Assignments in Mathematics Class X (Term II)

10. CIRCLES

IMPORTANT TERMS, DEFINITIONS, AND RESULTS

- A **circle** may be regarded as a collection of points in a plane at a fixed distance from a **fixed point**. The fixed point is called the centre of the circle. The fixed distance between the centre of the circle and the circumference, is called **radius.**
- The perimeter of the circle is referred to as the **circumference** of the circle.
- A **chord** of a circle is a line segment joining any two points on the circumference.
- An **arc** of a circle is a part of the circumference.
- A **diameter** of a circle is a chord which passes through the centre of the circle.
- A line, which intersects the circle in two distinct points, is called a **secant.**
- A line which has only one point common to the circle is called a **tangent** to the **circle.**
- There is one and only one tangent at a point of the circle.
- The tangent at any point of a circle is perpendicular to the radius through the point of contact.
- No tangent can be drawn from a point inside the circle.
- The lengths of tangents drawn from an external point to a circle are equal.
- The perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.
- Tangents drawn at the end points of a diameter of a circle are parallel.

SUMMATIVE ASSESSMENT

MULTIPLE CHOICE QUESTIONS [1 Mark]

A. Important Questions

1. If tangent PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°, then ∠POA is equal to :

(a) 50° (b) 60° (c) 70° (d) 80°

2. From a point T, the length of the tangent to a circle is 24 cm and the distance of T from the centre is 25 cm. The radius of the circle is :

(a) 7 cm (b) 12 cm

(c) 15 cm (d) 24.5 cm

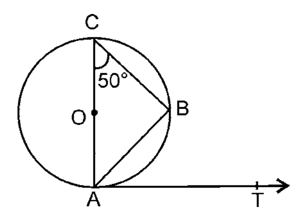
3. At one end of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord, parallel to XY and at a distance of 8 cm from A is :

(a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm

4. If angle between two radii of a circle is 130°, the angle between the tangents at the ends of the redii is :

(a) 90° (b) 50° (c) 70° (d) 40°

5. In the figure, AB is a chord of the circle and AOC is its diameter such that ∠ACB = 50°. If AT is the tangent to the circle at the point A, then ∠BAT is equal to :



(a) 65° (b) 60° (c) 50° (d) 40°

6. A tangent AB at a point A of a circle of radius 5 cm meets a line through the centre O at a point B so that OB = 12 cm. Length PB is :

(a) 10 cm (b) 12 cm

(c) 9 cm (d) $\sqrt{119}$ cm

7. The length of the tangent drawn from a point, whose distance from the centre of a circle is 20 cm and radius of the circle is 16 cm, is :

(a) 12 cm (b) 144 cm

(c) 169 cm (d) 25 cm

8. A tangent PQ at a point P of a circle of radius 15 cm meets a line through the centre O at a point Q so that OQ = 25 cm. Length of PQ is :

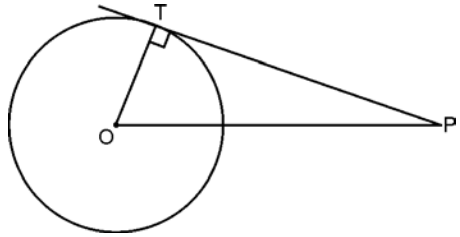
(a) 5 cm (b) 25 cm

(c) 16 cm (d) 20 cm

9. In a circle of radius 7 cm, tangent LM is drawn from a point L such that LM = 24 cm. If O is the centre of the circle, then length of OL is :

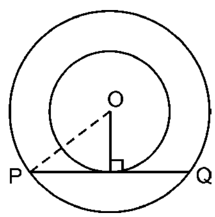
(a) 20 cm (b) 24 cm (c) 25 cm (d) 26 cm

10. In the figure, PT is a tangent to a circle with centre O. If OT = 6 cm, and OP = 10 cm, then the length of tangent PT is :



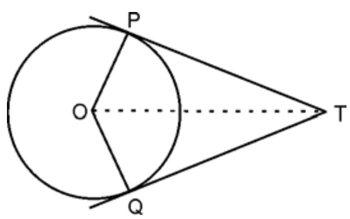
(a) 8 cm (b) 12 cm (c) 10 cm (d) 16 cm

11. In the given figure, O is the centre of two concentric circles of radii 3 cm and 5 cm. PQ is a chord of outer circle which touches the inner circle. The length of chord PQ is :



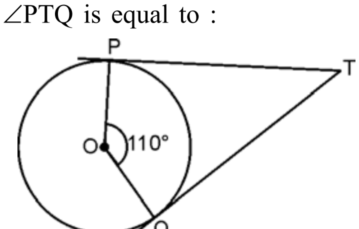
(a) 5 cm (b) 8 cm (c) 10 cm (d) $\sqrt{34}$ cm

12. In the figure, TP and TQ are two tangents to a circle with centre O, so that ∠POQ = 140°. ∠PTO is equal to :



(a) 40° (b) 50° (c) 60° (d) 70°

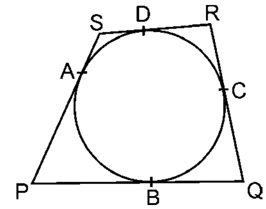
13. In the figure, if TP and TQ are the two tangents to a circle with centre O, so that ∠POQ = 110°, then ∠PTQ is equal to :



(a) 60° (b) 70° (c) 80° (d) 90°

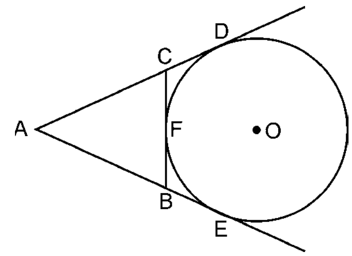
14. In the figure, quadrilateral PQRS is circumscribed, touching the circle at A, B, C and D. If

AP = 5 cm, QR = 7 cm and DR = 3 cm, then length PQ is equal to :



(a) 9 cm (b) 8 cm (c) 13 cm (d) 14 cm

15. In the figure, if AD, AE and BC are tangents to the circle at D, E and F respectively, then:



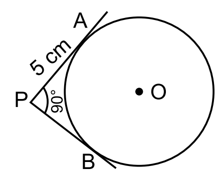
(a) AD = AB + BC + CA

(b) 2AD = AB + BC + CA

(c) $\frac{AD}{2}$ = AB + BC + CA

(d) $\frac{AD}{4}$ = AB + BC + CA

16. In the figure, the pair of tangents PA and PB drawn from an external point P to a circle with centre O are perpendicular to each other and length of each tangent is 5 cm. The radius of the circle is:



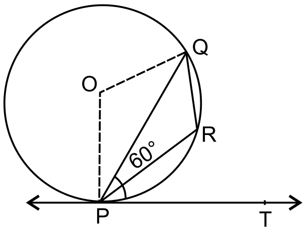
(a) 10 cm (b) 7.5 cm (c) 5 cm (d) 2.5 cm

17. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. The area of the quadrilateral PQOR is:

(a) 60 cm^2 (b) 65 cm^2

(c) 30 cm^2 (d) 32.5 cm^2

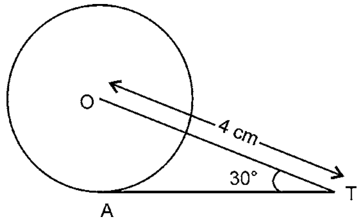
18. In the figure, PQ is a chord of a circle and PT is the tangent at P such that ∠QPT = 60°. Then ∠PRQ is equal to:



(a) 135° (b) 150° (c) 120° (d) 110°

19. In the figure, AT is a tangent to the circle with centre O such that OT = 4 cm and ∠OTA = 30°.

Then AT is equal to:



(a) 4 cm (b) 2 cm

(c) $2\sqrt{3}$ cm (d) $4\sqrt{3}$ cm

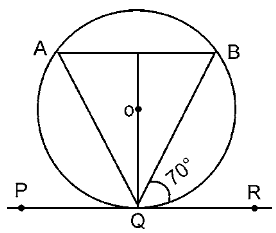
20. Two circles touch each other externally at C and AB is a common tangent to the circles. ∠ACB is:

(a) 60° (c) 45° (c) 90° (d) 180°

21. PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that ∠POR = 120°. ∠OPQ is:

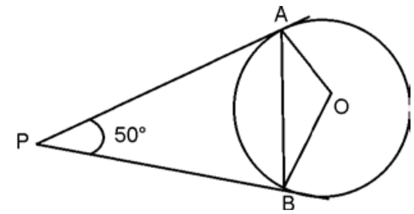
(a) 50° (b) 40° (c) 30° (d) 25°

22. In the figure, PQR is the tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and ∠BQR = 70°; ∠AQB is equal to:



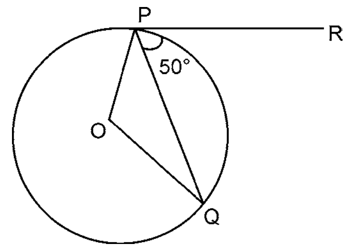
(a) 20° (b) 40° (c) 35° (d) 45°

23. In the figure, PA and PB are tangents to the circle with centre O such that ∠APB = 50°; ∠OAB is equal to:



(a) 25° (b) 30° (c) 40° (d) 50°

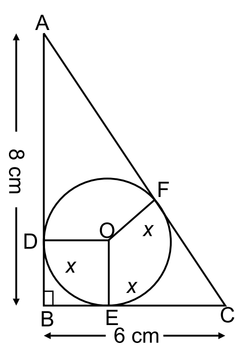
24. In the figure, O is the centre of the circle, PQ is a chord, and tangent PR at P makes and angle of 50° with PQ, then ∠POQ is equal to:



(a) 100° (b) 80° (c) 90° (d) 75°

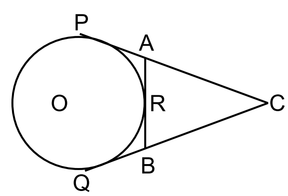
B. Questions From CBSE Examination Papers

1. ABC is a triangle right angled at B with BC = 6 cm and AB = 8 cm. A circle with centre O and radius x cm has been inscribed in ΔABC as shown in the figure. The value of x is : **[2011 (T-II)]**



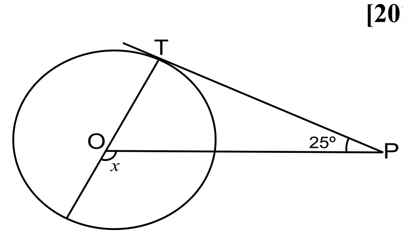
(a) 2 cm (b) 3 cm (c) 4 cm (d) 5 cm

2. CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm, BC = 7 cm, then the length BR is : **[2011 (T-II)]**



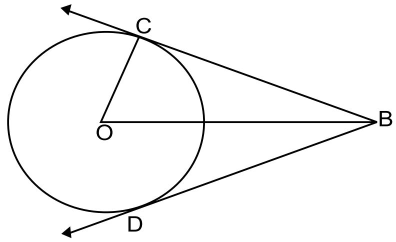
(a) 11 cm (b) 7 cm (c) 3 cm (d) 4 cm

3. In the figure, if PT is a tangent of the circle with centre O and ∠TPO = 25º, then the measure of x is : **[2011 (T-II)]**



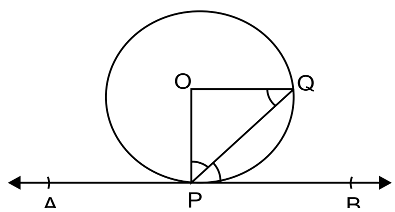
(a) 120º (b) 125º (c) 110º (d) 115º

4. In the figure, if OC = 9 cm and OB = 15 cm, then BC + BD is equal to : **[2011 (T-II)]**



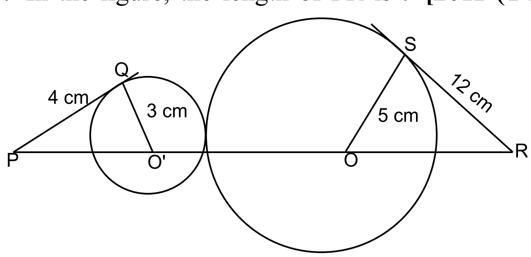
(a) 18 cm (b) 12 cm (c) 24 cm (d) 36 cm

5. In the figure, APB is a tangent to a circle with centre, O, at point P. If ∠QPB = 50º, then the measure of ∠POQ is : **[2011 (T-II)]**



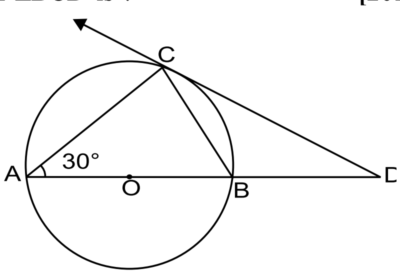
(a) 120º (b) 100º (c) 140º (d) 150º

6. In the figure, the length of PR is : **[2011 (T-II)]**



(a) 20 cm (b) 26 cm (c) 24 cm (d) 28 cm

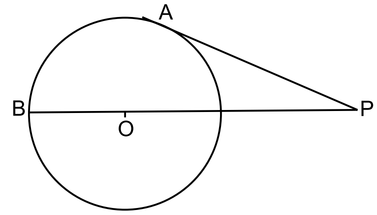
7. In the figure, AB is a diameter and AC is chord of a circle such that ∠BAC = 30º. If DC is tangent, then ΔBCD is : **[2011 (T-II)]**



(a) isosceles (b) equilateral

(c) right angled (d) acute angled

8. In the figure, PA is a tangent to a circle of radius 6 cm and PA = 8 cm, then length of PB is : **[2011 (T-II)]**

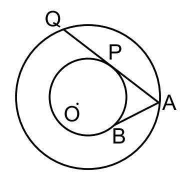


(a) 10 cm (b) 18 cm (c) 16 cm (d) 12 cm

9. PQ and PT are tangents drawn from a point P to a circle with centre O such that ∠QPT = 120º, then ∠QOT is equal to **[2011 (T-II)]**

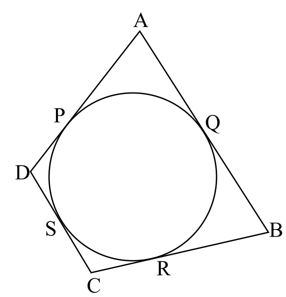
(a) 60° (b) 30° (c) 90° (d) 120°

10. The figure, shows two concentric circles with centre O. AB and APQ are tangents to the inner circle from point A lying on the outer circle. If AB = 7.5 cm, then AQ is equal to : **[2011 (T-II)]**



(a) 18 cm (b) 15 cm (c) 12 cm (d) 10 cm

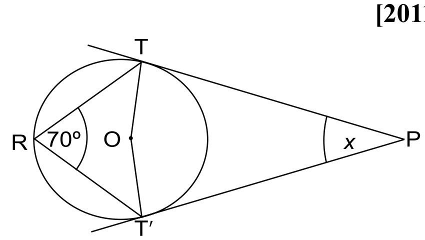
11. Quadrilateral ABCD circumscribes a circle as shown below. The side of the quadrilateral which is equal to AP + BR is : **[2011 (T-II)]**



(a) AD (b) AC (c) AB (d) BC

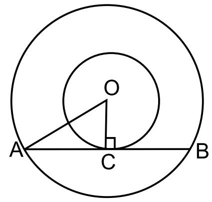
12. In the figure, PT and PT′ are tangents to the circle with centre O. If ∠TRT′ = 70º, then x equals :

[2011 (T-II)]



(a) 30º (b) 35º (c) 40º (d) 50º

13. In the figure, two concentric circles of radii a and b ($a > b$) are given. The chord AB of larger circle touchers the smaller circle at C. The length of AB is : **[2011 (T-II)]**



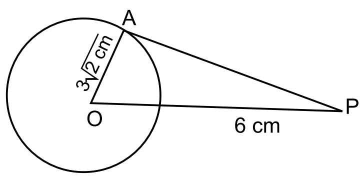
(a) $\sqrt{a^2-b^2}$ (b) $\sqrt{a^2+b^2}$

(c) $2\sqrt{a^2+b^2}$ (d) $2\sqrt{a^2-b^2}$

14. The length of tangent drawn from an external point P to a circle with centre O, is 8 cm. If the radius of the circle is 6 cm, then the length of OP (in cm) is : **[2011 (T-II)]**

(a) $2\sqrt{7}$ (b) $4\sqrt{7}$ (c) 10 (d) 10.5

15. A tangent PA is drawn from an external point P to a circle of radius $3\sqrt{2}$ cm such that the distance of the point point P from O is 6 cm as shown in figure. The value of $\angle$APO is : **[2011 (T-II)]**

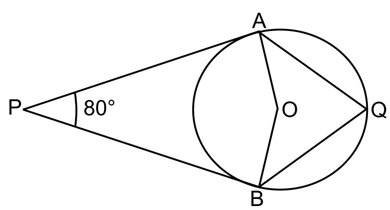


(a) 30º (b) 45º (c) 60º (d) 75º

16. The maximum number of common tangents that can be drawn to two circles intersecting at two distinct points is : **[2011 (T-II)]**

(a) 1 (b) 2 (c) 3 (d) 4

17. In the given figure, O is the centre of the circle. If PA and PB are tangents from an external point P to the circle, then $\angle$AQB is equal to : **[2011 (T-II)]**



(a) 100º (b) 80º (c) 70º (d) 50º

18. Number of tangents to a circle which are parallel to a secant is : **[2011 (T-II)]**

(a) 1 (b) 2 (c) 3 (d) infinite

19. A line which is perpendicular to the radius of the circle through the point of contact is called a : **[2011 (T-II)]**

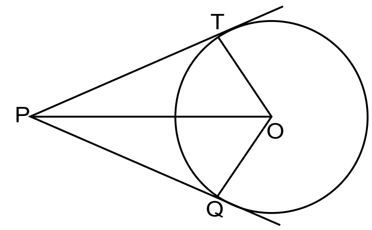
(a) tangent (b) chord

(c) normal (d) segment

20. If the angle between two radii of a circle is 140º, then the angle between the tangents at the ends of the radii is : **[2011 (T-II)]**

