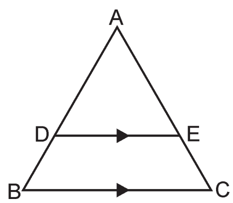
Assignments in Mathematics Class X (Term I)

6. TRIANGLES

IMPORTANT TERMS, DEFINITIONS AND RESULTS

- Two figures having the same shape but not necessarily the same size are called similar figures.
- All the congruent figures are similar but the converse is not true.
- Two polygons of the same number of sides are similar, if (*i*) their corresponding angles are equal and (*ii*) their corresponding sides are in the same ratio (*i.e.,* proportion).
- Two triangles are similar, if
  - (i) their corresponding angles are equal
  - (ii) their corresponding sides are in the same ratio (or proportion).
- **Basic Proportionality Theorem (B.P.T.) (Thales Theorem)**

  In a triangle, a line drawn parallel to one side, to intersect the other sides in distinct points, divides the two sides in the same ratio.

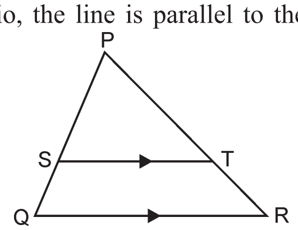


In ΔABC, if $DE \parallel BC$ then (*i*) $\frac{AD}{DB}=\frac{AE}{EC}$

(*ii*) $\frac{AB}{AD}=\frac{AC}{AE}$ (*iii*) $\frac{AB}{DB}=\frac{AC}{EC}$

- **Converse of Basic Proportionality Theorem**

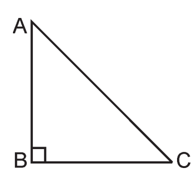
If a line divides any two sides of a triangle in the same ratio, the line is parallel to the third side.



In ΔPQR, if $\frac{PS}{SQ}=\frac{PT}{TR}$, , then $ST \parallel QR$.

- If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar (*AAA similarity criterion*).
- If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are similar (*AA similarity criterion*).
- If in two triangles, corresponding sides are in the same ratio, then their corresponding angles are equal and hence the triangles are similar (*SSS similarity criterion*).
- If one angle of a triangle is equal to the one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are similar (*SAS similarity criterion*).
- If a perpendicular is drawn from the vertex of the right angle of a right triangle to hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.
- The ratio of the areas of two similar triangles are equal to the ratio of the squares of any two corresponding sides.
- The areas of two similar triangles are in the ratio of the squares of the corresponding altitudes.
- The areas of two similar triangles are in the ratio of the squares of the corresponding medians.
- If the areas of two similar triangles are equal, then the triangles are congruent, *i.e.*, equal and similar triangles are congruent.
- **The Pythagoras Theorem**

  In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. In figure, $\angle B = 90°$, so, $AC^2 = AB^2 + BC^2$.



- **Converse of the Pythagoras Theorem**

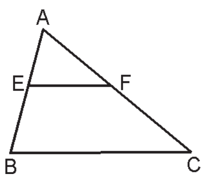
  In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

SUMMATIVE ASSESSMENT

MULTIPLE CHOICE QUESTIONS **[1 Mark]**

A. Important Questions

1. In the figure, if $\frac{AE}{EB} = \frac{AF}{FC}$, then we can conclude that :



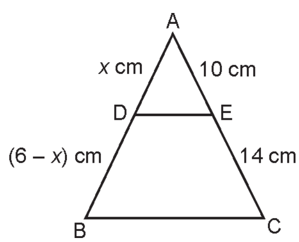
(a) *E* and *F* are the mid-points of *AB* and *AC* respectively

(b) $EF \parallel BC$

(c) $\frac{EF}{BC} = \frac{AB}{AC}$

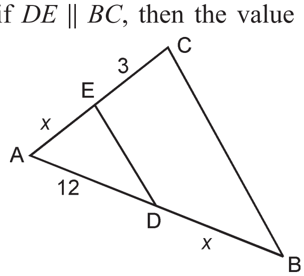
(d) none of the above

2. In the triangle *ABC*, $DE \parallel BC$, then the length of *DB* is :



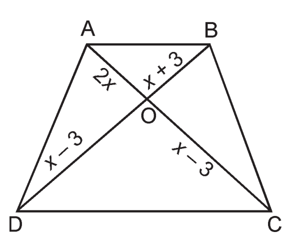
(a) 2.5 cm (b) 5 cm (c) 3.5 cm (d) 3 cm

3. In ΔABC, if $DE \parallel BC$, then the value of *x* is :



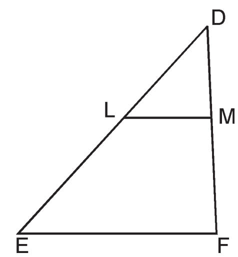
(a) 4 (b) 6 (c) 8 (d) 9

4. In the trapezium *ABCD*, $AB \parallel CD$, then the value of *x* is :



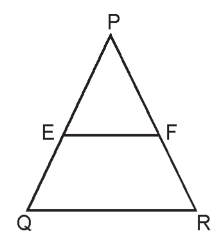
(a) 2 (b) 3 (c) –2 (d) –3

5. In the ΔDEF, $LM \parallel EF$ and $\frac{DM}{MF} = \frac{2}{3}$. If $DE = 5.5$ cm, then *DL* is :



(a) 2.5 cm (b) 2.4 cm (c) 2.2 cm (d) 2 cm

6. In the given figure, $PQ = 1.28$ cm, $PR = 2.56$ cm, $PE = 0.18$ cm and $PF = 0.36$ cm, then :



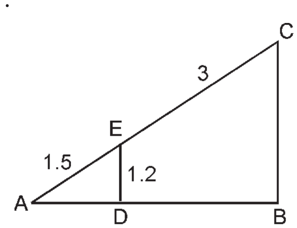
(a) *EF* is not parallel to *QR*

(b) $EF \parallel QR$

(c) cannot say anything

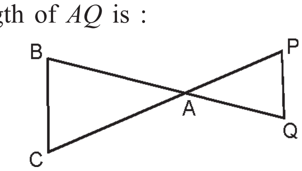
(d) none of the above

7. In the given figure, if ΔADE ~ ΔABC, then BC is equal to :



(a) 4.5` (b) 3 (c) 3.6 (d) 2.4

8. In the given figure. $\Delta ACB \sim \Delta APQ$. If $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm and AP = 2.8 cm, then the length of *AQ* is :



(a) 3.25 cm (b) 4 cm (c) 4.25 cm (d) 3 cm

9. If $\Delta ABC \sim \Delta PQR$ and $\angle P = 50°$, $\angle B = 60°$, then $\angle R$ is :

(a) 100° (b) 80° (c) 70° (d) cannot be determined

10. $\Delta ABC \sim \Delta DEF$ and the perimeters of ΔABC and ΔDEF are 30 cm and 18 cm respectively.

If BC = 9 cm, then EF is equal to :

(a) 6.3 cm (b) 5.4 cm (c) 7.2 cm (d) 4.5 cm

11. ΔABC ~ ΔDEF such that AB = 9.1 cm and DE = 6.5 cm. If the perimeter of ΔDEF is 25 cm, then perimeter of ΔABC is :

(a) 35 cm (b) 28 cm (c) 42 cm (d) 40 cm

12. If $\Delta ABC \sim \Delta EDF$ and ΔABC is not similar to ΔDEF, then which of the following is not true?

(a) $BC.\ EF = AC.\ FD$ (b) $AB.\ EF = AC.\ DE$

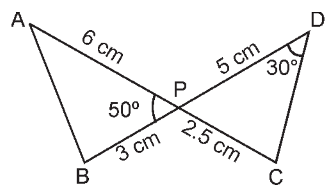
(c) $BC.\ DE = AB.\ EF$ (d) $BC.\ DE = AB.\ FD$

13. If in two triangles ABC and PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then :

(a) $\Delta PQR \sim \Delta CAB$ (b) $\Delta PQR \sim \Delta ABC$

(c) $\Delta CBA \sim \Delta PQR$ (d) $\Delta BCA \sim \Delta PQR$

14. In the given figure, two line segments, AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, $\angle APB$ = 50º and $\angle CDP$ = 30º. Then $\angle PBA$ is equal to:



(a) 50° (b) 30° (c) 60° (d) 100°

15. If in triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when :

(a) ∠B = ∠E (b) ∠A = ∠D

(c) ∠B = ∠D (d) ∠A = ∠F

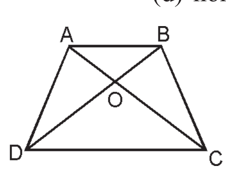
16. The areas of two similar triangles are 169 cm^2 and 121 cm^2, if the longest side of the larger triangle is 26 cm, then the longest side of the other triangle is :

(a) 12 cm (b) 14 cm (c) 19 cm (d) 22 cm

17. In the following trapezium $ABCD$, $AB \parallel CD$ and $CD = 2AB$. If area (ΔAOB) = 84 cm^2, then area (ΔCOD) is :

(a) 168 cm^2 (b) 336 cm^2

(c) 252 cm^2 (d) none of these



18. If $\Delta ABC \sim \Delta PQR$, area ($\Delta ABC$) = 80 cm^2 and area (ΔPQR) = 245 cm^2, then $\frac{AB}{PQ}$ is equal to :

(a) 16 : 49 (b) 4 : 7

(c) 2 : 5 (d) none of these

19. In the similar triangles, ΔABC and ΔDEF, $\frac{ar\,(\Delta ABC)}{ar\,(\Delta DEF)} = \frac{3}{4}$. If the median AL = 6 cm, then the median DM of ΔDEF is :

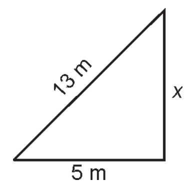
(a) $3\sqrt{2}\ cm$ (b) $4\sqrt{3}\ cm$

(c) $4\sqrt{2}\ cm$ (d) $3\sqrt{3}\ cm$

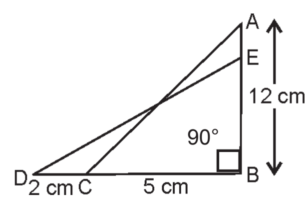
20. If a ladder of length 13 m is placed against a wall such that its foot is at a distance of 5 m from the wall, then the height of the top of the ladder from the ground is :

(a) 10 m (b) 11 m

(c) 12 m (d) none of these



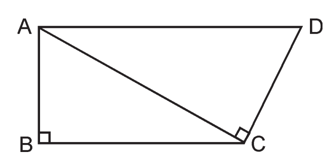
21. In the figure, if $AC = DE$, then the value of EB is :



(a) $3\sqrt{30}$ cm (b) $2\sqrt{30}$ cm

(c) $3\sqrt{15}$ cm (d) $4\sqrt{15}$ cm

22. In the quadrilateral $ABCD$, if $\angle B = 90°$ and $\angle ACD = 90°$, then AD^2 is:



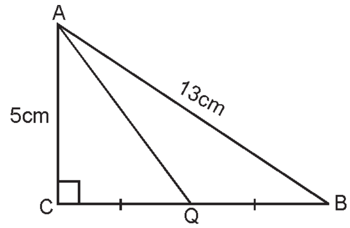
(a) $AC^2 - AB^2 + BC^2$ (b) $AC^2 + DC^2 + AB^2$

(c) $AB^2 + BC^2 + CD^2$ (d) $AB^2 + BC^2 + AC^2$

23. If diagonals of a rhombus are 12 cm and 16 cm, then the perimeter of the rhombus is :

(a) 20 cm (b) 40 cm (c) 28 cm (d) 56 cm

24. In the figure, ΔABC is right angled at C and Q is the mid-point of BC, then the length of AQ is :



(a) 6 cm (b) 12 cm

(c) $\sqrt{61}$ cm (d) $6\sqrt{3}$ cm

25. In triangle ABC and DEF, $\angle A \neq \angle C$, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = EF$. Then, the two triangle are :

(a) neither congruent nor similar
(b) congruent as well as similar
(c) congruent but not similar
(d) similar but not congruent

26. If in triangle ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, if :

(a) $\angle B = \angle E$ (b) $\angle A = \angle D$
(c) $\angle B = \angle D$ (d) $\angle A = \angle F$

27. If $\Delta ABC \sim \Delta QRP$, $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{9}{4}$, perimeter $\Delta ABC = 48$ cm, $AB = 18$ cm and $BC = 18$ cm, then PQ is equal to :

(a) 8 cm (b) 10 cm (c) 12 cm (d) $\frac{20}{3}$ cm

28. D and E are respectively the points on the sides AB and AC of a triangle ABC such that $AD = 2$ cm, $BD = 4$ cm, $BC = 9$ cm and $DE \parallel BC$. Then, length of DE (in cm) is :

(a) 6 (b) 5 (c) 3 (d) 2.5

29. It is given that $\Delta ABC \sim \Delta PQR$, with $\frac{BC}{QR} = \frac{1}{4}$, then, $\frac{\text{ar}(PRQ)}{\text{ar}(BCA)}$ is equal to :

(a) $\frac{1}{4}$ (b) $\frac{1}{16}$ (c) 4 (d) 16

B. Questions From CBSE Examination Papers

1. The lengths of the diagonals of a rhombus are 24 cm and 32 cm. The perimeter of the rhombus is : **[2010 (T-I)]**

(a) 9 cm (b) 128 cm (c) 80 cm (d) 56 cm

2. Which of the following cannot be the sides of a right triangle ? **[2010 (T-I)]**

(a) 9 cm, 15 cm, 12 cm
(b) 2 cm, 1 cm, $\sqrt{5}$ cm
(c) 400 mm, 300 mm, 500 mm
(d) 9 cm, 5 cm, 7 cm

3. $\Delta ABC \sim \Delta PQR$, M is the mid-point of BC and N is the mid point of QR. If the area of $\Delta ABC = 100$ sq. cm, the area of $\Delta PQR = 144$ sq. cm and $AM = 4$ cm, then PN is : **[2010 (T-I)]**

(a) 4.8 cm (b) 12 cm (c) 4 cm (d) 5.6 cm

4. ΔABC is such that $AB = 3$ cm, $BC = 2$ cm and $CA = 2.5$ cm. If $\Delta DEF \sim \Delta ABC$ and $EF = 4$ cm, then perimeter of ΔDEF is : **[2010 (T-I)]**

(a) 15 cm (b) 22.5 cm (c) 7.5 cm (d) 30 cm

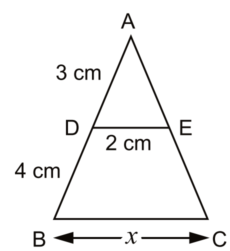
5. A vertical stick 30 m long casts a shadow 15 m long on the ground. At the same time, a tower casts a shadow 75 m long on the ground. The height of the tower is : **[2010 (T-I)]**

(a) 150 m (b) 100 m (c) 25 m (d) 200 m

6. $\Delta ABC \sim \Delta PQR$. If ar(ABC) = 2.25 m^2, *ar* (PQR) = 6.25 m^2, $PQ = 0.5$ m, then length of AB is : **[2010 (T-I)]**

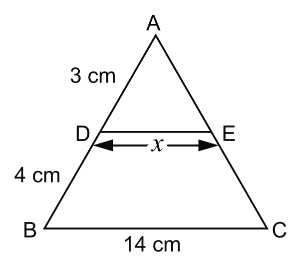
(a) 30 cm (b) 0.5 m (c) 50 m (d) 3 m

7. In figure, if $DE \parallel BC$ then x equals to :



(a) 3 cm (b) 4 cm (c) 7 cm (d) 4.7 cm

8. In figure, if $DE \parallel BC$, then x equals : **[2010 (T-I)]**

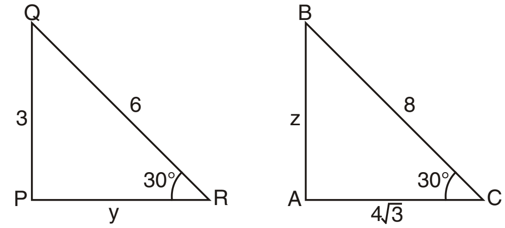


(a) 6 cm (b) 7 cm (c) 3 cm (d) 4 cm

9. ΔABC and ΔPQR are similar triangles such that $\angle A = 32°$ and $\angle R = 65°$, then $\angle B$ is : **[2010 (T-I)]**

(a) 83° (b) 32° (c) 65° (d) 97°

10. In the figure $\Delta ABC \sim \Delta PQR$, then $y + z$ is :



(a) $2+\sqrt{3}$ (b) $4+3\sqrt{3}$ (c) $4+\sqrt{3}$ (d) $3+4\sqrt{3}$

11. The perimeters of two similar triangles *ABC* and *LMN* are 60 cm and 48 cm respectively. **[2010 (T-I)]**

If $LM = 8$ cm, length of *AB* is :

(a) 10 cm (b) 8 cm (c) 5 cm (d) 6 cm

12. If in ΔABC and ΔDEF $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar if : **[2010 (T-I)]**

(a) $\angle B = \angle E$ (b) $\angle A = \angle D$

(c) $\angle B = \angle D$ (d) $\angle A = \angle F$

13. In an isosceles ΔABC, if $AC = BC$ and $AB^2 = 2AC^2$, then $\angle C$ is equal to : **[2010 (T-I)]**

(a) 45° (b) 60° (c) 30° (d) 90°

14. If $\Delta ABC \sim \Delta DEF$, $BC = 4$ cm, $EF = 5$ cm and *ar* $(\Delta ABC) = 80$ cm^2, then *ar* (ΔDEF) is : **[2010 (T-I)]**

(a) 100 cm^2 (b) 125 cm^2

(c) 150 cm^2 (d) 200 cm^2

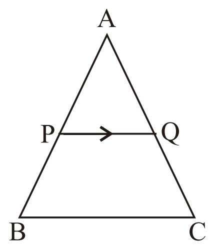
15. The areas of two similar triangles *ABC* and *PQR* are 25 cm^2 and 49 cm^2 respectively. If $QR = 9.8$ cm, then *BC* is : **[2010 (T-I)]**

(a) 9.8 cm (b) 7 cm (c) 49 cm (d) 25 cm

16. If the ratio of the corresponding sides of two similar triangles is 2 : 3, then the ratio of their corresponding altitude is : **[2010 (T-I)]**

(a) 3 : 2 (b) 16 : 81 (c) 4 : 9 (d) 2 : 3

17. In the figure, $PQ \parallel BC$ and $AP : PB = 1 : 2$. Find $\frac{\text{ar}(\Delta APQ)}{\text{ar}(\Delta ABC)}$: **[2010 (T-I)]**

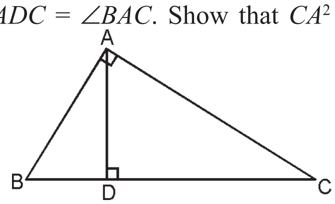


(a) 1 : 4 (b) 4 : 1 (c) 1 : 9 (d) 2 : 9

SHORT ANSWER TYPE QUESTIONS **[2 Marks]**

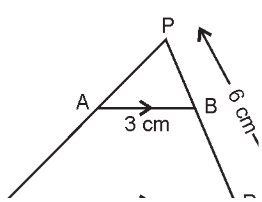
A. Important Questions

1. *D* is a point on the side *BC* of a triangle *ABC* such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB.CD$.



2. *S* and *T* are points on sides *PR* and *QR* of ΔPQR such that $\angle P = \angle RTS$.
Show that $\Delta RPQ \sim \Delta RTS$.

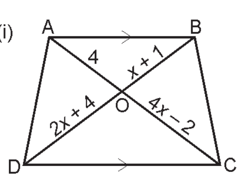
3. In the given figure, AB || QR. Find the length of PB.



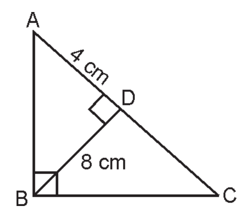
4. If the sides of a triangle are 3 cm, 4 cm and 6 cm long, determine whether the triangle is a right-angled triangle.

