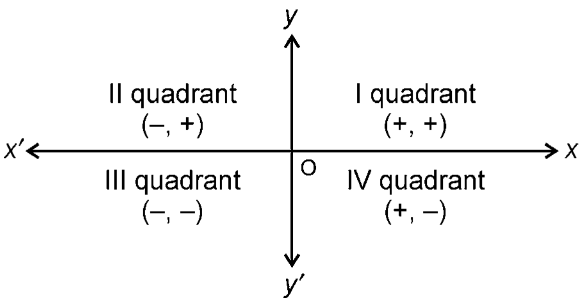
Assignments in Mathematics Class X (Term II)

7. COORDINATE GEOMETRY

IMPORTANT TERMS, DEFINITIONS AND RESULTS

- In the rectangular coordinate system, two number lines are drawn at right angles to each other. The point of intersection of these two number lines is called the **origin** whose coordinates are taken as (0, 0). The horizontal number line is known as the x-axis and the vertical one as the y-axis.
- In the ordered pair (p, q), p is called the *x-coordinate* or *abscissa* and q is known as *y-coordinate* or *ordinate* of the point.
- The coordinate plane is divided into four **quadrants**.



- The abscissa of a point is its perpendicular distance from y-axis.
- The ordinate of a point is its perpendicular distance from x-axis.
- The abscissa of every point situated on the right side of y-axis is **positive** and the abscissa of every point situated on the left side of y-axis is **negative**.
- The ordinate of every point situated above x-axis is **positive** and that of every point below x-axis is **negative.**
- The abscissa of every point on y-axis is zero.
- The ordinate of every point on x-axis is zero.
- The distance between any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or $$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$\Rightarrow$ PQ =

$$\sqrt{(\text{Difference of absissae})^2 + (\text{Difference of ordinates})^2}$$

- If O(0, 0) is the origin and $P(x, y)$ is any point, then from the above formula, we have :

$$OP = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$$

- In order to prove that a given figure is a :
  - (i) square, prove that four sides are equal and the diagonals are equal.
  - (ii) rhombus, prove that the four sides are equal.
  - (iii) rectangle, prove the opposite sides are equal and the diagonals are also equal.
  - (iv) parallelogram, prove that the opposite sides are equal.
  - (v) parallelogram but not a rectangle, prove that its opposite sides are equal but diagonals are not equal.
- Three points A, B and C are said to be collinear, if they lie on the same straight line.
- For three points to be *collinear,* the sum of the distances between two pairs of points is equal to the third pair of points.
- Three points will make :
  - (i) a scalene triangle, if no two sides of the triangle are equal.
  - (ii) an isosceles triangle, if any two sides are equal.
  - (iii) an equilateral triangle, if all the three sides are equal.
  - (iv) a right triangle, if sum of the squares of any two sides is equal to the square of the third side.
- The coordinates of the point $P(x, y)$ which divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$, are given by :

$$x = \frac{mx_2 + nx_1}{m+n},\ y = \frac{my_2 + ny_1}{m+n}.$$

- The coordinates of the mid-point M of a line segment AB with end points $A(x_1, y_1)$ and $B(x_2, y_2)$ are :

$$\left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right]$$

- The point of intersection of the medians of a triangle is called its *centroid*.
- The coordiantes of the centroid of the triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) are given by $\left[\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right]$.
- The area of a ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by :

area (ΔABC) =

$$\left|\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}\right|.$$

Since area of a triangle cannot be negative, we consider the absolute or numerical value of the area.

- Three given points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, are collinear if

$\Leftrightarrow$ area of $\Delta ABC = 0$

$\Leftrightarrow \frac{1}{2} [x_1 (y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Leftrightarrow x_1 (y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

SUMMATIVE ASSESSMENT

MULTIPLE CHOICE QUESTIONS [1 Mark]

A. Important Questions

1. Three points A, B, C are said to be collinear, if :

(a) they lie on the same straight line

(b) they do not lie on the same straight line

(c) they lie on three different straight lines

(d) none of these

2. The point (–3, 5) lies in :

(a) 1st quadrant (b) 2nd quadrant

(c) 3rd quadrant (d) 4th quadrant

3. The points A(0, –2), B(3, 1), C(0, 4) and D(–3, 1) are the vertices of a :

(a) parallelogram (b) rectangle

(c) square (d) rhombus

4. S is a point on x-axis at a distance of 4 units from y-axis to its right. The coordinates of S are :

(a) (4, 0) (b) (0, 4)

(c) (4, 4) (d) (–4, 4)

5. The distance between the points P(0, y) and Q(x, 0) is given by :

(a) $x^2 + y^2$ (b) $\sqrt{x^2 - y^2}$

(c) $\sqrt{x^2 + y^2}$ (d) $\sqrt{xy}$

6. The distance of the point P(2, 3) from the x-axis is :

(a) 2 (b) 3 (c) 1 (d) 5

7. The distance of the point P(–6, 8) from the origin is :

(a) 8 (b) $2\sqrt{7}$ (c) 10 (d) 6

8. If the distance between the points (2, –2) and (–1, x) is 5, one of the values of x is :

(a) –2 (b) 2 (c) –1 (d) 1

9. The distance between the points (0, 5) and (5, 0) is :

(a) 5 (b) $5\sqrt{2}$ (c) $2\sqrt{5}$ (d) 10

10. The point on the x-axis which is equidistant from P(–2, 9) and Q (2, –5) is :

(a) (0, 7) (b) (–7, 0)

(c) (7, 0) (d) (7, –7)

11. The distance between the points P(2, –3) and Q(2, 2) is :

(a) 2 units (b) 3 units

(c) 4 units (d) 5 units

12. If the points P(2, 3), Q(5, k) and R(6, 7) are collinear, then the value of k is :

(a) 4 (b) 6 (c) $-\frac{3}{2}$ (d) $\frac{1}{4}$

13. The points M(0, 6), N(–5, 3) and P(3, 1) are the vertices of a triangle, which is :

(a) isosceles (b) equilateral

(c) scalene (d) right angled

14. A is a point on y-axis at a distance of 4 units from x-axis lying below x-axis. The coordinates of A are:

(a) (4, 0) (b) (0, 4)

(c) (–4, 0) (d) (0, –4)

15. The mid-point of the line segment joining the points A(–2, 8) and B(–6, –4) is :

(a) (–6, –4) (b) (2, 6)

(c) (–4, 2) (d) (4, 2)

16. The point which divides the line segment joining the points (7, –6) and (3, 4) in the ratio 1 : 2 internally lies in the :

(a) 1st quadrant (b) 2nd quadrant

(c) 3rd quadrant (d) 4th quadrant

17. If the point P(2, 1) lies on the line segment joining points A(4, 2) and B(8, 4), then :

(a) $AP = \frac{AB}{3}$ (b) AP = AB

(c) $PB = \frac{AB}{3}$ (d) $AP = \frac{AB}{2}$

18. If the points (k, $2k$), ($3k$, $3k$) and (3, 1) are collinear, then k is :

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

19. The points (0, 6), (–5, 3) and (3, 1) are the vertices of a triangle which is :

(a) equilateral (b) isosceles
(c) scalene (d) right angled

20. If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the point Q(–6, 5) and R(–2, 3), then the value of a is :

(a) –4 (b) –12 (c) 12 (d) –6

21. The perpendicular bisector of the line segment joining the points A(1, 5) and B(4, 6) cuts the y-axis at:

(a) (0, 13) (b) (0, –13)
(c) (0, 12) (d) (13, 0)

22. The ratio in which (4, 5) divides the join of (2, 3) and (7, 8) is :

(a) 4 : 3 (b) 5 : 2
(c) 3 : 2 (d) 2 : 3

23. The y-axis divides the join of P(–4, 2) and Q(8, 3) in the ratio :

(a) 3 : 1 (b) 1 : 3
(c) 2 : 1 (d) 1 : 2

24. Two vertices of ΔPQR are P(–1, 4) and Q(5, 2) and its centroid is G(0, –3). The coordinates of R are:

(a) (4, 3) (b) (4, 15)
(c) (–4, –15) (d) (–15, – 4)

25. The x-axis divides the join of A(2, –3) and B(5, 6) in the ratio :

(a) 1 : 2 (b) 2 : 1
(c) 3 : 5 (d) 2 : 3

26. A point A divides the join of X(5, –2) and Y(9, 6) in the ratio 3 : 1. The coordinates of A are :

(a) (4, 7) (b) (8, 4)
(c) $\left(\frac{11}{2}, 5\right)$ (d) (12, 8)

27. If M(–1, 1) is the mid-point of the line segment joining P(–3, y) and Q(1, y + 4), then the value of y is:

(a) 1 (b) –1 (c) 2 (d) 0

28. If (x, 2), (–3, –4) and (7, –5) are collinear, then x is equal to :

(a) 60 (b) 63 (c) –63 (d) –60

29. If the area of the triangle formed by the points (a, 2a), (–2, 6) and (3, 1) is 5 square units, then a is equal to :

(a) 2 (b) $\frac{3}{5}$ (c) 3 (d) 5

30. If the distance between the points (4, p) and (1, 0) is 5, then the value of p is :

(a) 4 only (b) ±4
(c) –4 only (d) 0

31. A line intersects x and y-axes at P and Q respectively. If (2, –5) is the mid-point of PQ, then the coordinates of P and Q are respectively:

(a) (4, 0) and (0, –10) (b) (2, 0) and (0, –5)
(c) (–4, 0) and (0, 10) (d) (–10, 0) and (0, 4)

32. The distance between the points $(\cos\theta, \sin\theta)$ and $(\sin\theta, \cos\theta)$ is :

(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 2 (d) 1

33. If the points (1, 2), (–5, 6) and (a, –2) are collinear, then a is equal to :

(a) –3 (b) 7 (c) 2 (d) –2

34. The points A(9, 0), B(9, 6), C(–9, 6) and D(–9, 0) are the vertices of a :

(a) square (b) rectangle
(c) rhombus (d) trapezium

35. AOBC is a rectangle whose three vertices are A(0, 3), O(0, 0) and B(5, 0). The length of its diagonal is :

(a) 5 (b) 3 (c) $\sqrt{34}$ (d) 4

36. The points (–4, 0), (4, 0) and (0, 3) are vertices of a:

(a) right triangle (b) isosceles triangle
(c) equilateral triangle (d) scalene triangle

37. The coordinates of the vertices of an equilateral triangle are A(3, y), B(3, $\sqrt{3}$) and C(0, 0). The value of y is :

(a) 4 (b) 5
(c) –1 (d) none of these

38. The distance between the points (a cos θ + b sin θ, 0) and (0, a sin θ – b cos θ) is :

(a) $a^2 + b^2$ (b) $a + b$
(c) $a^2 - b^2$ (d) $\sqrt{a^2+b^2}$

39. (–1, 2), (2, –1) and (3, 1) are three vertices of a parallelogram. The coordinates of the fourth vertex are:

(a) (4, 0) (b) (– 2, 0)
(c) (–2, 6) (d) (6, 2)

40. If A(5, 3), B(11, –5) and C(12, a) are the vertices of a right angled triangle, right angled at C, then the value of a is :

(a) – 2, 4 (b) –2, –4
(c) 2, –4 (d) 2, 4

41. The perimeter of the triangle with vertices (0, 4), (0, 0) and (3, 0) is :

(a) 5 (b) 12

(c) 11 (d) $7+\sqrt{5}$

42. The point which lies on the perpendicular bisector of the line segment joining the points A(–2, –5) and B(2, 5) is :

(a) (0, 0) (b) (0, 2)

(c) (2, 0) (d) (–2, 0)

43. The area of the triangle with vertices $(a, b + c)$, $(b, c + a)$ and $(c, a + b)$ is :

(a) $(a + b + c)^2$ (b) 0

(c) $(a + b + c)$ (d) abc

44. If the centroid of the triangle formed by the points (a, b), (b, c) and (c, a) is at the origin, then $a^3 + b^3 + c^3$ is equal to :