(a) 90º (b) 40º (c) 70º (d) 60º

21. PQ and PT are tangents to a circle with centre O and radius 5 cm. If PQ = 12, then perimeter of quadrilateral PQOT is : **[2011 (T-II)]**



(a) 24 cm (b) 34 cm (c) 17 cm (d) 20 cm

22. How many parallel tangens can a circle have? **[2011 (T-II)]**

(a) 1 (b) 2

(c) infinite (d) none of these

23. The length of the tangents from a point A at a circle of radius 3 cm is 4 cm. The distance (in cm) of A from the centre of the circle is : **[2011 (T-II)]**

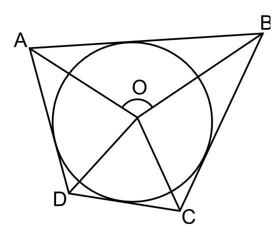
(a) $\sqrt{7}$ (b) 7 (c) 5 (d) 25

24. If two tangetns inclined at an angle of 60º are drawn to a circle of radius 3 cm, then length of tangent is equal to : **[2011 (T-II)]**

(a) $\sqrt{3}$ cm (b) $2\sqrt{3}$ cm

(c) $\frac{2}{\sqrt{3}}$ cm (d) $3\sqrt{3}$ cm

25. In the figure, if $\angle$AOB = 125º, then $\angle$COD is equal: **[2011 (T-II)]**



(a) 62.5º (b) 45º (c) 125º (d) 55º

26. In a right triangle ABC, right angled at B, BC = 15 cm, and AB = 8 cm. A circle is inscribed in triangle ABC. The radius of the circle is : **[2011 (T-II)]**

(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm

27. The area of a square inscribed in a circle of radius 8 cm is : **[2011 (T-II)]**

(a) 64 cm^2 (b) 100 cm^2

(c) 125 cm^2 (d) 128 cm^2

28. A line that intersects a circle in two distinct points is called : **[2011 (T-II)]**

(a) a diameter (b) a secant

(c) a tangent (d) a radius

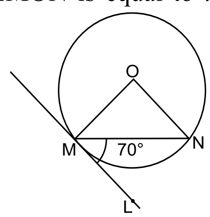
29. Two circles touch each other externally at C and AB is a common tangent to the circles, then ∠ACB is : **[2011 (T-II)]**

(a) 60º (b) 45º (c) 30º (d) 90º

30. The angle between two tangents drawn from an external point to a circle is 110º. The angle subtended at the centre by the segments joining the points of contact to the centre of circle is : **[2011 (T-II)]**

(a) 70° (b) 110° (c) 90° (d) 55°

31. In the figure, O is centre of a circle. MN is a chord and the tangent ML at M makes an angle of 70º with MN. ∠MON is equal to : **[2011 (T-II)]**



(a) 120° (b) 90° (c) 140° (d) 70°

32. The distance between two parallel tangents of a circle of radius 5 cm is : **[2011 (T-II)]**

(a) 5 cm (b) 10 cm (c) 15 cm (d) 2.5 cm

33. A quadrilateral ABCD is drawn to circumscribe a circle. If AB = 12 cm, BC = 15 cm and CD = 14 cm, then AD is : **[2011 (T-II)]**

(a) 10 cm (b) 11 cm (c) 12 cm (d) 14 cm

34. If tangents PA and PB from an external point P to a circle with centre O are inclined to each other at an angle of 80°, then ∠POA is equal to : **[2011 (T-II)]**

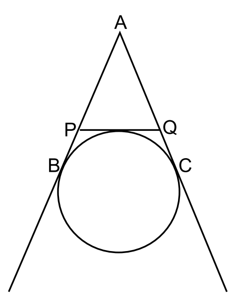
(a) 50° (b) 60° (c) 70° (d) 80°

35. If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then the length of each tangent is equal to : **[2011 (T-II)]**

(a) $\frac{3\sqrt{3}}{2}$ cm (b) $2\sqrt{3}$ cm

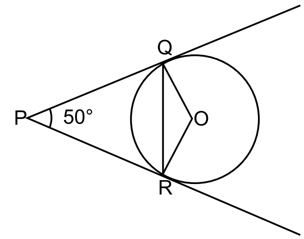
(c) $3\sqrt{3}$ cm (d) 6 cm

36. In the figure, AB, AC, PQ are tangents. If AB = 5 cm, then perimeter of ΔAPQ is : **[2011 (T-II)]**



(a) 10 cm (b) 15 cm (c) 12.5 cm (d) 20 cm

37. In the figure, PQ and PR are the tangents to the circle with centre O such that ∠QPR = 50°. Then ∠OQR is equal to : **[2011 (T-II)]**



(a) 25° (b) 30º (c) 40º (d) 50º

38. In two concentric circles, if chords are drawn in the outer circle which touch the inner circle, then : **[2011 (T-II)]**

(a) all chords are of different lengths

(b) all chords are of same length

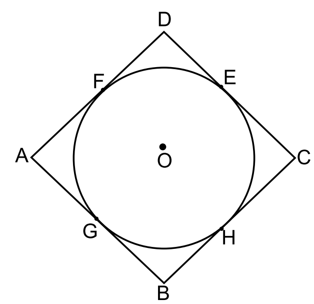
(c) only parallel chords are of same length

(d) only perpendicular chords are of same length

39. A tagnent PQ at the point P of a circle meets a line through the centre O at a point Q, so that OQ =12 cm and PQ = $\sqrt{119}$ cm, the diameter of circle is : **[2011 (T-II)]**

(a) 13 cm (b) 26 cm (c) 10 cm (d) 5 cm

40. In the figure, a quadrilateral ABCD is drawn to circumscribe a circle. Then **[2011 (T-II)]**



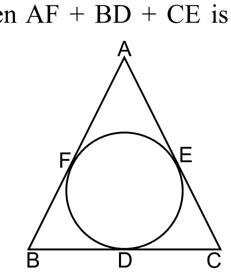
(a) AD + BC = AB + CD

(b) AB + BC = AD + CD

(c) BC + CD = AD + AB

(d) AB + BC + CD + AD = AC + BD

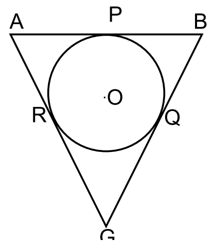
41. In the figure, if the semiperimeter of ΔABC is 23 cm, then AF + BD + CE is : **[2011 (T-II)]**



(a) 46 cm (b) 11.5 cm

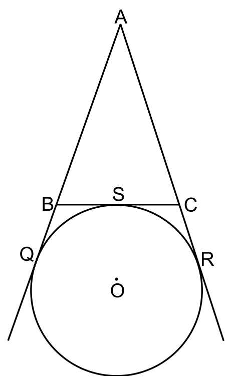
(c) 23 cm (d) 34.5 cm

42. In the figure, AP = 2 cm, BQ = 3 cm and RC = 4 cm, then the perimeter of ΔABC (in cm) is : **[2011 (T-II)]**



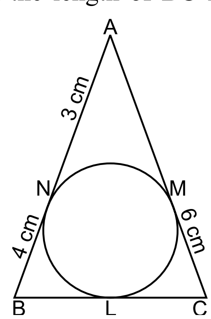
(a) 16 (b) 18 (c) 20 (d) 21

43. In the figure, AQ, AR and BC are tangents to circle with centre O. If AB = 7 cm, BC = 5 cm, AC = 5 cm, then length of the tangent AQ is : **[2011 (T-II)]**



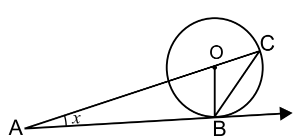
(a) 5 cm (b) 7 cm (c) 8.5 cm (d) 17 cm

44. In the figure, triangle ABC is circumscribing a circle. Then the length of BC is : **[2011 (T-II)]**



(a) 7 cm (b) 8 cm (c) 9 cm (d) 10 cm

45. In the figure, angle OBC = 30°, then value of x is : **[2011 (T-II)]**



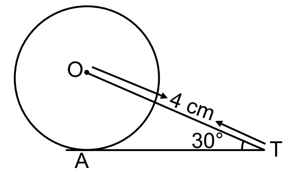
(a) 100° (b) 110° (c) 30° (d) 15°

46. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is : **[2011 (T-II)]**

(a) 60 cm^2 (b) 65 cm^2

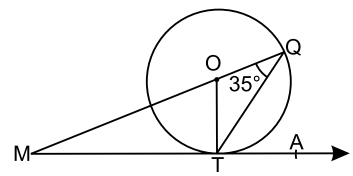
(c) 30 cm^2 (d) 32.5 cm^2

47. In the figure, AT is the tangent to the circle with centre O such that OT = 4 cm and $\angle$OTA = 30°. Then AT is equal to : **[2011 (T-II)]**



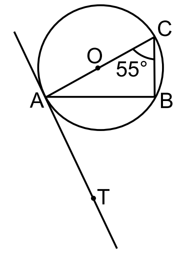
(a) 4 cm (b) 2 cm (c) $2\sqrt{3}$ cm (d) 8 cm

48. In the figure, O is the centre of the circle with $\angle$TQM = 35°, then angle ATQ would be equal to : **[2011 (T-II)]**