5. In the given figure, if $AB \parallel CD$, find the value of *x*.

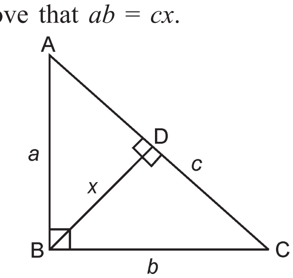
(i)



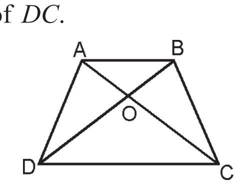
6. In the given figure, $\angle ABC = 90°$ and $BD \perp AC$. If $BD = 8$ cm and $AD = 4$ cm, find *CD*.



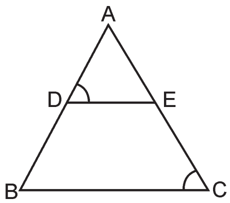
7. In a right angled triangle with sides a and b and hypotenuse *c*, the altitude drawn on the hypotenuse is *x*. Prove that $ab = cx$.



8. In the figure, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and $AB = 8$ cm. Find the value of *DC*.



9. In the given figure, $\frac{AD}{DB} = \frac{AE}{EC}$ and $\angle ADE = \angle ACB$. Prove that ΔABC is an isosceles triangle.



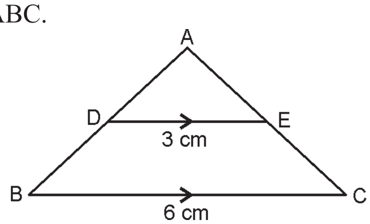
10. E is a point on the side AD produced of a parallelogram $ABCD$ and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$.

11. P and Q are points on sides AB and AC respectively of ΔABC. If $AP = 3$ cm, $PB = 6$ cm, $AQ = 5$ cm and $QC = 10$ cm, show that $BC = 3\,PQ$.

12. In a triangle ABC, altitudes AL and BM intersect in O. Prove that $\frac{AO}{BO} = \frac{OM}{OL}$.

13. A wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

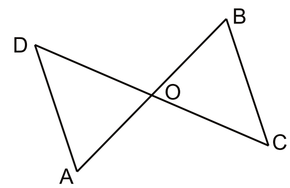
14. In the given figure, $DE \parallel BC$. If $DE = 3$ cm, $BC = 6$ cm and area $(\Delta ADE) = 15\text{ cm}^2$, find the area of ΔABC.



15. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

16. $ABCD$ is a trapezium with $AB \parallel DC$. If E and F are on non-parallel sides AD and BC respectively such that EF is parallel to AB, then show that $\frac{AE}{ED} = \frac{BF}{FC}$.

17. In the given figure, $OC.OD = OA.OB$. Show that $\angle A = \angle C$ and $\angle B = \angle D$.



18. D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Then find ar ΔDEF : ar ΔABC.

19. Prove that the area of the semicircle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the semicircles drawn on the other two sides of the triangle.

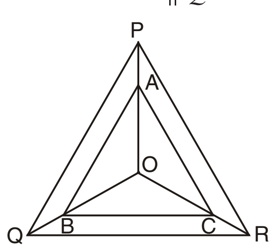
20. A ladder is placed against a wall such that its foot is at a distance of 3.5 m from the wall and its top reaches a window 12 m above the ground. Find the length of the ladder.

21. If ABC is an equilateral triangle of side $2a$, then find each of its altitude.

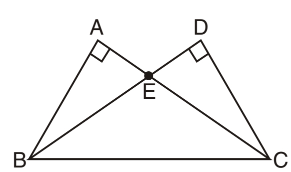
22. An aeroplane leaves an airport and flies due north at a speed of 500 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after 2 hours?

B. Questions From CBSE Examination Papers

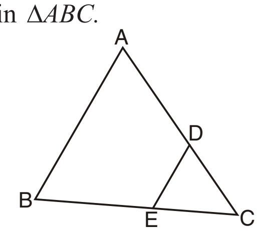
1. In the figure, A, B and C are points on OP, OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$. **[2010 (T-I)]**



2. In the figure, two triangles ABC and DBC are on the same base BC in which $\angle A = \angle D = 90°$. If CA and BD meet each other at E, show that $AE \times CE - BE \times DE$. **[2010 (T-I)]**

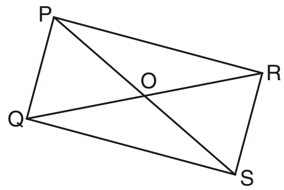


3. In the figure, if $\angle A = \angle B$ and $AD = BE$, show that $DE \parallel AB$ in ΔABC. **[2010 (T-I)]**

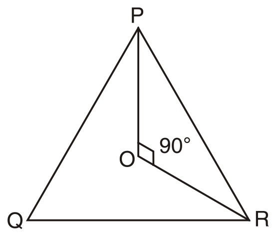


4. In the figure, PQR and SQR are two triangles on the same base QR. If PS intersect QR at O, then

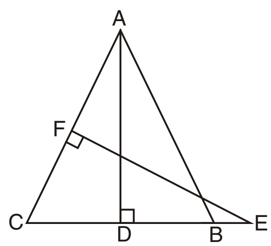
show that : $\dfrac{ar(PQR)}{ar(SQR)} = \dfrac{PO}{SO}$. **[2010 (T-I)]**



5. In the figure, O is a point inside ΔPQR such that $\angle POR = 90°$, $OP = 6$ cm and $OR = 8$. If $PQ = 24$ cm, $QR = 26$ cm, prove that ΔQPR is a right angled triangle. **[2010 (T-I)]**



6. In the given figure, E is a point on side CB produced of an isosceles ΔABC with AB = BC. If $AD \perp BC$ and $EF \perp AC$, prove that $\Delta ABD \sim \Delta ECF$. **[2010 (T-I)]**

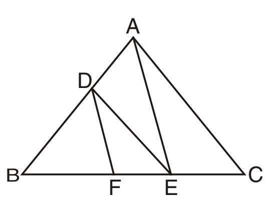


7. Prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal. **[2010 (T-I)]**

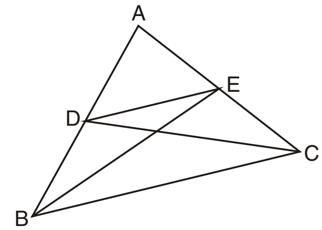
8. In a right angled triangle if hypotenuse is 20 cm and the ratio of other two sides is 4 : 3, find the sides. **[2010 (T-I)]**

9. In an isosceles triangle ABC, if $AB = AC = 13$ cm and the altitude from A on BC is 5 cm, find BC.

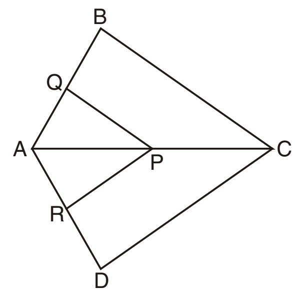
10. In the figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that $\dfrac{EF}{BF} = \dfrac{EC}{BE}$. **[2010 (T-I)]**



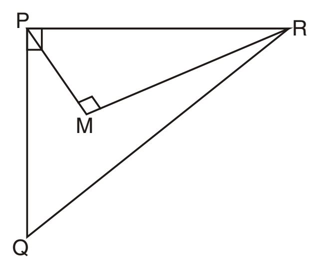
11. In the figure, $\Delta ABE \cong \Delta ACD$. Prove that $\Delta ADE \sim \Delta ABC$. **[2010 (T-I)]**



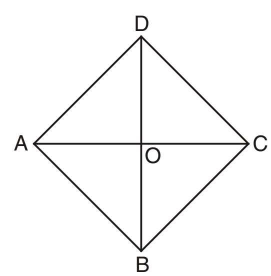
12. In the figure, if $PQ \parallel CB$ and $PR \parallel CD$, prove that $\dfrac{AR}{AD} = \dfrac{AQ}{AB}$ **[2010 (T-I)]**



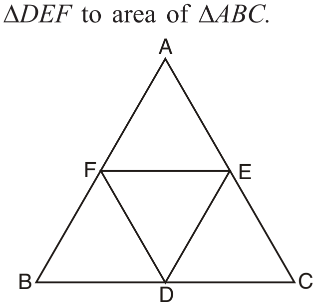
13. In the figure, $PM = 6$ cm, $MR = 8$ cm and $QR = 26$ cm, find the length of PQ. **[2010 (T-I)]**



14. In the figure, $ABCD$ is a rhombus. Prove that $4AB^2 = AC^2 + BD^2$. **[2010 (T-I)]**



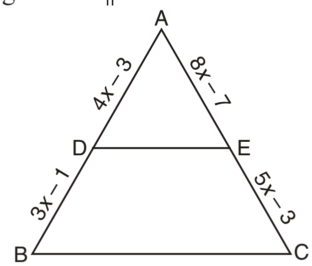
15. In the figure, $D, E, F,$ are mid-points of sides BC, CA, AB respectively of ΔABC. Find the ratio of area of ΔDEF to area of ΔABC. **[2010 (T-I)]**



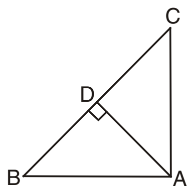
16. If one diagonal of a trapezium divides the other diagonal in the ratio 1 : 2, prove that one of the parallel sides is double the other. **[2010 (T-I)]**

17. In ΔABC, $AB = AC$ and D is a point on side AC such that $BC^2 = AC.CD$. Prove that $BD = BC$. **[2010 (T-I)]**

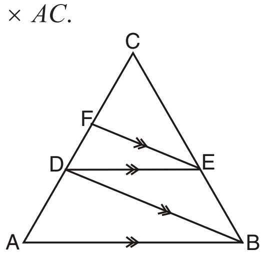
18. In the figure $DE \parallel BC$. Find x. **[2010 (T-I)]**



19. In the figure, $\angle BAC = 90°$, $AD \perp BC$. Prove that : $AB^2 + CD^2 = BD^2 + AC^2$. **[2010 (T-I)]**



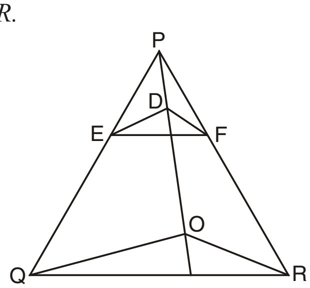
20. In the figure, $AB \parallel DE$ and $BD \parallel EF$. Prove that $DC^2 = CF \times AC$. **[2010 (T-I)]**



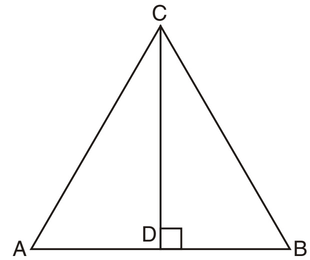
21. If D, E are points on the sides AB and AC of a ΔABC such that $AD = 6$ cm, $BD = 9$ cm, $AE = 8$ cm $EC = 12$ cm. Prove that $DE \parallel BC$. **[2010 (T-I)]**

22. Prove that the line joining the mid points of any two sides of a triangle is parallel to the third side. **[2010 (T-I)]**

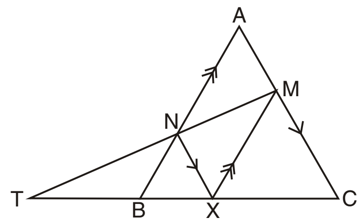
23. In the figure, $DE \parallel OQ$ and $DF \parallel OR$. Show that $EF \parallel QR$. **[2010 (T-I)]**



24. In the figure, $\angle ACB = 90°$ and CD $\perp AB$. Prove that $\dfrac{BC^2}{AC^2} = \dfrac{BD}{AD}$. **[2010 (T-I)]**

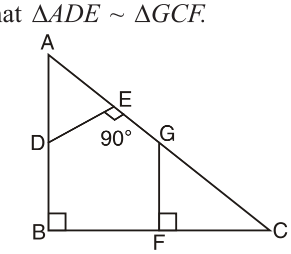


25. In the figure, $XN \parallel CA$ and $XM \parallel BA$. T is a point on CB produced. Prove that $TX^2 = TB.TC$. **[2010 (T-I)]**



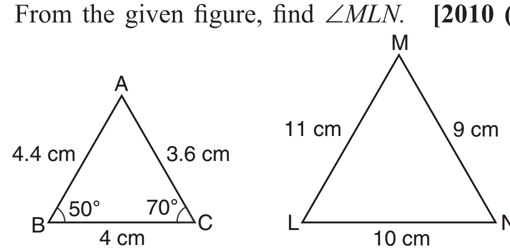
26. Two poles of height 10 m and 15 m stand vertically on a plane ground. If the distance between their feet is $5\sqrt{3}$ m, find the distance between their tops. **[2010 (T-I)]**

27. In the figure, $AB \perp BC$, $DE \perp AC$ and $GF \perp BC$. Prove that $\Delta ADE \sim \Delta GCF$. **[2010 (T-I)]**

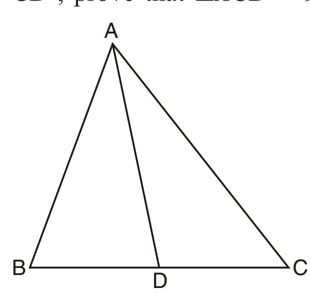


28. In ΔABC, $AD \perp BC$ such that $AD^2 = BD \times CD$. Prove that ΔABC is right angled at A. **[2010 (T-I)]**

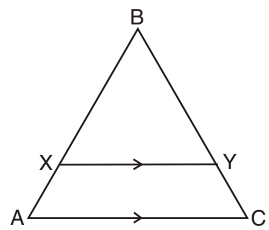
29. From the given figure, find $\angle MLN$. **[2010 (T-I)]**



30. In a quadrilateral $ABCD$, $\angle B = 90°$. If $AD^2 = AB^2 + BC^2 + CD^2$, prove that $\angle ACD = 90°$.

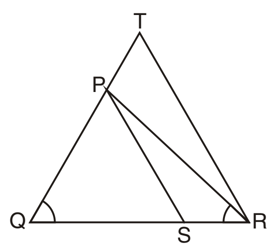


31. In the figure, D is point on the side BC of ΔABC such that $\angle ADC = \angle BAC$. Prove that $\dfrac{CA}{CD} = \dfrac{CB}{CA}$.



32. In the figure, $XY \parallel AC$ and XY divides triangular region ABC into two parts equal in area. Find the ratio of $\dfrac{AX}{AB}$. **[2010 (T-I)]**

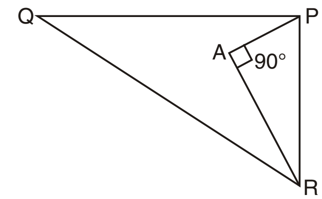
33. ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2AC^2$, prove that ABC is a right angled triangle. **[2010 (T-I)]**



34. In the figure, $\dfrac{QR}{QS} = \dfrac{QT}{PR}$ and $\angle TQR = \angle PRS$. Show that $\Delta PQS \sim \Delta PQR$. **[2010 (T-I)]**

35. D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Determine the ratio of the areas of ΔDEF and ΔABC. **[2008]**

36. In the given figure, $PQ = 24$ cm, $QR = 26$ cm, $\angle PAR = 90°$, $PA = 6$ cm and $AR = 8$ cm. Find $\angle QPR$. **[2008]**

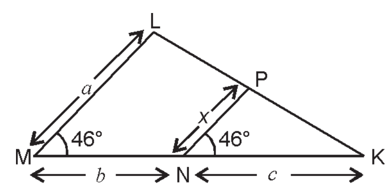


37. E is a point on the side AD produced of a || gm $ABCD$ and BE intersects CD at F. Show that $\Delta ABE \sim CFB$. **[2008]**

38. The lengths of the diagonals of a rhombus are 30 cm and 40 cm. Find the side of the rhombus. **[2008]**

39. A man goes 15 meters due west and then 8 metres due north. How far is he from the starting point? **[2008]**

40. In the figure, $\angle M = \angle N = 46°$. Express x in terms of a, b and c, where a, b and c are lengths of LM, MN and NK respectively. **[2009]**

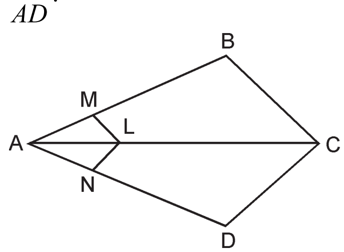


41. In a ΔABC, $DE \parallel BC$. If $DE = \dfrac{2}{3} BC$ and area of $\Delta ABC = 81$ cm^2, find the area of ΔADE. **[2009]**

SHORT ANSWER TYPE QUESTIONS [3 Marks]

A. Important Questions

1. In the figure, $LM \parallel CB$ and $LN \parallel CD$, prove that $\dfrac{AM}{AB} = \dfrac{AN}{AD}$.

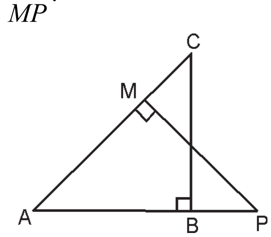


2. The diagonals of a trapezium divide each other in the same ratio. Prove

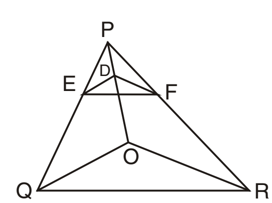
3. In the given figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that :

(i) $\Delta ABC \sim \Delta AMP$

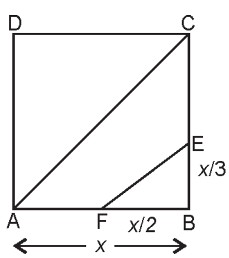
(ii) $\dfrac{CA}{PA} = \dfrac{BC}{MP}$.



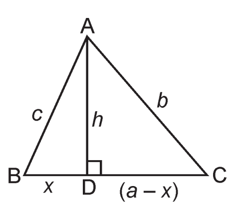
4. In the figure, $DE \parallel OQ$ and $DF \parallel OR$. Show that $EF \parallel QR$.



5. The diagonals of a quadrilateral *ABCD* intersect each other at the point *O* such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that *ABCD* is a trapezium.
6. By using the converse of the basic proportionality theorem, show that the line joining mid-points of non-parallel sides of a trapezium is parallel to the parallel sides.
7. If three or more parallel lines are intersected by two transversals, the intercepts made by them on the transversal are proportional. Prove.
8. *PQR* is triangle right angled at *P* and *M* is a point on *QR* such that $PM \perp QR$. Show that $PM^2 = QM.MR$.
9. *ABCD* is a trapezium in which $AB \parallel CD$. The diagonals *AC* and *BD* intersect at *O*. Prove that : (i) $\Delta AOB \sim \Delta COD$ (ii) If OA = 6 cm and OC = 8 cm, find : $\frac{\text{area}(\Delta ABO)}{\text{area}(\Delta COD)}$.
10. *D* and *E* are points on the sides *CA* and *CB* respectively of a triangle *ABC* right angled at *C*. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.
11. *ABC* is a triangle in which $\angle A = 90°$, $AN \perp BC$, AB = 12 cm and AC = 5 cm. Find the ratio of the areas of ΔANC and ΔABC.
12. *ABCD* is a square. *F* is the mid-point of *AB*. *BE* is the one-third of *BC*. If the area of the ΔBFE is 108 cm², find the length of *AC*. **[HOTS]**



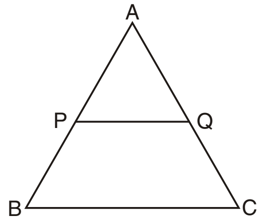
13. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its tip reach?
14. In the given figure, $\angle B < 90°$ and segment $AD \perp BC$, show that $b^2 = h^2 + a^2 + x^2 - 2ax$



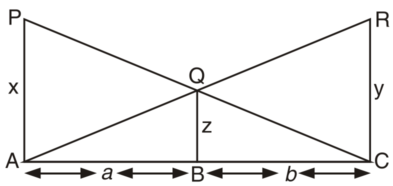
15. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.
16. Using BPT, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.
17. Using converse of BPT, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side.
18. Line *l* drawn parallel to side *AB* of quadrilateral *ABCD* meets *AD* at *E* and *BC* at *F* such that $\frac{AE}{ED} = \frac{BF}{FC}$. Then show that *ABCD* is a trapezium.
19. A girl of height 120 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 4.8 m above the ground, find the length of her shadow after 4 seconds.
20. If $\Delta ABC \sim \Delta DEF$ and, *AL* and *DM* are their corresponding medians, then show that $\frac{AL}{DM} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.
21. If $\Delta ABC \sim \Delta DEF$ and, *AL* and *DM* are their corresponding altitudes, then show that $\frac{AL}{DM} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.
22. If $\Delta ABC \sim DEF$, then show that $\frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta DEF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.
23. Diagonals of a trapezium *ABCD* with $AB \parallel DC$ intersect each other at the point *O*. If $AB = 3CD$, find the ratio of the areas of triangles *AOB* and *COD*.
24. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.
25. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding altitudes.
26. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding perimeters.