(a) abc (b) 0

(c) $a + b + c$ (d) $3abc$

45. The line segment joining points (–3, –4) and (1, –2) is divided by y-axis in the ratio :

(a) 1 : 3 (b) 2 : 3

(c) 3 : 1 (d) 2 : 3

46. If points $(x, 0)$, $(0, y)$ are (1, 1) and collinear, then $\frac{1}{x}+\frac{1}{y}$ is equal to :

(a) 1 (b) 2 (c) 0 (d) –1

47. If the ratio in which P divides the line segment joining (x_1, y_1) and (x_2, y_2) be k : 1, then the coordinates of the point P are :

(a) $\left(\frac{kx_1+x_2}{k+1}, \frac{ky_2+y_1}{k+1}\right)$

(b) $\left(\frac{kx_2+x_1}{k+1}, \frac{ky_2+y_1}{k+1}\right)$

(c) $\left(\frac{x_1+x_2}{k+1}, \frac{y_1+y_2}{k+1}\right)$

(d) none of these

48. Area of a triangle is taken :

(a) always positive

(b) always negative

(c) sometimes positive sometimes negative

(d) none of these

49. The area of the triangle formed by $(x, y + z)$, $(y, z + x)$ and $(z, x + y)$ is :

(a) $x + y + z$ (b) xyz

(c) $(x + y + z)^2$ (d) 0

50. The fourth vertex D of a parallelogram ABCD whose three vertices are A(–2, 3), B(6, 7) and C(8, 3) is:

(a) (0, 1) (b) (0, –1)

(c) (–1, 0) (d) (1, 0)

51. If the points A(1, 2), B(0, 0) and C(a, b) are collinear, then :

(a) $a = b$ (b) $a = 2b$

(c) $2a = b$ (d) $a = -b$

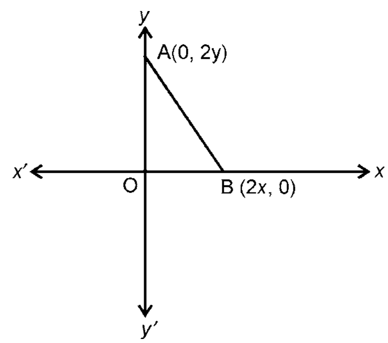
52. In the figure, OAB is a triangle. The coordinates of the point which is equidistant from the three vertices are :

(a) (x, y)

(b) (y, x)

(c) $\left(\frac{x}{2}, \frac{y}{2}\right)$

(d) $\left(\frac{y}{2}, \frac{x}{2}\right)$



53. A circle drawn with origin as the centre passes through $\left(\frac{13}{2}, 0\right)$ The point which does not lie in the interior of the circle is :

(a) $\left(-\frac{3}{4}, 1\right)$ (b) $\left(2, \frac{7}{3}\right)$

(c) $\left(5, \frac{1}{2}\right)$ (d) $\left(-6, \frac{5}{2}\right)$

54. The coordinates of the centroid of the triangle with vertices (a, 0), (0, b) and (a, b) are :

(a) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (b) $\left(\frac{a}{3}, \frac{b}{3}\right)$

(c) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (d) none of these

55. Area of the triangle with vertices $(x, 0)$, $(0, y)$ and (x, y) is :

(a) $x + y$ (b) $x - y$ (c) $\frac{x}{y}$ (d) $\frac{xy}{z}$

56. If the area of a quadrilateral ABCD is zero, then the four points A, B, C and D are :

(a) collinear (b) not collinear

(c) nothing can be said (d) none of these

B. Questions From CBSE Examination Papers

1. If the points (0, 0), (1, 2) and (x, y) are collinear then : **[2011 (T-II]**

   (a) $x = y$ (b) $2x = y$
   (c) $x = 2y$ (d) $2x = -y$

2. The perpendicular distance of A (5, 12) from the y-axis is : **[2011 (T-II]**

   (a) 13 units (b) 5 units
   (c) 12 units (d) 17 units

3. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is : **[2011 (T-II]**

   (a) 8 (b) 10 (c) 12 (d) 15

SHORT ANSWER TYPE QUESTIONS [2 Marks]

A. Important Questions

1. Find the distance between the points $(a\cos 35^\circ, 0)$ and $(0, a\cos 55^\circ)$

2. Find the value of x which is an integer such that the distance between the points P(x, 2) and Q(3, – 6) is 10 units.

3. Is the point (4, 4) equidistant from the points P(–1, 4) and Q(1, 0)?

4. A is a point on the x-axis and B is a point on the y-axis. If the abscissa of A be a and the ordinate of B be $-a$, then find the length of segment AB.

5. What is the distance between A on the x-axis whose abscissa is 11 and B(7, 3)?

6. Find the coordinates of the other end of a diameter of a circle whose one end is A(2, 1) and centre is P $\left(\frac{3}{2}, \frac{-5}{2}\right)$.

7. Find the coordinates of the centroid of the triangle whose vertices are (0, 6), (8, 12) and (8, 0).

8. If the point (a, b) is equidistant from the points (7, 1) and (3, 5), find the relation between a and b.

9. Two vertices of a triangle are A(–7, 4) and B(3, –5). If its centroid is (2, –1), then find the coordinates of the third vertex C.

10. The mid-point of the line segment joining (3, 6) and (x, 2) is (2, y). Find the values of x and y.

11. What point on the x-axis is equidistant from (7, 6) and (–3, 4) ?

12. Find the ratio in which the line segment joining the points (6, 4) and (1, –7) is divided by the x-axis.

13. Find the distance between the points A$(x + y, x - y)$ and B$(x - y, -x - y)$.

14. Are the points A$(a, b + c)$, B$(b, c + a)$ and C$(c, a + b)$ collinear ?

15. Find the value of x if the points (x, 8), (–4, 2) and (5, –1) are collinear.

16. Is ΔABC with vertices A(–2, 0), B(2, 0), C(0, 2) similar to ΔDEF with vertices D(–4, 0), E(4, 0) and F(0, 4) ? Justify your answer.

17. Find the points on the y-axis, each of which is at a distance of 13 units from the point (–5, 7).

18. Are the points A(4, 5), B(7, 6) and C(6, 3) collinear?

19. If the points P(a, –11), Q(5, b), R(2, 15) and S(1, 1) are the vertices of a parallelogram PQRS, find the values of a and b.

20. Find all possible values of a for which the distance between the points A(a, –1) and B(5, 3) is 5 units.

21. Show that the points A(3, 1), B(12, –2) and C(0, 2) cannot be the vertices of a triangle.

22. If the points A(– 6, 10), B(– 4, 6) and C(3, –8) are collinear, then show that AB = $\frac{2}{9}$ AC.

23. Find the coordinates of the point R which divides the line segment joining the points P(–2, 3) and Q(4, 7) internally in the ratio 4 : 7.

24. If the point C(–1, 2) divides internally the line segment joining A(2, 5) and B in the ratio 3 : 4, find the coordinates of B.

25. Check whether the point P(–2, 4) lies on a circle of radius 6 units and centre C(3, 5).

26. Does the point A(– 4, 2) lie on the line segment joining the points X(– 4, 6) and Y(– 4, –6) ? Justify your answer.

27. In what ratio does the point P(2, –5) divide the line segment joining A(– 3, 5) and B(4, –9) ?

28. Show that the points A(–1, –2), B(4, 3), C(2, 5) and D(–3, 0) are the vertices of the rectangle ABCD.

29. Show that A(–2, 3), B(8, 3) and C(6, 7) are the vertices of a right angled triangle.

30. Find the value of m if the point (0, 2) is equidistant from (3, m) and (m, 5).

B. Questions From CBSE Examination Papers

1. If the point A (4, 3) and B(x, 5) are on the circle with centre O(2, 3), find the value of x. **[2011 (T-II)]**

2. Three consecutive vertices of a parallelogram ABCD are A(1, 2), B(1, 0) and C(4, 0). Find the foruth vertex D. **[2011 (T-II)]**

3. Which point on x-axis is equidistant from (7, 6) and (–3, 4)? **[2011 (T-II)]**

4. Find the ratio in which the line $2x + y = 4$ divides the join of A(2, –2) and B(3, 7). Also, find the coordinates of the point of their intersection. **[2011 (T-II)]**

5. Find the value of k for which the distance between (9, 2) and (3, k) is 10 units. **[2011 (T-II)]**

6. Find the ratio in which the point $\left(\frac{-2}{7}, \frac{-20}{7}\right)$ divides the join of (–2, –2) and (2, – 4). **[2011 (T-II)]**

7. Find the coordinates of a point on x-axis which divides the line segment joining the points (–2, –3) and (1, 6) in the ratio 1 : 2. **[2011 (T-II)]**

8. The line segment joining the points P(3, 3) and Q(6, – 6) is trisected at the points A and B such that A is nearer to P. If A also lies on the line given by $2x + y + k = 0$, find the value of k.

9. Find the type of triangle formed by points A(–5, 6), B(– 4, –2), C(7, 5). **[2011 (T-II)]**

10. In what ratio does the point (– 4, 6) divide the line segment joining the points A(– 6, 10) and B(3, –8)? **[2011 (T-II)]**

11. Find a relation between x and y such that the point P(x, y) is equidistant from the points A(7, 1) and B(3, 5). **[2011 (T-II)]**

12. If the points A(6, 1), B(8, 2), C(9, 4) and D(P, 3) are the vertices of a parallelogram taken in order, find the value of P. **[2011 (T-II)]**

13. Find the co-ordinates of the points of trisection of the line segment joining the points A(2, –2) and B(–7, 4). **[2011 (T-II)]**

14. Check whether the points (2, 2), (4, 0) and (–6, 10) are collinear. **[2011 (T-II)]**

15. Find the ratio in which the y-axis divides the join of (5, – 6) and (–1, – 4). **[2011 (T-II)]**

16. Find the co-ordinates of a point A, where AB is the diameter of a circle whose centre is O(2, –3) and B is (1, 4). **[2011 (T-II)]**

17. Using section formula, show that the points A(–3, 1), B (1, 3) and C(–1, 1) are collinear.

18. Find the value of p, for which the points (1, 3), (3, p) and (5, –1) are collinear. **[2011 (T-II)]**

19. Find points on the x-axis, which are at a distance of 5 units from the point A (5, –3).**[2011 (T-II)]**

20. Show that the points (a, $b + c$), (b, $c + a$) and (c, $a + b$) are collinear. **[2011 (T-II)]**

21. Prove that the points (0, 0), (5, 5) and (–5, 5) are the vertices of a right angled isosceles triangle.

22. Find the value of x, if the distance between the points (x, –1) and (3, –2) is $x + 5$. **[2011 (T-II)]**

23. If the point C (–1, 2) divides internally the line segment joining A (2, 5) and B(x, y) in the ratio 3 : 4, then find the coordinates of B. **[2011 (T-II)]**

24. Show that the point P (–4, 2) lies on the line segment joining the points A (– 4, 6) and B (– 4, –6). **[2011 (T-II)]**

25. Show that the point (1, –1) is the centre of the circle circumscribing the triangle whose vertices are (4, 3) and (–2, 3) and (6, –1). **[2011 (T-II)]**

26 If A(1, 2), B (4, y), C(x, 6) and D(3, 5) are the vertices of a parallelogram ABCD taken in order, find the values of x and y. **[2011 (T-II)]**

27. If the point P(x, y) is equidistant from the points A(5, 1) and B (–1, 5) then prove that $3x = 2y$. **[2011 (T-II)]**

28. Determine the ratio in which the point P(x, –2) divides the join of A(–4, 3) and B(2, –4). Also find the value of x. **[2011 (T-II)]**

