Wisgrow Math A – Plus
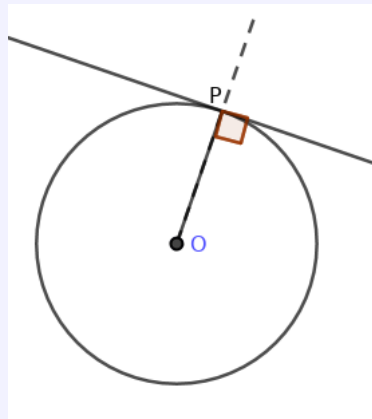
Session 76 │Tangents 1│ Worksheet 76

1) Construct a tangent to a circle by the steps given below

    a) Draw a circle of radius 3cm and mark a point P on the circle.

    b) Mark O as the centre of the circle and draw the radius OP

    c) Draw the tangent to the circle at P

    d) Draw another tangent to this circle parallel to the first tangent.

Answers

Extent the radius to the diametre. Draw tangent at the other end of the diametre also

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2) Draw suitable figure find the lengths asked in the question.

    a) A tangent of length 12cm is drawn to a circle from a point outside the circle.If the radius of the circle is 5cm find the distance from centre to the exterior point from which the tangent is drawn.

    b) What is the length of tangent drawn from a point at the distance 10 cm away from centre of a circle of radius 6cm

    c) A tangent is drawn from a point at the distance 26 cm away from the centre of a circle. If the length of the tangent is 24cm find the radius of the circle.

Answers

Draw suitable figuresIf l is the length of tangent, r is the radius of the circle and d is the distance from the center to the outer point

$d^2 = l^2 + r^2$

$d^2 = 12^2 + 5^2 = 144 + 25 = 169, d = \sqrt{169} = 13$cm If l is the length of tangent, r is the radius of the circle and d is the distance from the center to the outer point

$d^2 = l^2 + r^2$

$10^2 = l^2 + 6^2$, $l^2 = 100 - 36 = 64, l = \sqrt{64} = 8$cm If l is the length of tangent, r is the radius of the circle and d is the distance from the center to the outer point

$d^2 = l^2 + r^2$

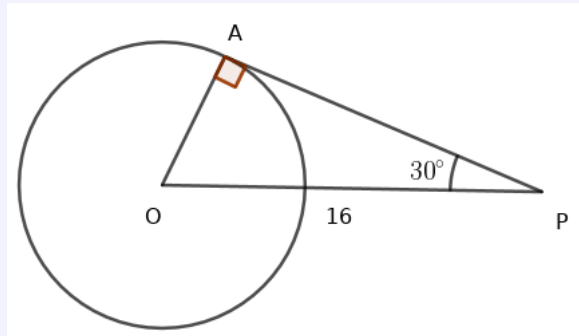$26^2 = 24^2 + r^2, r^2 = 26^2 - 24^2 = 676 - 576 = 100, r = \sqrt{100} = 10$cm

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3) O is the center of the circle, $\angle OPA = 30°, OP = 16$, PA is a tangent from the outer point P,then

    a) Draw a rough diagram

b) What are the angles of $\triangle OAP$

c) What is the radius of the circle?

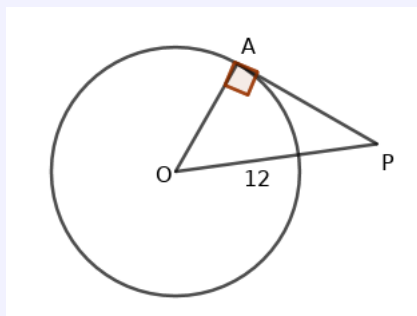d) What is the length of the tangent?

Answers

a) see the diagarm

b) $\angle OPA = 30°, \angle OAP = 90°, \angle AOP = 60°$

c) This is a $30° - 60° - 90°$ triangle.
Side opposite to $90°$ is 16cm . Therefore the side opposite to $30°$ is 8cm, side opposite to $60°$ is $8\sqrt{3}$cm
Length of tangent $PA = 8\sqrt{3}$cm, Radius $OA = 8$cm

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4) In the figure O is the centre of the circle. A tangent PA is drawn from P outside the circle at the distance 12cm from the centre. If the length of the tangent and radius are equal then

a) Draw a rough diagram

b) What are the angles of $\triangle OAP$?

c) What is the length of tangent and radius?