B. Questions From CBSE Examination Papers

1. In ΔABC, in figure, PQ meets AB in P and AC in Q. If $AP = 1$ cm, PB = 3 cm, $AQ = 1.5$ cm, $QC = 4.5$ cm, prove that area of ΔAPQ is one sixteenth of the area of ΔABC. **[2010 (T-I)]**



2. In the figure, PA, QB and RC are prepending to AC. Prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$ **[2010 (T-I)]**



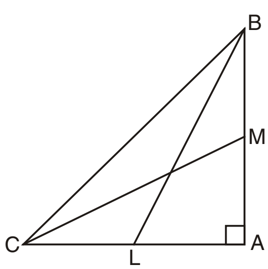
3. In an equilateral triangle ABC, D is a point on side BC such that $3BD = BC$. Prove that $9AD^2 = 7AB^2$ **[2010 (T-I)]**

4. In ΔPQR, $PD \perp QR$ such that D lies on QR. If $PQ = a$, $PR = b$, $QD = c$ and $DR = d$ and a, b, c, d are positive units, prove that $(a + b)(a - b) = (c + d)(c - d)$. **[2010 (T-I)]**

5. In ΔABC, $AD \perp BC$ such that $AB^2 = BD.CD$. Prove that ABC is a right triangle, right angle at A. **[2010 (T-I)]**

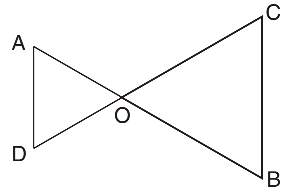
6. P and Q are points on sides AB and AC respectively of ΔABC. If $AP = 3$ cm, $PB = 6$ cm, $AQ = 5$ cm and $QC = 10$ cm, show that $BC = 3PQ$. **[2010 (T-I)]**

7. In figure, BL and CM are medians of ΔABC right angled at A. Prove that $4(BL^2 + CM^2) = 5BC^2$ **[2010 (T-I)]**



8. The perpendicular AD on the base BC of ΔABC intersects BC in D such that $BD = 3CD$. Prove that $2AB^2 = 2AC^2 + BC^2$. **[2010 (T-I)]**

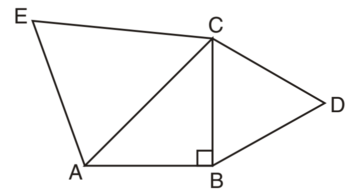
9. In the figure, $OA.OB = OC.OD$. Show that $\angle A = \angle C$ and $\angle B = \angle D$. **[2010 (T-I)]**



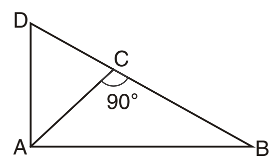
10. In the figure, ABC is an isosceles triangle right angled at B. Two equilateral triangles are constructed with side BC and AC. Prove that :

$ar\ \Delta BCD = \frac{1}{2} \text{ar}\ \Delta ACE$

[2010 (T-I)]



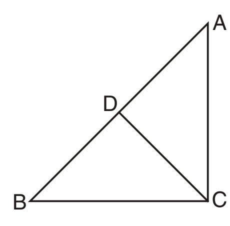
11. In the figure, ABD is a triangle in which $\angle DAB = 90°$ and $AC \perp BD$. Prove that $AB^2 = BC \times BD$. **[2010 (T-I)]**



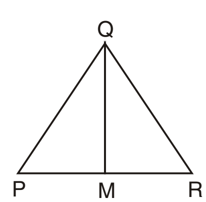
12. In ABC, if AD is the median, then show that $AB^2 + AC^2 = 2[AD^2 + BD^2]$. **[2010 (T-I)]**

13. In figure, ABC is right triangle right angled at C. Let $BC = a$, $CA = b$, $AB = c$ and let p be the length of perpendicular from C on AB. Prove that :

(i) $cp = ab$ (ii) $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$ **[2010 (T-I)]**

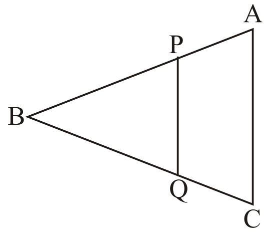


14. In the figure, PQR is a triangle in which $QM \perp PR$ and $PR^2 - PQ^2 = QR^2$, prove that $QM^2 = PM \times MR$. **[2010 (T-I)]**



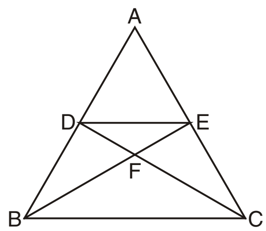
15. In figure, the line segment PQ is parallel to AC of triangle ABC and it divides the triangle into two parts of equal area. Find the ratio $\frac{AP}{AB}$.

[2010 (T-I)]



16. In figure, DE || BC and AD : DB = 5 : 1. Find $\frac{\text{ar}(\Delta DFE)}{\text{ar}(\Delta CFB)}$.

[2010 (T-I)]

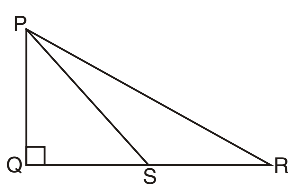


17. Two isosceles triangles have equal vertical angles and their areas are in the ratio 16 : 25. Find the ratio of their corresponding heights. **[2010 (T-I)]**

18. Prove that the equilateral triangles described on the two sides of a right-angled triangle are together equal to the equilateral triangle described on the hypotenuse in terms of their areas. **[2010 (T-I)]**

19. In the figure, PQR is a right angled triangle in which $Q = 90°$. If $QS = SR$, show that $PR^2 - 4PS^2 = 3PQ^2$

[2010 (T-I)]



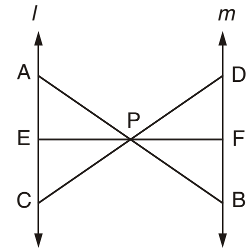
20. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals. **[2010 (T-I)]**

21. If the areas of two similar triangles are equal, prove that they are congruent. **[2010 (T-I)]**

22. In an isosceles triangle ABC with $AB = AC$, $BD \perp AC$, prove that $BD^2 - CD^2 = 2CD.AD$.

[2010 (T-I)]

23. In the figure, $l \parallel m$ and line segments AB, CD and EF are concurrent at P. Prove that : $\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$.



24. Triangle ABC is right angled at B and D is mid point of BC. Prove that : $AC^2 = 4AD^2 - 3AB^2$.

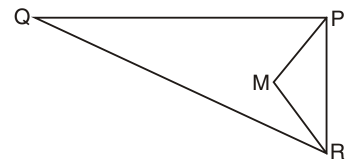
[2010 (T-I)]

25. Prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal.

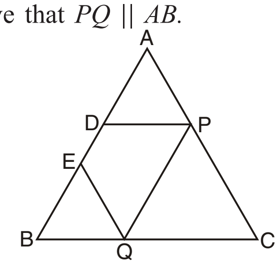
[2010 (T-I)]

26. In the figure, $\angle QPR = 90°$, $\angle PMR = 90°$, $QR = 26$ cm, $PM = 8$ cm, $MR = 6$ cm. Find area (ΔPQR).

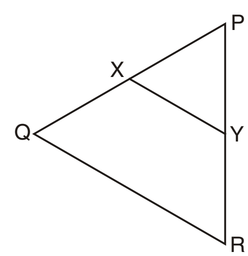
[2010 (T-I)]



27. In ΔABC, D and E are two points lying on side AB such that $AD = BE$. If $DP \parallel BC$ and $EQ \parallel AC$, then prove that $PQ \parallel AB$. **[2010 (T-I)]**



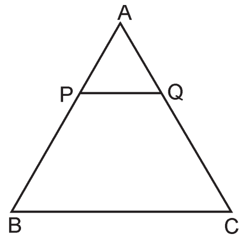
28. In figure, $XY \parallel QR$, $\frac{PQ}{XQ} = \frac{7}{3}$ and $PR = 6.3$ cm. Find YR. **[2010 (T-I)]**



29. In ΔABC, AD is a median and E is mid-point of AD. If BE is produced, it meets AC at F. Show that $AF = \frac{1}{3}AC$. **[2006]**

30. In the given figure, $PQ \parallel BC$ and **[2008]**

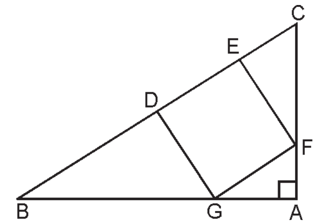
$AP : PB = 1 : 2$. Find $\frac{ar\,(\Delta APQ)}{ar\,(\Delta ABC)}$.



31. If D is a point on the side AB of ΔABC such that $AD : DB = 3 : 2$ and E is a point on BC such that $DE \parallel AC$. Find the ratio of areas of ΔABC and ΔBDE. **[2001, 2006 C]**

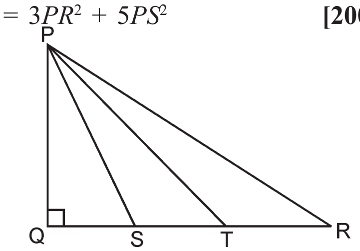
32. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals. **[2005, 2006]**

33. In the figure, $DEFG$ is a square and $\angle BAC = 90°$. Show that $DE^2 = BD \times EC$. **[2009]**



34. O is a point in the interior of rectangle $ABCD$. If O is joined to each of the vertices of the rectangle, prove that $OB^2 + OD^2 = OA^2 + OC^2$. **[2006 C]**

35. In the figure, ΔPQR is right angled at Q and the points S and T trisect the side QR. Prove that $8PT^2 = 3PR^2 + 5PS^2$ **[2006 C, 2009]**



LONG ANSWER TYPE QUESTIONS [4 Marks]

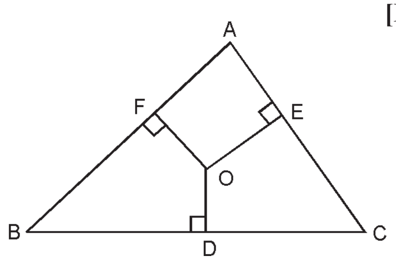
A. Important Questions

1. In the given figure, O is a point in the interior of a triangle ABC, $OD \perp BC$, $OE \perp AC$ and $OF \perp AB$. Show that

(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

[HOTS]



2. $ABCD$ is a quadrilateral. P, Q, R, S are the points of trisection of the sides AB, BC, CD and DA respectively. Prove that $PQRS$ is a parallelogram.

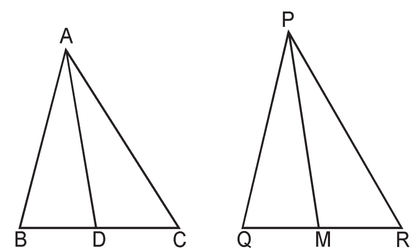
3. In a right triangle ABC, right angled at C. P and Q are points on the sides CA and CB respectively which divide these sides in the ratio 1 : 2. Prove that :

(i) $9AQ^2 = 9AC^2 + 4BC^2$

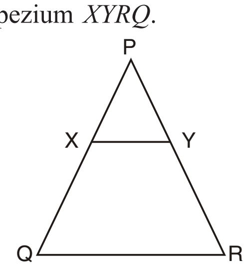
(ii) $9BP^2 = 9BC^2 + 4AC^2$

(iii) $9(AQ^2 + BP^2) = 13AB^2$

4. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see figure). Show that $\Delta ABC \sim \Delta PQR$.



5. In the given figure, in ΔPQR, $XY \parallel QR$, $PX = 1$ cm, $XQ = 3$ cm, $YR = 4.5$ cm and $QR = 9$ cm. Find PY and XY. Further if the area of ΔPXY is *'A'* cm^2, find in terms of A the area of ΔPQR and the area of trapezium $XYRQ$.



6. If A be the area of a right triangle and a one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\dfrac{2Aa}{\sqrt{a^4 + 4A^2}}$.

[HOTS]

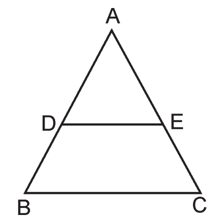
B. Questions From CBSE Examination Papers

1. Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. **[2010 (T-I)]**
2. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides. **[2010 (T-I)]**
3. State and prove Basic Proportionality theorem. **[2010 (T-I)]**
4. Prove that in a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle. **[2010 (T-I)]**
5. Prove that if a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, the other two sides are divided in the same ratio. **[2010 (T-I)]**
6. State and prove converse of Pythagoras theorem. **[2010 (T-I)]**
7. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

   Using the above result do the following :

   In the figure, $DE \parallel BC$ and $BD = CE$.

   Prove that ΔABC is an isosceles triangle. **[2009]**



8. Through the mid-point *M* of the side *CD* of a parallelogram *ABCD*, the line *BM* is drawn intersecting *AC* in *L* and *AD* produced in *E*. Prove that $EL = 2BL$. **[2009]**

FORMATIVE ASSESSMENT

Activity-1

Objective : To verify the Basic Proportionality Theorem by activity method.

Materials Required : Ruled paper, white sheets of paper, colour pencils, a pair of scissors, geometry box, etc.

Procedure :

1. On a white sheet of paper, draw an acute angled triangle ABC and an obtuse angled triangle PQR. Using a pair of scissors, cut them out.

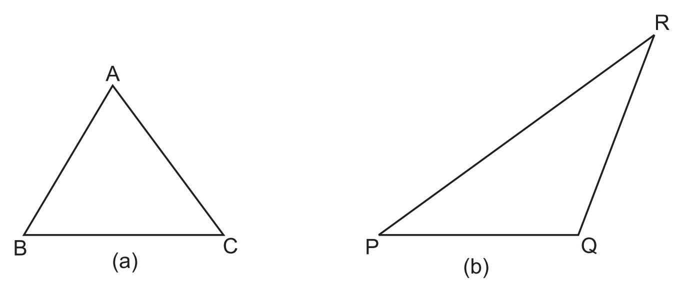


Figure-1

2. Take a ruled paper. Place ΔABC over the ruled paper such that any one side of the triangle is placed on one of the lines of the ruled paper. Place the ΔPQR over the ruled paper in the same manner as discussed above. Mark points X_1, X_2, X_3, X_4 on ΔABC and Y_1, Y_2, Y_3, Y_4 on ΔPQR. Join X_1X_2, X_3X_4, Y_1Y_2, and Y_3Y_4.

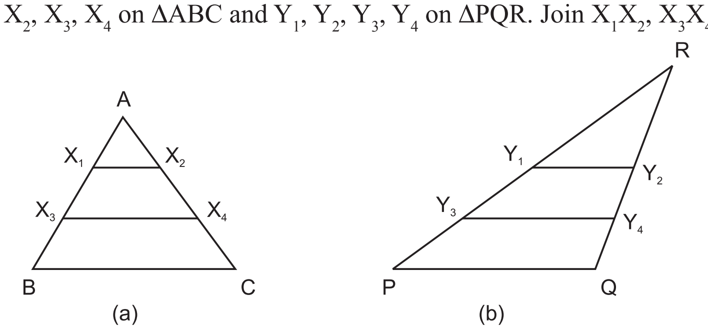


Figure-2

3. Using a ruler, measure AX_1, X_1B, AX_2, X_2C, AX_3, X_3B, AX_4, X_4C and record the 1engths in the following table.

AX_1	X_1B	AX_2	X_2C	$\frac{AX_1}{X_1B}$	$\frac{AX_2}{X_2C}$	AX_3	X_3B	AX_4	X_4C	$\frac{AX_3}{X_3B}$	$\frac{AX_4}{X_4C}$

4. Similarly, measure RY_1, Y_1P, RY_2, Y_2Q, RY_3, Y_3P, RY_4 and Y_4Q.

R_1Y	Y_1P	RY_2	Y_2Q	$\frac{RY_1}{Y_1P}$	$\frac{RY_2}{Y_2Q}$	RY_3	Y_3P	RY_4	Y_4Q	$\frac{RY_3}{Y_3P}$	$\frac{RY_4}{Y_4Q}$

Observations :

1. In figure 2(a), we see that X_1X_2 BC and X_3X_4 BC

   Similarly, in figure 2(b), we see that Y_1Y_2 PQ and Y_3Y_4 PQ.

2. From the tables, we see that $\frac{AX_1}{X_1B} = \frac{AX_2}{X_2C}, \frac{AX_3}{X_3B} = \frac{AX_4}{X_4C}$

   And $\frac{RY_1}{Y_1P} = \frac{RY_2}{Y_2Q}, \frac{RY_3}{Y_3P} = \frac{RY_4}{Y_4Q}$

   Thus, we have :

   If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Conclusion : From the above activity, the Basic Proportionality Theorem is verified.

Do Yourself : Draw two different triangles and verify the Basic Proportionality Theorem by the activity method.

Activity-2

Objective : To verify the Pythagoras Theorem by the method of paper folding, cutting and pasting.

Materials Required : White sheets of paper, tracing paper, a pair of scissors, gluestick, geometry box, etc.

Procedure :

1. On a white sheet of paper, draw a right triangle ABC, right angled at B and AB = x, BC = y and AC = z.

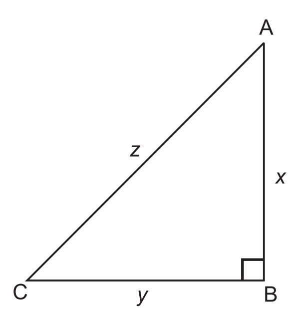


Figure-1

2. Using tracing paper, trace the triangle ABC and make 4 replicas of it. Shade each triangle using different colour. Cut out each triangle.

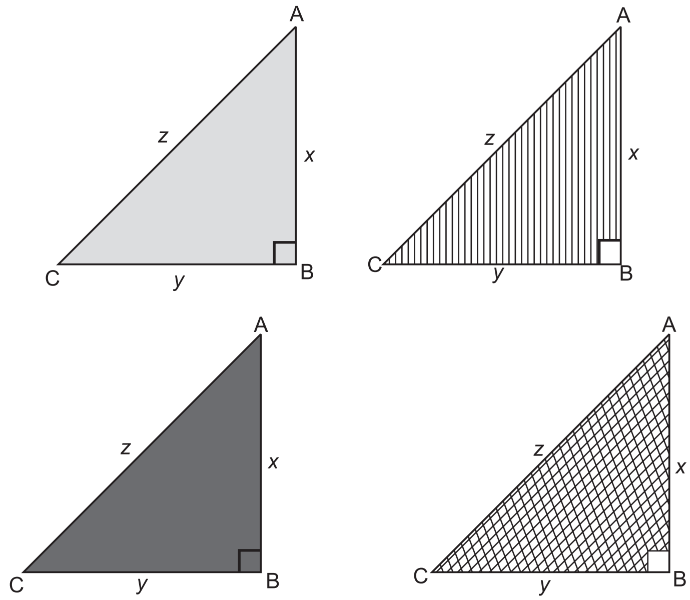


Figure-2

3. On a white sheet of paper, draw a square of side *z*. Shade it and cut it out.

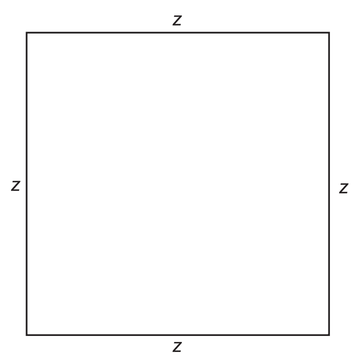


Figure-3

4. Paste the 4 right triangles and the square cut out obtained in steps 2 and 3 above as shown on next page.

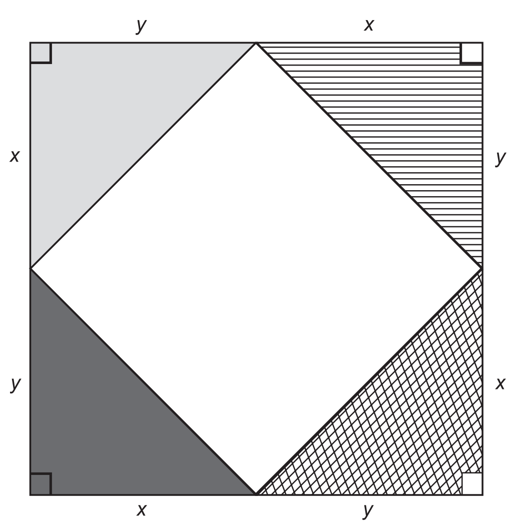


Figure-4

Observations :

1. In the figure 4, we see that it is a square of side $(x + y)$.

   So its area = $(x + y)^2$.