29. If points A(–2, –1), B (a, 0), C (4, b) and D(1, 2) are the vertices of a parallelogram ABCD, find the values of a and b. **[2011 (T-II)]**

30. Find the value of x such that PQ = QR, where the coordinates of P, Q and R are (6, –1), (1, 3) and (x, 8) respectively. **[2011 (T-II)]**

31. Find the value of k for which the point (8, 1), (k, –4) and (2, –5) are collinear. **[2011 (T-II)]**

32. Show that P(1, –1) is the centre of the circle circumscribing the triangle whose angular points are A (4, 3), B(–2, 3) and C (6, –1). **[2011 (T-II)]**

33. One end of a diameter of a circle is at (2, 3) and the centre is (–2, 5). What are the cooridnates of the other end of the diameter? **[2011 (T-II)]**

34. A point P is at a distance of $\sqrt{10}$ from the point (2, 3). Find the coordinates of the point P if its y coordinate is twice its x coordinate. **[2011 (T-II)]**

35. Find the coordinates of the point B, if the point

P(– 4, 1) divides the line segment joining the points A(2, –2) and B in the ratio 3 : 5. **[2011 (T-II)]**

36. Find the third vertex of the triangle ABC if two of its vertices are at A(–3, 1) and B (0, 2) and the mid-point of BC is at $D\left(\frac{3}{2}, -\frac{1}{2}\right)$. **[2011 (T-II)]**

37. Find the value of s if the point P(0, 2) is equidistant from Q (3, s) and R(s, 5).

38. Find the perimeter of the triangle formed by the points (0, 0), (1, 0), (0, 1). **[2011 (T-II)]**

39. Find the points on the x-axis which are at a distnace of $2\sqrt{5}$ from the point (7, –4). How many such points are there? **[2011 (T-II)]**

40. If A and B are the points (–2, –2) and (2, –4) respectively, find the coordinates of P on the line segment AB such that AP = $\frac{3}{7}$ AB. **[2011 (T-II)]**

41. Find a point on x-axis which is equidistant from (–3, –4) and (4, –3). **[2011 (T-II)]**

42. Find the value of a so that the point (3, a) lies on the line represented by $2x - 3y = 5$. **[2009]**

SHORT ANSWER TYPE QUESTIONS [3 Marks]

A. Important Questions

1. If P(x, y) is the mid-point of the line segment joining the points A(3, 4) and B(k, 6) and $x + y = 10$, then find the value of k.

2. Find the coordinates of the centre of a circle passing through the points A(2, 1), B(5, –8) and C(2, –9). Also, find the radius of the circle.

3. Find the points on the x-axis which are at a distance of $2\sqrt{5}$ units from the point (7, –4). How many such points are there ?

4. Find the area of the triangle ABC with A(1, –4) and the mid-points of the sides through A being (2, –1) and (0, –1).

5. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

6. If three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) lie on the same line, prove that **[HOTS]**

$$\frac{y_2 - y_3}{x_2 x_3} + \frac{y_3 - y_1}{x_3 x_1} + \frac{y_1 - y_2}{x_1 x_2} = 0$$

7. The centre of a circle is ($2a$, a – 7). Find the value of a, if the circle passes through the point (11, –9) and has diameter $10\sqrt{2}$ units.

8. What type of triangle is formed by the points $A\left(\sqrt{2}, \sqrt{2}\right)$ $B\left(\sqrt{2}, -\sqrt{2}\right)$ and $C\left(-\sqrt{6}, \sqrt{6}\right)$.

9. Find the ratio in which the point $M\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and B(2, –5).

10. If the mid-points of the sides of a triangle are (2, 3), $\left(\frac{3}{2}, 4\right)$ and $\left(\frac{11}{2}, 5\right)$, find the centroid of the triangle.

11. Using coordinate geometry, prove that the diagonals of a rectangle bisect each other and are equal.

12. Find the coordinates of the points C on the line segment joining the points A(–1, 3) and B(2, 5) such that AC = $\frac{3}{5}$ AB.

13. Show that (4, –1), (6, 0), (7, 2) and (5, 1) are the vertices of a rhombus. Is it a square?

14. The coordinates of the three vertices A, B and C of a parallelogram ABCD are (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively. Find the coordinates of the fourth vertex D in terms of $x_1, x_2, x_3, y_1, y_2, y_3$. **[HOTS]**

15. If $D\left(-\frac{1}{2}, \frac{5}{2}\right)$, E(7, 3) and $F\left(\frac{7}{2}, \frac{7}{2}\right)$ are the mid-points of sides of ΔABC, find the area of ΔABC.

16. If the line segment joining the points A($3a$ + 1, –3) and B($8a$, 5) is divided by the point P($9a$ – 2, –b) in the ratio 3 : 1, find the values of a and b.

B. Questions From CBSE Examination Papers

1. Find the value(s) of x for which distance between the points P(2, –3) and Q(x, 5) is 10 units. **[2011 (T-II)]**

2. Find a relation between x and y such that the point P(x, y) is equidistant from the points A(2, 5) and B(–3, 7). **[2011 (T-II)]**

3. Find the area of a triangle ABC whose vertices are A(1, –1), B (–4, 6) and C(–3, –5).

4. If the points (1, 4), (r, –2) and (–3, 16) are collinear, find the value of 'r'. **[2011 (T-II)]**

5. If A (3, 0), B (4, 5), C (–1, 4) and D (–2, –1) be four points in a plane, show that ABCD is a rhombus but not a square. **[2011 (T-II)]**

6. The mid-point of the line segment joining points A(x, y + 1) and B(x + 1, y + 2) is C. Find the value of x and y if the coordinates of C are (3/2, 5/2). **[2011 (T-II)]**

7. Find the area of rhombus ABCD if its vertices are A(3, 0), B(4, 5), C(–1, 4), D(–2, –1). **[2011 (T-II)]**

8. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). **[2011 (T-II)]**

9. The area of a triangle whose vertices are (–2, –2), (–1, –3) and (x, 0) is 3 square units. Find the value of x. **[2011 (T-II)]**

10. If P(2, 1), Q(4, 2), R(5, 4) and S(3, 3) are vertices of a quadrilateral PQRS, find the area of the quadrilateral PQRS. **[2011 (T-II)]**

11. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

12. Show that the points P(0, –2), Q(3,1), R(0, 4) and S(–3, 1) are the vertices of a square. PQRS. **[2011 (T-II)]**

13. The points A (2, 9), B (a, 5) and C (5, 5) are the vertices of a triangle ABC, right angled at B. Find the value of a and hence the area of triangle ABC. **[2011 (T-II)]**

14. Find the value of k for which the points A(–1, 3), B(2, k) and C(5, –1) are collinear. **[2011 (T-II)]**

15. Find the lengths of the medians AD and BE of the triangle ABC whose vertices are A(1, –1), B(0, 4) and C(–5, 3). **[2011 (T-II)]**

16. The mid-points of the sides AB, BC and CA of a triangle ABC are D(2, 1), E(1, 0) and F(–1, 3) respectively. Find the coordinates of the vertices of the triangle ABC. **[2011 (T-II)]**

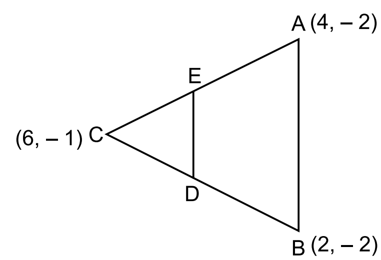
17. ABCD is a rectangle formed by joining the points A(–1, –1), B(–1, 4), C(5, 4) and D(5, –1). P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square, a rectangle or a rhombus? Justify your answer. **[2011 (T-II)]**

18. The line segment joining the points A(2, 1) and B(5, –8) is trisected at the points P and Q, where P is nearer to A. If point P lies on the line $2x - y + k = 0$, find the value of k. **[2011 (T-II)]**

19. If P(x, y) is any point on the line segment joining the points A(a, 0) and B(0, b), then show that $\frac{x}{a}+\frac{y}{b}=1$ **[2011 (T-II)]**

20. If C is a point lying on the line segment AB joining A(1, 1), B(2, 3) such that 3AC = BC, then find co-ordinates of C. **[2011 (T-II)]**

21. In the figure, in ΔABC, D and E are the mid-points of the sides BC and AC respectively. Find the length of DE. Prove that DE = $\frac{1}{2}$ AB. **[2011 (T-II)]**



22. Show that the points (–4, 0), (4, 0) and (0, 3) are vertices of an isosceles triangle. **[2011 (T-II)]**

23. Find the value of p so that the points with coordinates (3, 5), (p, 6) and $\left(\frac{1}{2}, \frac{15}{2}\right)$ are collinear. **[2011 (T-II)]**

24. The base BC of an equilateral triangle ABC lies on y-axis. The co-ordinates of the point C are (0, –3). If origin is the mid-point of BC, find the coordinates of points A and B. **[2011 (T-II)]**

25. Determine the ratio in which the point (x, 2) divides the line segment joining the points (–3, –4) and (3, 5). Also, find x. **[2011 (T-II)]**

26. Show that the points A(a, a), B (–a, –a) and C $\left(-a\sqrt{3},\, a\sqrt{3}\right)$ form an equilateral triangle. **[2011 (T-II)]**

27. Point P divides the line segment joining the points A(2, 1) and B(5, –8) such that $\frac{AP}{PB} = \frac{1}{3}$. If P lies on the line $2x - y + k = 0$, find the value of k. **[2011 (T-II)]**

28. If the vertices of a triangle are (2, 4) (5, k), (3, 10) and its area is 15 square units, find value of k. **[2011 (T-II)]**

29. Find the coordinates of centre of the circle passing through the point (0, 0), (–2, 1) and (–3, 2). Also find its radius. **[2011 (T-II)]**

30. If mid-points of sides ΔPQR are (1, 2), (0, 1), (1, 0), then find the coordintes of the three vertices of triangle PQR. **[2011 (T-II)]**

31. Prove that the points (0, 0), (5, 5), (–5, 5) are the vertices of a right angled triangle. **[2011 (T-II)]**

32. Find the co-ordinates of the points which divide the line segment joining A(–2, 2) and B(2, 8) into four equal parts. **[2011 (T-II)]**

33. If A (4, –6), B (3, –2) and C(5, 2) are the vertices of a ΔABC, then verify the fact that a median of ΔABC divides it into two triangles of equal areas. **[2011 (T-II)]**

34. If the points (10, 5), (8, 4) and (6, 6) are the mid-points of the sides of a triangle, find its vertices. **[2006]**

35. In what ratio does the line $x - y - 2 = 0$ divide the line segment joining (3, –1) and (8, 9) ? **[2007]**

36. Three consecutive vertices of a parallelogram are (– 2, 1); (1, 0) and (4, 3). Find the coordinate of the fourth vertex. **[2007]**

37. The point R divides the line segment AB where A(– 4, 0), B(0, 6) are such that AR = $\frac{3}{4}$ AB. Find the coordinates of R. **[2008]**

38. The coordinates of A and B are (1, 2) and (2, 3) respectively. If P lies on AB, find the coordinates of P such that $\frac{AP}{PB} = \frac{4}{3}$. **[2008]**

39. If A(4, –8), B(3, 6) and C(5, – 4) are the vertices of ΔABC, D is the mid-point of BC and P is a point on AD joined such that $\frac{AP}{PD}$ = 2. Find the coordinates of P. **[2008]**

40. The line segment joining the points A(2, 1) and B(5, –8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by $2x - y + k = 0$, find the value of k. **[2009]**

41. If P(x, y) is any point on the line joining the points A(a, 0) and B(0, b), then show that $\frac{x}{a} + \frac{y}{b} = 1$. **[2009]**

42. If the points (p, q), (m, n) and ($p - m$, $q - n$) are collinear, show that $pn = qm$. **[2010]**

43. Point P divides the line segment joining the points A(–1, 3) and B(9, 8) such that $\frac{AP}{PB} = \frac{k}{1}$. If P lies on the line $x - y + 2 = 0$, find the value of k. **[2010]**

LONG ANSWER TYPE QUESTIONS [4 Marks]

A. Important Questions

1. The points A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of ΔABC.

(i) The median from A meets BC at D. What are the coordinates of the point D?