(a) 55° (b) 56° (c) 35° (d) 54°

49. In the figure, AB is a chord of circle and AOC is diameter such that $\angle$ACB = 55°. If AT is a tangent to the circle at point A, then angle BAT is : **[2011 (T-II)]**



(a) 65° (b) 40° (c) 50° (d) 55°

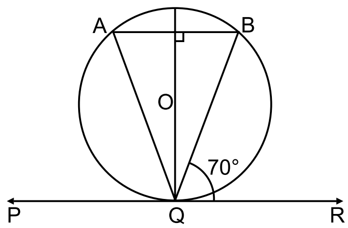
50. A circle touches all the four sides of quadrilateral ABCD whose sides are AB = 6 cm, BC = 7 cm, CD = 4 cm. The length of side AD is : **[2011 (T-II)]**

(a) 5 cm (b) 7 cm (c) 6.5 cm (d) 3 cm

51. PQ is a tangent to a circle with centre O at point P. If ΔOPQ is an isosceles triangle, then $\angle$OQP is equal to : **[2011 (T-II)]**

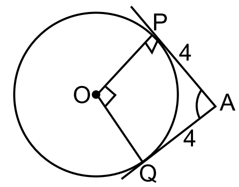
(a) 15° (b) 30° (c) 45° (d) 60°

52. In the figure, PQR is the tangent to a circle at Q whose centre is O. AB is a chord parallel to PR and $\angle$BQR = 70°, then angle $\angle$AQB is equal to : **[2011 (T-II)]**



(a) 20° (b) 40° (c) 35° (d) 45°

53. In the figure, the pair of tangents AP and AQ, drawn from an external point A to a circle with centre O, are perpendicular to each other and length of each tangent is 4 cm, then the radius of the circle is : **[2011 (T-II)]**

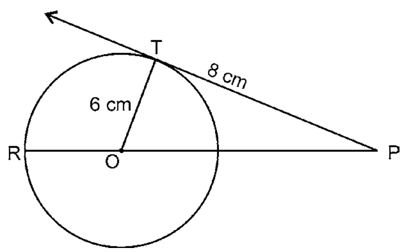


(a) 10 cm (b) 7.5 cm (c) 2.5 cm (d) 4 cm

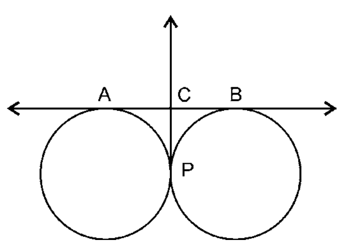
SHORT ANSWER TYPE QUESTIONS [2 Marks]

A. Important Questions

1. In the figure, find the length of PR.

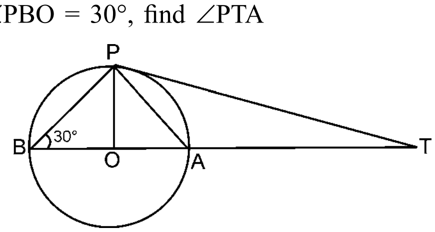


2. In the figure, if BC = 4.5 cm, find the length of AB.



3. A pair of tangents PA and PB are drawn from an external point P to a circle with centre O. If ∠APB = 90° and PA = 6 cm, find the radius of the circle.

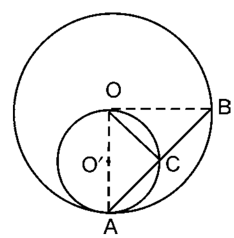
4. In the figure, BOA is a diameter of a circle and the tangent at point P meets BA produced at T. If ∠PBO = 30°, find ∠PTA



5. The length of tangent from an external point on a circle is always greater than the radius of the circle. Is it true?

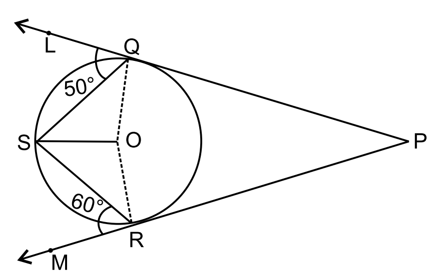
6. If the angle between two tangents drawn from a point P to a circle of radius *a* and centre O is 90°, then find OP.

7. In the figure, circles C(O, *r*) and C′ (O′, *r*/2) touch internally at a point A and AB is a chord of the circle C (O, *r*), intersecting C′ (O′, *r*) at C. Prove that AC = CB. **[HOTS]**

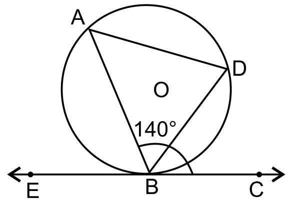


8. P is the mid-point of an arc QPR of a circle. Show that the tangent at P is parallel to the chord QR.

9. In the figure, PQL and PRM are tangents to the circle with centre O at the points Q and R respectively and S is a point on the circle such that ∠SQL = 50° and ∠SRM = 60°. Find ∠QSR. **[HOTS]**



10. In the figure, if ∠CBA = 140°, find ∠ADB. **[HOTS]**

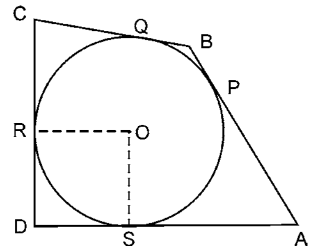


11. Show that the tangent to the circumcircle of an isosceles triangle ABC at A, in which AB = AC, is parrallel to BC.

12. PQ and PR are tangent segments to a circle with centre O. If ∠QPR = 80°, find ∠QOR.

13. AB is a diameter of a circle and AC is its chord such that ∠BAC = 30°. If the tangent at C intersects AB at D, then prove that BC = BD.

14. In the figure, ∠ADC = 90°, BC = 38 cm, CD = 28 cm and BP = 25 cm. Find the radius of the circle.

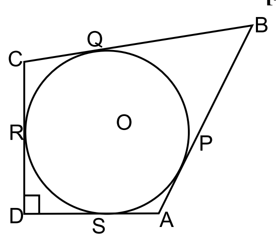


15. If a number of circles pass through the end points P and Q of a line segment PQ, then prove that their centres lie on the perpendicular bisector of PQ.

16. Two tangent segments BC and BD are drawn to a circle with centre O such that ∠CBD = 120º. Show that OB = 2BC.

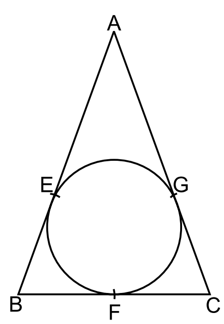
B. Questions From CBSE Examination Papers

1. In the figure, ∠ADC = 90°, BC = 38 cm, CD = 28 cm and BP = 25 cm. Find the radius of the circle. **[2011 (T-II)]**



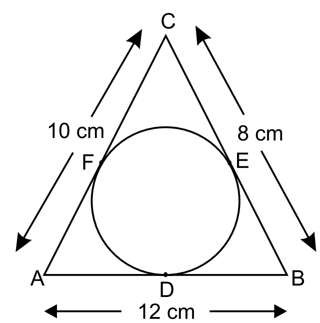
2. Out of the two concentric circles, the radius of the outer is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle. **[2011 (T-II)]**

3. In the isosceles triangle ABC shown below, AB = AC, show that BF = FC **[2011 (T-II)]**



4. In the figure, a circle is inscribed in a ΔABC with sides AB = 12 cm, BC = 8 cm and AC = 10 cm. Find the lengths of AD, BE and CF.

[2011 (T-II)]



5. Prove that opposite sides of quadrilateral circumscribing a circle subtend supplementary angles at the centre of circle. **[2011 (T-II)]**

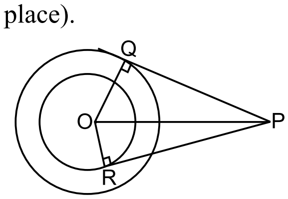
6. If all sides of a parallelogram touch a circle, then prove that it is a rhombus. **[2011 (T-II)]**

7. Two tangents PA and PB are drawn from an external point P to a circle with centre. O. Prove that AOBP is a cyclic quadrialteral.