2. Also, area of this square = area of the square of side z + 4 × area of ΔABC

   $= z^2 + 4 \times \frac{1}{2} \times xy = z^2 + 2xy$

3. From 1 and 2 above, $(x + y)^2 = z^2 + 2xy$

   $\Rightarrow x^2 + y^2 + 2xy = z^2 + 2xy$

   $\Rightarrow x^2 + y^2 = z^2$

Conclusion : From the above activity, it is verified that in a right triangle the square of the hypotenuse is equal to the sum of the squares of other two sides.

Or in other words, we can say that the Pythagoras Theorem is verified.

Do Yourself : Draw two different right triangles and verify the Pythagoras Theorem by Activity method in each case.

Activity-3

Objective : To illustrate that the medians of a triangle concur at a point (called the centroid) which always lies inside the triangle.

Materials Required : White sheets of paper, colour pencils, geometry box, a pair of scissors, gluestick, etc.

Procedure :

1. On a white sheet of paper, draw an acute angled triangle, a right triangle and an obtuse angled triangle. Using a pair of scissors, cut out these triangles.

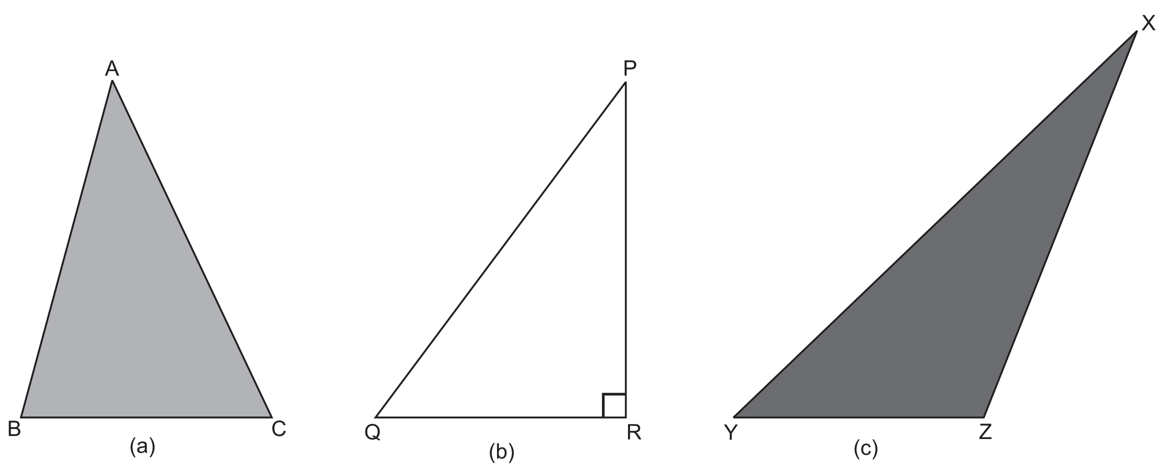


Figure-1

2. Take the cut out of the acute angled triangle ABC and find the mid-points of its sides by paper folding method.

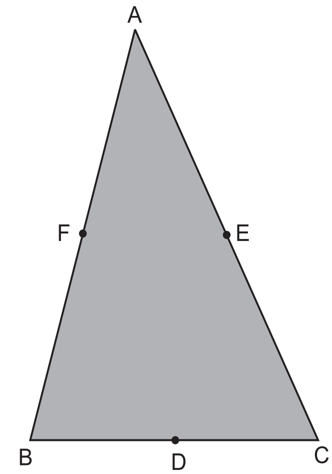


Figure-2

3. Let the mid-points of BC, AC and AB be D, E and F respectively.
4. Join AD, BE and CF.

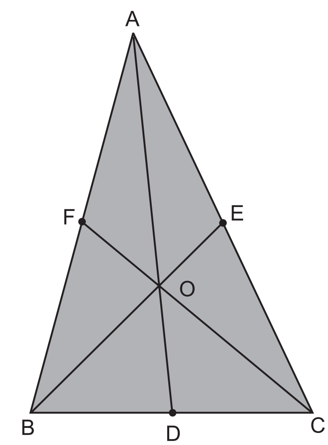


Figure-3

5. Repeat the steps 2, 3 and 4 for the right triangle PQR and the obtuse angled triangle XYZ.

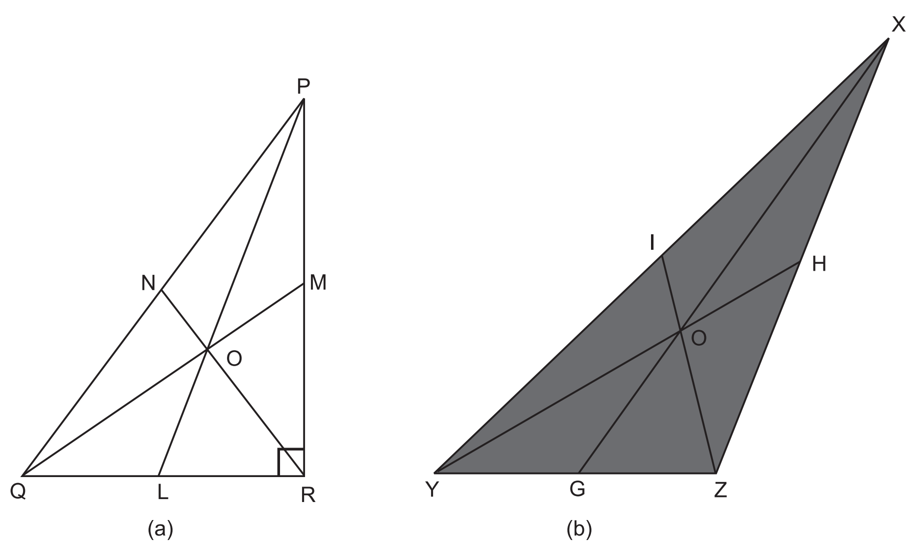


Figure-4

Observations :

1. In figure 3, D, E and F are mid-points of BC, AC and AB respectively.
   So, AD, BE and CF are medians of ΔABC.
2. Similarly, PL, QM and RN are the medians of ΔPQR [figure 4(a)] and XG, YH and ZI are medians of ΔXYZ [figure 4(b)].
3. We see that in each case the medians pass through a common point, or the medians are concurrent. The point of concurrence of the medians of a triangle is called its centroid.
4. We also observe that in each case the centroid lies in the interior of the triangle.

Conclusion : From the above activity, it is observed that :

(a) the medians of a triangle are concurrent. This point of concurrence of the medians is called the centroid of the triangle.

(b) centroid of a triangle always lies in its interior. Or the medians of a triangle always intersect in the interior of the triangle.

Activity-4

Objective : To verify by activity method that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangle on each side of the perpendicular are similar to the whole triangle and to each other.

Materials Required : White sheets of paper, tracing paper, colour pencils, a pair of scissors, gluestick, geometry box, etc.

Procedure :

1. On a white sheet of paper, draw a right triangle ABC, right angled at B.

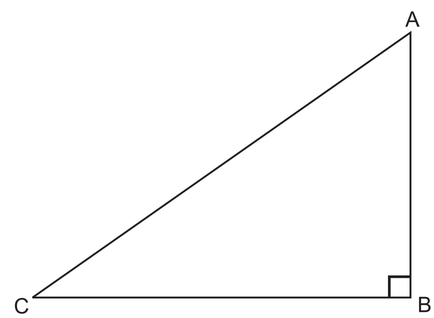


Figure-1

2. On a tracing paper, trace ΔABC. Using paper folding method, draw BD⊥AC. Cut the triangles ABD and CBD as shown below. Shade both sides of each triangular cut outs.

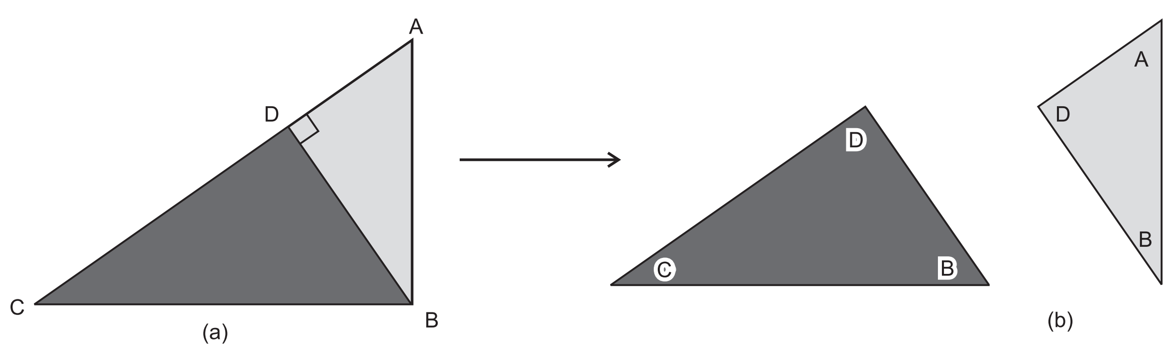


Figure-2

3. Flip over the triangular cut out BCD and place it over ΔABC in figure 1, as shown below.

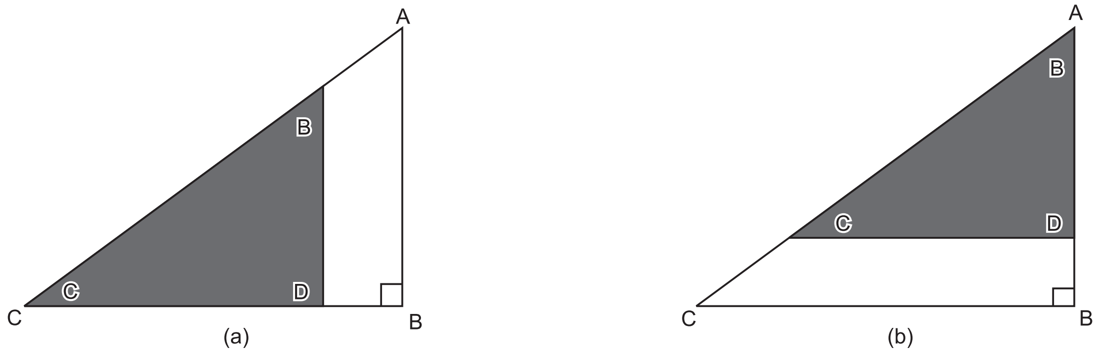


Figure-3

4. Flip over the triangular cut out ABD and place it over ΔABC in figure 1, as shown below.

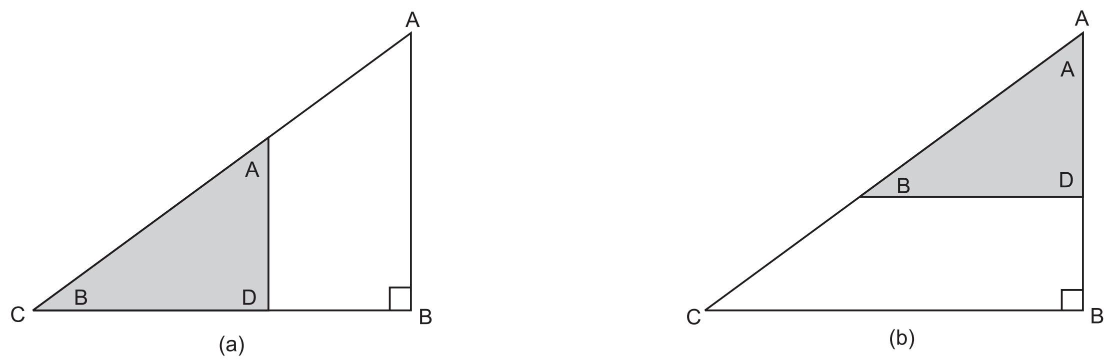


Figure-4

5. Now, take the triangular cut outs BCD and ABD and place them one over another as shown below.

Figure-5

Observations :

1. In figure 2(a), $\angle ABC = 90°$ and in figure 2(b), $\angle CDB = \angle ADB = 90°$.

2. In figure 3(a), and 3(b), we see that $\angle ACB = \angle BCD$ and $\angle BAC = \angle DBC$

So, $\Delta ABC \sim \Delta BDC$

3. In figure 4(a) and 4(b), we see that $\angle ACB = \angle ABD$ and $\angle BAC = \angle DAB$

So, $\Delta ABC \sim \Delta ADB$

4. In figure 5(a) and 5(b), we see that $\angle BCD = \angle ABD$ and $\angle CBD = \angle BAD$

So, $\Delta CBD \sim \Delta BAD$

Conclusion : From the activity, it is verified that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.

Do Yourself : Draw two different right triangles and verify by activity method that if a perpendicular is drawn from the vertex of the right triangle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.

Exercise 6.1

Question 1:

Fill in the blanks using correct word given in the brackets:−

(i) All circles are __________. (congruent, similar)

(ii) All squares are __________. (similar, congruent)

(iii) All __________ triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are __________ and (b) their corresponding sides are __________. (equal, proportional)

Answer:

(i) Similar

(ii) Similar

(iii) Equilateral

(iv) (a) Equal

(b) Proportional

Question 2:

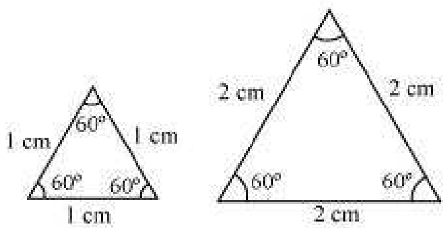
Give two different examples of pair of

(i) Similar figures

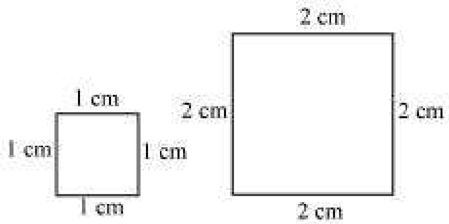
(ii)Non-similar figures

Answer:

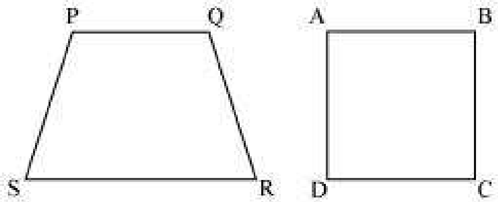
(i) Two equilateral triangles with sides 1 cm and 2 cm



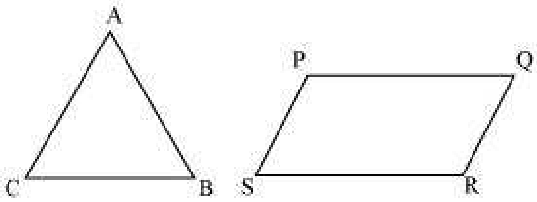
Two squares with sides 1 cm and 2 cm



(ii) Trapezium and square

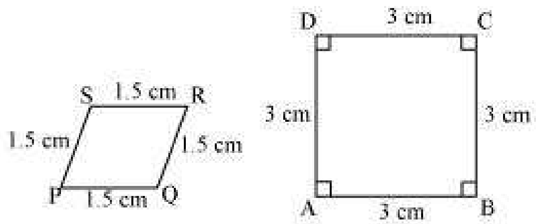


Triangle and parallelogram



Question 3:

State whether the following quadrilaterals are similar or not:



Answer:

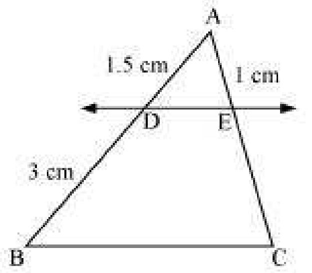
Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional, i.e. 1:2, but their corresponding angles are not equal.

Exercise 6.2

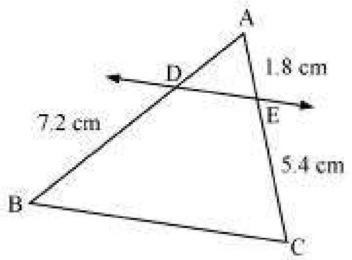
Question 1:

In figure.6.17. (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).

(i)

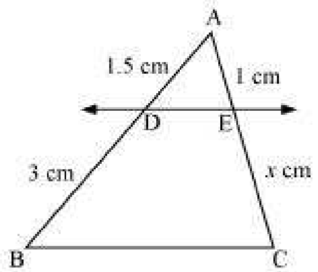


(ii)



Answer:

(i)



Let EC = x cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$\frac{AD}{DB}=\frac{AE}{EC}$

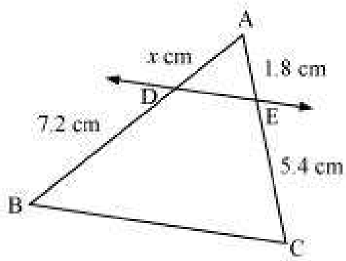
$\frac{1.5}{3}=\frac{1}{x}$

$x=\frac{3\times1}{1.5}$

$x=2$

$\therefore EC=2\text{ cm}$

(ii)



Let AD = *x* cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$\frac{AD}{DB}=\frac{AE}{EC}$

$\frac{x}{7.2}=\frac{1.8}{5.4}$

$x=\frac{1.8\times7.2}{5.4}$

$x=2.4$

$\therefore AD=2.4\text{ cm}$

Question 2:

E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR.

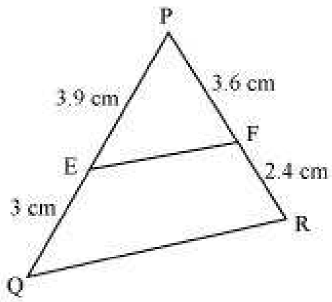
(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

(iii)PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.63 cm

Answer:

(i)



Given that, PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm, FR = 2.4 cm

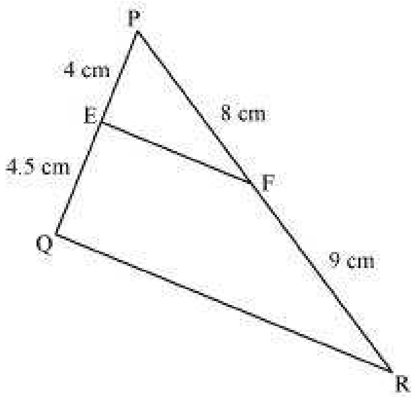
$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

Hence, $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Therefore, EF is not parallel to QR.

(ii)

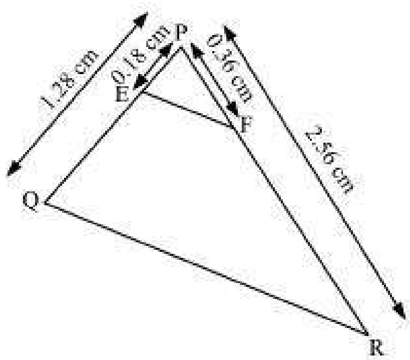


PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm

$$\frac{PE}{EQ}=\frac{4}{4.5}=\frac{8}{9}$$
$$\frac{PF}{FR}=\frac{8}{9}$$
Hence, $\frac{PE}{EQ}=\frac{PF}{FR}$

Therefore, EF is parallel to QR.