(ii) Find the coordinates of the point P on AD such that AP : PD = 2 : 1.

(iii) Find the coordinates of points Q and R on medians BE and CF respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1.

(iv) What are the coordinates of the centroid of the triangle ABC? **[HOTS]**

2. If P and Q are two points whose coordinates are (at^2, $2at$) and $\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$ respectively and S is the point (a, 0), show that $\frac{1}{SP} + \frac{1}{SQ}$ is independent of t. **[HOTS]**

3. If (– 4, 3) and (4, 3) are two vertices of an equilateral triangle, find the coordinates of the third vertex given that the origin lies in the interior of the triangle.

4. If the points (x, y), (x_1, y_1) and ($x - x_1$, $y - y_1$) are collinear, show that $xy_1 = x_1y$. Also, show that the line joining the given points passes through the origin. **[HOTS]**

5. Mr. Aggarwal starts walking from his home to office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office. What is the extra distance travelled by Mr. Aggarwal in reaching his office ? Assume that all distances covered are in straight lines. If the house is situated at (2, 4), bank at (5, 8), school at (13, 14) and office at (13, 26) and coordinates are in km. **[HOTS]**
6. Find the centre of a circle passing through the points (6, –6), (3, –7) and (3, 3).
7. If the coordinates of the mid-points of the sides of a triangle are (1, 1), (2, –3) and (3, 4), find its centroid.

FORMATIVE ASSESSMENT

Activity-1

Objective : To verify the distance formula and section formula. Or to verify the following :

(*i*) The distance between the points (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(*ii*) The coordinates of the point P, which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$ are $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$.

(*iii*) The coordinates of the mid-points P of the line segment joining the points A (x_1, y_1) and B (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Materials Required : Graph paper/squared paper, geometry box, etc.

Procedure :

1. Take a 1 cm squared paper and on it draw the coordinate axes XOX' and YOY'.
2. Plot the points A (–3, 1), B (5, 1) and C (1, 8) on the squared paper and join AB, BC and AC to get a ΔABC.
3. Now using a pair of compasses, find the mid-points of AC and BC. Mark these mid-points as P and Q respectively.
4. Mark the point R on AB such that AR = $\frac{1}{4}$ AB or R divides AB in the ratio 1 : 3. Similarly, mark S on AB such that AS = $\frac{3}{4}$ AB or S divides AB in the ratio 3 : 1.
5. From the graph paper, write the coordinates of P, Q, R and S.

Coordinates of	
P	(–1, 9/2)
Q	(3, 9/2)
R	(–1, 1)
S	(3, 1)

Observations :

1. Using a ruler, measure the length of AB, BC and CA

   We have AB = BC = CA = 8 cm.
2. Now, using formula

   $$AB = \sqrt{(5+3)^2 + (1-1)^2} \text{ cm} = 8 \text{ cm}$$

   $$BC = \sqrt{(1-5)^2 + (8-1)^2} \text{ cm}$$
   $$= \sqrt{16 + 49} \text{ cm} = \sqrt{65} \text{ cm} = 8.06 \text{ cm}$$
   $$= 8 \text{ cm}$$

   $$CA = \sqrt{(1+3)^2 + (8-1)^2} \text{ cm}$$
   $$= \sqrt{16 + 49} \text{ cm} = \sqrt{65} \text{ cm} = 8.06 \text{ cm} = 8 \text{ cm}$$

   We see that in both the cases, the length of each side comes out to be 8 cm.
3. Since P and Q are mid-points of AC and BC respectively, therefore, using formula, the coordinates of P are $\left(\frac{1-3}{2}, \frac{8+1}{2}\right)$ or $\left(-1, \frac{9}{2}\right)$ the coordinate of Q are $\left(\frac{1+5}{2}, \frac{8+1}{2}\right)$ or $\left(3, \frac{9}{2}\right)$

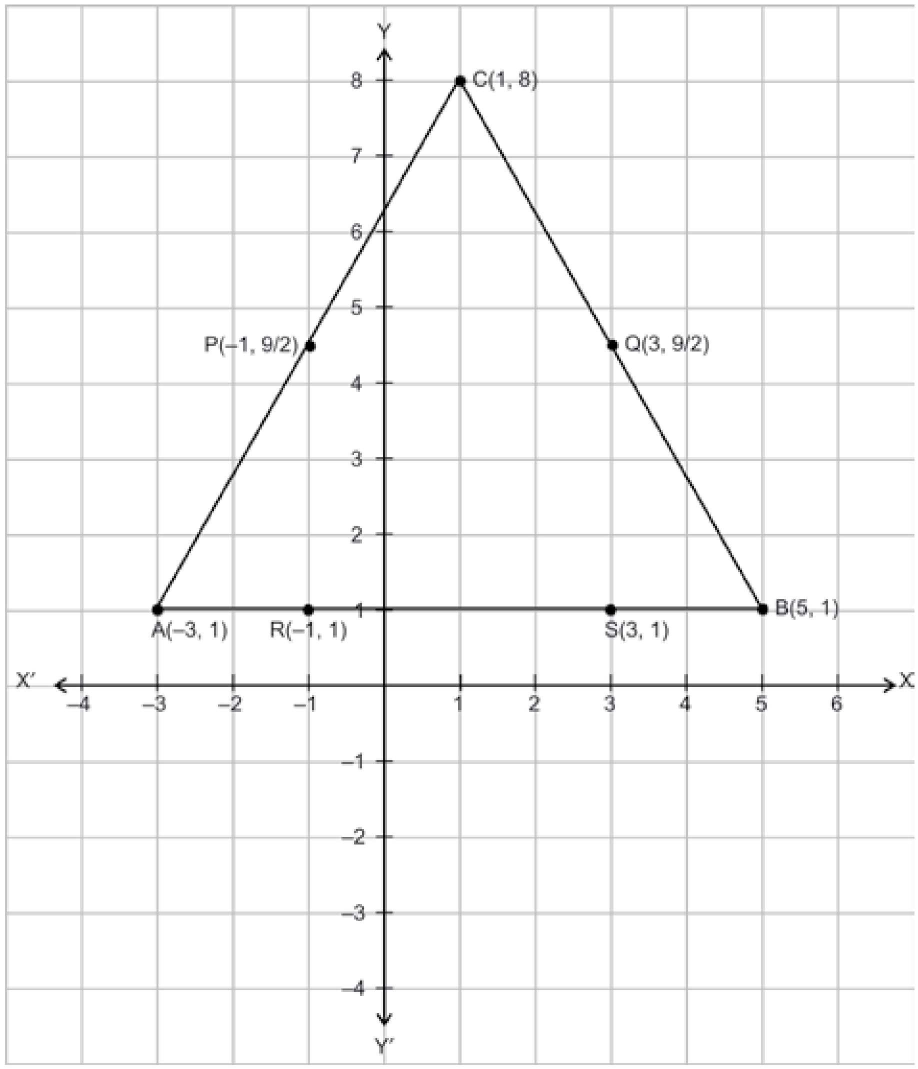
   Also, from the table : the coordinates of P and Q are same as obtained above.
4. Since R divides AB in the ratio 1 : 3, therefore,

coordinates of R (using formula) are $\left(\frac{1\times5+3\times(-3)}{1+3}, \frac{1\times1+3\times1}{1+3}\right)$ or (–1, 1).

Similarly, S divides AB in the ratio 3 : 1, therefore, coordinates of S (using formula) are:

$\left(\frac{3\times5+1\times(-3)}{3+1}, \frac{3\times1+1\times1}{3+1}\right)$ or (3, 1).

Also, from the table, the coordinates of R and S are same as obtained above.



Conclusion : From the above activity it is verified that :

(*i*) the distance of the line segments joining the points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

(*ii*) If a point P divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$, then the coordinates of P are $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$.

(*iii*) The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Do Yourself : Draw three different triangles on a paper and in each case verify the above formulae.

Activity-2

Objective : To verify the following formula :

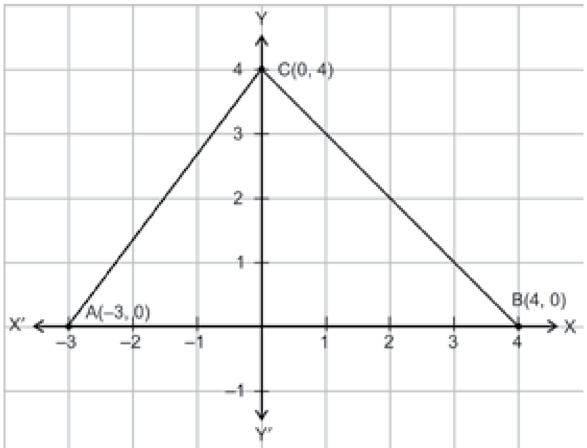
The area of a triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by $\frac{1}{2}\,[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

Materials Required : Graph paper/squared paper, geometry box etc.

Procedure : 1. Take a 1 cm squared paper and on it draw the coordinate axes XOX' and YOY'.

Case I.

1. Plot the points A (–3, 0), B (4, 0) and C(0, 4) on the squared paper and join AB, BC and AC to get a scalene triangle ABC.



2. Find AB, BC and AC using the distance formula.

$$AB = \sqrt{(-3-4)^2 + (0-0)^2} \text{ cm}$$
$$= \sqrt{49} \text{ cm} = 7 \text{ cm}$$
$$BC = \sqrt{(4-0)^2 + (0-4)^2} \text{ cm}$$
$$= \sqrt{16+16} \text{ cm} = 4\sqrt{2} \text{ cm}$$
$$AC = \sqrt{(-3-0)^2 + (0-4)^2} \text{ cn}$$
$$= \sqrt{9+16} \text{ cm} = 5 \text{ cm}$$

3. Find the area of ΔABC using Heron's formula.

Here, $a = 7$, $b = 4\sqrt{2}$, $c = 5$

$$\therefore \quad s = \frac{7+4\sqrt{2}+5}{2} = 6+2\sqrt{2}$$

$\therefore$ Area of ΔABC

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{\left(6+2\sqrt{2}\right)\left(6+2\sqrt{2}-7\right)\left(6+2\sqrt{2}-4\sqrt{2}\right)\left(6+2\sqrt{2}-5\right)} \text{ cm}^2$$
$$= \sqrt{\left(6+2\sqrt{2}\right)\left(2\sqrt{2}-1\right)\left(6-2\sqrt{2}\right)\left(2\sqrt{2}+1\right)} \text{ cm}^2$$
$$= \sqrt{\left(6+2\sqrt{2}\right)\left(6-2\sqrt{2}\right)\left(2\sqrt{2}-1\right)\left(2\sqrt{2}+1\right)} \text{ cm}^2$$
$$= \sqrt{(36-8)(8-1)} \text{ cm}^2$$
$$= \sqrt{28\times 7} \text{ cm}^2 = 14 \text{ cm}^2$$

4. Now, find the area of the triangle ABC, using the formula

$$\frac{1}{2}[x_1(y_2-y_3) - x_2(y_3-y_1) + x_3(y_1-y_2)]$$

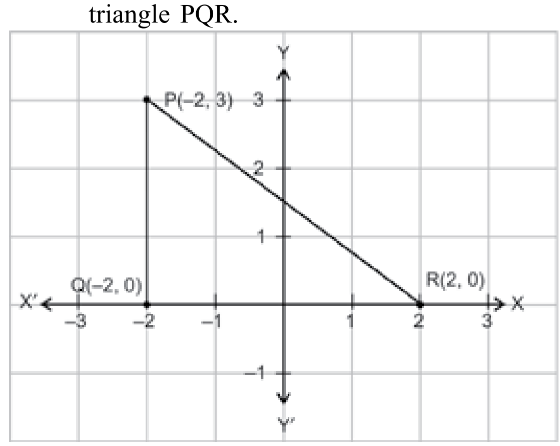
Area of $\Delta ABC = \frac{1}{2}$ [–3(0 – 4) + 4(4 – 0) + 0(0 – 0)] cm^2

$= \frac{1}{2}$ [12 + 16] cm^2 = 14 cm^2.