[2011 (T-II)]

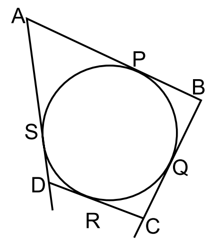
8. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord. **[2011 (T-II)]**

9. In the figure, is the centre of two concentric circles of radii 6 cm and 4 cm. PQ and PR are tangents to the two circles from an external point P. If PQ = 10 cm, find the length of PR (upto one decimal place). **[2011 (T-II)]**



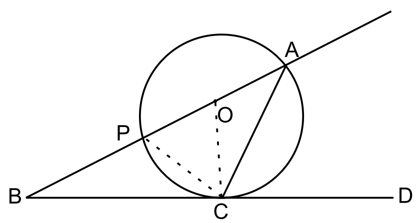
10. Prove that the tangents drawn at the end-points of a diameter of a circle are parallel. **[2011 (T-II)]**

11. In the figure, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC. **[2011 (T-II)]**

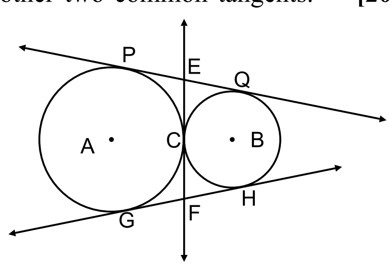


12. Prove that the line segment joining the points of contact of two parallel tangents to a circle is a diameter of the circle. **[2011 (T-II)]**

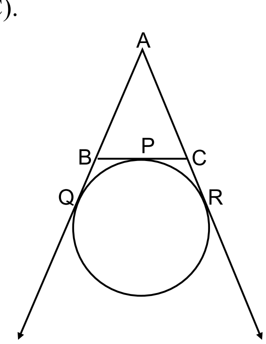
13. In the figure, O is the centre of a circle and BCD is tangent to it at C. Prove that $\angle BAC + \angle ACD = 90°$. **[2011 (T-II)]**



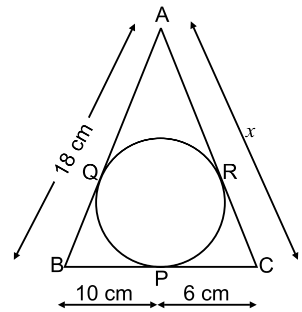
14. In the figure, two circles touch each other externally at C. Prove that the common tangent at C bisects the other two common tangents. **[2011 (T-II)]**



15. In the figure, a circle touches the side BC of triangle ABC at P and touches AB and AC produced at Q and R respectively. Show that AQ = $\frac{1}{2}$ (Perimeter of ΔABC). **[2011 (T-II)]**

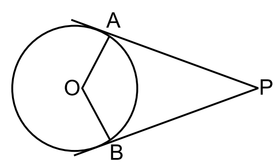


16. In the figure, all three sides of a triangle touch the circle. Find the value of x. **[2011 (T-II)]**

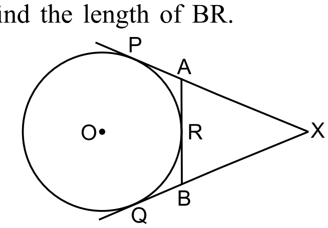


17. Two tangents PA and PB are drawn to a circle with centre O, such that $\angle APB = 120°$. Prove that OP = 2 AP. **[2011 (T-II)]**

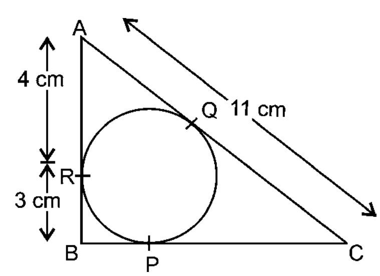
18. In the figure, O is the centre of the circle. PA and PB are tangents to the circle from the point P. Prove that AOBP is a cyclic quadrilateral. **[2011 (T-II)]**



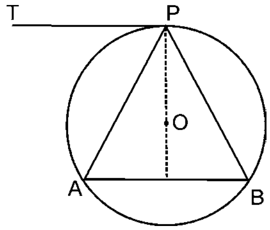
19. In the figure, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm, and BC = 7 cm, then find the length of BR. **[2009]**



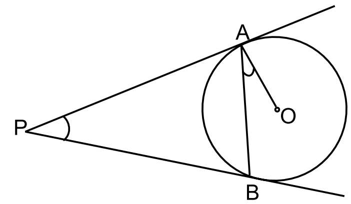
20. In the figure, ΔABC is circumscribing a circle. Find the length of BC. **[2009]**



21. A tangent PT is drawn parallel to a chord AB as shown in figure. Prove that APB is an isosceles triangle. **[2000]**



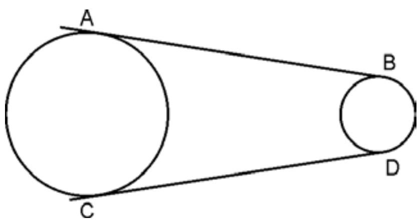
22. Two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that $\angle APB = 2\angle OAB$. **[2003]**



SHORT ANSWER TYPE QUESTIONS **[3 Marks]**

A. Important Questions

1. In the figure, AB and CD are common tangents to two circles of unequal radii. Prove that AB = CD. **[HOTS]**



2. If d_1, d_2 ($d_2 > d_1$) be the diameters of two concentric circles and c be the length of a chord of a circle, which is tangent to the other circle, prove that $d_2^2 = c^2 + d_1^2$ **[HOTS]**

3. ABC is a right triangle, right angled at A. A circle is inscribed in it. The lengths of the sides containing the right angle are 12 cm and 5 cm. Find the radius of the incircle. **[HOTS]**

4. a, b, c are the sides of a right triangle, where c is the hypotenune. Prove that the radius r of the circle which touches the sides of the triangle is given by $r = \frac{a+b-c}{2}$ **[HOTS]**

5. AB is a diameter of a circle APB. AH and BK are perpendicular from A and B respectively to the tangent at P. Prove that AH + BR = AB. **[HOTS]**

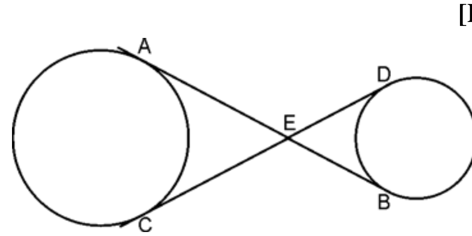
6. Show that the tangents drawn at the ends of chord of a circle make equal angles with the chord.

7. Let A be the one point of intersection of two intersecting circles with centre O and Q. The tangents at A to the two circles meet the circles again at B and C respectively. Let the point P be located such that AOP is a parallelogram. Prove that P is the circumcentre of ΔABC.

8. A circle with centre O and radius 5 cm has been inscribed in an equilateral triangle ABC. Find the perimeter of ΔABC.

9. A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.

10. In the figure, common tangents AB and CD to two circles intersect at E. Prove that AB = CD. **[HOTS]**



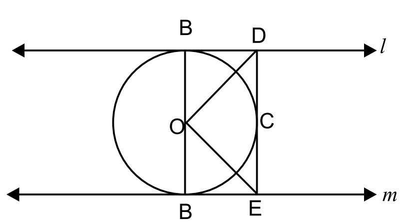
11. Prove that the centre of a circle touching two intersecting lines lies on the angles biscetor of the lines.

12. Prove that there is one and only one tangent at any point on the circumference of a circle.

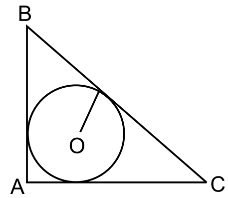
B. Questions From CBSE Examination Papers

1. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. **[2011 (T-II)]**

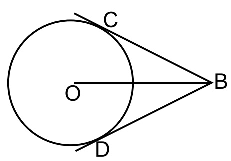
2. In the figure, l and m are two parallel tangents at A and B. The tangent at C makes an intercept DE btween l and m. Prove that DE subtends a right angle at the centre of the circle. **[2011 (T-II)]**



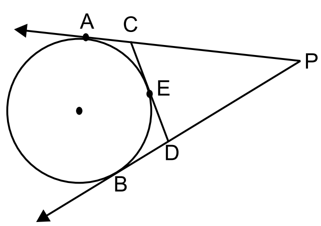
3. In the figure, triangle ABC is a right angled triangle with AB = 6 cm, AC = 8 cm $\angle A = 90°$. A circle with centre O is inscribed inside the triangle. Find the radius *r*. **[2011 (T-II)]**



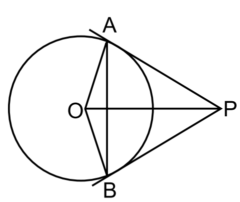
4. If from an external point B of the circle with centre O, two tangents BC and BD are drawn such that $\angle DBC = 120°$, prove that BO = 2BC. **[2011 (T-II)]**



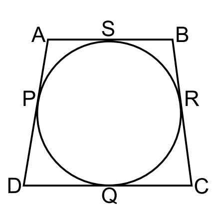
5. In the figure 4, from an external point P, PA and PB are tangents to the circle with centre O. If CD is another tangent at point E to the circle and PA = 12 cm, find the perimeter of Δ PCD. **[2011 (T-II)]**



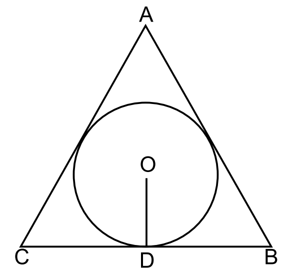
6. In the figure, OP is equal to the diameter of the circle. Prove that ABP is an equilateral triangle. **[2011 (T-II)]**



7. In the figure, a circle touches all the four sides of a quadrilateral ABCD with sides AB = 6 cm, BC = 7 cm and CD = 4 cm. Find AD. **[2011 (T-II)]**

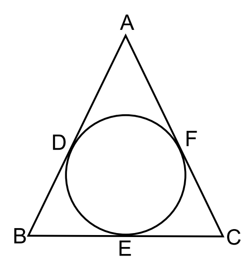


8. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact are of lengths 8 cm and 6 cm respectively. If area of ΔABC is 84 cm^2, then find the sides AB and AC. **[2011 (T-II)]**