(iii)



PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

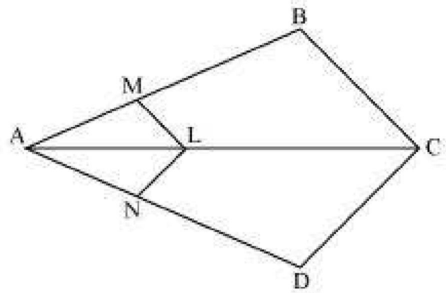
$$\frac{PE}{PQ}=\frac{0.18}{1.28}=\frac{18}{128}=\frac{9}{64}$$
$$\frac{PF}{PR}=\frac{0.36}{2.56}=\frac{9}{64}$$
Hence, $\frac{PE}{PQ}=\frac{PF}{PR}$

Therefore, EF is parallel to QR.

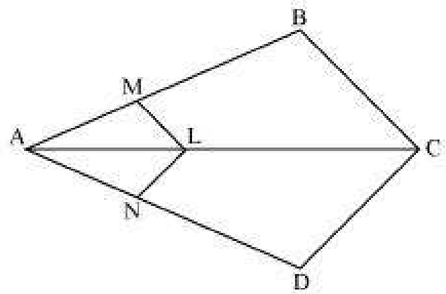
Question 3:

In the following figure, if LM || CB and LN || CD, prove that

$$\frac{AM}{AB}=\frac{AN}{AD}.$$



Answer:



In the given figure, LM || CB

By using basic proportionality theorem, we obtain

$$\frac{AM}{AB}=\frac{AL}{AC} \qquad (i)$$

Similarly, LN || CD

$$\therefore \frac{AN}{AD}=\frac{AL}{AC} \qquad (ii)$$

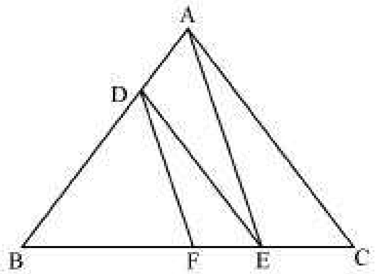
From (i) and (ii), we obtain

$$\frac{AM}{AB}=\frac{AN}{AD}$$

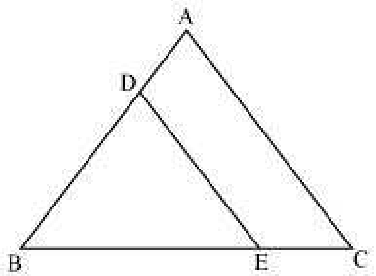
Question 4:

In the following figure, DE || AC and DF || AE. Prove that

$\frac{BF}{FE} = \frac{BE}{EC}$.

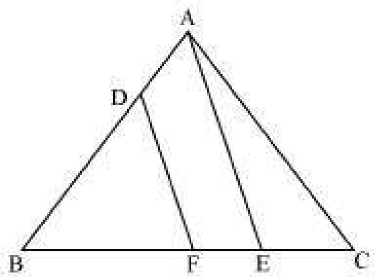


Answer:



In ΔABC, DE || AC

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ (Basic Proportionality Theorem) (i)



In ΔBAE, $DF \parallel AE$

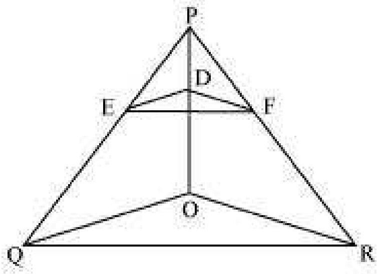
$$\therefore \frac{BD}{DA} = \frac{BF}{FE} \qquad \text{(Basic Proportionality Theorem)} \qquad (ii)$$

From (i) and (ii), we obtain

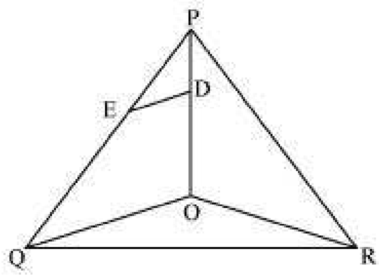
$$\frac{BE}{EC} = \frac{BF}{FE}$$

Question 5:

In the following figure, DE || OQ and DF || OR, show that EF || QR.

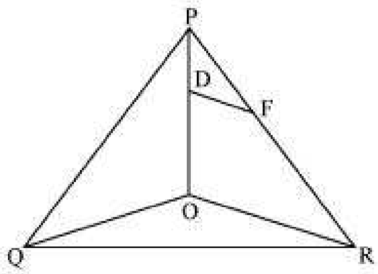


Answer:



In Δ POQ, DE || OQ

$$\therefore \frac{PE}{EQ} = \frac{PD}{DO} \qquad \text{(Basic proportionality theorem)} \qquad (i)$$



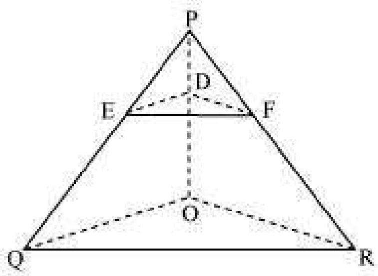
In ΔPOR, $DF \parallel OR$

$$\therefore \frac{PF}{FR} = \frac{PD}{DO} \qquad \text{(Basic proportionality theorem)} \qquad (ii)$$

From (i) and (ii), we obtain

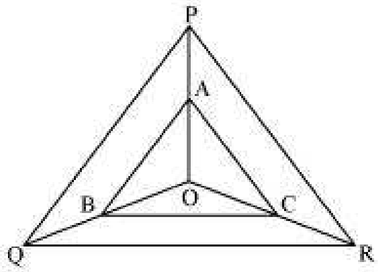
$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$$\therefore EF \parallel QR \qquad \text{(Converse of basic proportionality theorem)}$$

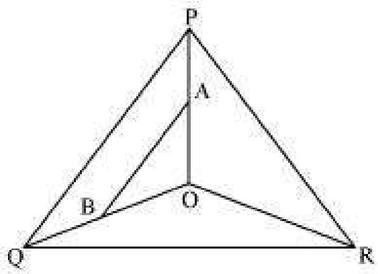


Question 6:

In the following figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

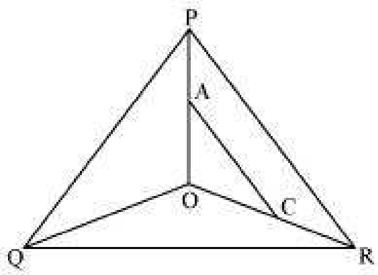


Answer:



In Δ POQ, AB || PQ

$$\therefore \frac{OA}{AP} = \frac{OB}{BQ} \qquad \text{(Basic proportionality theorem)} \qquad (i)$$



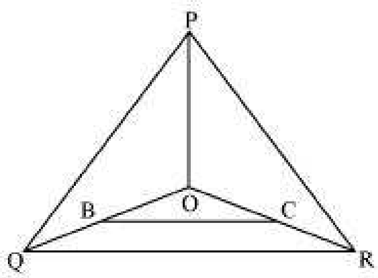
In ΔPOR, $AC \parallel PR$

$\therefore \frac{OA}{AP} = \frac{OC}{CR}$ (By basic proportionality theorem) (ii)

From (i) and (ii), we obtain

$\frac{OB}{BQ} = \frac{OC}{CR}$

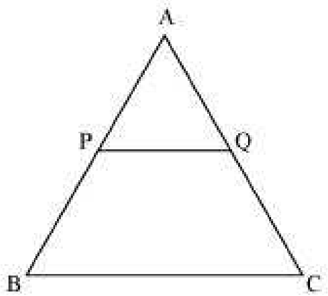
$\therefore BC \parallel QR$ (By the converse of basic proportionality theorem)



Question 7:

Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer:



Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that $PQ \parallel BC$

By using basic proportionality theorem, we obtain

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = \frac{1}{1} \qquad (\text{P is the mid-point of AB. } \therefore \text{ AP} = \text{PB})$$

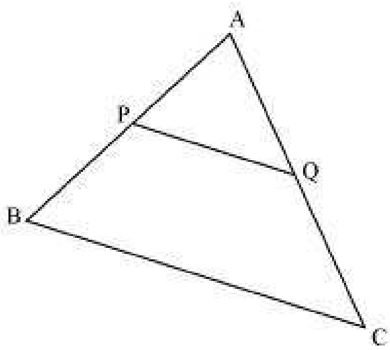
$$\Rightarrow AQ = QC$$

Or, Q is the mid-point of AC.

Question 8:

Using Converse of basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer:



Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.

i.e., AP = PB and AQ = QC

It can be observed that

$\frac{AP}{PB} = \frac{1}{1}$

and $\frac{AQ}{QC} = \frac{1}{1}$

$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$

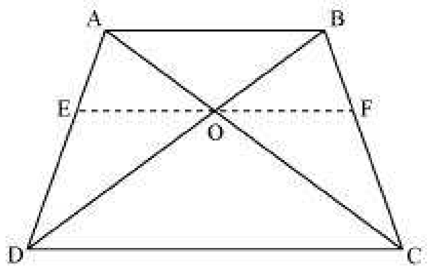
Hence, by using basic proportionality theorem, we obtain

$PQ \| BC$

Question 9:

ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Answer:



Draw a line EF through point O, such that $EF \| CD$

In ΔADC, $EO \| CD$

By using basic proportionality theorem, we obtain

$\frac{AE}{ED} = \frac{AO}{OC} \quad (1)$

In ΔABD, $OE \| AB$

So, by using basic proportionality theorem, we obtain

$$\frac{ED}{AE} = \frac{OD}{BO}$$

$$\Rightarrow \frac{AE}{ED} = \frac{BO}{OD} \qquad (2)$$

From equations (1) and (2), we obtain

$$\frac{AO}{OC} = \frac{BO}{OD}$$

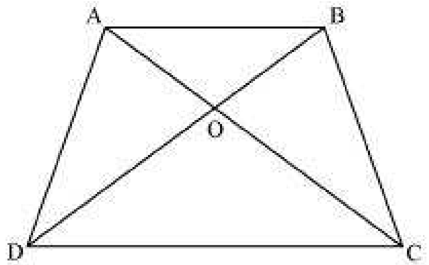
$$\Rightarrow \frac{AO}{BO} = \frac{OC}{OD}$$

Question 10:

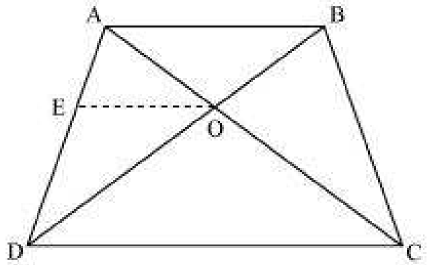
The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Answer:

Let us consider the following figure for the given question.



Draw a line OE || AB



In ΔABD, OE || AB

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \qquad (1)$$

However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \qquad (2)$$

From equations (1) and (2), we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

⇒ EO || DC [By the converse of basic proportionality theorem]

⇒ AB || OE || DC

⇒ AB || CD

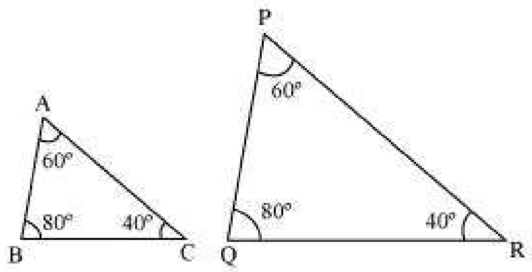
∴ ABCD is a trapezium.

Exercise 6.3

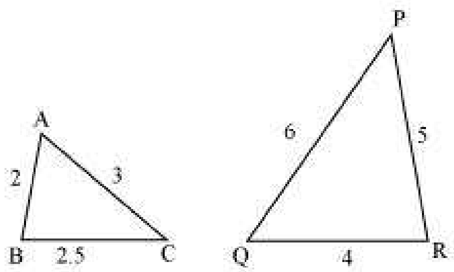
Question 1:

State which pairs of triangles in the following figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

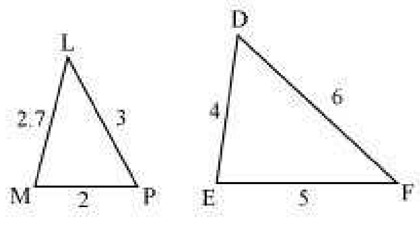
(i)



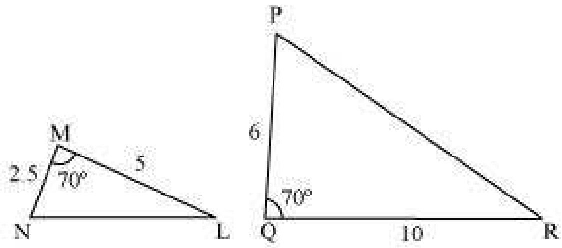
(ii)



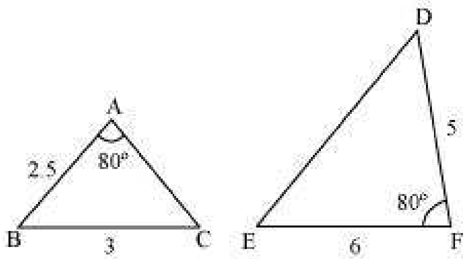
(iii)



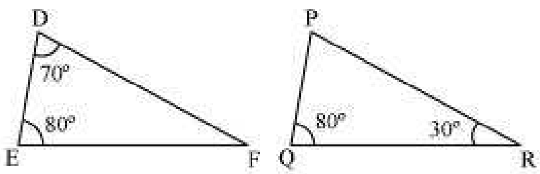
(iv)



(v)



(vi)



Answer:

(i) ∠A = ∠P = 60°

∠B = ∠Q = 80°

∠C = ∠R = 40°

Therefore, ΔABC ~ ΔPQR [By AAA similarity criterion]

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$

(ii)

$\therefore\ \Delta ABC \sim \Delta QRP$ [By SSS similarity criterion]

(iii)The given triangles are not similar as the corresponding sides are not proportional.

(iv)The given triangles are not similar as the corresponding sides are not proportional.

(v)The given triangles are not similar as the corresponding sides are not proportional.

(vi) In ΔDEF,

$\angle D + \angle E + \angle F = 180^\circ$

(Sum of the measures of the angles of a triangle is 180º.)

$70^\circ + 80^\circ + \angle F = 180^\circ$

$\angle F = 30^\circ$

Similarly, in ΔPQR,

$\angle P + \angle Q + \angle R = 180^\circ$

(Sum of the measures of the angles of a triangle is 180º.)

$\angle P + 80^\circ + 30^\circ = 180^\circ$

$\angle P = 70^\circ$

In ΔDEF and ΔPQR,

$\angle D = \angle P$ (Each 70°)

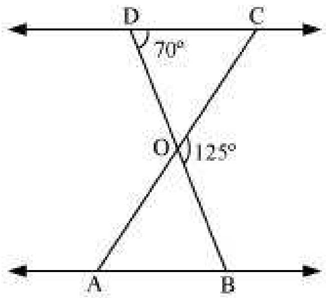
$\angle E = \angle Q$ (Each 80°)

$\angle F = \angle R$ (Each 30°)

∴ ΔDEF ~ ΔPQR [By AAA similarity criterion]

Question 2:

In the following figure, ΔODC ~ ΔOBA, ∠BOC = 125° and ∠CDO = 70°. Find ∠DOC, ∠DCO and ∠OAB



Answer:

DOB is a straight line.

∴ $\angle DOC + \angle COB = 180°$

⇒ $\angle DOC = 180° - 125°$

$= 55°$

In ΔDOC,

$\angle DCO + \angle CDO + \angle DOC = 180°$

(Sum of the measures of the angles of a triangle is 180º.)

⇒ $\angle DCO + 70° + 55° = 180°$

⇒ $\angle DCO = 55°$

It is given that ΔODC ~ ΔOBA.

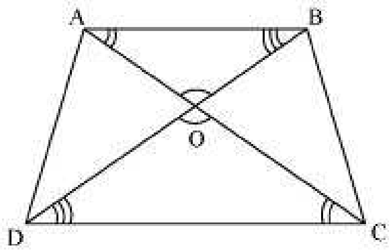
∴ $\angle OAB = \angle OCD$ [Corresponding angles are equal in similar triangles.]

⇒ $\angle OAB = 55°$

Question 3:

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{AO}{OC} = \frac{OB}{OD}$

Answer:



In ΔDOC and ΔBOA,

$\angle CDO = \angle ABO$ [Alternate interior angles as AB || CD]

$\angle DCO = \angle BAO$ [Alternate interior angles as AB || CD]

$\angle DOC = \angle BOA$ [Vertically opposite angles]

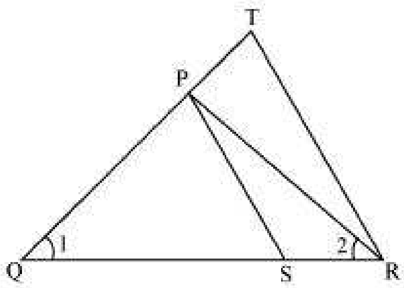
$\therefore$ ΔDOC ~ ΔBOA [AAA similarity criterion]

$$\therefore \frac{DO}{BO} = \frac{OC}{OA} \qquad \text{[Corresponding sides are proportional]}$$

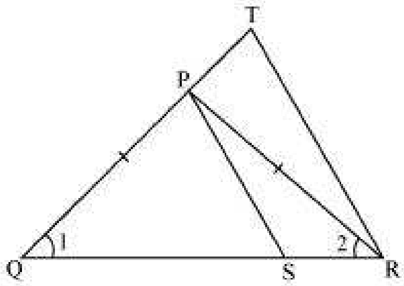
$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$$

Question 4:

In the following figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2.$ Show that $\Delta PQS \sim \Delta TQR$



Answer:



In ΔPQR, ∠PQR = ∠PRQ

∴ PQ = PR (i)

Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i), we obtain

$$\frac{QR}{QS} = \frac{QT}{QP} \qquad (ii)$$

In ΔPQS and ΔTQR,

$$\frac{QR}{QS} = \frac{QT}{QP} \qquad [\text{Using } (ii)]$$

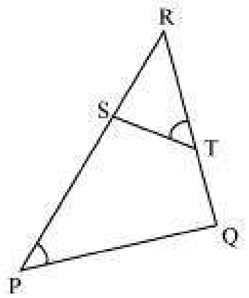
$$\angle Q = \angle Q$$

$$\therefore\ \Delta PQS \sim \Delta TQR \qquad [\text{SAS similarity criterion}]$$

Question 5:

S and T are point on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS.

Answer:



In ΔRPQ and ΔRST,

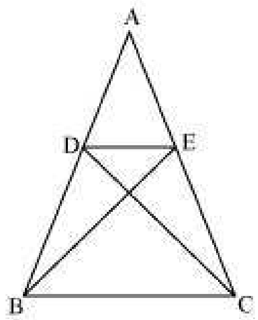
∠RTS = ∠QPS (Given)

∠R = ∠R (Common angle)

∴ ΔRPQ ~ ΔRTS (By AA similarity criterion)

Question 6:

In the following figure, if ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC.



Answer:

It is given that ΔABE ≅ ΔACD.