Answers

a)

b) $\angle OAP = 90°, OA = PA.$ The angles opposite to equal sides are equal. Each of them is $45°$
$\triangle OAP$ is a $45° - 45° - 90°$ triangle.

c) Length of tangent $= \frac{12}{\sqrt{2}} = 6\sqrt{2}$cm, Radius $= 6\sqrt{2}$cm

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5) Choose the correct answer

a) Radius of a circle and the length of tangent from an outer point to the circle are equal to 1. What is the distance from the center of the circle to the outer point?

b) The tangent from an outer point to the circle has length 12cm.The tangent makes an angle $30°$ with the line joining the center and outer point. What is the radius of the circle?

(a) $4\sqrt{3}$ (b) $2\sqrt{2}$ (c) $3\sqrt{3}$ (d) $5\sqrt{2}$

c) A tangent is drawn from an outer point to a circle of radius 5cm. If the length of tangent is 12cm then what is the distance from center to the outer point?

(a) 13cm   (b) 12cm   (c) 18cm   (d) 10cm

Answers

a) $\sqrt{2}$

b) $4\sqrt{3}$cm

c) 13cm

[1]John P A, jpavpz@gmail.com, sjpuzzles@gmail.com ,9847307721

Session 77 │Tangents 2 │ **Worksheet 77**

1) In the figure $\angle OPA = 40°, OP = 18$cm then

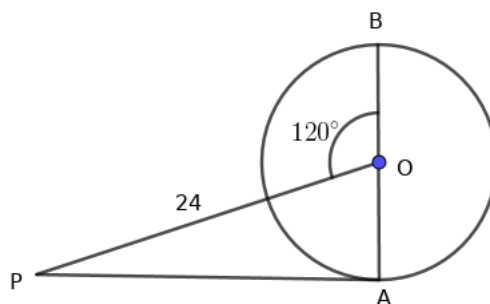   **a)** What is the measure of $\angle AOP$?

    b) What is the radius of the circle?

    c) What is the length of the tangent?
    $[\sin 40 = 0.6428, \cos 40° = 0.7660, \tan 40 = 0.8391]$

Answers

    a) $\angle AOP = 90 - 40 = 50°$

    b) $\sin 40° = \frac{OA}{OP} = \frac{OA}{18}$
        $OA = 18 \times 0.6428 = 11.57$ cm

    c) $\cos 40° = \frac{PA}{18}$
        $PA = 0.7660 \times 18 = 13.788$cm

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2) In the figure $\angle POB = 120°, OP = 24$cm , AB is the diametre of the circle.

    a) What are the angles of $\triangle POA$?

    b) What is the diametre of the circle?
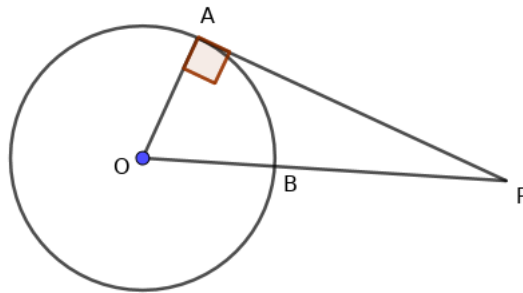
    c) What is the length of the tangent from P

3) The length of tangent drawn from a point at a distance 8 cm from the centre to a circle is 4cm. Construct the tangent. Measure the radius of the circle and write aside.

4) In the figure the length of tangent PA is 12cm and $PB = 8$cm . what is the radius of the circle?

5) In the figure O is the centre of the circle and PA is a tangent. If the area of the triangle is $OPA$is 6 sq.cm and $OP = 5$cm

a) What is the radius of the circle?

b) What is the length of tangent?