Observations : From (3) and (4), we see that the area of ΔABC comes out to be same on both the cases.

Case II.

1. Plot the points P (–2, 3), Q (–2, 0) and R (2, 0) and join PQ, QR and PR to get the right triangle PQR.



2. Find PQ, QR and PR using the distance formula.

$$PQ = \sqrt{(-2+2)^2 + (3-0)^2} \text{ cm}$$
$$= \sqrt{9} \text{ cm} = 3 \text{ cm}$$
$$QR = \sqrt{(-2-2)^2 + (0-0)^2} \text{ cm}$$
$$= \sqrt{16} \text{ cm} = 4 \text{ cm}$$
$$PR = \sqrt{(-2-2)^2 + (3-0)^2} \text{ cm}$$
$$= \sqrt{16+9} \text{ cm} = 5 \text{ cm}$$

3. Find the area of ΔPQR using Heros's formula:

Here, a = 3 cm, b = 4 cm, c = 5 cm

$$\therefore \quad s = \frac{3+4+5}{2} \text{ cm} = 6 \text{ cm}$$

$$\therefore \text{ Area of } \Delta PQR = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{6(6-3)(6-4)(6-5)} \text{ cm}^2$$
$$= \sqrt{6\times 3\times 2\times 1} \text{ cm}^2 = 6 \text{ cm}^2$$

4. Now, find the area of ΔPQR using the formula $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$\therefore$ Area of ΔPQR $= \frac{1}{2}$ [–2 (0 – 0) – 2 (0 – 3) + 2 (3 – 0)] cm^2 $= \frac{1}{2}$ [6 + 6] = 6 cm^2

Observations : From (3) and (4) above, we see that area of ΔPQR comes out to be same in both the cases.

Conclusion : From the above activity it is verified that the area of a triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Investigation

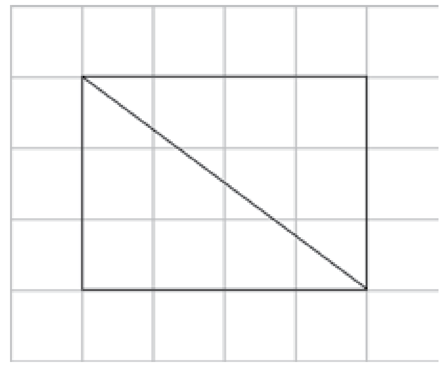
On a squared / graph paper, draw a rectangle of dimensions 4 × 3. Draw one of the diagonals of the rectangle. The diagonal passes through 6 squares.

Now on a squared paper draw several rectangles of different sizes. The length (*l*) and breadth (*b*) of each rectangle must be a whole number of squares with a common factor of 1 only. For example, sides 4 squares by 9 squares is acceptable but 4 squares by 6 squares is not because 4 and 6 have a common factor 2.

For each rectangle, draw a diagonal and count the number of squares through which the diagonal passes.

The dimensions of a rectangle is 120 × 91. How many squares will the diagonal pass through?

If the dimensions of a rectangle are $m \times n$, where m and n are co-primes, then how many squares will the diagonal pass through?



Puzzle

A merchant has nine gold coins which look identical but in fact one of the coins is an underweight fake. Investigate how the merchant can use only a balance to find the fake coin in just two weighings.

Exercise 7.1

Question 1:

Find the distance between the following pairs of points:

(i) (2, 3), (4, 1) (ii) (−5, 7), (−1, 3) (iii) (*a*, *b*), (− *a*, − *b*)

Answer:

(i) Distance between the two points is given by

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Therefore, distance between $(2, 3)$ and $(4, 1)$ is given by

$$l=\sqrt{(2-4)^2+(3-1)^2}=\sqrt{(-2)^2+(2)^2}$$
$$=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$$

(ii) Distance between $(-5,7)$ and $(-1,3)$ is given by

$$l=\sqrt{(-5-(-1))^2+(7-3)^2}=\sqrt{(-4)^2+(4)^2}$$
$$=\sqrt{16+16}=\sqrt{32}=4\sqrt{2}$$

(iii) Distance between (a,b) and $(-a,-b)$ is given by

$$l=\sqrt{(a-(-a))^2+(b-(-b))^2}$$
$$=\sqrt{(2a)^2+(2b)^2}=\sqrt{4a^2+4b^2}=2\sqrt{a^2+b^2}$$

Question 2:

Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.

Answer:

Distance between points $(0,0)$ and $(36,15)$

$= \sqrt{(36-0)^2+(15-0)^2} = \sqrt{36^2+15^2}$

$= \sqrt{1296+225} = \sqrt{1521} = 39$

Yes, we can find the distance between the given towns A and B.

Assume town A at origin point (0, 0).

Therefore, town B will be at point (36, 15) with respect to town A.

And hence, as calculated above, the distance between town A and B will be 39 km.

Question 3:

Determine if the points (1, 5), (2, 3) and (− 2, − 11) are collinear.

Answer:

Let the points (1, 5), (2, 3), and (−2, −11) be representing the vertices A, B, and C of the given triangle respectively.

Let $A = (1,5), B = (2,3), C = (-2,-11)$

$\therefore AB = \sqrt{(1-2)^2+(5-3)^2} = \sqrt{5}$

$BC = \sqrt{(2-(-2))^2+(3-(-11))^2} = \sqrt{4^2+14^2} = \sqrt{16+196} = \sqrt{212}$

$CA = \sqrt{(1-(-2))^2+(5-(-11))^2} = \sqrt{3^2+16^2} = \sqrt{9+256} = \sqrt{265}$

Since $AB + BC \neq CA$,

Therefore, the points (1, 5), (2, 3), and (−2, −11) are not collinear.

Question 4:

Check whether (5, − 2), (6, 4) and (7, − 2) are the vertices of an isosceles triangle.

Answer:

Let the points (5, −2), (6, 4), and (7, −2) are representing the vertices A, B, and C of the given triangle respectively.

$$AB = \sqrt{(5-6)^2 + (-2-4)^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$$

$$BC = \sqrt{(6-7)^2 + (4-(-2))^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37}$$

$$CA = \sqrt{(5-7)^2 + (-2-(-2))^2} = \sqrt{(-2)^2 + 0^2} = 2$$

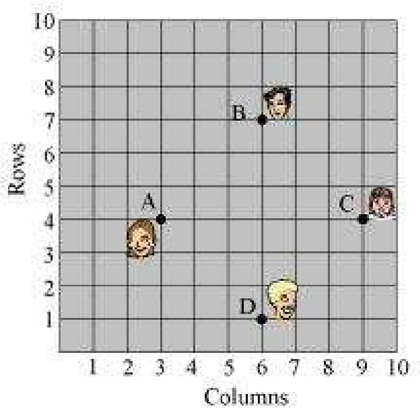
Therefore, $AB = BC$

As two sides are equal in length, therefore, ABCis an isosceles triangle.

Question 5:

In a classroom, 4 friends are seated at the points A, B, C and D as shown in the following figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees.

Using distance formula, find which of them is correct.



Answer:

It can be observed that A (3, 4), B (6, 7), C (9, 4), and D (6, 1) are the positions of these 4 friends.

$$AB = \sqrt{(3-6)^2 + (4-7)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

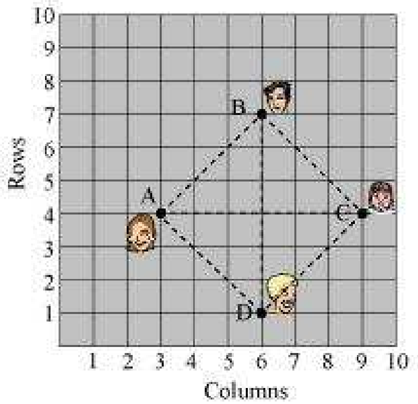
$$BC = \sqrt{(6-9)^2 + (7-4)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CB = \sqrt{(9-6)^2 + (4-1)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$AD = \sqrt{(3-6)^2 + (4-1)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$\text{Diagonal AC} = \sqrt{(3-9)^2 + (4-4)^2} = \sqrt{(-6)^2 + 0^2} = 6$$

$$\text{Diagonal BD} = \sqrt{(6-6)^2 + (7-1)^2} = \sqrt{0^2 + (6)^2} = 6$$



It can be observed that all sides of this quadrilateral ABCD are of the same length and also the diagonals are of the same length.

Therefore, ABCD is a square and hence, Champa was correct

Question 6:

Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) (− 1, − 2), (1, 0), (− 1, 2), (− 3, 0)

(ii) (− 3, 5), (3, 1), (0, 3), (− 1, − 4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Answer:

(i) Let the points (−1, −2), (1, 0), (−1, 2), and (−3, 0) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$\therefore AB = \sqrt{(-1-1)^2+(-2-0)^2} = \sqrt{(-2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{(1-(-1))^2+(0-2)^2} = \sqrt{(2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$CD = \sqrt{(-1-(-3))^2+(2-0)^2} = \sqrt{(2)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$AD = \sqrt{(-1-(-3))^2+(-2-0)^2} = \sqrt{(2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\text{Diagonal AC} = \sqrt{(-1-(-1))^2+(-2-2)^2} = \sqrt{0^2+(-4)^2} = \sqrt{16} = 4$$

$$\text{Diagonal BD} = \sqrt{(1-(-3))^2+(0-0)^2} = \sqrt{(4)^2+0^2} = \sqrt{16} = 4$$

It can be observed that all sides of this quadrilateral are of the same length and also, the diagonals are of the same length. Therefore, the given points are the vertices of a square.

(ii)Let the points (− 3, 5), (3, 1), (0, 3), and (−1, −4) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$AB = \sqrt{(-3-3)^2+(5-1)^2} = \sqrt{(-6)^2+(4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$

$$BC = \sqrt{(3-0)^2+(1-3)^2} = \sqrt{(3)^2+(-2)^2} = \sqrt{9+4} = \sqrt{13}$$

$$CD = \sqrt{(0-(-1))^2+(3-(-4))^2} = \sqrt{(1)^2+(7)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$AD = \sqrt{(-3-(-1))^2+(5-(-4))^2} = \sqrt{(-2)^2+(9)^2} = \sqrt{4+81} = \sqrt{85}$$

It can be observed that all sides of this quadrilateral are of different lengths. Therefore, it can be said that it is only a general quadrilateral, and not specific such as square, rectangle, etc.

(iii)Let the points (4, 5), (7, 6), (4, 3), and (1, 2) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$AB = \sqrt{(4-7)^2 + (5-6)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$BC = \sqrt{(7-4)^2 + (6-3)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$CD = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$AD = \sqrt{(4-1)^2 + (5-2)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$\text{Diagonal AC} = \sqrt{(4-4)^2 + (5-3)^2} = \sqrt{(0)^2 + (2)^2} = \sqrt{0+4} = 2$$

$$\text{Diagonal CD} = \sqrt{(7-1)^2 + (6-2)^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 13\sqrt{2}$$

It can be observed that opposite sides of this quadrilateral are of the same length. However, the diagonals are of different lengths. Therefore, the given points are the vertices of a parallelogram.

Question 7:

Find the point on the *x*-axis which is equidistant from (2, − 5) and (− 2, 9).

Answer:

We have to find a point on *x*-axis. Therefore, its *y*-coordinate will be 0.