∴ AB = AC [By CPCT] (1)

And, AD = AE [By CPCT] (2)

In ΔADE and ΔABC,

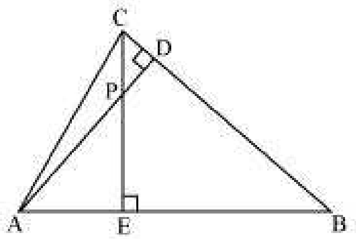
$\frac{AD}{AB} = \frac{AE}{AC}$ [Dividing equation (2) by (1)]

∠A = ∠A [Common angle]

∴ ΔADE ~ ΔABC [By SAS similarity criterion]

Question 7:

In the following figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:



(i) ΔAEP ~ ΔCDP

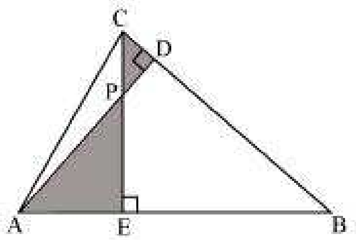
(ii) ΔABD ~ ΔCBE

(iii) ΔAEP ~ ΔADB

(v) ΔPDC ~ ΔBEC

Answer:

(i)



In ΔAEP and ΔCDP,

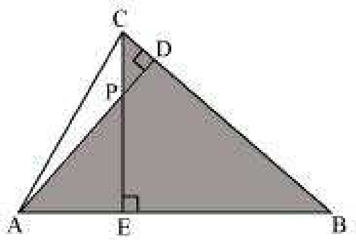
∠AEP = ∠CDP (Each 90°)

∠APE = ∠CPD (Vertically opposite angles)

Hence, by using AA similarity criterion,

ΔAEP ~ ΔCDP

(ii)



In ΔABD and ΔCBE,

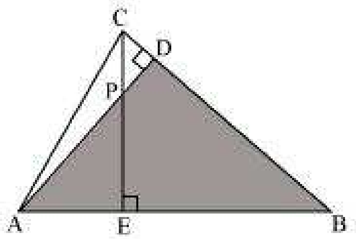
∠ADB = ∠CEB (Each 90°)

∠ABD = ∠CBE (Common)

Hence, by using AA similarity criterion,

ΔABD ~ ΔCBE

(iii)



In ΔAEP and ΔADB,

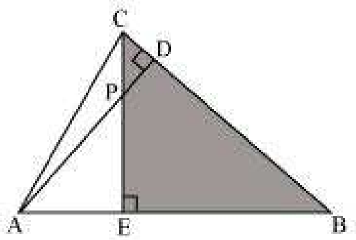
∠AEP = ∠ADB (Each 90°)

∠PAE = ∠DAB (Common)

Hence, by using AA similarity criterion,

ΔAEP ~ ΔADB

(iv)



In ΔPDC and ΔBEC,

∠PDC = ∠BEC (Each 90°)

∠PCD = ∠BCE (Common angle)

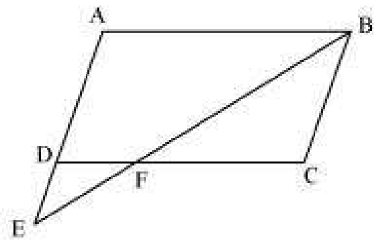
Hence, by using AA similarity criterion,

ΔPDC ~ ΔBEC

Question 8:

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB

Answer:



In ΔABE and ΔCFB,

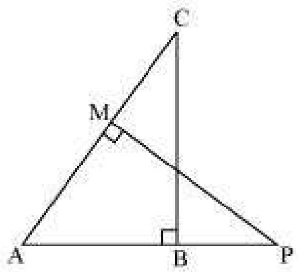
∠A = ∠C (Opposite angles of a parallelogram)

∠AEB = ∠CBF (Alternate interior angles as AE || BC)

∴ ΔABE ~ ΔCFB (By AA similarity criterion)

Question 9:

In the following figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:



(i) ΔABC ~ ΔAMP

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Answer:

In ΔABC and ΔAMP,

∠ABC = ∠AMP (Each 90°)

∠A = ∠A (Common)

∴ ΔABC ~ ΔAMP (By AA similarity criterion)

$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP}$ (Corresponding sides of similar triangles are proportional)

Question 10:

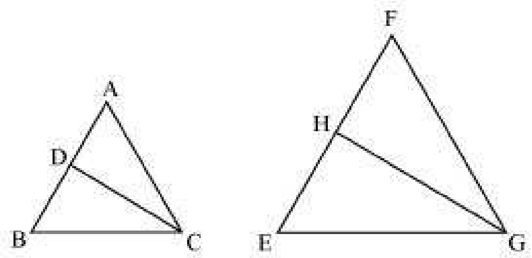
CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, Show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) ΔDCB ~ ΔHGE

(iii) ΔDCA ~ ΔHGF

Answer:



It is given that ΔABC ~ ΔFEG.

∴ ∠A = ∠F, ∠B = ∠E, and ∠ACB = ∠FGE

∠ACB = ∠FGE

∴ ∠ACD = ∠FGH (Angle bisector)

And, ∠DCB = ∠HGE (Angle bisector)

In ΔACD and ΔFGH,

∠A = ∠F (Proved above)

∠ACD = ∠FGH (Proved above)

∴ ΔACD ~ ΔFGH (By AA similarity criterion)

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

In ΔDCB and ΔHGE,

∠DCB = ∠HGE (Proved above)

∠B = ∠E (Proved above)

∴ ΔDCB ~ ΔHGE (By AA similarity criterion)

In ΔDCA and ΔHGF,

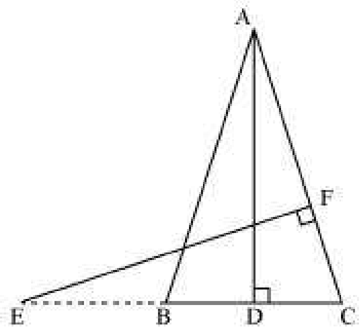
∠ACD = ∠FGH (Proved above)

∠A = ∠F (Proved above)

∴ ΔDCA ~ ΔHGF (By AA similarity criterion)

Question 11:

In the following figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that ΔABD ~ ΔECF



Answer:

It is given that ABC is an isosceles triangle.

∴ AB = AC

⇒ ∠ABD = ∠ECF

In ΔABD and ΔECF,

∠ADB = ∠EFC (Each 90°)

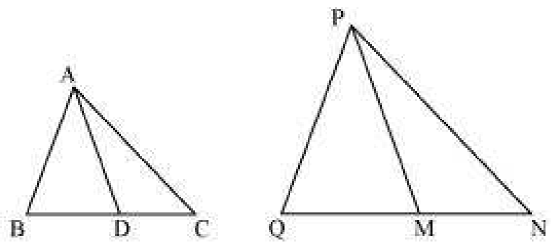
∠BAD = ∠CEF (Proved above)

∴ ΔABD ~ ΔECF (By using AA similarity criterion)

Question 12:

Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see the given figure). Show that ΔABC ~ ΔPQR.

Answer:



Median divides the opposite side.

∴ $BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$

Given that,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

In ΔABD and ΔPQM,

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$ (Proved above)

∴ ΔABD ~ ΔPQM (By SSS similarity criterion)

⇒ ∠ABD = ∠PQM (Corresponding angles of similar triangles)

In ΔABC and ΔPQR,

∠ABD = ∠PQM (Proved above)

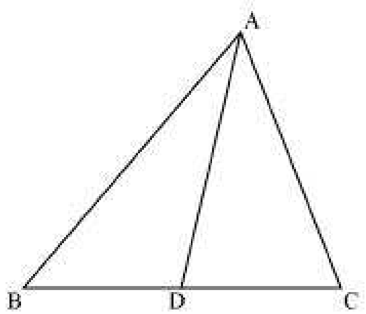
$\frac{AB}{PQ} = \frac{BC}{QR}$

∴ ΔABC ~ ΔPQR (By SAS similarity criterion)

Question 13:

D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. Show that $CA^2 = CB.CD.$

Answer:



In ΔADC and ΔBAC,

∠ADC = ∠BAC (Given)

∠ACD = ∠BCA (Common angle)

∴ ΔADC ~ ΔBAC (By AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.

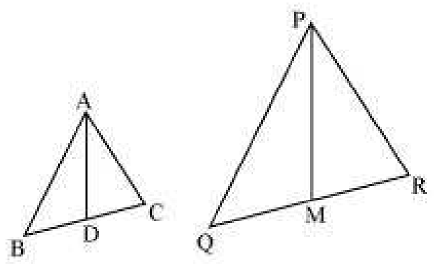
$\therefore \frac{CA}{CB} = \frac{CD}{CA}$

$\Rightarrow CA^2 = CB \times CD$

Question 14:

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$

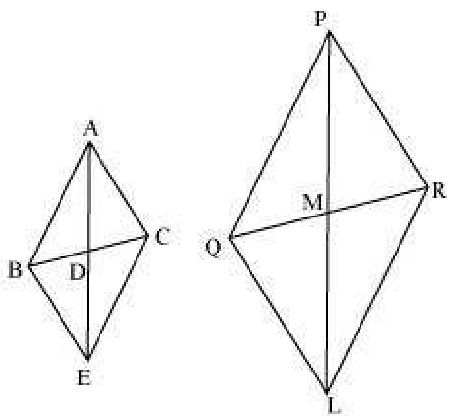
Answer:



Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then, join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.

Therefore, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

∴ AC = BE and AB = EC (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR

It was given that

$$\frac{AB}{PQ}=\frac{AC}{PR}=\frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ}=\frac{BE}{QL}=\frac{2AD}{2PM}$$

$$\Rightarrow \frac{AB}{PQ}=\frac{BE}{QL}=\frac{AE}{PL}$$

∴ ΔABE ~ ΔPQL (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

∴ ∠BAE = ∠QPL … (1)

Similarly, it can be proved that ΔAEC ~ ΔPLR and

∠CAE = ∠RPL … (2)

Adding equation (1) and (2), we obtain

∠BAE + ∠CAE = ∠QPL + ∠RPL

⇒ ∠CAB = ∠RPQ … (3)

In ΔABC and ΔPQR,

$\frac{AB}{PQ}=\frac{AC}{PR}$ (Given)

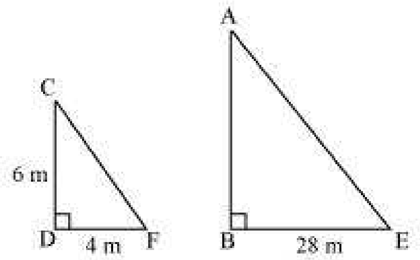
∠CAB = ∠RPQ [Using equation (3)]

∴ ΔABC ~ ΔPQR (By SAS similarity criterion)

Question 15:

A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer:



Let AB and CD be a tower and a pole respectively.

Let the shadow of BE and DF be the shadow of AB and CD respectively.

At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.

Therefore, ∠DCF = ∠BAE

And, ∠DFC = ∠BEA

∠CDF = ∠ABE (Tower and pole are vertical to the ground)

∴ ΔABE ~ ΔCDF (AAA similarity criterion)

$$\Rightarrow \frac{AB}{CD} = \frac{BE}{DF}$$

$$\Rightarrow \frac{AB}{6 \text{ m}} = \frac{28}{4}$$

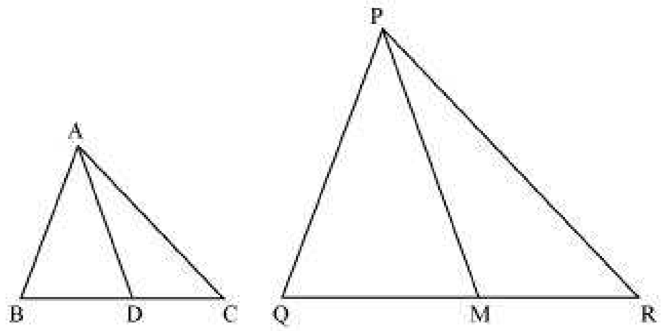
$$\Rightarrow AB = 42 \text{ m}$$

Therefore, the height of the tower will be 42 metres.

Question 16:

If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$ prove tha t $\frac{AB}{PQ} = \frac{AD}{PM}$

Answer:



It is given that ΔABC ~ ΔPQR

We know that the corresponding sides of similar triangles are in proportion.

$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ … (1)

Also, ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R … (2)

Since AD and PM are medians, they will divide their opposite sides.

$\therefore BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$ … (3)

From equations (1) and (3), we obtain

$\frac{AB}{PQ} = \frac{BD}{QM}$ … (4)

In ΔABD and ΔPQM,

∠B = ∠Q [Using equation (2)]

$\frac{AB}{PQ} = \frac{BD}{QM}$ [Using equation (4)]

∴ ΔABD ~ ΔPQM (By SAS similarity criterion)

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

Exercise 6.4

Question 1:

Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.

Answer:

It is given that $\Delta ABC \sim \Delta DEF$.

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

Given that,

$EF = 15.4$ cm,

$ar(\Delta ABC) = 64\ cm^2$,

$ar(\Delta DEF) = 121\ cm^2$

$$\therefore \frac{ar(ABC)}{ar(DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \left(\frac{64\ cm^2}{121\ cm^2}\right) = \frac{BC^2}{(15.4\ cm)^2}$$

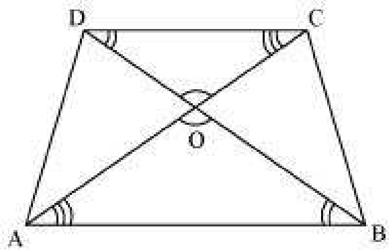
$$\Rightarrow \frac{BC}{15.4} = \left(\frac{8}{11}\right) cm$$

$$\Rightarrow BC = \left(\frac{8 \times 15.4}{11}\right) cm = (8 \times 1.4)\ cm = 11.2\ cm$$

Question 2:

Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.

Answer:



Since AB || CD,

∴ ∠OAB = ∠OCD and ∠OBA = ∠ODC (Alternate interior angles)

In ΔAOB and ΔCOD,

∠AOB = ∠COD (Vertically opposite angles)

∠OAB = ∠OCD (Alternate interior angles)

∠OBA = ∠ODC (Alternate interior angles)

∴ ΔAOB ~ ΔCOD (By AAA similarity criterion)

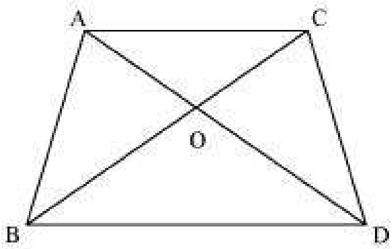
$$\therefore \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \left(\frac{AB}{CD}\right)^2$$

Since AB = 2 CD,

$$\therefore \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \left(\frac{2\ CD}{CD}\right)^2 = \frac{4}{1} = 4:1$$

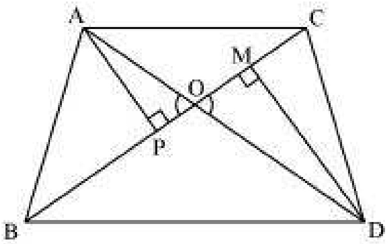
Question 3:

In the following figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{area(\Delta ABC)}{area(\Delta DBC)} = \frac{AO}{DO}$



Answer:

Let us draw two perpendiculars AP and DM on line BC.



We know that area of a triangle = $\frac{1}{2}\times \text{Base}\times \text{Height}$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DBC)}=\frac{\frac{1}{2}BC\times AP}{\frac{1}{2}BC\times DM}=\frac{AP}{DM}.$$

In ΔAPO and ΔDMO,

∠APO = ∠DMO (Each = 90°)

∠AOP = ∠DOM (Vertically opposite angles)

∴ ΔAPO ~ ΔDMO (By AA similarity criterion)

$$\therefore \frac{AP}{DM}=\frac{AO}{DO}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DBC)}=\frac{AO}{DO}$$

Question 4:

If the areas of two similar triangles are equal, prove that they are congruent.

Answer:

Let us assume two similar triangles as ΔABC ~ ΔPQR.

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \qquad (1)$$

Given that, ar $(\Delta ABC) =$ ar (ΔPQR)

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = 1$$

Putting this value in equation (1), we obtain

$$1 = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

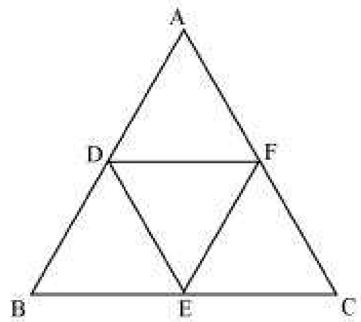
$\Rightarrow AB = PQ, BC = QR,$ and $AC = PR$

$\therefore \Delta ABC \cong \Delta PQR$ (By SSS congruence criterion)

Question 5:

D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the area of ΔDEF and ΔABC.

Answer:



D and E are the mid-points of ΔABC.

$\therefore DE \parallel AC$ and $DE = \frac{1}{2}AC$

In ΔBED and ΔBCA,

$\angle BED = \angle BCA$ (Corresponding angles)

$\angle BDE = \angle BAC$ (Corresponding angles)

$\angle EBD = \angle CBA$ (Common angles)

$\therefore \Delta BED \sim \Delta BCA$ (AAA similarity criterion)

$$\frac{ar(\Delta BED)}{ar(\Delta BCA)} = \left(\frac{DE}{AC}\right)^2$$

$$\Rightarrow \frac{ar(\Delta BED)}{ar(\Delta BCA)} = \frac{1}{4}$$

$$\Rightarrow ar(\Delta BED) = \frac{1}{4}ar(\Delta BCA)$$

Similarly, $ar(\Delta CFE) = \frac{1}{4}ar(CBA)$ and $ar(\Delta ADF) = \frac{1}{4}ar(\Delta ABC)$

Also, $ar(\Delta DEF) = ar(\Delta ABC) - \left[ar(\Delta BED) + ar(\Delta CFE) + ar(\Delta ADF)\right]$

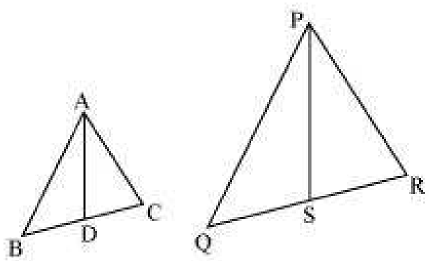
$$\Rightarrow ar(\Delta DEF) = ar(\Delta ABC) - \frac{3}{4}ar(\Delta ABC) = \frac{1}{4}ar(\Delta ABC)$$

$$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

Question 6:

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer:



Let us assume two similar triangles as ΔABC ~ ΔPQR. Let AD and PS be the medians of these triangles.

∵ ΔABC ~ ΔPQR

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ...(1)

∠A = ∠P, ∠B = ∠Q, ∠C = ∠R ... (2)

Since AD and PS are medians,

∴ BD = DC = $\frac{BC}{2}$

And, QS = SR = $\frac{QR}{2}$

Equation (1) becomes

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR}$ ... (3)

In ΔABD and ΔPQS,

∠B = ∠Q [Using equation (2)]

And, $\frac{AB}{PQ} = \frac{BD}{QS}$ [Using equation (3)]

∴ ΔABD ~ ΔPQS (SAS similarity criterion)

Therefore, it can be said that

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS}$ ... (4)

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

From equations (1) and (4), we may find that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$

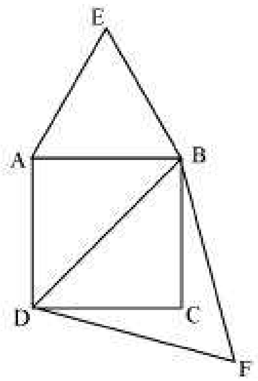
And hence,

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AD}{PS}\right)^2$$

Question 7:

Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Answer:



Let ABCD be a square of side a.

Therefore, its diagonal $= \sqrt{2}a$

Two desired equilateral triangles are formed as ΔABE and ΔDBF.

Side of an equilateral triangle, ΔABE, described on one of its sides = *a*

Side of an equilateral triangle, ΔDBF, described on one of its diagonals $= \sqrt{2}a$

We know that equilateral triangles have all its angles as 60º and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

$$\frac{\text{Area of } \Delta \text{ABE}}{\text{Area of } \Delta \text{DBF}} = \left(\frac{a}{\sqrt{2}a}\right)^2 = \frac{1}{2}$$

Question 8:

ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is

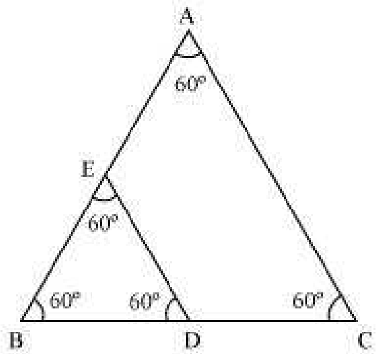
(A) 2 : 1

(B) 1 : 2

(C) 4 : 1

(D) 1 : 4

Answer:



We know that equilateral triangles have all its angles as 60º and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

Let side of ΔABC = x

Therefore, side of $\Delta \text{BDE} = \frac{x}{2}$

$$\therefore \frac{\text{area}(\Delta ABC)}{\text{area}(\Delta BDE)} = \left(\frac{x}{\frac{x}{2}}\right)^2 = \frac{4}{1}$$

Hence, the correct answer is (C).