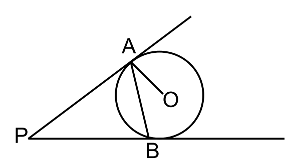
9. A circle touches the side BC of ΔABC at P and sides AB and AC produced at Q and R respectively. Prove that AQ = $\frac{1}{2}$ (Perimeter opf ΔABC). **[2011 (T-II)]**

10. In the figure, triangle ABC is isosceles in which AB = AC, circumscribed about a circle. Prove that base is bisected by the point of contact. **[2011 (T-II)]**

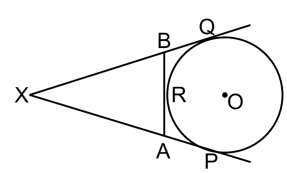


11. Prove that the angle between the two tangents to a circle drawn from an external point, is supplementary to the angle subtended by the line segment joining the points of contact at the centre. **[2011 (T-II)]**

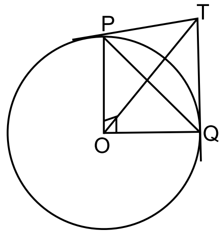
12. Two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that $\angle APB = 2\angle OAB$. **[2011 (T-II)]**



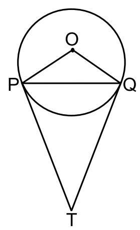
13. In the figure, XP and XQ are tangents from an external point X to the circle with centre O. R is a point on the circle where another tangent ARB is drawn to the circle. Prove that XA + AR = XB + BR. **[2011 (T-II)]**



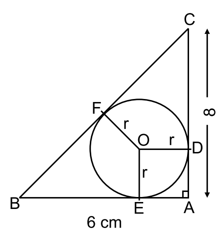
14. In the figure, PO ⊥ QO. The tangents to the circle with centre O at P and Q intersect at a point T. Prove that PQ and OT are right bisectors of each other. **[2011 (T-II)]**



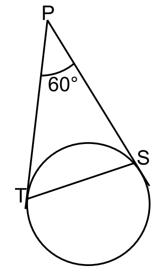
15. In the figure, PQ is a chrod of length 8 cm of a circle of radius 5 cm. The tangents at point P and Q intersect at point T. Find the length of tangent TP. **[2011 (T-II)]**



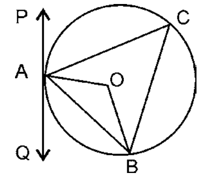
16. In the figure, ABC is a right triangle, right angled at A, with AB = 6 cm and AC = 8 cm. A circle with centre O has been inscribed inside the triangle. Calculate the radius of the inscribed circle. **[2011 (T-II)]**



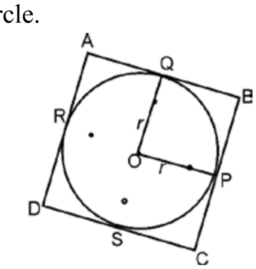
17. In the figure, PT and PS are tangents to a circle from a point P such that PT = 5 cm and ∠TPS = 60°. Find the length of chord TS. **[2011 (T-II)]**



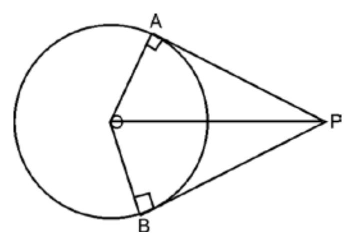
18. PAQ is a tangent to the circle with centre O at a point A as shown in the figure. If ∠OBA = 35°, find the value of ∠BAQ and ∠ACB. **[2001]**



19. In the figure, a circle is inscribed in a quadrilateral ABCD in which ∠B = 90°. If AD = 23 cm, AB = 29 cm and DS = 5 cm, find the radius (r) of the circle. **[2008]**



20. In the figure, OP is equal to diameter of the circle. Prove that ABP is an equilateral triangle. **[2008]**



LONG ANSWER TYPE QUESTIONS [4 Marks]

A. Important Questions

1. Two circles with centres O and O′ of radii 3 cm and 4 cm respectively intersect at two points P and Q such that OP and O′P are tangents to the two circles. Find the length of the common chord PQ.

2. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

3. In a right triangle ABC, in which ∠B = 90°, a circle is drawn with AB as diameter intersecting the hypotenunes AC at P. Prove that the tangent to the circle at P bisects BC.

4. From a point P, two tangents PA and PB are drawn to a circle C (O, r). If OP = 2r, show that ΔAPB is equilateral.

5. O is the centre of a circle, PA and PB are tangents to the circle from a point P. Prove that (i) PAOB is a cyclic quadrilateral (ii) PO is the bisector of ∠APB. (iii) ∠OAB = ∠OPA.

6. If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R respectively, prove that

AQ = $\frac{1}{2}$ (AB + BC + CA).

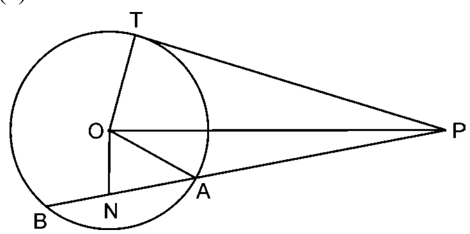
7. If an isosceles triangle ABC, in which AB = AC = 6 cm is inscribed in a circle of radius 9 cm, find the area of the triangle.

8. In the figure, from an external point P, a tangent PT and a line segment PAB is drawn to a circle with centre O. ON is perpendicular to the chord AB. Prove that :

(a) PA. PB = $PN^2 - AN^2$

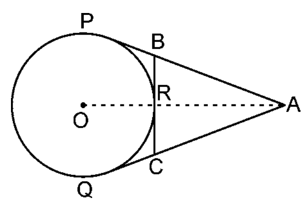
(b) $OP^2 - OT^2 = PN^2 - AN^2$

(c) PA. PB = PT^2.



9. If a hexagon ABCDEF circumscribes a circle, prove that AB + CD + EF = BC + DE + FA.

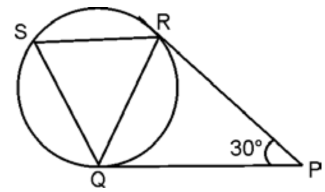
10. In the figure, O is the centre of the circle. If OR = 5 cm and OA = 13 cm, find the perimeter of ΔABC.



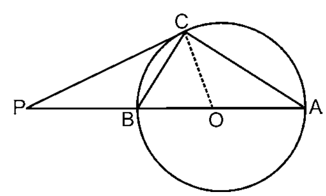
11. The transverse common tangents AB and CD of two circles with centre O and O′ intersect at E. Prove that the points O, E and O′ are collinear. **[HOTS]**

12. Let *s* denote the semi perimeter of a ΔABC in which BC = *a*, CA = *b* and AB = *c*. If a circle touches BC, CA and AB at D, E and F respectively, prove that BD = $s - b$.

13. In the figure, tangents PQ and PR are drawn to a circle such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find ∠RQS.



14. In the figure, the tangent at a point C of a circle and a diameter AB when extended intersect at P. If ∠PCA = 110°, find ∠CBA.



B. Questions From CBSE Examination Papers

1. OABC is a rhombus whose three vertices A, B and C lie on a circle with centre O. If the radius of the circle is 10 cm, find the area of the rhombus.

[2011 (T-II)]

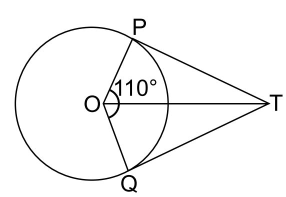
2. Prove that the lengths of the tangents drawn from an external point to a circle are equal.

[2011 (T-II)]

3. Prove that the angle between the two tangents drawn from any external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre. **[2011 (T-II)]**

4. A circle touches the sides of a quadrilateral ABCD at P, Q R, S respectively. Show that angle subtended at the centre by pairs of opposite sides are supplementary.

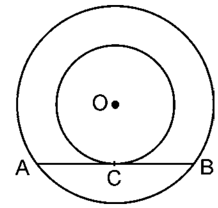
Using the above, find ∠PTQ in the figure, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°. **[2011 (T-II)]**



5. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Using the above , do the following :

In figure, O is the centre of the two concentric circles. AB is a chord of the larger circle touching the smaller circle at C. Prove that AC = BC. **[2009]**



FORMATIVE ASSESSMENT

Activity

Objective : To show the following with the help of an activity.

(*a*) Lengths of tangents drawn from an external point are equal.

(*b*) Tangents are equally inclined to the segment joining the centre to that point.

Materials Required : White sheet of paper (or tracing paper), geometry box, sketch pens, a pair of scissors.

Preparation for the Activity

1. Draw a circle of any radius with centre O on a tracing paper.
2. Take a point, say P, outside of the circle.
3. From point P, draw a line segment touching the circle at A (the point of contact), which is the required tangent. [See figure 1]

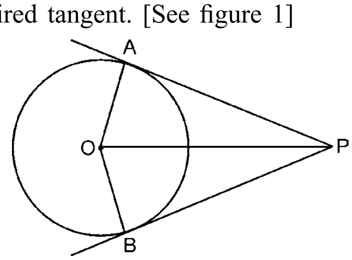


Figure 1

4. Following step 3, draw one more tangent PB, B being the point of contact. [See figure 1]
5. Join OA, OP and OB. [See figure 1]

(a) Lengths of tangents drawn from an external point are equal.