Let the point on *x*-axis be $(x,0)$.

$$\text{Distance between } (x,0) \text{ and } (2,-5) = \sqrt{(x-2)^2 + (0-(-5))^2} = \sqrt{(x-2)^2 + (5)^2}$$

$$\text{Distance between } (x,0) \text{ and } (-2,9) = \sqrt{(x-(-2))^2 + (0-(-9))^2} = \sqrt{(x+2)^2 + (9)^2}$$

By the given condition, these distances are equal in measure.

$$\sqrt{(x-2)^2 + (5)^2} = \sqrt{(x+2)^2 + (9)^2}$$

$$(x-2)^2 + 25 = (x+2)^2 + 81$$

$$x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$$

$$8x = 25 - 81$$

$$8x = -56$$

$$x = -7$$

Therefore, the point is (− 7, 0).

Question 8:

Find the values of y for which the distance between the points P (2, − 3) and Q (10, y) is 10 units.

Answer:

It is given that the distance between (2, −3) and (10, y) is 10.

$$\text{Therefore, } \sqrt{(2-10)^2+(-3-y)^2}=10$$
$$\sqrt{(-8)^2+(3+y)^2}=10$$
$$64+(y+3)^2=100$$
$$(y+3)^2=36$$
$$y+3=\pm 6$$
$$y+3=6 \text{ or } y+3=-6$$
$$\text{Therefore, } y=3 \text{ or } -9$$

Question 9:

If Q (0, 1) is equidistant from P (5, − 3) and R (x, 6), find the values of x. Also find the distance QR and PR.

Answer:

$$\text{PQ}=\text{QR}$$
$$\sqrt{(5-0)^2+(-3-1)^2}=\sqrt{(0-x)^2+(1-6)^2}$$
$$\sqrt{(5)^2+(-4)^2}=\sqrt{(-x)^2+(-5)^2}$$
$$\sqrt{25+16}=\sqrt{x^2+25}$$
$$41=x^2+25$$
$$16=x^2$$
$$x=\pm 4$$

Therefore, point R is (4, 6) or (−4, 6).

When point R is (4, 6),

$$PR=\sqrt{(5-4)^2+(-3-6)^2}=\sqrt{1^2+(-9)^2}=\sqrt{1+81}=\sqrt{82}$$

$$QR=\sqrt{(0-4)^2+(1-6)^2}=\sqrt{(-4)^2+(-5)^2}=\sqrt{16+25}=\sqrt{41}$$

When point R is (−4, 6),

$$PR=\sqrt{(5-(-4))^2+(-3-6)^2}=\sqrt{(9)^2+(-9)^2}=\sqrt{81+81}=9\sqrt{2}$$

$$QR=\sqrt{(0-(-4))^2+(1-6)^2}=\sqrt{(4)^2+(-5)^2}=\sqrt{16+25}=\sqrt{41}$$

Question 10:

Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (− 3, 4).

Answer:

Point (x, y) is equidistant from (3, 6) and (−3, 4).

$$\therefore \sqrt{(x-3)^2+(y-6)^2}=\sqrt{(x-(-3))^2+(y-4)^2}$$

$$\sqrt{(x-3)^2+(y-6)^2}=\sqrt{(x+3)^2+(y-4)^2}$$

$$(x-3)^2+(y-6)^2=(x+3)^2+(y-4)^2$$

$$x^2+9-6x+y^2+36-12y=x^2+9+6x+y^2+16-8y$$

$$36-16=6x+6x+12y-8y$$

$$20=12x+4y$$

$$3x+y=5$$

$$3x+y-5=0$$

Exercise 7.2

Question 1:

Find the coordinates of the point which divides the join of (− 1, 7) and (4, − 3) in the ratio 2:3.

Answer:

Let P(x, y) be the required point. Using the section formula, we obtain

$$x=\frac{2\times 4+3\times(-1)}{2+3}=\frac{8-3}{5}=\frac{5}{5}=1$$

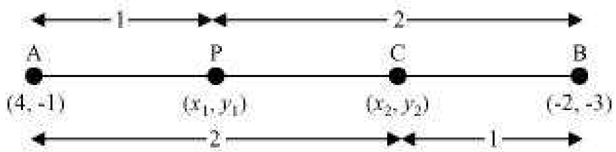
$$y=\frac{2\times(-3)+3\times 7}{2+3}=\frac{-6+21}{5}=\frac{15}{5}=3$$

Therefore, the point is (1, 3).

Question 2:

Find the coordinates of the points of trisection of the line segment joining (4, − 1) and (− 2, − 3).

Answer:



Let P (x_1, y_1) and Q (x_2, y_2) are the points of trisection of the line segment joining the given points i.e., AP = PQ = QB

Therefore, point P divides AB internally in the ratio 1:2.

$$x_1=\frac{1\times(-2)+2\times 4}{1+2},\quad y_1=\frac{1\times(-3)+2\times(-1)}{1+2}$$

$$x_1=\frac{-2+8}{3}=\frac{6}{3}=2,\quad y_1=\frac{-3-2}{3}=\frac{-5}{3}$$

$$\text{Therefore, } P(x_1, y_1)=\left(2,-\frac{5}{3}\right)$$

Point Q divides AB internally in the ratio 2:1.

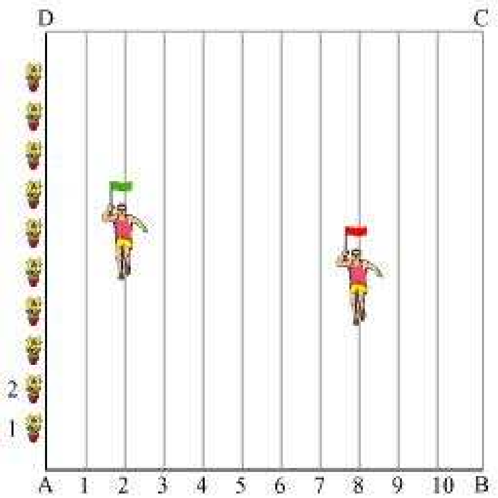
$$x_2 = \frac{2\times(-2)+1\times 4}{2+1},\quad y_2 = \frac{2\times(-3)+1\times(-1)}{2+1}$$

$$x_2 = \frac{-4+4}{3} = 0,\qquad y_2 = \frac{-6-1}{3} = \frac{-7}{3}$$

$$Q(x_2, y_2) = \left(0, -\frac{7}{3}\right)$$

Question 3:

To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in the following figure. Niharika runs $\frac{1}{4}$th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Answer:

It can be observed that Niharika posted the green flag at $\frac{1}{4}$ of the distance AD i.e., $\left(\frac{1}{4}\times 100\right)\text{m} = 25$ m from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25).

Similarly, Preet posted red flag at $\frac{1}{5}$ of the distance AD i.e., $\left(\frac{1}{5}\times 100\right)\text{m} = 20$ m from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20).

Distance between these flags by using distance formula = GR

$= \sqrt{(8-2)^2+(25-20)^2} = \sqrt{36+25} = \sqrt{61}\text{ m}$

The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A (x, y).

$$x = \frac{2+8}{2},\quad y = \frac{25+20}{2}$$

$$x = \frac{10}{2} = 5,\ y = \frac{45}{2} = 22.5$$

$$\text{Hence, A}(x, y) = (5, 22.5)$$

Therefore, Rashmi should post her blue flag at 22.5m on 5th line

Question 4:

Find the ratio in which the line segment joining the points (− 3, 10) and (6, − 8) is divided by (− 1, 6).

Answer:

Let the ratio in which the line segment joining (−3, 10) and (6, −8) is divided by point (−1, 6) be k : 1.

Therefore, $-1 = \frac{6k-3}{k+1}$

$-k-1 = 6k-3$

$7k = 2$

$k = \frac{2}{7}$

Therefore, the required ratio is $2:7$.

Question 5:

Find the ratio in which the line segment joining A (1, − 5) and B (− 4, 5) is divided by the *x*-axis. Also find the coordinates of the point of division.

Answer:

Let the ratio in which the line segment joining A (1, −5) and B (−4, 5) is divided by *x*-axisbe $k:1$.

Therefore, the coordinates of the point of division is $\left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$.

We know that *y*-coordinate of any point on *x*-axis is 0.

$\therefore \frac{5k-5}{k+1} = 0$

$k = 1$

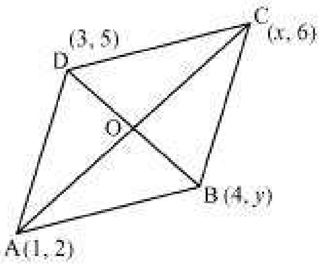
Therefore, *x*-axis divides it in the ratio 1:1.

Division point = $\left(\frac{-4(1)+1}{1+1}, \frac{5(1)-5}{1+1}\right) = \left(\frac{-4+1}{2}, \frac{5-5}{2}\right) = \left(\frac{-3}{2}, 0\right)$

Question 6:

If (1, 2), (4, *y*), (*x*, 6) and (3, 5) are the vertices of a parallelogram taken in order, find *x* and *y*.

Answer:



Let (1, 2), (4, *y*), (*x*, 6), and (3, 5) are the coordinates of A, B, C, D vertices of a parallelogram ABCD. Intersection point O of diagonal AC and BD also divides these diagonals.

Therefore, O is the mid-point of AC and BD.

If O is the mid-point of AC, then the coordinates of O are

$$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) \Rightarrow \left(\frac{x+1}{2}, 4\right)$$

If O is the mid-point of BD, then the coordinates of O are

$$\left(\frac{4+3}{2}, \frac{5+y}{2}\right) \Rightarrow \left(\frac{7}{2}, \frac{5+y}{2}\right)$$

Since both the coordinates are of the same point O,

$$\therefore \frac{x+1}{2} = \frac{7}{2} \text{ and } 4 = \frac{5+y}{2}$$
$$\Rightarrow x+1 = 7 \text{ and } 5+y = 8$$
$$\Rightarrow x = 6 \text{ and } y = 3$$

Question 7:

Find the coordinates of a point A, where AB is the diameter of circle whose centre is (2, − 3) and B is (1, 4)

Answer:

Let the coordinates of point A be (*x*, *y*).

Mid-point of AB is (2, −3), which is the center of the circle.

$$\therefore (2,-3)=\left(\frac{x+1}{2},\frac{y+4}{2}\right)$$

$$\Rightarrow \frac{x+1}{2}=2 \text{ and } \frac{y+4}{2}=-3$$

$$\Rightarrow x+1=4 \text{ and } y+4=-6$$

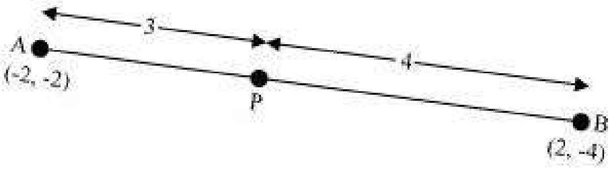
$$\Rightarrow x=3 \text{ and } y=-10$$

Therefore, the coordinates of A are $(3,-10)$.

Question 8:

If A and B are (− 2, − 2) and (2, − 4), respectively, find the coordinates of P such that $AP=\frac{3}{7}AB$ and P lies on the line segment AB.

Answer:



The coordinates of point A and B are (−2, −2) and (2, −4) respectively.