Question 9:

Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(A) 2 : 3

(B) 4 : 9

(C) 81 : 16

(D) 16 : 81

Answer:

If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides of these triangles.

It is given that the sides are in the ratio 4:9.

Therefore, ratio between areas of these triangles = $\left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Hence, the correct answer is (D).

Exercise 6.5

Question 1:

Sides of triangles are given below. Determine which of them are right triangles? In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

Answer:

(i) It is given that the sides of the triangle are 7 cm, 24 cm, and 25 cm.

Squaring the lengths of these sides, we will obtain 49, 576, and 625.

49 + 576 = 625

Or, $7^2 + 24^2 = 25^2$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 25 cm.

(ii) It is given that the sides of the triangle are 3 cm, 8 cm, and 6 cm.

Squaring the lengths of these sides, we will obtain 9, 64, and 36.

However, 9 + 36 ≠ 64

Or, $3^2 + 6^2 \neq 8^2$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iii)Given that sides are 50 cm, 80 cm, and 100 cm.

Squaring the lengths of these sides, we will obtain 2500, 6400, and 10000.

However, 2500 + 6400 ≠ 10000

Or, $50^2 + 80^2 \neq 100^2$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iv)Given that sides are 13 cm, 12 cm, and 5 cm.

Squaring the lengths of these sides, we will obtain 169, 144, and 25.

Clearly, 144 +25 = 169

Or, $12^2 + 5^2 = 13^2$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

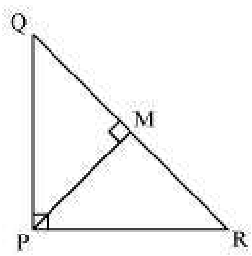
We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 13 cm.

Question 2:

PQR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR. Show that $PM^2 = QM \times MR$.

Answer:



Let $\angle MPR = x$

InΔMPR,

$\angle MRP = 180^\circ - 90^\circ - x$

$\angle MRP = 90^\circ - x$

Similarly, inΔMPQ,

$\angle MPQ = 90^\circ - \angle MPR$

$= 90^\circ - x$

$\angle MQP = 180^\circ - 90^\circ - (90^\circ - x)$

$\angle MQP = x$

InΔ QMP and ΔPMR,

$\angle MPQ = \angle MRP$

$\angle PMQ = \angle RMP$

$\angle MQP = \angle MPR$

$\therefore \Delta QMP \sim \Delta PMR$ (By AAA similarity criterion)

$\Rightarrow \dfrac{QM}{PM} = \dfrac{MP}{MR}$

$\Rightarrow PM^2 = QM \times MR$

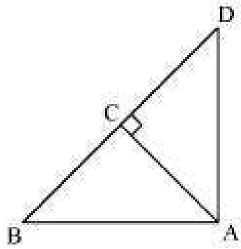
Question 3:

In the following figure, ABD is a triangle right angled at A and AC ⊥ BD. Show that

(i) AB^2 = BC × BD

(ii) AC^2 = BC × DC

(iii) AD^2 = BD × CD



Answer:

(i) In ΔADB and ΔCAB,

$\angle DAB = \angle ACB \quad (\text{Each } 90°)$

$\angle ABD = \angle CBA \quad (\text{Common angle})$

$\therefore \Delta ADB \sim \Delta CAB$ (AA similarity criterion)

$\Rightarrow \frac{AB}{CB} = \frac{BD}{AB}$

$\Rightarrow AB^2 = CB \times BD$

(ii) Let $\angle CAB = x$

$\text{In} \Delta CBA,$

$\angle CBA = 180° - 90° - x$

$\angle CBA = 90° - x$

$\text{Similarly, in} \Delta CAD,$

$\angle CAD = 90° - \angle CAB$

$= 90° - x$

$\angle CDA = 180° - 90° - (90° - x)$

$\angle CDA = x$

$\text{In} \Delta CBA \text{ and } \Delta CAD,$

$\angle CBA = \angle CAD$

$\angle CAB = \angle CDA$

$\angle ACB = \angle DCA \quad (\text{Each } 90°)$

$\therefore \Delta CBA \sim \Delta CAD \quad (\text{By AAA rule})$

$\Rightarrow \frac{AC}{DC} = \frac{BC}{AC}$

$\Rightarrow AC^2 = DC \times BC$

(iii) In ΔDCA and ΔDAB,

∠DCA = ∠ DAB (Each 90º)

∠CDA = ∠ ADB (Common angle)

$\therefore \Delta DCA \sim \Delta DAB$ (AA similarity criterion)

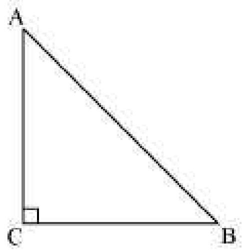
$\Rightarrow \frac{DC}{DA} = \frac{DA}{DB}$

$\Rightarrow AD^2 = BD \times CD$

Question 4:

ABC is an isosceles triangle right angled at C. prove that $AB^2 = 2\ AC^2$.

Answer:



Given that ΔABC is an isosceles triangle.

∴ AC = CB

Applying Pythagoras theorem in ΔABC (i.e., right-angled at point C), we obtain

$AC^2 + CB^2 = AB^2$

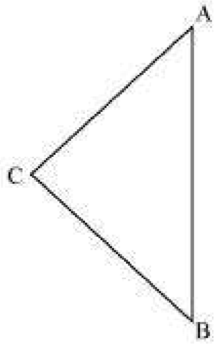
$\Rightarrow AC^2 + AC^2 = AB^2$ $(AC = CB)$

$\Rightarrow 2AC^2 = AB^2$

Question 5:

ABC is an isosceles triangle with AC = BC. If $AB^2 = 2\ AC^2$, prove that ABC is a right triangle.

Answer:



Given that,

$AB^2 = 2AC^2$

$\Rightarrow AB^2 = AC^2 + AC^2$

$\Rightarrow AB^2 = AC^2 + BC^2 \quad (\text{As } AC = BC)$

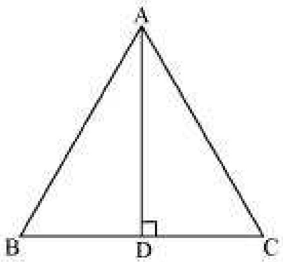
The triangle is satisfying the pythagoras theorem.

Therefore, the given triangle is a right - angled triangle.

Question 6:

ABC is an equilateral triangle of side 2*a*. Find each of its altitudes.

Answer:



Let AD be the altitude in the given equilateral triangle, ΔABC.

We know that altitude bisects the opposite side.

∴ BD = DC = *a*

In ΔADB,

$\angle ADB = 90°$

Applying pythagoras theorem, we obtain

$AD^2 + DB^2 = AB^2$

$\Rightarrow AD^2 + a^2 = (2a)^2$

$\Rightarrow AD^2 + a^2 = 4a^2$

$\Rightarrow AD^2 = 3a^2$

$\Rightarrow AD = a\sqrt{3}$

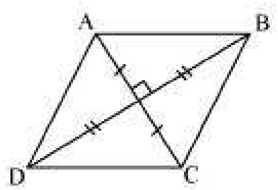
In an equilateral triangle, all the altitudes are equal in length.

Therefore, the length of each altitude will be $\sqrt{3}a$.

Question 7:

Prove that the sum of the squares of the sides of rhombus is equal to the sum of the squares of its diagonals.

Answer:



In ΔAOB, ΔBOC, ΔCOD, ΔAOD,

Applying Pythagoras theorem, we obtain

$AB^2 = AO^2 + OB^2$ ... (1)

$BC^2 = BO^2 + OC^2$ ... (2)

$CD^2 = CO^2 + OD^2$ ... (3)

$AD^2 = AO^2 + OD^2$ ... (4)

Adding all these equations, we obtain

$$AB^2 + BC^2 + CD^2 + AD^2 = 2\left(AO^2 + OB^2 + OC^2 + OD^2\right)$$

$$= 2\left(\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right)$$

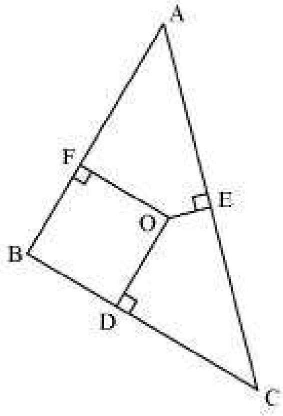
(Diagonals bisect each other)

$$= 2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right)$$

$$= (AC)^2 + (BD)^2$$

Question 8:

In the following figure, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that

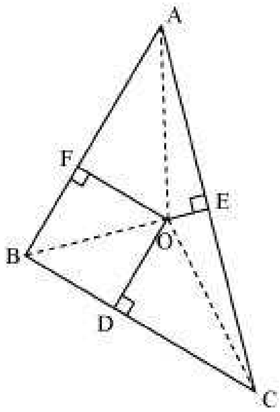


(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Answer:

Join OA, OB, and OC.



(i) Applying Pythagoras theorem in ΔAOF, we obtain

$$OA^2 = OF^2 + AF^2$$

Similarly, in ΔBOD,

$$OB^2 = OD^2 + BD^2$$

Similarly, in ΔCOE,

$$OC^2 = OE^2 + EC^2$$

Adding these equations,

$$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$$

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2$$

(ii) From the above result,

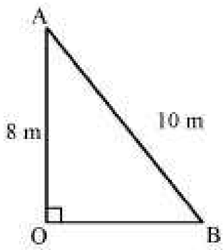
$$AF^2 + BD^2 + EC^2 = \left(OA^2 - OE^2\right) + \left(OC^2 - OD^2\right) + \left(OB^2 - OF^2\right)$$

$$\therefore AF^2 + BD^2 + EC^2 = AE^2 + CD^2 + BF^2$$

Question 9:

A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Answer:



Let OA be the wall and AB be the ladder.

Therefore, by Pythagoras theorem,

$$AB^2 = OA^2 + BO^2$$

$$(10\text{ m})^2 = (8\text{ m})^2 + OB^2$$

$$100\text{ m}^2 = 64\text{ m}^2 + OB^2$$

$$OB^2 = 36\text{ m}^2$$

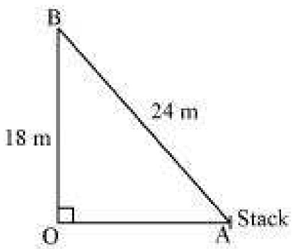
$$OB = 6\text{ m}$$

Therefore, the distance of the foot of the ladder from the base of the wall is
6 m.

Question 10:

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Answer:



Let OB be the pole and AB be the wire.

By Pythagoras theorem,

$AB^2 = OB^2 + OA^2$

$(24\ \text{m})^2 = (18\ \text{m})^2 + OA^2$

$OA^2 = (576 - 324)\text{m}^2 = 252\ \text{m}^2$

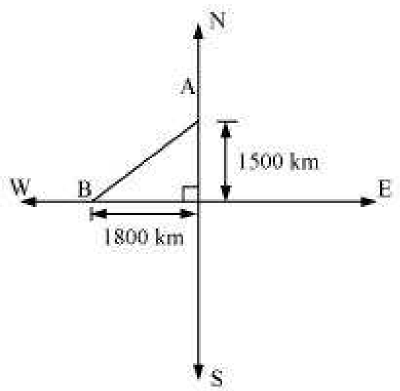
$OA = \sqrt{252}\ \text{m} = \sqrt{6\times6\times7}\ \text{m} = 6\sqrt{7}\ \text{m}$

Therefore, the distance from the base is $6\sqrt{7}$ m.

Question 11:

An aeroplane leaves an airport and flies due north at a speed of 1,000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1,200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Answer:



Distance travelled by the plane flying towards north in $1\frac{1}{2}\text{hrs}$

$= 1{,}000 \times 1\frac{1}{2} = 1{,}500\ \text{km}$

Similarly, distance travelled by the plane flying towards west in $1\frac{1}{2}\text{hrs}$

$= 1{,}200 \times 1\frac{1}{2} = 1{,}800\ \text{km}$

Let these distances be represented by OA and OB respectively.

Applying Pythagoras theorem,

Distance between these planes after $1\frac{1}{2}$ hrs, AB = $\sqrt{OA^2 + OB^2}$

$$= \left(\sqrt{(1,500)^2 + (1,800)^2}\right) \text{km} = \left(\sqrt{2250000 + 3240000}\right) \text{km}$$

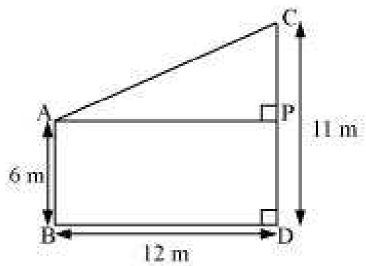
$$= \left(\sqrt{5490000}\right) \text{km} = \left(\sqrt{9 \times 610000}\right) \text{km} = 300\sqrt{61} \text{ km}$$

Therefore, the distance between these planes will be $300\sqrt{61}$ km after $1\frac{1}{2}$ hrs.

Question 12:

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Answer:



Let CD and AB be the poles of height 11 m and 6 m.

Therefore, CP = 11 − 6 = 5 m

From the figure, it can be observed that AP = 12m

Applying Pythagoras theorem for ΔAPC, we obtain

$AP^2 + PC^2 = AC^2$

$(12\text{ m})^2 + (5\text{ m})^2 = AC^2$

$AC^2 = (144 + 25)\text{m}^2 = 169\text{ m}^2$

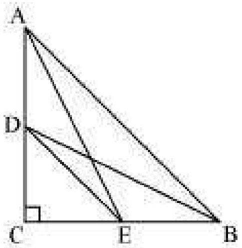
$AC = 13\text{ m}$

Therefore, the distance between their tops is 13 m.

Question 13:

D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$

Answer:



Applying Pythagoras theorem in ΔACE, we obtain

$AC^2 + CE^2 = AE^2 \quad \ldots (1)$

Applying Pythagoras theorem in ΔBCD, we obtain

$BC^2 + CD^2 = BD^2 \quad \ldots (2)$

Using equation (1) and equation (2), we obtain

$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2 \quad \ldots (3)$

Applying Pythagoras theorem in ΔCDE, we obtain

$DE^2 = CD^2 + CE^2$

Applying Pythagoras theorem in ΔABC, we obtain

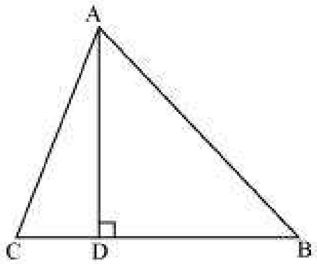
$AB^2 = AC^2 + CB^2$

Putting the values in equation (3), we obtain

$DE^2 + AB^2 = AE^2 + BD^2$

Question 14:

The perpendicular from A on side BC of a ΔABC intersect BC at D such that DB = 3 CD. Prove that 2 AB^2 = 2 AC^2 + BC^2



Answer:

Applying Pythagoras theorem for ΔACD, we obtain

$$AC^2 = AD^2 + DC^2$$
$$AD^2 = AC^2 - DC^2 \quad \ldots (1)$$

Applying Pythagoras theorem in ΔABD, we obtain

$$AB^2 = AD^2 + DB^2$$
$$AD^2 = AB^2 - DB^2 \quad \ldots (2)$$

From equation (1) and equation (2), we obtain

$$AC^2 - DC^2 = AB^2 - DB^2 \quad \ldots (3)$$

It is given that 3DC = DB

$$\therefore DC = \frac{BC}{4} \text{ and } DB = \frac{3BC}{4}$$

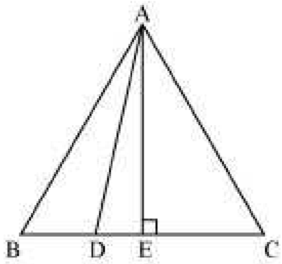
Putting these values in equation (3), we obtain

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$$
$$AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$$
$$16AC^2 - BC^2 = 16AB^2 - 9BC^2$$
$$16AB^2 - 16AC^2 = 8BC^2$$
$$2AB^2 = 2AC^2 + BC^2$$

Question 15:

In an equilateral triangle ABC, D is a point on side BC such that BD = $\frac{1}{3}$ BC. Prove that 9 AD^2 = 7 AB^2.

Answer:



Let the side of the equilateral triangle be *a*, and AE be the altitude of ΔABC.

$\therefore$ BE = EC = $\frac{BC}{2} = \frac{a}{2}$

And, AE = $\frac{a\sqrt{3}}{2}$

Given that, BD = $\frac{1}{3}$ BC

$\therefore$ BD = $\frac{a}{3}$

DE = BE − BD = $\frac{a}{2} - \frac{a}{3} = \frac{a}{6}$

Applying Pythagoras theorem in ΔADE, we obtain

$AD^2 = AE^2 + DE^2$

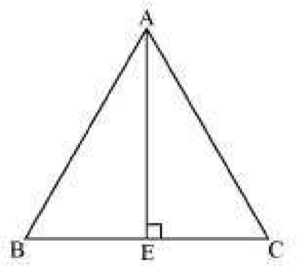
$$\mathrm{AD}^2 = \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2$$
$$= \left(\frac{3a^2}{4}\right) + \left(\frac{a^2}{36}\right)$$
$$= \frac{28a^2}{36}$$
$$= \frac{7}{9}\mathrm{AB}^2$$

⇒ 9 AD^2 = 7 AB^2

Question 16:

In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Answer:



Let the side of the equilateral triangle be *a*, and AE be the altitude of ΔABC.

∴ BE = EC = $\frac{\mathrm{BC}}{2} = \frac{a}{2}$

Applying Pythagoras theorem in ΔABE, we obtain

$AB^2 = AE^2 + BE^2$

$$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$$

$$AE^2 = a^2 - \frac{a^2}{4}$$

$$AE^2 = \frac{3a^2}{4}$$

$4AE^2 = 3a^2$

⇒ 4 × (Square of altitude) = 3 × (Square of one side)

Question 17:

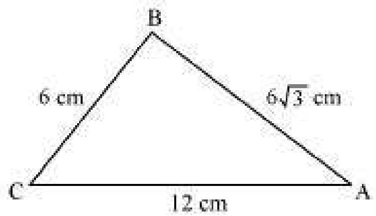
Tick the correct answer and justify: In ΔABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.

The angle B is:

(A) 120° (B) 60°

(C) 90° (D) 45°

Answer:



Given that, AB = $6\sqrt{3}$ cm, AC = 12 cm, and BC = 6 cm

It can be observed that

$AB^2 = 108$

$AC^2 = 144$

And, $BC^2 = 36$

$AB^2 + BC^2 = AC^2$

The given triangle, ΔABC, is satisfying Pythagoras theorem.

Therefore, the triangle is a right triangle, right-angled at B.

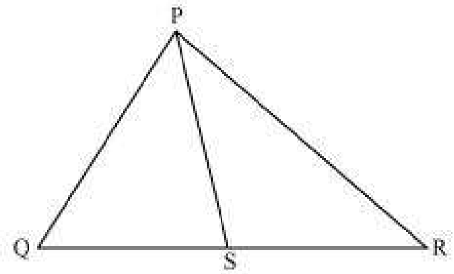
∴ ∠B = 90°

Hence, the correct answer is (C).

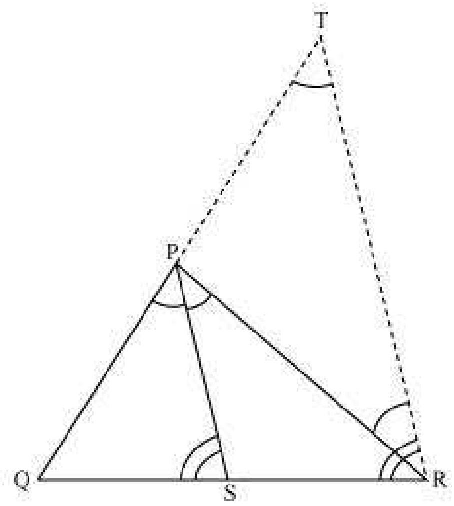
Exercise 6.6

Question 1:

In the given figure, PS is the bisector of ∠QPR of ΔPQR. Prove that $\frac{QS}{SR} = \frac{PQ}{PR}$.