Fold the paper along OP.

Observation :

You will observe that point A coincides with point B and line segment PA (the tangent from point P) concides with line segment PB (another tangent from the same point on the circle). [See figure 2]

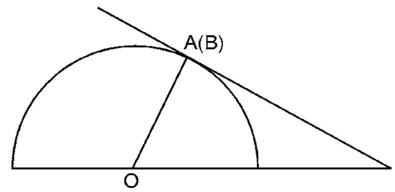


Figure 2

Hence, lengths of tangents drawn from an external point are equal.

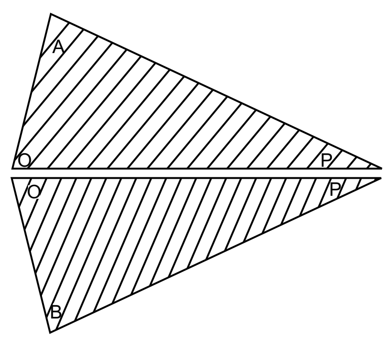
Result : Lengths of tangents drawn from an external point are equal.

(b) Tangents are equally inclined to the line segment joining the centre with the external point.

1. Cut out the two triangles, ΔOPA and ΔOPB, so formed [figure 3].
2. Colour the two triangles with different colours.
3. Put one triangle on the other.

Observation :

You will observe that two triangles are congruent to each other (i.e., one triangle exactly superimposes the other) with the following (angle) correspondence.



Cut outs of ΔOAP and DOBP

Figure 3

∠OPA = ∠OPB, (Proves the required result)

∠PAO = ∠PBO,

∠AOP = ∠BOP [See figure 4]

Hence, tangents drawn from an external point, are equally inclined to the line segment joining the centre with that point.

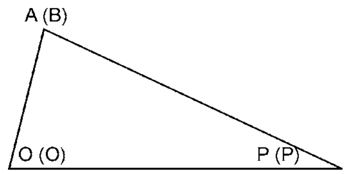


Figure 4

Result : Tangents drawn from an external point are equally inclined to the line segment joining the centre with the external point.

Note : The tangent at any point of a circle is perpendicular to the radius through the point of contact, i.e., $\angle OAP = \angle OBP = 90^\circ$.

Exercise 10.2

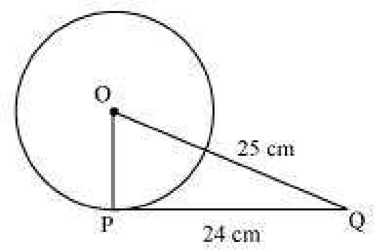
Question 1:

From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

(A) 7 cm (B) 12 cm

(C) 15 cm (D) 24.5 cm

Answer:



Let O be the centre of the circle.

Given that,

OQ = 25cm and PQ = 24 cm

As the radius is perpendicular to the tangent at the point of contact,

Therefore, OP $\perp$ PQ

Applying Pythagoras theorem in ΔOPQ, we obtain

$OP^2 + PQ^2 = OQ^2$

$OP^2 + 24^2 = 25^2$

$OP^2 = 625 - 576$

$OP^2 = 49$

OP = 7

Therefore, the radius of the circle is 7 cm.

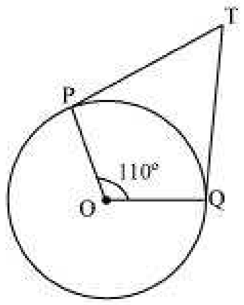
Hence, alternative (A) is correct

Question 2:

In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to

(A) 60° (B) 70°

(C) 80° (D) 90°



Answer:

It is given that TP and TQ are tangents.

Therefore, radius drawn to these tangents will be perpendicular to the tangents.

Thus, OP ⊥ TP and OQ ⊥ TQ

∠OPT = 90º

∠OQT = 90º

In quadrilateral POQT,

Sum of all interior angles = 360°

∠OPT + ∠POQ +∠OQT + ∠PTQ = 360°

⇒ 90° + 110º + 90° +∠PTQ = 360°

⇒ ∠PTQ = 70°

Hence, alternative (B) is correct

Question 3:

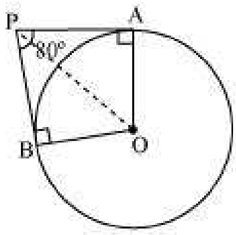
If tangents PA and PB from a point P to a circle with centre O are inclined to each other an angle of 80°, then ∠POA is equal to

(A) 50° (B) 60°

(C) 70° (D) 80°

Answer:

It is given that PA and PB are tangents.



Therefore, the radius drawn to these tangents will be perpendicular to the tangents.

Thus, OA ⊥ PA and OB ⊥ PB

∠OBP = 90º

∠OAP = 90º

In AOBP,

Sum of all interior angles = 360°

∠OAP + ∠APB +∠PBO + ∠BOA = 360°

90° + 80° +90º +∠BOA = 360°

∠BOA = 100°

In ΔOPB and ΔOPA,

AP = BP (Tangents from a point)

OA = OB (Radii of the circle)

OP = OP (Common side)

Therefore, ΔOPB ≅ ΔOPA (SSS congruence criterion)

A ↔ B, P ↔ P, O ↔ O

And thus, ∠POB = ∠POA

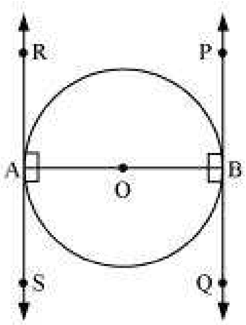
$$\angle POA = \frac{1}{2}\angle AOB = \frac{100^\circ}{2} = 50^\circ$$

Hence, alternative (A) is correct.

Question 4:

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Answer:



Let AB be a diameter of the circle. Two tangents PQ and RS are drawn at points A and B respectively.

Radius drawn to these tangents will be perpendicular to the tangents.

Thus, OA ⊥ RS and OB ⊥ PQ

∠OAR = 90º

∠OAS = 90º

∠OBP = 90º

∠OBQ = 90º

It can be observed that

∠OAR = ∠OBQ (Alternate interior angles)

∠OAS = ∠OBP (Alternate interior angles)

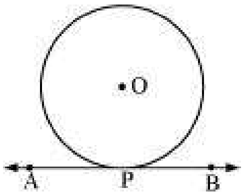
Since alternate interior angles are equal, lines PQ and RS will be parallel.

Question 5:

Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

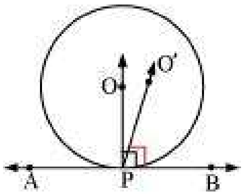
Answer:

Let us consider a circle with centre O. Let AB be a tangent which touches the circle at P.



We have to prove that the line perpendicular to AB at P passes through centre O. We shall prove this by contradiction method.

Let us assume that the perpendicular to AB at P does not pass through centre O. Let it pass through another point O'. Join OP and O'P.



As perpendicular to AB at P passes through O', therefore,

$\angle O'PB = 90°$ … (1)

O is the centre of the circle and P is the point of contact. We know the line joining the centre and the point of contact to the tangent of the circle are perpendicular to each other.

$\therefore\ \angle OPB = 90°$ … (2)

Comparing equations (1) and (2), we obtain

$\angle O'PB = \angle OPB$ … (3)

From the figure, it can be observed that,

$\angle O'PB < \angle OPB$ … (4)

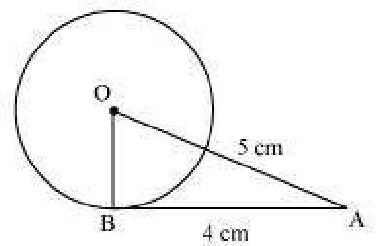
Therefore, $\angle O'PB = \angle OPB$ is not possible. It is only possible, when the line O'P coincides with OP.

Therefore, the perpendicular to AB through P passes through centre O.

Question 6:

The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Answer:



Let us consider a circle centered at point O.

AB is a tangent drawn on this circle from point A.

Given that,

OA = 5cm and AB = 4 cm

In ΔABO,

OB ⊥ AB (Radius ⊥ tangent at the point of contact)

Applying Pythagoras theorem in ΔABO, we obtain

$AB^2 + BO^2 = OA^2$

$4^2 + BO^2 = 5^2$

$16 + BO^2 = 25$

$BO^2 = 9$

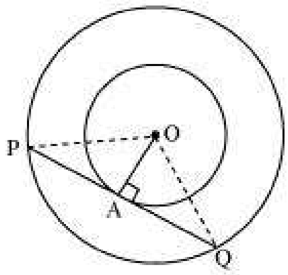
BO = 3

Hence, the radius of the circle is 3 cm.

Question 7:

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer:



Let the two concentric circles be centered at point O. And let PQ be the chord of the larger circle which touches the smaller circle at point A. Therefore, PQ is tangent to the smaller circle.