Since $AP=\frac{3}{7}AB$,

Therefore, AP: PB = 3:4

Point P divides the line segment AB in the ratio 3:4.

$$\text{Coordinates of P}=\left(\frac{3\times2+4\times(-2)}{3+4},\frac{3\times(-4)+4\times(-2)}{3+4}\right)$$

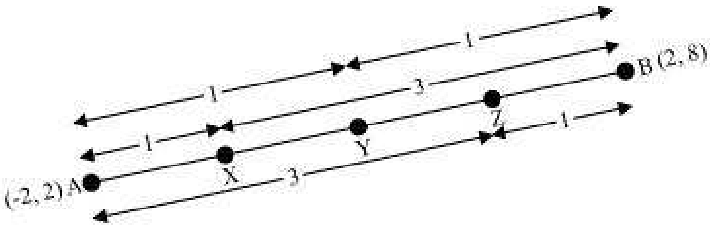
$$=\left(\frac{6-8}{7},\frac{-12-8}{7}\right)$$

$$=\left(-\frac{2}{7},-\frac{20}{7}\right)$$

Question 9:

Find the coordinates of the points which divide the line segment joining A (– 2, 2) and B (2, 8) into four equal parts.

Answer:



From the figure, it can be observed that points P, Q, R are dividing the line segment in a ratio 1:3, 1:1, 3:1 respectively.

$$\text{Coordinates of P} = \left(\frac{1\times2+3\times(-2)}{1+3}, \frac{1\times8+3\times2}{1+3}\right)$$

$$= \left(-1, \frac{7}{2}\right)$$

$$\text{Coordinates of Q} = \left(\frac{2+(-2)}{2}, \frac{2+8}{2}\right)$$

$$= (0, 5)$$

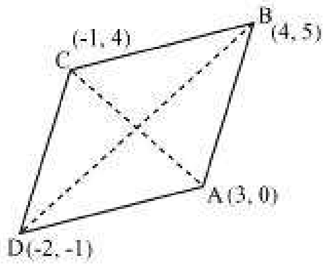
$$\text{Coordinates of R} = \left(\frac{3\times2+1\times(-2)}{3+1}, \frac{3\times8+1\times2}{3+1}\right)$$

$$= \left(1, \frac{13}{2}\right)$$

Question 10:

Find the area of a rhombus if its vertices are (3, 0), (4, 5), (– 1, 4) and (– 2, −1) taken in order. [**Hint:** Area of a rhombus = $\frac{1}{2}$ (product of its diagonals)]

Answer:



Let (3, 0), (4, 5), (−1, 4) and (−2, −1) are the vertices A, B, C, D of a rhombus ABCD.

$$\text{Length of diagonal AC} = \sqrt{\left[3-(-1)\right]^2 + (0-4)^2}$$
$$= \sqrt{16+16} = 4\sqrt{2}$$
$$\text{Length of diagonal BD} = \sqrt{\left[4-(-2)\right]^2 + \left[5-(-1)\right]^2}$$
$$= \sqrt{36+36} = 6\sqrt{2}$$
$$\text{Therefore, area of rhombus ABCD} = \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$$
$$= 24 \text{ square units}$$

Exercise 7.3

Question 1:

Find the area of the triangle whose vertices are:

(i) (2, 3), (− 1, 0), (2, − 4) (ii) (− 5, − 1), (3, − 5), (5, 2)

Answer:

(i) Area of a triangle is given by

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of the given triangle} = \frac{1}{2}\left[2\{0-(-4)\} + (-1)\{(-4)-(3)\} + 2(3-0)\right]$$

$$= \frac{1}{2}\{8+7+6\}$$

$$= \frac{21}{2} \text{ square units}$$

(ii) $$\text{Area of the given triangle} = \frac{1}{2}\left[(-5)\{(-5)-(2)\} + 3(2-(-1)) + 5\{-1-(-5)\}\right]$$

$$= \frac{1}{2}\{35+9+20\}$$

$$= 32 \text{ square units}$$

Question 2:

In each of the following find the value of '*k*', for which the points are collinear.

(i) (7, − 2), (5, 1), (3, − *k*) (ii) (8, 1), (*k*, − 4), (2, − 5)

Answer:

(i) For collinear points, area of triangle formed by them is zero.

Therefore, for points (7, −2) (5, 1), and (3, *k*), area = 0

$$\frac{1}{2}\left[7\{1-k\}+5\{k-(-2)\}+3\{(-2)-1\}\right]=0$$
$$7-7k+5k+10-9=0$$
$$-2k+8=0$$
$$k=4$$

(ii) For collinear points, area of triangle formed by them is zero.

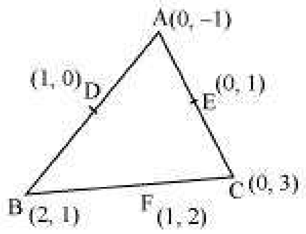
Therefore, for points (8, 1), (k, −4), and (2, −5), area = 0

$$\frac{1}{2}\left[8\{-4-(-5)\}+k\{(-5)-(1)\}+2\{1-(-4)\}\right]=0$$
$$8-6k+10=0$$
$$6k=18$$
$$k=3$$

Question 3:

Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, − 1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Answer:



Let the vertices of the triangle be A (0, −1), B (2, 1), C (0, 3).

Let D, E, F be the mid-points of the sides of this triangle. Coordinates of D, E, and F are given by

$$D = \left(\frac{0+2}{2}, \frac{-1+1}{2}\right) = (1, 0)$$

$$E = \left(\frac{0+0}{2}, \frac{3-1}{2}\right) = (0, 1)$$

$$F = \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$$

$$\text{Area of a triangle} = \frac{1}{2}\left\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right\}$$

$$\text{Area of } \Delta DEF = \frac{1}{2}\left\{1(2-1) + 1(1-0) + 0(0-2)\right\}$$

$$= \frac{1}{2}(1+1) = 1 \text{ square units}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left[0(1-3) + 2\{3-(-1)\} + 0(-1-1)\right]$$

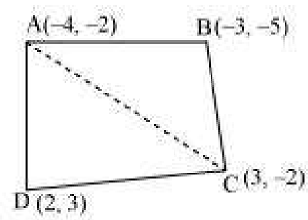
$$= \frac{1}{2}\{8\} = 4 \text{ square units}$$

Therefore, required ratio $= 1:4$

Question 4:

Find the area of the quadrilateral whose vertices, taken in order, are (− 4, − 2), (− 3, − 5), (3, − 2) and (2, 3)

Answer:



Let the vertices of the quadrilateral be A (−4, −2), B (−3, −5), C (3, −2), and D (2, 3). Join AC to form two triangles ΔABC and ΔACD.

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left[(-4)\{(-5)-(-2)\} + (-3)\{(-2)-(-2)\} + 3\{(-2)-(-5)\}\right]$$

$$= \frac{1}{2}(12+0+9) = \frac{21}{2} \text{square units}$$

$$\text{Area of } \Delta ACD = \frac{1}{2}\left[(-4)\{(-2)-(3)\} + 3\{(3)-(-2)\} + 2\{(-2)-(-2)\}\right]$$

$$= \frac{1}{2}\{20+15+0\} = \frac{35}{2} \text{ square units}$$

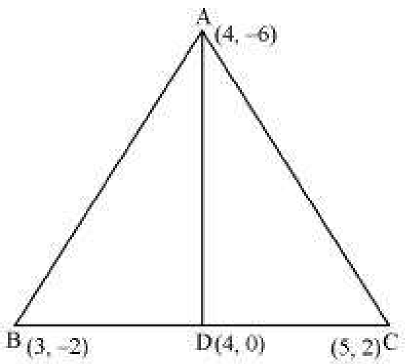
$$\text{Area of } \square ABCD = \text{Area of } \Delta ABC + \text{Area of } \Delta ACD$$

$$= \left(\frac{21}{2} + \frac{35}{2}\right) \text{square units} = 28 \text{ square units}$$

Question 5:

You have studied in Class IX that a median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A (4, − 6), B (3, − 2) and C (5, 2)

Answer:



Let the vertices of the triangle be A (4, −6), B (3, −2), and C (5, 2).

Let D be the mid-point of side BC of ΔABC. Therefore, AD is the median in ΔABC.

$$\text{Coordinates of point D} = \left(\frac{3+5}{2}, \frac{-2+2}{2}\right) = (4,0)$$

$$\text{Area of a triangle} = \frac{1}{2}\left\{x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right\}$$

$$\text{Area of } \Delta\text{ABD} = \frac{1}{2}\left[(4)\{(-2)-(0)\} + (3)\{(0)-(-6)\} + (4)\{(-6)-(-2)\}\right]$$

$$= \frac{1}{2}(-8+18-16) = -3 \text{square units}$$

However, area cannot be negative. Therefore, area of ΔABD is 3 square units.

$$\text{Area of } \Delta\text{ADC} = \frac{1}{2}\left[(4)\{0-(2)\} + (4)\{(2)-(-6)\} + (5)\{(-6)-(0)\}\right]$$

$$= \frac{1}{2}(-8+32-30) = -3 \text{square units}$$

However, area cannot be negative. Therefore, area of ΔADC is 3 square units.

Clearly, median AD has divided ΔABC in two triangles of equal areas.

Exercise 7.4

Question 1:

Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, − 2) and B(3, 7)

Answer:

Let the given line divide the line segment joining the points A(2, −2) and B(3, 7) in a ratio k : 1.

Coordinates of the point of division $= \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$

This point also lies on $2x + y - 4 = 0$

$$\therefore 2\left(\frac{3k+2}{k+1}\right)+\left(\frac{7k-2}{k+1}\right)-4=0$$

$$\Rightarrow \frac{6k+4+7k-2-4k-4}{k+1}=0$$

$$\Rightarrow 9k-2=0$$

$$\Rightarrow k=\frac{2}{9}$$

Therefore, the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, −2) and B(3, 7) is 2:9.

Question 2:

Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear.

Answer:

If the given points are collinear, then the area of triangle formed by these points will be 0.

$$\text{Area of a triangle} = \frac{1}{2}\left\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right\}$$

$$\text{Area} = \frac{1}{2}\left[x(2-0)+1(0-y)+7(y-2)\right]$$

$$0 = \frac{1}{2}\left[2x-y+7y-14\right]$$

$$0 = \frac{1}{2}\left[2x+6y-14\right]$$

$$2x+6y-14=0$$

$$x+3y-7=0$$

This is the required relation between *x* and *y*.

Question 3:

Find the centre of a circle passing through the points (6, − 6), (3, − 7) and (3, 3).

Answer:

Let O (*x*, *y*) be the centre of the circle. And let the points (6, −6), (3, −7), and (3, 3) be representing the points A, B, and C on the circumference of the circle.

$$\therefore \text{OA} = \sqrt{(x-6)^2+(y+6)^2}$$

$$\text{OB} = \sqrt{(x-3)^2+(y+7)^2}$$

$$\text{OC} = \sqrt{(x-3)^2+(y-3)^2}$$

$$\text{However, OA} = \text{OB} \qquad \text{(Radii of the same circle)}$$

$$\Rightarrow \sqrt{(x-6)^2+(y+6)^2} = \sqrt{(x-3)^2+(y+7)^2}$$

$$\Rightarrow x^2+36-12x+y^2+36+12y = x^2+9-6x+y^2+49+14y$$

$$\Rightarrow -6x-2y+14=0$$

$$\Rightarrow 3x+y=7 \qquad \ldots(1)$$

$$\text{Similarly, OA} = \text{OC} \qquad \text{(Radii of the same circle)}$$

$$\Rightarrow \sqrt{(x-6)^2+(y+6)^2} = \sqrt{(x-3)^2+(y-3)^2}$$

$$\Rightarrow x^2+36-12x+y^2+36+12y = x^2+9-6x+y^2+9-6y$$

$$\Rightarrow -6x+18y+54=0$$

$$\Rightarrow -3x+9y=-27 \qquad \ldots(2)$$

On adding equation (1) and (2), we obtain

$10y = -20$

$y = -2$

From equation (1), we obtain

$3x - 2 = 7$

$3x = 9$

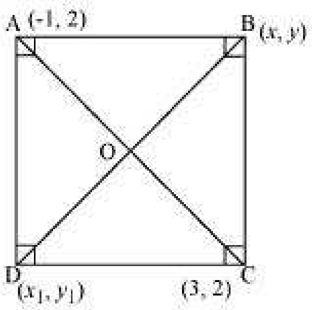
$x = 3$

Therefore, the centre of the circle is (3, −2).