Answer:



Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T.

Given that, PS is the angle bisector of ∠QPR.

∠QPS = ∠SPR … (1)

By construction,

∠SPR = ∠PRT (As PS || TR) … (2)

∠QPS = ∠QTR (As PS || TR) … (3)

Using these equations, we obtain

∠PRT = ∠QTR

∴ PT = PR

By construction,

PS || TR

By using basic proportionality theorem for ΔQTR,

$$\frac{QS}{SR} = \frac{QP}{PT}$$

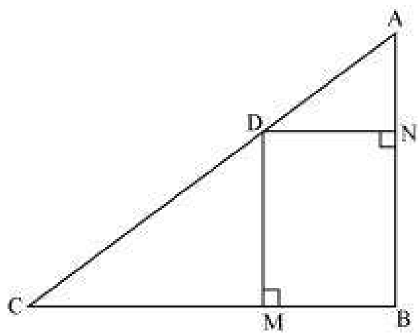
$$\Rightarrow \frac{QS}{SR} = \frac{PQ}{PR} \qquad (PT = TR)$$

Question 2:

In the given figure, D is a point on hypotenuse AC of ΔABC, DM ⊥ BC and DN ⊥ AB, Prove that:

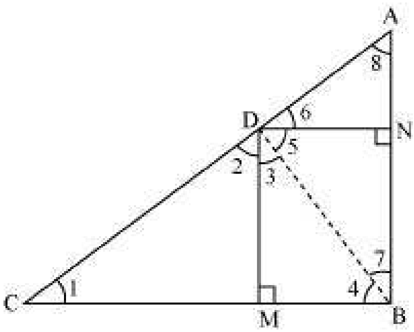
(i) $DM^2 = DN.MC$

(ii) $DN^2 = DM.AN$



Answer:

(i)Let us join DB.



We have, DN || CB, DM || AB, and ∠B = 90°

∴ DMBN is a rectangle.

∴ DN = MB and DM = NB

The condition to be proved is the case when D is the foot of the perpendicular drawn from B to AC.

∴ ∠CDB = 90°

⇒ ∠2 + ∠3 = 90° … (1)

In ΔCDM,

∠1 + ∠2 + ∠DMC = 180°

⇒ ∠1 + ∠2 = 90° … (2)

In ΔDMB,

∠3 + ∠DMB + ∠4 = 180°

⇒ ∠3 + ∠4 = 90° … (3)

From equation (1) and (2), we obtain

∠1 = ∠3

From equation (1) and (3), we obtain

∠2 = ∠4

In ΔDCM and ΔBDM,

∠1 = ∠3 (Proved above)

∠2 = ∠4 (Proved above)

∴ ΔDCM ~ ΔBDM (AA similarity criterion)

$$\Rightarrow \frac{BM}{DM} = \frac{DM}{MC}$$

$$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC} \qquad (BM = DN)$$

$\Rightarrow DM^2 = DN \times MC$

(ii) In right triangle DBN,

$\angle 5 + \angle 7 = 90°$ … (4)

In right triangle DAN,

$\angle 6 + \angle 8 = 90°$ … (5)

D is the foot of the perpendicular drawn from B to AC.

$\therefore \angle ADB = 90°$

$\Rightarrow \angle 5 + \angle 6 = 90°$ … (6)

From equation (4) and (6), we obtain

$\angle 6 = \angle 7$

From equation (5) and (6), we obtain

$\angle 8 = \angle 5$

In ΔDNA and ΔBND,

$\angle 6 = \angle 7$ (Proved above)

$\angle 8 = \angle 5$ (Proved above)

$\therefore \Delta DNA \sim \Delta BND$ (AA similarity criterion)

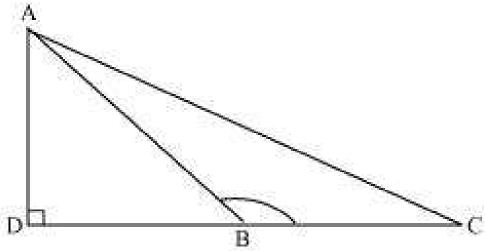
$$\Rightarrow \frac{AN}{DN} = \frac{DN}{NB}$$

$\Rightarrow DN^2 = AN \times NB$

$\Rightarrow DN^2 = AN \times DM$ (As NB = DM)

Question 3:

In the given figure, ABC is a triangle in which $\angle ABC > 90°$ and AD ⊥ CB produced. Prove that $AC^2 = AB^2 + BC^2 + 2BC.BD$.



Answer:

Applying Pythagoras theorem in ΔADB, we obtain

$AB^2 = AD^2 + DB^2$ … (1)

Applying Pythagoras theorem in ΔACD, we obtain

$AC^2 = AD^2 + DC^2$

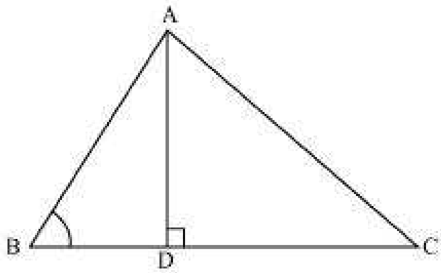
$AC^2 = AD^2 + (DB + BC)^2$

$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$

$AC^2 = AB^2 + BC^2 + 2DB \times BC$ [Using equation (1)]

Question 4:

In the given figure, ABC is a triangle in which ∠ABC < 90° and AD ⊥ BC. Prove that $AC^2 = AB^2 + BC^2 - 2BC.BD$.



Answer:

Applying Pythagoras theorem in ΔADB, we obtain

$AD^2 + DB^2 = AB^2$

$\Rightarrow AD^2 = AB^2 - DB^2$ … (1)

Applying Pythagoras theorem in ΔADC, we obtain

$AD^2 + DC^2 = AC^2$

$AB^2 - BD^2 + DC^2 = AC^2$ [Using equation (1)]

$AB^2 - BD^2 + (BC - BD)^2 = AC^2$

$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$

$= AB^2 + BC^2 - 2BC \times BD$

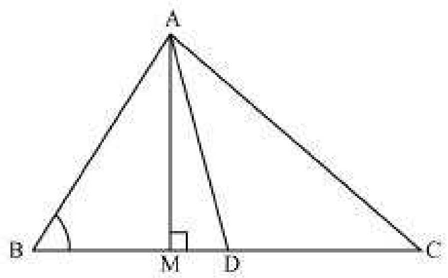
Question 5:

In the given figure, AD is a median of a triangle ABC and AM ⊥ BC. Prove that:

(i) $AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

(iii) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$



Answer:

(i) Applying Pythagoras theorem in ΔAMD, we obtain

$AM^2 + MD^2 = AD^2$ … (1)

Applying Pythagoras theorem in ΔAMC, we obtain

$AM^2 + MC^2 = AC^2$

$AM^2 + (MD + DC)^2 = AC^2$

$(AM^2 + MD^2) + DC^2 + 2MD.DC = AC^2$

$AD^2 + DC^2 + 2MD.DC = AC^2$ [Using equation (1)]

Using the result, $DC = \frac{BC}{2}$, we obtain

$$AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD.\left(\frac{BC}{2}\right) = AC^2$$

$$AD^2 + \left(\frac{BC}{2}\right)^2 + MD \times BC = AC^2$$

(ii) Applying Pythagoras theorem in ΔABM, we obtain

$AB^2 = AM^2 + MB^2$

$= (AD^2 - DM^2) + MB^2$

$= (AD^2 - DM^2) + (BD - MD)^2$

$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$

$= AD^2 + BD^2 - 2BD \times MD$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right) \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - BC \times MD$$

(iii)Applying Pythagoras theorem in ΔABM, we obtain

$AM^2 + MB^2 = AB^2$ … (1)

Applying Pythagoras theorem in ΔAMC, we obtain

$AM^2 + MC^2 = AC^2$ … (2)

Adding equations (1) and (2), we obtain

$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$

$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$

$2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$

$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$

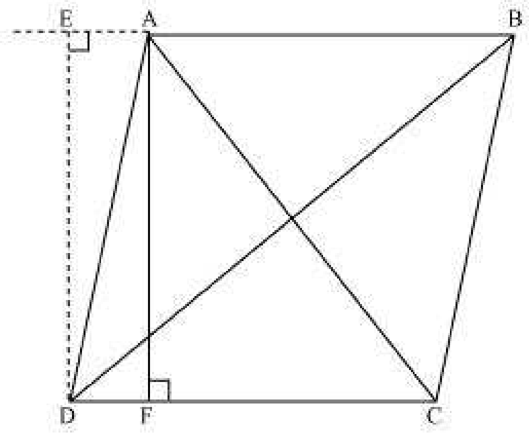
$$2\left(AM^2 + MD^2\right) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$

$$2AD^2 + \frac{BC^2}{2} = AB^2 + AC^2$$

Question 6:

Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Answer:



Let ABCD be a parallelogram.

Let us draw perpendicular DE on extended side AB, and AF on side DC.

Applying Pythagoras theorem in ΔDEA, we obtain

$DE^2 + EA^2 = DA^2$ … (*i*)

Applying Pythagoras theorem in ΔDEB, we obtain

$DE^2 + EB^2 = DB^2$

$DE^2 + (EA + AB)^2 = DB^2$

$(DE^2 + EA^2) + AB^2 + 2EA \times AB = DB^2$

$DA^2 + AB^2 + 2EA \times AB = DB^2$ … (*ii*)

Applying Pythagoras theorem in ΔADF, we obtain

$AD^2 = AF^2 + FD^2$

Applying Pythagoras theorem in ΔAFC, we obtain

$AC^2 = AF^2 + FC^2$

$= AF^2 + (DC - FD)^2$

$= AF^2 + DC^2 + FD^2 - 2DC \times FD$

$= (AF^2 + FD^2) + DC^2 - 2DC \times FD$

$AC^2 = AD^2 + DC^2 - 2DC \times FD$ … (*iii*)

Since ABCD is a parallelogram,

AB = CD … (*iv*)

And, BC = AD … (*v*)

In ΔDEA and ΔADF,

∠DEA = ∠AFD (Both 90°)

∠EAD = ∠ADF (EA || DF)

AD = AD (Common)

∴ ΔEAD ≅ ΔFDA (AAS congruence criterion)

⇒ EA = DF … (vi)

Adding equations (*i*) and (*iii*), we obtain

$DA^2 + AB^2 + 2EA \times AB + AD^2 + DC^2 - 2DC \times FD = DB^2 + AC^2$

$DA^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2DC \times FD = DB^2 + AC^2$

$BC^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2AB \times EA = DB^2 + AC^2$

[Using equations (*iv*) and (*vi*)]

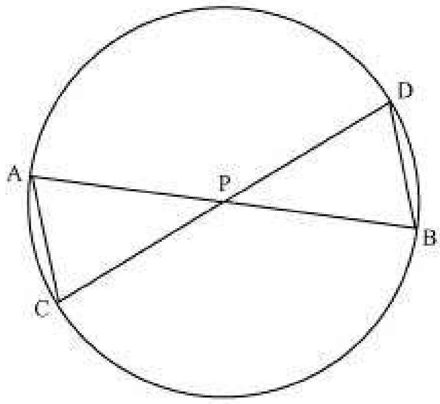
$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Question 7:

In the given figure, two chords AB and CD intersect each other at the point P. prove that:

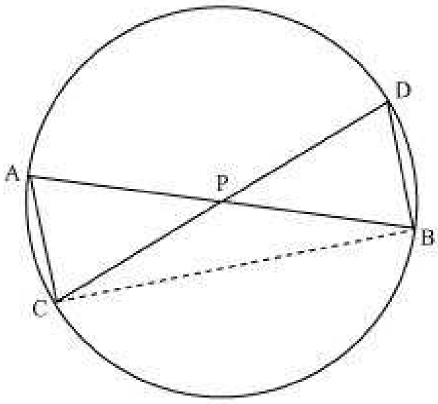
(i) ΔAPC ~ ΔDPB

(ii) AP.BP = CP.DP



Answer:

Let us join CB.



(i) In ΔAPC and ΔDPB,

∠APC = ∠DPB (Vertically opposite angles)

∠CAP = ∠BDP (Angles in the same segment for chord CB)

ΔAPC ~ ΔDPB (By AA similarity criterion)

(ii) We have already proved that

ΔAPC ~ ΔDPB

We know that the corresponding sides of similar triangles are proportional.

$$\therefore \frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD}$$

$$\Rightarrow \frac{AP}{DP} = \frac{PC}{PB}$$

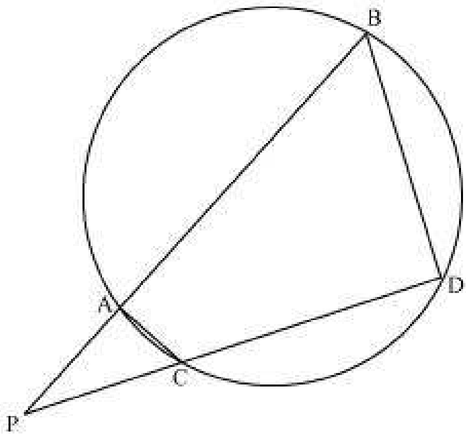
∴ AP. PB = PC. DP

Question 8:

In the given figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that

(i) ΔPAC ~ ΔPDB

(ii) PA.PB = PC.PD



Answer:

(i) In ΔPAC and ΔPDB,

∠P = ∠P (Common)

∠PAC = ∠PDB (Exterior angle of a cyclic quadrilateral is ∠PCA = ∠PBD equal to the opposite interior angle)

∴ ΔPAC ~ ΔPDB

(ii)We know that the corresponding sides of similar triangles are proportional.

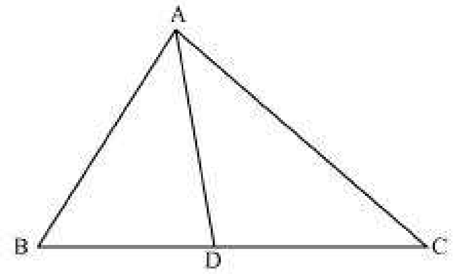
$$\therefore \frac{PA}{PD} = \frac{AC}{DB} = \frac{PC}{PB}$$

$$\Rightarrow \frac{PA}{PD} = \frac{PC}{PB}$$

∴ PA.PB = PC.PD

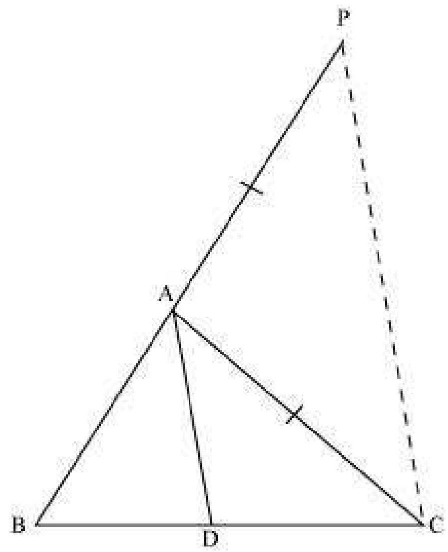
Question 9:

In the given figure, D is a point on side BC of ΔABC such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove that AD is the bisector of ∠BAC.



Answer:

Let us extend BA to P such that AP = AC. Join PC.



It is given that,

$$\frac{BD}{CD}=\frac{AB}{AC}$$

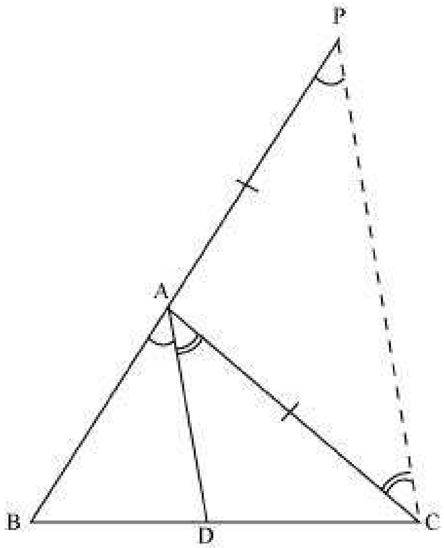
$$\Rightarrow \frac{BD}{CD}=\frac{AP}{AC}$$

By using the converse of basic proportionality theorem, we obtain

AD || PC

⇒ ∠BAD = ∠APC (Corresponding angles) … (1)

And, ∠DAC = ∠ACP (Alternate interior angles) … (2)



By construction, we have

AP = AC

⇒ ∠APC = ∠ACP … (3)

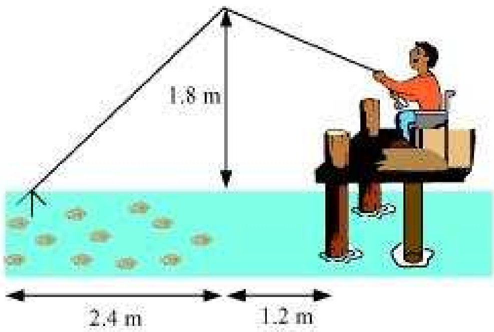
On comparing equations (1), (2), and (3), we obtain

∠BAD = ∠APC

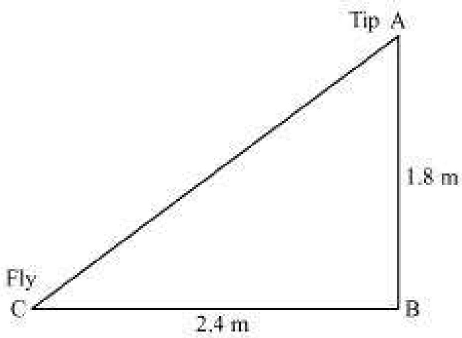
⇒ AD is the bisector of the angle BAC

Question 10:

Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, ho much string does she have out (see Fig. 6.64)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



Answer:



Let AB be the height of the tip of the fishing rod from the water surface. Let BC be the horizontal distance of the fly from the tip of the fishing rod.

Then, AC is the length of the string.

AC can be found by applying Pythagoras theorem in ΔABC.

$AC^2 = AB^2 + BC^2$

$AB^2 = (1.8\text{ m})^2 + (2.4\text{ m})^2$

$AB^2 = (3.24 + 5.76)\text{ m}^2$

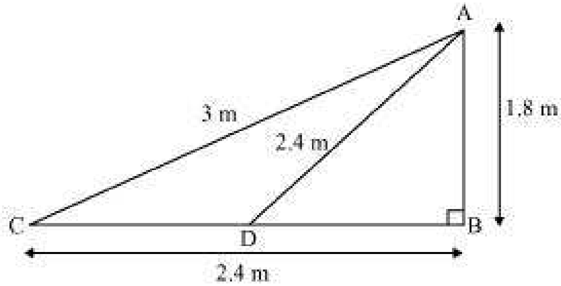
$AB^2 = 9.00\text{ m}^2$

$\Rightarrow AB = \sqrt{9}\text{ m} = 3\text{ m}$

Thus, the length of the string out is 3 m.

She pulls the string at the rate of 5 cm per second.

Therefore, string pulled in 12 seconds = 12 × 5 = 60 cm = 0.6 m



Let the fly be at point D after 12 seconds.

Length of string out after 12 seconds is AD.

AD = AC − String pulled by Nazima in 12 seconds

= (3.00 − 0.6) m

= 2.4 m

In ΔADB,

$AB^2 + BD^2 = AD^2$

$(1.8 \text{ m})^2 + BD^2 = (2.4 \text{ m})^2$

$BD^2 = (5.76 - 3.24) \text{ m}^2 = 2.52 \text{ m}^2$

BD = 1.587 m

Horizontal distance of fly = BD + 1.2 m

= (1.587 + 1.2) m

= 2.787 m

= 2.79 m