$OA \perp PQ$ (As OA is the radius of the circle)

Applying Pythagoras theorem in ΔOAP, we obtain

$OA^2 + AP^2 = OP^2$

$3^2 + AP^2 = 5^2$

$9 + AP^2 = 25$

$AP^2 = 16$

AP = 4

In ΔOPQ,

Since $OA \perp PQ$,

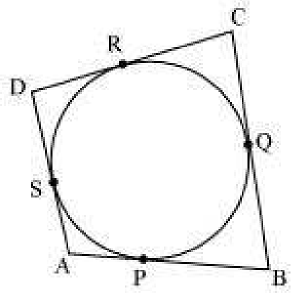
AP = AQ (Perpendicular from the center of the circle bisects the chord)

$\therefore PQ = 2AP = 2 \times 4 = 8$

Therefore, the length of the chord of the larger circle is 8 cm.

Question 8:

A quadrilateral ABCD is drawn to circumscribe a circle (see given figure) Prove that AB + CD = AD + BC



Answer:

It can be observed that

DR = DS (Tangents on the circle from point D) … (1)

CR = CQ (Tangents on the circle from point C) … (2)

BP = BQ (Tangents on the circle from point B) … (3)

AP = AS (Tangents on the circle from point A) … (4)

Adding all these equations, we obtain

DR + CR + BP + AP = DS + CQ + BQ + AS

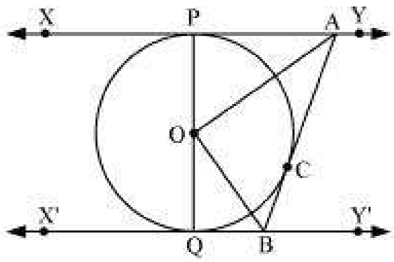
(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC

Question 9:

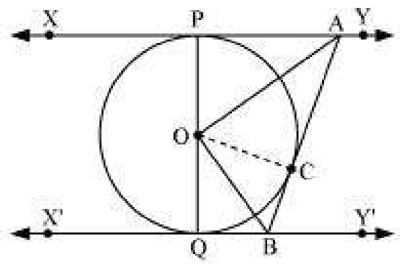
In the given figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B.

Prove that $\angle AOB = 90^\circ$.



Answer:

Let us join point O to C.



In ΔOPA and ΔOCA,

OP = OC (Radii of the same circle)

AP = AC (Tangents from point A)

AO = AO (Common side)

ΔOPA ≅ ΔOCA (SSS congruence criterion)

Therefore, P ↔ C, A ↔ A, O ↔ O

∠POA = ∠COA … (*i*)

Similarly, ΔOQB ≅ ΔOCB

∠QOB = ∠COB … (*ii*)

Since POQ is a diameter of the circle, it is a straight line.

Therefore, ∠POA + ∠COA + ∠COB + ∠QOB = 180º

From equations (*i*) and (*ii*), it can be observed that

2∠COA + 2 ∠COB = 180º

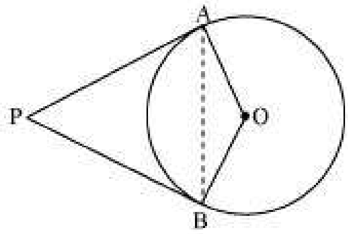
∠COA + ∠COB = 90º

∠AOB = 90°

Question 10:

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Answer:



Let us consider a circle centered at point O. Let P be an external point from which two tangents PA and PB are drawn to the circle which are touching the circle at point A and B respectively and AB is the line segment, joining point of contacts A and B together such that it subtends ∠AOB at center O of the circle.

It can be observed that

OA (radius) ⊥ PA (tangent)

Therefore, ∠OAP = 90°

Similarly, OB (radius) ⊥ PB (tangent)

∠OBP = 90°

In quadrilateral OAPB,

Sum of all interior angles = 360°

∠OAP +∠APB+∠PBO +∠BOA = 360°

90° + ∠APB + 90° + ∠BOA = 360°

∠APB + ∠BOA = 180°

Hence, it can be observed that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Question 11:

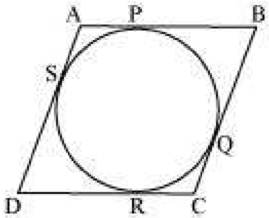
Prove that the parallelogram circumscribing a circle is a rhombus.

Answer:

Since ABCD is a parallelogram,

AB = CD …(1)

BC = AD …(2)



It can be observed that

DR = DS (Tangents on the circle from point D)

CR = CQ (Tangents on the circle from point C)

BP = BQ (Tangents on the circle from point B)

AP = AS (Tangents on the circle from point A)

Adding all these equations, we obtain

DR + CR + BP + AP = DS + CQ + BQ + AS

(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC

On putting the values of equations (1) and (2) in this equation, we obtain

2AB = 2BC

AB = BC …(3)

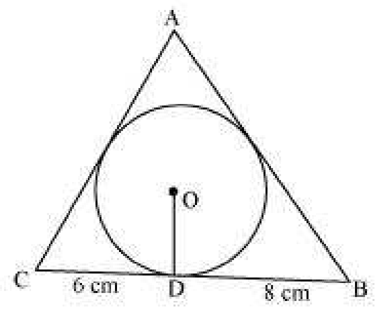
Comparing equations (1), (2), and (3), we obtain

AB = BC = CD = DA

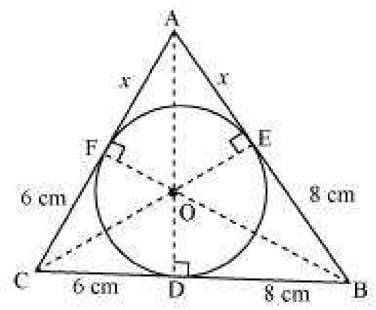
Hence, ABCD is a rhombus.

Question 12:

A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see given figure). Find the sides AB and AC.



Answer:



Let the given circle touch the sides AB and AC of the triangle at point E and F respectively and the length of the line segment AF be *x.*

In ΔABC,

CF = CD = 6cm (Tangents on the circle from point C)

BE = BD = 8cm (Tangents on the circle from point B)

AE = AF = x (Tangents on the circle from point A)

AB = AE + EB = $x + 8$

BC = BD + DC = 8 + 6 = 14

CA = CF + FA = $6 + x$

2s = AB + BC + CA

= $x + 8 + 14 + 6 + x$

= $28 + 2x$

$s = 14 + x$

Area of ΔABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{\{14+x\}\{(14+x)-14\}\{(14+x)-(6+x)\}\{(14+x)-(8+x)\}}$$

$$= \sqrt{(14+x)(x)(8)(6)}$$

$$= 4\sqrt{3(14x+x^2)}$$

Area of ΔOBC = $\frac{1}{2}\times OD\times BC = \frac{1}{2}\times 4\times 14 = 28$

Area of ΔOCA = $\frac{1}{2}\times OF\times AC = \frac{1}{2}\times 4\times(6+x) = 12+2x$

Area of ΔOAB = $\frac{1}{2}\times OE\times AB = \frac{1}{2}\times 4\times(8+x) = 16+2x$

Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$$4\sqrt{3(14x+x^2)} = 28 + 12 + 2x + 16 + 2x$$

$$\Rightarrow 4\sqrt{3(14x+x^2)} = 56+4x$$

$$\Rightarrow \sqrt{3(14x+x^2)} = 14+x$$

$$\Rightarrow 3(14x+x^2) = (14+x)^2$$

$$\Rightarrow 42x+3x^2 = 196+x^2+28x$$

$$\Rightarrow 2x^2+14x-196 = 0$$

$$\Rightarrow x^2+7x-98=0$$

$$\Rightarrow x^2+14x-7x-98=0$$

$$\Rightarrow x(x+14)-7(x+14)=0$$

$$\Rightarrow (x+14)(x-7)=0$$

Either $x+14 = 0$ or $x - 7 = 0$

Therefore, $x = -14$ and 7

However, $x = -14$ is not possible as the length of the sides will be negative.

Therefore, $x = 7$

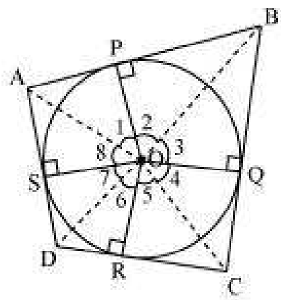
Hence, AB = $x + 8 = 7 + 8 = 15$ cm

CA = $6 + x = 6 + 7 = 13$ cm

Question 13:

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Answer:



Let ABCD be a quadrilateral circumscribing a circle centered at O such that it touches the circle at point P, Q, R, S. Let us join the vertices of the quadrilateral ABCD to the center of the circle.

Consider ΔOAP and ΔOAS,

AP = AS (Tangents from the same point)

OP = OS (Radii of the same circle)

OA = OA (Common side)

ΔOAP ≅ ΔOAS (SSS congruence criterion)

Therefore, A ↔ A, P ↔ S, O ↔ O

And thus, ∠POA = ∠AOS

∠1 = ∠8

Similarly,

∠2 = ∠3

∠4 = ∠5

∠6 = ∠7

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^\circ$

$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^\circ$

$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^\circ$

$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180^\circ$

$\angle AOB + \angle COD = 180^\circ$

Similarly, we can prove that $\angle BOC + \angle DOA = 180^\circ$

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle