$$x=2,\quad y=1$$

$\therefore$ x = 2, y = 1

(iii) $3x-5y=20$

$6x-10y=40$

$$\frac{a_1}{a_2}=\frac{3}{6}=\frac{1}{2},\quad \frac{b_1}{b_2}=\frac{-5}{-10}=\frac{1}{2},\quad \frac{c_1}{c_2}=\frac{-20}{-40}=\frac{1}{2}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Therefore, the given sets of lines will be overlapping each other i.e., the lines will be coincident to each other and thus, there are infinite solutions possible for these equations.

(iv) $x-3y-7=0$

$3x-3y-15=0$

$$\frac{a_1}{a_2}=\frac{1}{3},\quad \frac{b_1}{b_2}=\frac{-3}{-3}=1,\quad \frac{c_1}{c_2}=\frac{-7}{-15}=\frac{7}{15}$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication,

$$\frac{x}{45-(21)}=\frac{y}{-21-(-15)}=\frac{1}{-3-(-9)}$$

$$\frac{x}{24}=\frac{y}{-6}=\frac{1}{6}$$

$$\frac{x}{24}=\frac{1}{6} \text{ and } \frac{y}{-6}=\frac{1}{6}$$

$x=4$ and $y=-1$

$\therefore \ x=4, y=-1$

Question 2:

(i) For which values of *a* and *b* will the following pair of linear equations have an infinite number of solutions?

$2x+3y=7$

$(a-b)x+(a+b)y=3a+b-2$

(ii) For which value of *k* will the following pair of linear equations have no solution?

$3x+y=1$

$(2k-1)x+(k-1)y=2k+1$

Answer:

(i) $2x+3y-7=0$

$(a-b)x+(a+b)y-(3a+b-2)=0$

$$\frac{a_1}{a_2}=\frac{2}{a-b}, \quad \frac{b_1}{b_2}=\frac{3}{a+b}, \quad \frac{c_1}{c_2}=\frac{-7}{-(3a+b-2)}=\frac{7}{(3a+b-2)}$$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{a-b} = \frac{7}{3a+b-2}$$

$$6a+2b-4 = 7a-7b$$

$$a-9b = -4 \qquad (1)$$

$$\frac{2}{a-b} = \frac{3}{a+b}$$

$$2a+2b = 3a-3b$$

$$a-5b = 0 \qquad (2)$$

Subtracting (1) from (2), we obtain

$$4b = 4$$

$$b = 1$$

Substituting this in equation (2), we obtain

$$a-5\times 1 = 0$$

$$a = 5$$

Hence, a = 5 and b = 1 are the values for which the given equations give infinitely many solutions.

(ii) $3x+y-1=0$

$$(2k-1)x+(k-1)y-2k-1=0$$

$$\frac{a_1}{a_2} = \frac{3}{2k-1}, \quad \frac{b_1}{b_2} = \frac{1}{k-1}, \quad \frac{c_1}{c_2} = \frac{-1}{-2k-1} = \frac{1}{2k+1}$$

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$$

$$\frac{3}{2k-1} = \frac{1}{k-1}$$

$$3k-3 = 2k-1$$

$k = 2$

Hence, for *k* = 2, the given equation has no solution.

Question 3:

Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$8x + 5y = 9$

$3x + 2y = 4$

Answer:

$8x + 5y = 9 \quad (i)$

$3x + 2y = 4 \quad (ii)$

From equation (*ii*), we obtain

$x = \frac{4 - 2y}{3} \quad (iii)$

Substituting this value in equation (*i*), we obtain

$8\left(\frac{4 - 2y}{3}\right) + 5y = 9$

$32 - 16y + 15y = 27$

$-y = -5$

$y = 5 \quad (iv)$

Substituting this value in equation (*ii*), we obtain

$3x + 10 = 4$

$x = -2$

Hence, $x = -2, y = 5$

Again, by cross-multiplication method, we obtain

$8x+5y-9=0$

$3x+2y-4=0$

$$\frac{x}{-20-(-18)}=\frac{y}{-27-(-32)}=\frac{1}{16-15}$$

$$\frac{x}{-2}=\frac{y}{5}=\frac{1}{1}$$

$\frac{x}{-2}=1$ and $\frac{y}{5}=1$

$x=-2$ and $y=5$

Question 4:

Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i)A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii)A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii)Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v)The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Answer:

(i)Let *x* be the fixed charge of the food and *y* be the charge for food per day.

According to the given information,

$$x+20y=1000 \qquad (1)$$

$$x+26y=1180 \qquad (2)$$

Subtracting equation (1) from equation (2), we obtain

$$6y=180$$

$$y=30$$

Substituting this value in equation (1), we obtain

$$x+20\times30=1000$$

$$x=1000-600$$

$$x=400$$

Hence, fixed charge = Rs 400

And charge per day = Rs 30

(ii)Let the fraction be $\frac{x}{y}$.

According to the given information,

$$\frac{x-1}{y}=\frac{1}{3} \quad \Rightarrow \quad 3x-y=3 \qquad (1)$$

$$\frac{x}{y+8}=\frac{1}{4} \quad \Rightarrow \quad 4x-y=8 \qquad (2)$$

Subtracting equation (1) from equation (2), we obtain

$$x=5 \qquad (3)$$

Putting this value in equation (1), we obtain

$$15-y=3$$

$$y=12$$

Hence, the fraction is $\frac{5}{12}$.

(iii)Let the number of right answers and wrong answers be *x* and *y* respectively.

According to the given information,

$3x - y = 40 \qquad (1)$

$4x - 2y = 50$

$\Rightarrow 2x - y = 25 \qquad (2)$

Subtracting equation (2) from equation (1), we obtain

x = 15 (3)

Substituting this in equation (2), we obtain

$30 - y = 25$

$y = 5$

Therefore, number of right answers = 15

And number of wrong answers = 5

Total number of questions = 20

(iv)Let the speed of 1st car and 2nd car be *u* km/h and *v* km/h.

Respective speed of both cars while they are travelling in same direction = ($u - v$) km/h

Respective speed of both cars while they are travelling in opposite directions i.e., travelling towards each other = ($u + v$) km/h

According to the given information,

$5(u - v) = 100$

$\Rightarrow u - v = 20 \quad ...(1)$

$1(u + v) = 100 \quad ...(2)$

Adding both the equations, we obtain

$2u = 120$

$u = 60 \text{ km/h} \quad (3)$

Substituting this value in equation (2), we obtain

v = 40 km/h

Hence, speed of one car = 60 km/h and speed of other car = 40 km/h

(v) Let length and breadth of rectangle be *x* unit and *y* unit respectively.

Area = *xy*

According to the question,

$(x-5)(y+3) = xy-9$

$\Rightarrow 3x-5y-6=0 \qquad (1)$

$(x+3)(y+2) = xy+67$

$\Rightarrow 2x+3y-61=0 \qquad (2)$

By cross-multiplication method, we obtain

$$\frac{x}{305-(-18)} = \frac{y}{-12-(-183)} = \frac{1}{9-(-10)}$$

$$\frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

$x = 17, y = 9$

Hence, the length and breadth of the rectangle are 17 units and 9 units respectively.

Exercise 3.6

Question 1:

Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x}+\frac{1}{3y}=2$

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

(iii) $\frac{4}{x}+3y=14$

$\frac{3}{x}-4y=23$

(iv) $\frac{5}{x-1}+\frac{1}{y-2}=2$

$\frac{6}{x-1}-\frac{3}{y-2}=1$

(v) $\frac{7x-2y}{xy}=5$

$\frac{8x+7y}{xy}=15$

(vi) $6x+3y=6xy$

$2x+4y=5xy$

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4$

$\frac{15}{x+y}-\frac{5}{x-y}=-2$

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Answer:

(i) $\frac{1}{2x}+\frac{1}{3y}=2$

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$, then the equations change as follows.

$$\frac{p}{2}+\frac{q}{3}=2 \quad \Rightarrow \quad 3p+2q-12=0 \quad (1)$$

$$\frac{p}{3}+\frac{q}{2}=\frac{13}{6} \quad \Rightarrow \quad 2p+3q-13=0 \quad (2)$$

Using cross-multiplication method, we obtain

$$\frac{p}{-26-(-36)}=\frac{q}{-24-(-39)}=\frac{1}{9-4}$$

$$\frac{p}{10}=\frac{q}{15}=\frac{1}{5}$$

$$\frac{p}{10}=\frac{1}{5} \text{ and } \frac{q}{15}=\frac{1}{5}$$

$$p=2 \text{ and } q=3$$

$$\frac{1}{x}=2 \text{ and } \frac{1}{y}=3$$

$$x=\frac{1}{2} \text{ and } y=\frac{1}{3}$$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

Putting $\frac{1}{\sqrt{x}}=p$ and $\frac{1}{\sqrt{y}}=q$ in the given equations, we obtain

$2p+3q=2 \quad (1)$

$4p-9q=-1 \quad (2)$

Multiplying equation (1) by 3, we obtain

$6p + 9q = 6$ (3)

Adding equation (2) and (3), we obtain

$10p = 5$

$p = \frac{1}{2}$ (4)

Putting in equation (1), we obtain

$2 \times \frac{1}{2} + 3q = 2$

$3q = 1$

$q = \frac{1}{3}$

$p = \frac{1}{\sqrt{x}} = \frac{1}{2}$

$\sqrt{x} = 2$

$x = 4$

and $q = \frac{1}{\sqrt{y}} = \frac{1}{3}$

$\sqrt{y} = 3$

$y = 9$

Hence, $x = 4, y = 9$

(iii) $\frac{4}{x} + 3y = 14$

$\frac{3}{x} - 4y = 23$

Substituting $\frac{1}{x} = p$ in the given equations, we obtain

$4p + 3y = 14 \Rightarrow 4p + 3y - 14 = 0$ (1)

$3p - 4y = 23 \Rightarrow 3p - 4y - 23 = 0$ (2)

By cross-multiplication, we obtain

$$\frac{p}{-69-56}=\frac{y}{-42-(-92)}=\frac{1}{-16-9}$$

$$\frac{p}{-125}=\frac{y}{50}=\frac{-1}{25}$$

$$\frac{p}{-125}=\frac{-1}{25} \text{ and } \frac{y}{50}=\frac{-1}{25}$$

$$p=5 \text{ and } y=-2$$

$$p=\frac{1}{x}=5$$

$$x=\frac{1}{5}$$

$$y=-2$$

(iv) $$\frac{5}{x-1}+\frac{1}{y-2}=2$$

$$\frac{6}{x-1}-\frac{3}{y-2}=1$$

Putting $\frac{1}{x-1}=p$ and $\frac{1}{y-2}=q$ in the given equation, we obtain

$5p+q=2 \qquad (1)$

$6p-3q=1 \qquad (2)$

Multiplying equation (1) by 3, we obtain

$15p+3q=6 \qquad (3)$

Adding (2) an (3), we obtain

$21p=7$

$$p=\frac{1}{3}$$

Putting this value in equation (1), we obtain

$5\times\frac{1}{3}+q=2$

$q=2-\frac{5}{3}=\frac{1}{3}$

$p=\frac{1}{x-1}=\frac{1}{3}$

$\Rightarrow x-1=3$

$\Rightarrow x=4$

$q=\frac{1}{y-2}=\frac{1}{3}$

$y-2=3$

$y=5$

$\therefore x=4, y=5$

(v) $\frac{7x-2y}{xy}=5$

$\frac{7}{y}-\frac{2}{x}=5 \quad (1)$

$\frac{8x+7y}{xy}=15$

$\frac{8}{y}+\frac{7}{x}=15 \quad (2)$

Putting $\frac{1}{x}=p$ and $\frac{1}{y}=q$ in the given equation, we obtain

$-2p+7q=5 \quad \Rightarrow \quad -2p+7q-5=0 \quad (3)$

$7p+8q=15 \quad \Rightarrow \quad 7p+8q-15=0 \quad (4)$

By cross-multiplication method, we obtain

$$\frac{p}{-105-(-40)}=\frac{q}{-35-30}=\frac{1}{-16-49}$$

$$\frac{p}{-65}=\frac{q}{-65}=\frac{1}{-65}$$

$$\frac{p}{-65}=\frac{1}{-65} \text{ and } \frac{q}{-65}=\frac{1}{-65}$$

$$p=1 \quad \text{and} \quad q=1$$

$$p=\frac{1}{x}=1 \qquad q=\frac{1}{y}=1$$

$$x=1 \qquad y=1$$

$$(\text{vi}) \quad 6x+3y=6xy$$

$$\Rightarrow \frac{6}{y}+\frac{3}{x}=6 \qquad (1)$$

$$2x+4y=5xy$$

$$\frac{2}{y}+\frac{4}{x}=5 \qquad (2)$$

Putting $\frac{1}{x}=p$ and $\frac{1}{y}=q$ in these equations, we obtain

$$3p+6q-6=0$$

$$4p+2q-5=0$$

By cross-multiplication method, we obtain

$$\frac{p}{-30-(-12)}=\frac{q}{-24-(-15)}=\frac{1}{6-24}$$

$$\frac{p}{-18}=\frac{q}{-9}=\frac{1}{-18}$$

$$\frac{p}{-18}=\frac{1}{-18} \text{ and } \frac{q}{-9}=\frac{1}{-18}$$

$$p=1 \quad \text{and} \quad q=\frac{1}{2}$$

$$p=\frac{1}{x}=1 \qquad q=\frac{1}{y}=\frac{1}{2}$$

$$x=1 \qquad y=2$$

Hence, $x = 1$, $y = 2$

(vii) $$\frac{10}{x+y}+\frac{2}{x-y}=4$$

$$\frac{15}{x+y}-\frac{5}{x-y}=-2$$

Putting $\frac{1}{x+y}=p$ and $\frac{1}{x-y}=q$ in the given equations, we obtain

$$10p+2q=4 \quad \Rightarrow \quad 10p+2q-4=0 \quad (1)$$

$$15p-5q=-2 \quad \Rightarrow \quad 15p-5q+2=0 \quad (2)$$

Using cross-multiplication method, we obtain

$$\frac{p}{4-20}=\frac{q}{-60-(20)}=\frac{1}{-50-30}$$

$$\frac{p}{-16}=\frac{q}{-80}=\frac{1}{-80}$$

$$\frac{p}{-16}=\frac{1}{-80} \text{ and } \frac{q}{-80}=\frac{1}{-80}$$

$$p=\frac{1}{5} \text{ and } q=1$$

$p = \frac{1}{x+y} = \frac{1}{5}$ and $q = \frac{1}{x-y} = 1$

$x + y = 5 \qquad (3)$

and $x - y = 1 \qquad (4)$

Adding equation (3) and (4), we obtain

$2x = 6$

$x = 3 \qquad (5)$

Substituting in equation (3), we obtain

$y = 2$

Hence, $x = 3$, $y = 2$

(viii) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$

Putting $\frac{1}{3x+y} = p$ and $\frac{1}{3x-y} = q$ in these equations, we obtain

$p + q = \frac{3}{4} \qquad (1)$

$\frac{p}{2} - \frac{q}{2} = \frac{-1}{8}$

$p - q = \frac{-1}{4} \qquad (2)$

Adding (1) and (2), we obtain

$2p = \frac{3}{4} - \frac{1}{4}$

$2p = \frac{1}{2}$

$p = \frac{1}{4}$

Substituting in (2), we obtain

$$\frac{1}{4}-q=\frac{-1}{4}$$

$$q=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$

$$p=\frac{1}{3x+y}=\frac{1}{4}$$

$$3x+y=4 \qquad (3)$$

$$q=\frac{1}{3x-y}=\frac{1}{2}$$

$$3x-y=2 \qquad (4)$$

Adding equations (3) and (4), we obtain

$$6x=6$$

$$x=1 \qquad (5)$$

Substituting in (3), we obtain

$$3(1)+y=4$$

$$y=1$$

Hence, x = 1, y = 1

Question 2:

Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Answer:

(i)Let the speed of Ritu in still water and the speed of stream be *x* km/h and *y* km/h respectively.

Speed of Ritu while rowing

Upstream = $(x-y)$ km/h

Downstream = $(x+y)$ km/h

According to question,

$2(x+y)=20$

$\Rightarrow x+y=10 \qquad (1)$

$2(x-y)=4$

$\Rightarrow x-y=2 \qquad (2)$

Adding equation (1) and (2), we obtain

$2x=12 \Rightarrow x=6$

Putting this in equation (1), we obtain

y = 4

Hence, Ritu's speed in still water is 6 km/h and the speed of the current is 4 km/h.

(ii)Let the number of days taken by a woman and a man be *x* and *y* respectively.

Therefore, work done by a woman in 1 day = $\frac{1}{x}$

Work done by a man in 1 day = $\frac{1}{y}$

According to the question,

$4\left(\frac{2}{x}+\frac{5}{y}\right)=1$

$\frac{2}{x}+\frac{5}{y}=\frac{1}{4}$

$3\left(\frac{3}{x}+\frac{6}{y}\right)=1$

$\frac{3}{x}+\frac{6}{y}=\frac{1}{3}$

Putting $\frac{1}{x}=p$ and $\frac{1}{y}=q$ in these equations, we obtain

$2p+5q=\frac{1}{4}$

$\Rightarrow \quad 8p+20q=1$

$3p+6q=\frac{1}{3}$

$\Rightarrow \; 9p+18q=1$

By cross-multiplication, we obtain

$\frac{p}{-20-(-18)}=\frac{q}{-9-(-8)}=\frac{1}{144-180}$

$\frac{p}{-2}=\frac{q}{-1}=\frac{1}{-36}$

$\frac{p}{-2}=\frac{-1}{36}$ and $\frac{q}{-1}=\frac{1}{-36}$

$p=\frac{1}{18}$ and $q=\frac{1}{36}$

$p=\frac{1}{x}=\frac{1}{18}$ and $q=\frac{1}{y}=\frac{1}{36}$

$x=18 \qquad y=36$

Hence, number of days taken by a woman = 18

Number of days taken by a man = 36

(iii) Let the speed of train and bus be *u* km/h and *v* km/h respectively.

According to the given information,

$$\frac{60}{u}+\frac{240}{v}=4 \qquad (1)$$

$$\frac{100}{u}+\frac{200}{v}=\frac{25}{6} \qquad (2)$$

Putting $\frac{1}{u}=p$ and $\frac{1}{v}=q$ in these equations, we obtain

$$60p+240q=4 \qquad (3)$$

$$100p+200q=\frac{25}{6}$$

$$600p+1200q=25 \qquad (4)$$

Multiplying equation (3) by 10, we obtain

$$600p+2400q=40 \qquad (5)$$

Subtracting equation (4) from (5), we obtain

$$1200q=15$$

$$q=\frac{15}{1200}=\frac{1}{80} \qquad (6)$$

Substituting in equation (3), we obtain

$$60p+3=4$$

$$60p=1$$

$$p=\frac{1}{60}$$

$$p=\frac{1}{u}=\frac{1}{60} \quad \text{and} \quad q=\frac{1}{v}=\frac{1}{80}$$

$$u=60\text{ km/h} \quad \text{and} \quad v=80\text{ km/h}$$

Hence, speed of train = 60 km/h

Speed of bus = 80 km/h

Exercise 3.7

Question 1:

The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differs by 30 years. Find the ages of Ani and Biju.

Answer:

The difference between the ages of Biju and Ani is 3 years. Either Biju is 3 years older than Ani or Ani is 3 years older than Biju. However, it is obvious that in both cases, Ani's father's age will be 30 years more than that of Cathy's age.

Let the age of Ani and Biju be x and y years respectively.

Therefore, age of Ani's father, Dharam = 2 × x = 2x years

And age of Biju's sister Cathy $= \frac{y}{2}$ years

By using the information given in the question,

Case (I) When Ani is older than Biju by 3 years,

$x - y = 3$ (i)

$$2x - \frac{y}{2} = 30$$

$4x - y = 60$ (ii)

Subtracting (i) from (ii), we obtain

$3x = 60 - 3 = 57$

$$x = \frac{57}{3} = 19$$

Therefore, age of Ani = 19 years

And age of Biju = 19 − 3 = 16 years

Case (II) When Biju is older than Ani,

$y - x = 3$ (i)

$$2x - \frac{y}{2} = 30$$

$4x - y = 60$ (*ii*)

Adding (*i*) and (*ii*), we obtain

$3x = 63$

$x = 21$

Therefore, age of Ani = 21 years

And age of Biju = 21 + 3 = 24 years

Question 2:

One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II)

[**Hint:** $x + 100 = 2(y - 100)$, $y + 10 = 6(x - 10)$]

Answer:

Let those friends were having Rs x and y with them.

Using the information given in the question, we obtain

$x + 100 = 2(y - 100)$

$x + 100 = 2y - 200$

$x - 2y = -300$ (*i*)

And, $6(x - 10) = (y + 10)$

$6x - 60 = y + 10$

$6x - y = 70$ (*ii*)

Multiplying equation (*ii*) by 2, we obtain

$12x - 2y = 140$ (*iii*)

Subtracting equation (*i*) from equation (*iii*), we obtain

$11x = 140 + 300$

$11x = 440$

$x = 40$

Using this in equation (*i*), we obtain

$40 - 2y = -300$

$40 + 300 = 2y$

$2y = 340$

$y = 170$

Therefore, those friends had Rs 40 and Rs 170 with them respectively.

Question 3:

A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Answer:

Let the speed of the train be x km/h and the time taken by train to travel the given distance be t hours and the distance to travel was d km. We know that,

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken to travel that distance}}$$

$$x = \frac{d}{t}$$

Or, $d = xt$ (i)

Using the information given in the question, we obtain

$$(x+10) = \frac{d}{(t-2)}$$

$$(x+10)(t-2) = d$$

$$xt + 10t - 2x - 20 = d$$

By using equation (i), we obtain

$-2x + 10t = 20$ (ii)

$$(x-10) = \frac{d}{(t+3)}$$

$$(x-10)(t+3) = d$$

$$xt - 10t + 3x - 30 = d$$

By using equation (i), we obtain

$3x - 10t = 30$ (iii)

Adding equations (ii) and (iii), we obtain

$x = 50$

Using equation (*ii*), we obtain

$(-2) \times (50) + 10t = 20$

$-100 + 10t = 20$

$10t = 120$

$t = 12$ hours

From equation (*i*), we obtain

Distance to travel = $d = xt$

$= 50 \times 12$

$= 600$ km

Hence, the distance covered by the train is 600 km.

Question 4:

The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Answer:

Let the number of rows be x and number of students in a row be y.

Total students of the class

= Number of rows × Number of students in a row

$= xy$

Using the information given in the question,

Condition 1

Total number of students = $(x - 1)(y + 3)$

$xy = (x - 1)(y + 3) = xy - y + 3x - 3$

$3x - y - 3 = 0$

$3x - y = 3$ (*i*)

Condition 2

Total number of students = $(x + 2)(y - 3)$

$xy = xy + 2y - 3x - 6$

$3x - 2y = -6$ (*ii*)

Subtracting equation (*ii*) from (*i*),

$(3x - y) - (3x - 2y) = 3 - (-6)$

$-y + 2y = 3 + 6$

$y = 9$

By using equation (*i*), we obtain

$3x - 9 = 3$

$3x = 9 + 3 = 12$

$x = 4$

Number of rows = x = 4

Number of students in a row = y = 9

Number of total students in a class = $xy = 4 \times 9 = 36$

Question 5:

In a ΔABC, ∠C = 3 ∠B = 2 (∠A + ∠B). Find the three angles.

Answer:

Given that,

$\angle C = 3\angle B = 2(\angle A + \angle B)$

$3\angle B = 2(\angle A + \angle B)$

$3\angle B = 2\angle A + 2\angle B$

$\angle B = 2\angle A$

$2\angle A - \angle B = 0$ … (*i*)

We know that the sum of the measures of all angles of a triangle is 180°. Therefore,

$\angle A + \angle B + \angle C = 180°$

$\angle A + \angle B + 3\angle B = 180°$

$\angle A + 4\angle B = 180°$ … (*ii*)

Multiplying equation (*i*) by 4, we obtain

$8\angle A - 4\angle B = 0$ … (*iii*)

Adding equations (*ii*) and (*iii*), we obtain

$9\angle A = 180°$

$\angle A = 20°$

From equation (*ii*), we obtain

$20° + 4\angle B = 180°$

$4\angle B = 160°$

$\angle B = 40°$

$\angle C = 3\angle B$

$= 3 \times 40° = 120°$

Therefore, $\angle A$, $\angle B$, $\angle C$ are 20°, 40°, and 120° respectively.

Question 6:

Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the y axis.

Answer:

$5x - y = 5$

Or, $y = 5x - 5$

The solution table will be as follows.

x	0	1	2
y	−5	0	5

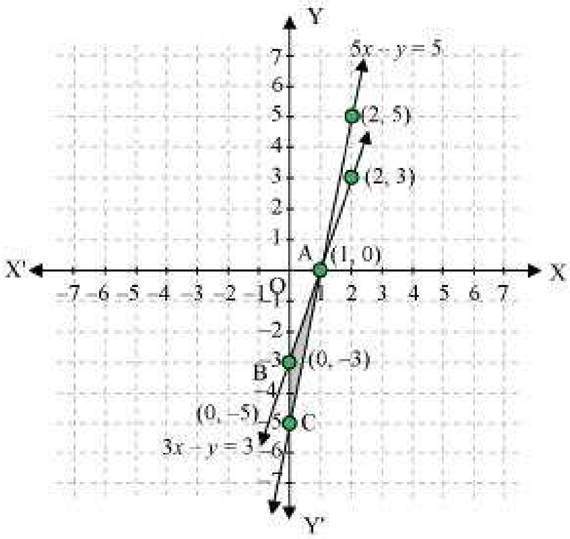
$3x - y = 3$

Or, $y = 3x - 3$

The solution table will be as follows.

x	0	1	2
y	− 3	0	3

The graphical representation of these lines will be as follows.



It can be observed that the required triangle is ΔABC formed by these lines and *y*-axis.

The coordinates of vertices are A (1, 0), B (0, − 3), C (0, − 5).

Question 7:

Solve the following pair of linear equations.

(i) $px + qy = p - q$

$qx - py = p + q$

(ii) $ax + by = c$

$bx + ay = 1 + c$

(iii) $\frac{x}{a} - \frac{y}{b} = 0$

$ax + by = a^2 + b^2$

(iv) $(a - b)x + (a + b)y = a^2 - 2ab - b^2$

$(a + b)(x + y) = a^2 + b^2$

(v) $152x - 378y = -74$

$-378x + 152y = -604$

Answer:

(i) $px + qy = p - q$ … (1)

$qx - py = p + q$ … (2)

Multiplying equation (1) by p and equation (2) by q, we obtain

$p^2x + pqy = p^2 - pq$ … (3)

$q^2x - pqy = pq + q^2$ … (4)

Adding equations (3) and (4), we obtain

$p^2x + q^2x = p^2 + q^2$

$(p^2 + q^2)x = p^2 + q^2$

$$x = \frac{p^2 + q^2}{p^2 + q^2} = 1$$

From equation (1), we obtain

$p(1) + qy = p - q$

$qy = -q$

$y = -1$

(ii) $ax + by = c$ … (1)

$bx + ay = 1 + c$ … (2)

Multiplying equation (1) by a and equation (2) by b, we obtain

$a^2x + aby = ac$ … (3)

$b^2x + aby = b + bc$ … (4)

Subtracting equation (4) from equation (3),

$(a^2 - b^2)x = ac - bc - b$

$$x = \frac{c(a-b) - b}{a^2 - b^2}$$

From equation (1), we obtain

$ax + by = c$

$$a\left\{\frac{c(a-b)-b}{a^2-b^2}\right\}+by=c$$

$$\frac{ac(a-b)-ab}{a^2-b^2}+by=c$$

$$by=c-\frac{ac(a-b)-ab}{a^2-b^2}$$

$$by=\frac{a^2c-b^2c-a^2c+abc+ab}{a^2-b^2}$$

$$by=\frac{abc-b^2c+ab}{a^2-b^2}$$

$$by=\frac{bc(a-b)+ab}{a^2-b^2}$$

$$y=\frac{c(a-b)+a}{a^2-b^2}$$

(iii) $\frac{x}{a}-\frac{y}{b}=0$

Or, $bx - ay = 0$ … (1)

$ax + by = a^2 + b^2$ … (2)

Multiplying equation (1) and (2) by b and a respectively, we obtain

$b^2x - aby = 0$ … (3)

$a^2x + aby = a^3 + ab^2$ … (4)

Adding equations (3) and (4), we obtain

$b^2x + a^2x = a^3 + ab^2$

$x (b^2 + a^2) = a (a^2 + b^2)$

$x = a$

By using (1), we obtain

$b (a) - ay = 0$

$ab - ay = 0$

$ay = ab$

$y = b$

(iv) $(a - b)x + (a + b)y = a^2 - 2ab - b^2$ … (1)

$(a + b)(x + y) = a^2 + b^2$

$(a + b)x + (a + b)y = a^2 + b^2$ … (2)

Subtracting equation (2) from (1), we obtain

$(a - b)x - (a + b)x = (a^2 - 2ab - b^2) - (a^2 + b^2)$

$(a - b - a - b)x = -2ab - 2b^2$

$-2bx = -2b(a + b)$

$x = a + b$

Using equation (1), we obtain

$(a - b)(a + b) + (a + b)y = a^2 - 2ab - b^2$

$a^2 - b^2 + (a + b)y = a^2 - 2ab - b^2$

$(a + b)y = -2ab$

$$y = \frac{-2ab}{a+b}$$

(v) $152x - 378y = -74$

$76x - 189y = -37$

$$x = \frac{189y - 37}{76} \quad \ldots (1)$$

$-378x + 152y = -604$

$-189x + 76y = -302$ … (2)

Substituting the value of x in equation (2), we obtain

$$-189\left(\frac{189y - 37}{76}\right) + 76y = -302$$

$-(189)^2 y + 189 \times 37 + (76)^2 y = -302 \times 76$

$189 \times 37 + 302 \times 76 = (189)^2 y - (76)^2 y$

$6993 + 22952 = (189 - 76)(189 + 76)y$

$29945 = (113)(265)y$

$y = 1$

From equation (1), we obtain

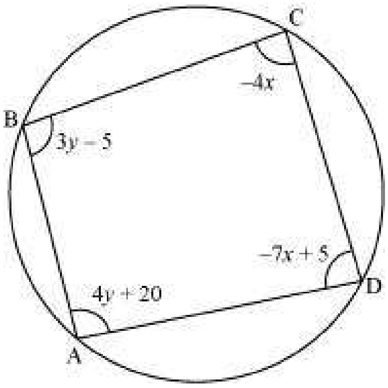
$x = \frac{189(1) - 37}{76}$

$x = \frac{189 - 37}{76} = \frac{152}{76}$

$x = 2$

Question 8:

ABCD is a cyclic quadrilateral finds the angles of the cyclic quadrilateral.



Answer:

We know that the sum of the measures of opposite angles in a cyclic quadrilateral is 180°.

Therefore, ∠A + ∠C = 180

$4y + 20 - 4x = 180$

$-4x + 4y = 160$

$x - y = -40$ (*i*)

Also, ∠B + ∠D = 180

$3y - 5 - 7x + 5 = 180$

$-7x + 3y = 180$ (*ii*)

Multiplying equation (*i*) by 3, we obtain

$3x - 3y = -120$ (*iii*)

Adding equations (*ii*) and (*iii*), we obtain

$-7x + 3x = 180 - 120$

$-4x = 60$

$x = -15$

By using equation (*i*), we obtain

$x - y = -40$

$-15 - y = -40$

$y = -15 + 40 = 25$

$\angle A = 4y + 20 = 4(25) + 20 = 120°$

$\angle B = 3y - 5 = 3(25) - 5 = 70°$

$\angle C = -4x = -4(-15) = 60°$

$\angle D = -7x + 5 = -7(-15) + 5 = 110°$