Question 4:

The two opposite vertices of a square are (− 1, 2) and (3, 2). Find the coordinates of the other two vertices.

Answer:



Let ABCD be a square having (−1, 2) and (3, 2) as vertices A and C respectively. Let (x, y), (x_1, y_1) be the coordinate of vertex B and D respectively.

We know that the sides of a square are equal to each other.

∴ AB = BC

$$\Rightarrow \sqrt{(x+1)^2 + (y-2)^2} = \sqrt{(x-3)^2 + (y-2)^2}$$

$$\Rightarrow x^2 + 2x + 1 + y^2 - 4y + 4 = x^2 + 9 - 6x + y^2 + 4 - 4y$$

$$\Rightarrow 8x = 8$$

$$\Rightarrow x = 1$$

We know that in a square, all interior angles are of 90°.

In ΔABC,

$AB^2 + BC^2 = AC^2$

$$\Rightarrow \left(\sqrt{(1+1)^2+(y-2)^2}\right)^2+\left(\sqrt{(1-3)^2+(y-2)^2}\right)^2=\left(\sqrt{(3+1)^2+(2-2)^2}\right)^2$$

$\Rightarrow 4 + y^2 + 4 - 4y + 4 + y^2 - 4y + 4 = 16$

$\Rightarrow 2y^2 + 16 - 8y = 16$

$\Rightarrow 2y^2 - 8y = 0$

$\Rightarrow y\,(y - 4) = 0$

$\Rightarrow y = 0$ or 4

We know that in a square, the diagonals are of equal length and bisect each other at 90°. Let O be the mid-point of AC. Therefore, it will also be the mid-point of BD.

$$\text{Coordinate of point O} = \left(\frac{-1+3}{2}, \frac{2+2}{2}\right)$$

$$\left(\frac{1+x_1}{2}, \frac{y+y_1}{2}\right) = (1, 2)$$

$$\frac{1+x_1}{2} = 1$$

$$1 + x_1 = 2$$

$$x_1 = 1$$

$$\text{and } \frac{y+y_1}{2} = 2$$

$\Rightarrow y + y_1 = 4$

If $y = 0$,

$y_1 = 4$

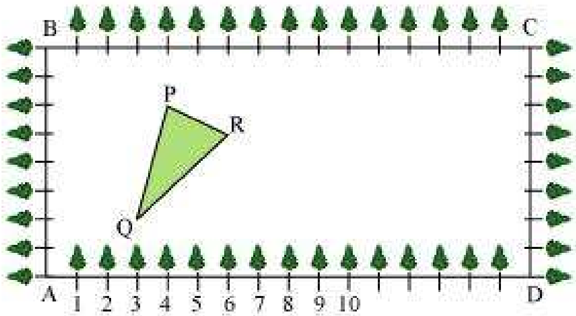
If $y = 4$,

$y_1 = 0$

Therefore, the required coordinates are (1, 0) and (1, 4).

Question 5:

The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the following figure. The students are to sow seeds of flowering plants on the remaining area of the plot.



(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of Δ PQR if C is the origin?

Also calculate the areas of the triangles in these cases. What do you observe?

Answer:

(i) Taking A as origin, we will take AD as *x*-axis and AB as *y*-axis. It can be observed that the coordinates of point P, Q, and R are (4, 6), (3, 2), and (6, 5) respectively.

$$\text{Area of triangle PQR} = \frac{1}{2}\left[x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right]$$

$$= \frac{1}{2}\left[4(2-5)+3(5-6)+6(6-2)\right]$$

$$= \frac{1}{2}\left[-12-3+24\right]$$

$$= \frac{9}{2}\text{ square units}$$

(ii) Taking C as origin, CB as *x*-axis, and CD as *y*-axis, the coordinates of vertices P, Q, and R are (12, 2), (13, 6), and (10, 3) respectively.

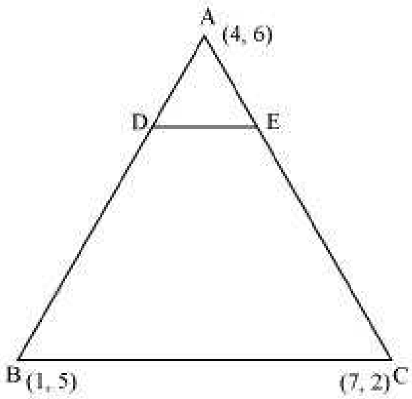
$$\text{Area of triangle PQR} = \frac{1}{2}\left[x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 + y_2\right)\right]$$
$$= \frac{1}{2}\left[12(6-3) + 13(3-2) + 10(2-6)\right]$$
$$= \frac{1}{2}[36 + 13 - 40]$$
$$= \frac{9}{2}\text{ square units}$$

It can be observed that the area of the triangle is same in both the cases.

Question 6:

The vertices of a ΔABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the ΔADE and compare it with the area of ΔABC. (Recall Converse of basic proportionality theorem and Theorem 6.6 related to ratio of areas of two similar triangles)

Answer:



Given that, $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$

$$\frac{AD}{AD+DB}=\frac{AE}{AE+EC}=\frac{1}{4}$$
$$\frac{AD}{DB}=\frac{AE}{EC}=\frac{1}{3}$$

Therefore, D and E are two points on side AB and AC respectively such that they divide side AB and AC in a ratio of 1:3.

$$\text{Coordinates of Point D}=\left(\frac{1\times1+3\times4}{1+3},\frac{1\times5+3\times6}{1+3}\right)$$
$$=\left(\frac{13}{4},\frac{23}{4}\right)$$

$$\text{Coordinates of point E}=\left(\frac{1\times7+3\times4}{1+3},\frac{1\times2+3\times6}{1+3}\right)$$
$$=\left(\frac{19}{4},\frac{20}{4}\right)$$

$$\text{Area of a triangle}=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$$\text{Area of }\Delta ADE=\frac{1}{2}\left[4\left(\frac{23}{4}-\frac{20}{4}\right)+\frac{13}{4}\left(\frac{20}{4}-6\right)+\frac{19}{4}\left(6-\frac{23}{4}\right)\right]$$
$$=\frac{1}{2}\left[3-\frac{13}{4}+\frac{19}{16}\right]=\frac{1}{2}\left[\frac{48-52+19}{16}\right]=\frac{15}{32}\text{ square units}$$

$$\text{Area of }\Delta ABC=\frac{1}{2}\left[4(5-2)+1(2-6)+7(6-5)\right]$$
$$=\frac{1}{2}[12-4+7]=\frac{15}{2}\text{ square units}$$

Clearly, the ratio between the areas of ΔADE and ΔABC is 1:16.

Alternatively,

We know that if a line segment in a triangle divides its two sides in the same ratio, then the line segment is parallel to the third side of the triangle. These two triangles so formed (here ΔADE and ΔABC) will be similar to each other.

Hence, the ratio between the areas of these two triangles will be the square of the ratio between the sides of these two triangles.

Therefore, ratio between the areas of ΔADE and ΔABC = $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$

Question 7:

Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of ΔABC.

(i) The median from A meets BC at D. Find the coordinates of point D.

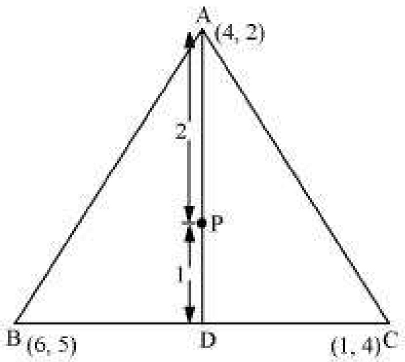
(ii) Find the coordinates of the point P on AD such that AP: PD = 2:1

(iii) Find the coordinates of point Q and R on medians BE and CF respectively such that BQ: QE = 2:1 and CR: RF = 2:1.

(iv) What do you observe?

(v) If A(x_1, y_1), B(x_2, y_2), and C(x_3, y_3) are the vertices of ΔABC, find the coordinates of the centroid of the triangle.

Answer:



(i) Median AD of the triangle will divide the side BC in two equal parts.

Therefore, D is the mid-point of side BC.

$$\text{Coordinates of D} = \left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$$

(ii) Point P divides the side AD in a ratio 2:1.

$$\text{Coordinates of P} = \left(\frac{2\times\frac{7}{2}+1\times4}{2+1}, \frac{2\times\frac{9}{2}+1\times2}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

(iii) Median BE of the triangle will divide the side AC in two equal parts.

Therefore, E is the mid-point of side AC.

$$\text{Coordinates of E} = \left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$$

Point Q divides the side BE in a ratio 2:1.

$$\text{Coordinates of Q} = \left(\frac{2\times\frac{5}{2}+1\times6}{2+1}, \frac{2\times3+1\times5}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

Median CF of the triangle will divide the side AB in two equal parts. Therefore, F is the mid-point of side AB.

$$\text{Coordinates of F} = \left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$$

Point R divides the side CF in a ratio 2:1.

$$\text{Coordinates of R} = \left(\frac{2\times5+1\times1}{2+1}, \frac{2\times\frac{7}{2}+1\times4}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

(iv) It can be observed that the coordinates of point P, Q, R are the same.

Therefore, all these are representing the same point on the plane i.e., the centroid of the triangle.

(v) Consider a triangle, ΔABC, having its vertices as A(x_1, y_1), B(x_2, y_2), and C(x_3, y_3).

Median AD of the triangle will divide the side BC in two equal parts. Therefore, D is the mid-point of side BC.

$$\text{Coordinates of D} = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$$

Let the centroid of this triangle be O.

Point O divides the side AD in a ratio 2:1.

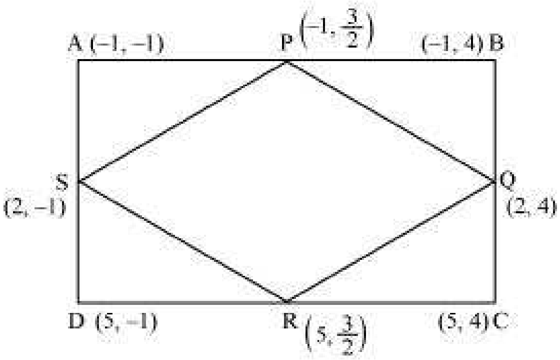
$$\text{Coordinates of O} = \left(\frac{2\times\frac{x_2+x_3}{2}+1\times x_1}{2+1}, \frac{2\times\frac{y_2+y_3}{2}+1\times y_1}{2+1}\right)$$

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Question 8:

ABCD is a rectangle formed by the points A (− 1, − 1), B (− 1, 4), C (5, 4) and D (5, − 1). P, Q, R and S are the mid-points of AB, BC, CD, and DA respectively. Is the quadrilateral PQRS is a square? a rectangle? or a rhombus? Justify your answer.

Answer:



P is the mid-point of side AB.

Therefore, the coordinates of P are $\left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$

Similarly, the coordinates of Q, R, and S are $(2, 4)$, $\left(5, \frac{3}{2}\right)$, and $(2, -1)$ respectively.

$$\text{Length of PQ} = \sqrt{(-1-2)^2 + \left(\frac{3}{2}-4\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of QR} = \sqrt{(2-5)^2 + \left(4-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of RS} = \sqrt{(5-2)^2 + \left(\frac{3}{2}+1\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of SP} = \sqrt{(2+1)^2 + \left(-1-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of PR} = \sqrt{(-1-5)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2} = 6$$

$$\text{Length of QS} = \sqrt{(2-2)^2 + (4+1)^2} = 5$$

It can be observed that all sides of the given quadrilateral are of the same measure. However, the diagonals are of different lengths. Therefore, PQRS is a rhombus.