Answers

a) Let $OA = r, PA = x$. $\frac{1}{2}rx = 6, rx = 12$
$r^2 + x^2 = 5^2$
$(r + x)^2 = r^2 + x^2 + 2rx, (r + x)^2 = 25 + 24 = 49, r + x = 7$
$(r - x)^2 = (r + x)^2 - 4rx = 49 - 48 = 1$
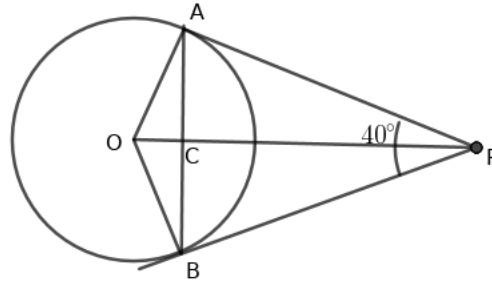$r + x = 7, r - x = 1 \rightarrow 2r = 8, r = 4$

b) Length of tangent $x = \sqrt{5^2 - 4^2} = 3$

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Wisgrow Math A – Plus

Session 78 │Tangents 3 │ Worksheet 78

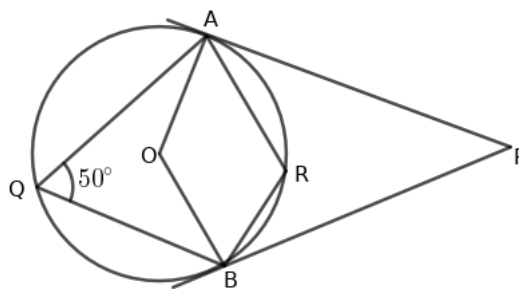1) In the figure PA, PB are tangents . O is the centre of the circle.

a) What are the measures of $\angle OAP, \angle OBP$?

b) If $\angle APB = 40°$ then what is the measure of $\angle AOB$

c) The lines AB and CD intersect at C .What is the relation between the length of lines CO, CP, CA and CB?

Answers

a) Tangent is perpendicular to the radius .
$\angle OAP = \angle OBP = 90°$

b) $OAPB$ is a cyclic quadrilateral .$\angle AOB = 180 - 40 = 140°$

c) $OAPB$ is a cyclic quadrilateral.A Circle passes through the vertices . The lines OP and AB are the chords of the circle. They intersect at C
$CO \times CP = CA \times CB$

2) In the figure PA and PB are tangents O is the centre of the circle ,$\angle AQB = 50°$ then

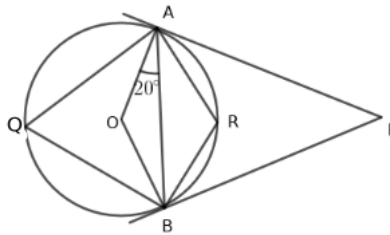a) What is the measure of $\angle AOB$?

b) What is the measure of angle $\angle ARB, \angle APB$?

Answers

a) $\angle AOB = 2 \times 50° = 100°$

b) $QARB$ is cyclic. $\angle ARB = 180 - 50 = 130°$

c) $OAPB$ is cyclic. $\angle APB = 180 - 100 = 80°$

3) In the figure O is the centre of the circle, PA, PB are tangents . If $\angle OAB = 20°$ then

a) What is the measure of $\angle AOB$ and , $\angle AQB$?

b) What is the measure of $\angle ARB$?

c) What is the measure of $\angle APB$?

Answers

a) $OA = OB, \therefore \angle OBA = 20°$
$\angle AOB = 180 - (20 + 20) = 140°$
$\angle AQB = \frac{1}{2} \times \angle AOB = 70°$

b) $AQBR$ is cyclic. $\angle ARB = 180 - 70 = 110°$

c) $OAPB$ is cyclic .$\angle APB = 180 - 140 = 40°$

4) Draw two tangents from an outer point of a circle of radius 3cm such that the angle between the tangents is $60°$

a) What is the distance from centre to the outer point?

b) What is the length of tangents ?

Answers

★ Draw a circle of radius 3cm. Draw two radii such that the angle between them is $180 - 60 = 120°$.Draw radii OA, OB

★ Draw tangents at A and B. They meet at P

★ $\angle APB = 60°$

a) Triangle OAP is a $30° - 60° - 90°$ triangle. Side opposite to $30°$ 3 cm , Side opposite to $90°$ is 6 cm
The side opposite to $60°$ is $3\sqrt{3}$cm
$OP = 6$cm

b) Length of tangent is $3\sqrt{3}$cm ,$PA = PB = 3\sqrt{3}$cm

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5) Two angles of a trinagle are $40°, 60°$.The sides of the triangle touches a circle of radius 3 cm

Answers

- ⋆ Draw a circle of radius 3 cm

- ⋆ Since two angles are $40°, 60°$ their supplementary angles are $180 - 40 = 140°, 180 - 60 = 120°$. Draw radii such that it divide the angle around the centre as $140°, 120°, 100°$

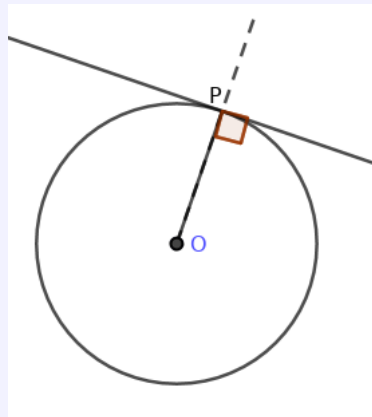- ⋆ Draw tangents to the circle at the ends of the radii.

Session 76 │Tangents 1│ Worksheet 76

1) Construct a tangent to a circle by the steps given below

   a) Draw a circle of radius 3cm and mark a point P on the circle.

   b) Mark O as the centre of the circle and draw the radius OP

   c) Draw the tangent to the circle at P

   d) Draw another tangent to this circle parallel to the first tangent.

Answers

Extent the radius to the diametre. Draw tangent at the other end of the diametre also

---

2) Draw suitable figure find the lengths asked in the question.

   a) A tangent of length 12cm is drawn to a circle from a point outside the circle.If the radius of the circle is 5cm find the distance from centre to the exterior point from which the tangent is drawn.

   b) What is the length of tangent drawn from a point at the distance 10 cm away from centre of a circle of radius 6cm

   c) A tangent is drawn from a point at the distance 26 cm away from the centre of a circle. If the length of the tangent is 24cm find the radius of the circle.

Answers

Draw suitable figuresIf l is the length of tangent, r is the radius of the circle and d is the distance from the center to the outer point
$d^2 = l^2 + r^2$
$d^2 = 12^2 + 5^2 = 144 + 25 = 169, d = \sqrt{169} = 13$cm If l is the length of tangent, r is the radius of the circle and d is the distance from the center to the outer point
$d^2 = l^2 + r^2$
$10^2 = l^2 + 6^2$, $l^2 = 100 - 36 = 64, l = \sqrt{64} = 8$cm If l is the length of tangent, r is the radius of the circle and d is the distance from the center to the outer point
$d^2 = l^2 + r^2$
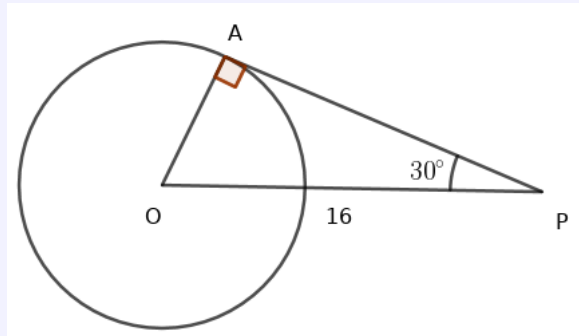$26^2 = 24^2 + r^2, r^2 = 26^2 - 24^2 = 676 - 576 = 100, r = \sqrt{100} = 10$cm

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3) O is the center of the circle, $\angle OPA = 30°, OP = 16$, PA is a tangent from the outer point P,then

   a) Draw a rough diagram

b) What are the angles of $\triangle OAP$

c) What is the radius of the circle?

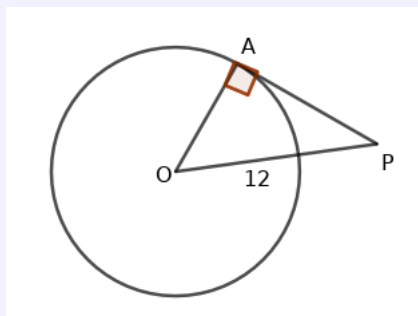d) What is the length of the tangent?

Answers

a) see the diagarm

b) $\angle OPA = 30°, \angle OAP = 90°, \angle AOP = 60°$

c) This is a $30° - 60° - 90°$ triangle.
Side opposite to $90°$ is 16cm . Therefore the side opposite to $30°$ is 8cm, side opposite to $60°$ is $8\sqrt{3}$cm
Length of tangent $PA = 8\sqrt{3}$cm, Radius $OA = 8$cm

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4) In the figure O is the centre of the circle. A tangent PA is drawn from P outside the circle at the distance 12cm from the centre. If the length of the tangent and radius are equal then

a) Draw a rough diagram

b) What are the angles of $\triangle OAP$?

c) What is the length of tangent and radius?

Answers

a)

b) $\angle OAP = 90°, OA = PA.$ The angles opposite to equal sides are equal. Each of them is $45°$
$\triangle OAP$ is a $45° - 45° - 90°$ triangle.

c) Length of tangent $= \frac{12}{\sqrt{2}} = 6\sqrt{2}$cm, Radius $= 6\sqrt{2}$cm

---

5) Choose the correct answer

a) Radius of a circle and the length of tangent from an outer point to the circle are equal to 1. What is the distance from the center of the circle to the outer point?

(a) 1 (b) $\sqrt{2}$ (c) 3 (d) 2

b) The tangent from an outer point to the circle has length 12cm.The tangent makes an angle $30°$ with the line joining the center and outer point. What is the radius of the circle?

(a) $4\sqrt{3}$ (b) $2\sqrt{2}$ (c) $3\sqrt{3}$ (d) $5\sqrt{2}$

c) A tangent is drawn from an outer point to a circle of radius 5cm. If the length of tangent is 12cm then what is the distance from center to the outer point?

(a) 13cm   (b) 12cm   (c) 18cm   (d) 10cm

Answers

a) $\sqrt{2}$

b) $4\sqrt{3}$cm

c) 13cm

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Session 77 │Tangents 2 │ Worksheet 77
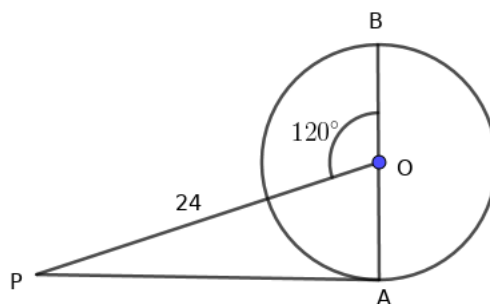
1) In the figure $\angle OPA = 40°, OP = 18$cm then

a) What is the measure of $\angle AOP$?

b) What is the radius of the circle?

c) What is the length of the tangent?
   $[\sin 40 = 0.6428, \cos 40° = 0.7660, \tan 40 = 0.8391]$

Answers

a) $\angle AOP = 90 - 40 = 50°$

b) $\sin 40° = \frac{OA}{OP} = \frac{OA}{18}$
   $OA = 18 \times 0.6428 = 11.57$ cm

c) $\cos 40° = \frac{PA}{18}$
   $PA = 0.7660 \times 18 = 13.788$cm

---

2) In the figure $\angle POB = 120°, OP = 24$cm , AB is the diametre of the circle.

a) What are the angles of $\triangle POA$?

b) What is the diametre of the circle?

c) What is the length of the tangent from P

3) The length of tangent drawn from a point at a distance 8 cm from the centre to a circle is 4cm. Construct the tangent. Measure the radius of the circle and write aside.

4) In the figure the length of tangent PA is 12cm and $PB = 8$cm . what is the radius of the circle?

5) In the figure O is the centre of the circle and PA is a tangent. If the area of the triangle is OPA is 6 sq.cm and $OP = 5$cm

a) What is the radius of the circle?

b) What is the length of tangent?

> **Answers**
>
> a) Let $OA = r, PA = x$. $\frac{1}{2}rx = 6, rx = 12$
> $r^2 + x^2 = 5^2$
> $(r+x)^2 = r^2 + x^2 + 2rx, (r+x)^2 = 25 + 24 = 49, r + x = 7$
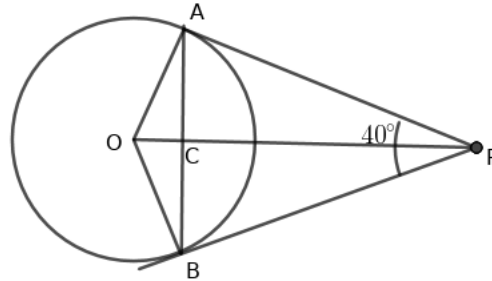> $(r-x)^2 = (r+x)^2 - 4rx = 49 - 48 = 1$
> $r + x = 7, r - x = 1 \rightarrow 2r = 8, r = 4$
>
> b) Length of tangent $x = \sqrt{5^2 - 4^2} = 3$

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Wisgrow Math A – Plus

Session 78 │Tangents 3 │ Worksheet 78

1) In the figure PA, PB are tangents . O is the centre of the circle.

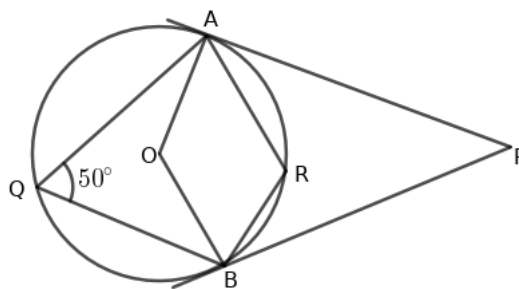   a) What are the measures of $\angle OAP, \angle OBP$?

   b) If $\angle APB = 40°$ then what is the measure of $\angle AOB$

   c) The lines AB and CD intersect at C .What is the relation between the length of lines CO, CP, CA and CB?

   Answers

   a) Tangent is perpendicular to the radius .
      $\angle OAP = \angle OBP = 90°$

   b) $OAPB$ is a cyclic quadrilateral . $\angle AOB = 180 - 40 = 140°$

   c) $OAPB$ is a cyclic quadrilateral. A Circle passes through the vertices . The lines OP and AB are the chords of the circle. They intersect at C
      $CO \times CP = CA \times CB$

2) In the figure PA and PB are tangents O is the centre of the circle , $\angle AQB = 50°$ then
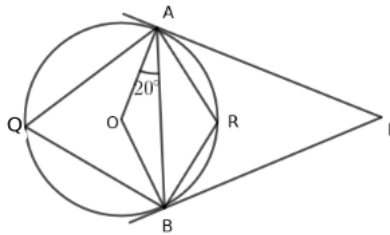
a) What is the measure of $\angle AOB$?

b) What is the measure of angle $\angle ARB, \angle APB$?

Answers

a) $\angle AOB = 2 \times 50° = 100°$

b) $QARB$ is cyclic. $\angle ARB = 180 - 50 = 130°$

c) $OAPB$ is cyclic. $\angle APB = 180 - 100 = 80°$

3) In the figure O is the centre of the circle, PA, PB are tangents . If $\angle OAB = 20°$ then

a) What is the measure of $\angle AOB$ and , $\angle AQB$?

b) What is the measure of $\angle ARB$?

c) What is the measure of $\angle APB$?

Answers

a) $OA = OB, \therefore \angle OBA = 20°$
$\angle AOB = 180 - (20 + 20) = 140°$
$\angle AQB = \frac{1}{2} \times \angle AOB = 70°$

b) $AQBR$ is cyclic. $\angle ARB = 180 - 70 = 110°$

c) $OAPB$ is cyclic .$\angle APB = 180 - 140 = 40°$

4) Draw two tangents from an outer point of a circle of radius 3 cm such that the angle between the tangents is $60°$

a) What is the distance from centre to the outer point?

b) What is the length of tangents ?

Answers

★ Draw a circle of radius 3 cm. Draw two radii such that the angle between them is $180 - 60 = 120°$ .Draw radii OA, OB

★ Draw tangents at A and B. They meet at P

★ $\angle APB = 60°$

a) Triangle OAP is a $30° - 60° - 90°$ triangle. Side opposite to $30°$ 3 cm , Side opposite to $90°$ is 6 cm
The side opposite to $60°$ is $3\sqrt{3}$ cm
$OP = 6$ cm

b) Length of tangent is $3\sqrt{3}$ cm ,$PA = PB = 3\sqrt{3}$ cm

5) Two angles of a trinagle are $40°, 60°$.The sides of the triangle touches a circle of radius 3 cm

Answers

* ⋆ Draw a circle of radius 3 cm

* ⋆ Since two angles are $40°, 60°$ their supplementary angles are $180 - 40 = 140°, 180 - 60 = 120°$. Draw radii such that it divide the angle around the centre as $140°, 120°, 100°$
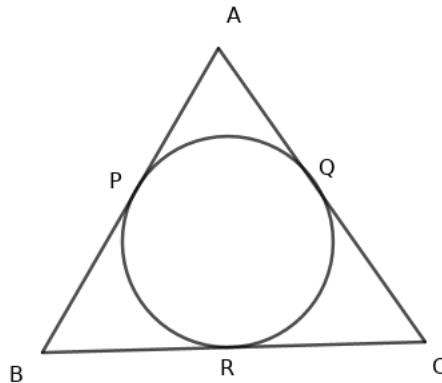
* ⋆ Draw tangents to the circle at the ends of the radii.

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Session 80 │Tangents 6 │ Worksheet 81

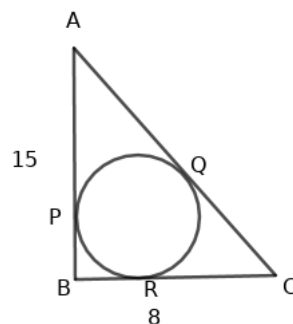1) In the figure $AB = AC$, the circle touches the sides at P, Q, R.

a) $AP = AQ$ why?

b) Prove that $BR = CR$

> **Answers**
>
> a) Two tangents from an outer point to a circle are equal . $\therefore AP = AQ$
>
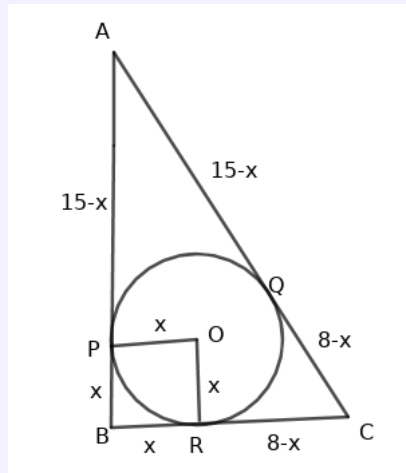> b) $AB = AC, AB - AP = AC - AQ$
> $BP = CQ \rightarrow BR = CR$

2) In the figure $\angle B = 90°$, $AB = 15$ cm , $BC = 8$ cm .

a) Draw a rough figure , mark O as the centre . Suggest a suitable name to $PORB$

b) If $PB = x$ then fin the length AP, AQ, CR, CQ

c) What is the radius of the circle.
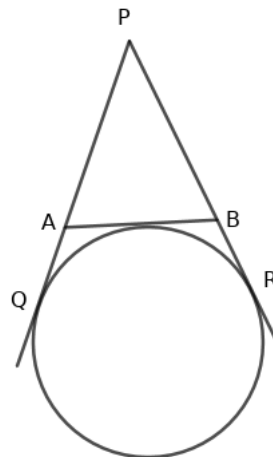
a) See the figure

$PORB$ is a square

b) $AP = 15 - x, AQ = 15 - x, CR = 8 - x, CQ = 8 - x$

c) Hypotenuse of the right triangle is $AC = \sqrt{15^2 + 8^2} = 17$

$15 - x + 8 - x = 17, 23 - 17 = 2x, 2x = 6, x = 3$

Radius of the circle is 3cm

3) In the figure PQ and PR are the tangents from P outside the circle. $PQ = 24$cm $AQ = 10$cm , $BR = 8$cm then


a) What is the length of PR

b) What is the length of AB?

c) What is the perimetre of $\triangle PAB$

d) Prove that $PQ + PR =$Perimeter of $\triangle PAB$.

a) $PQ = PR$(Tangents from the outer point to the circle are equal)
∴ $PR = 24$cm

b) Mark C , the point where the line AB touches the circle.
$AQ = AC = 10$
$BR = BC = 8$
$AB = 10 + 8 = 18$

c) $PA + AB + PB = 14 + 18 + 16 = 48$cm

d) $PQ + PR = (PA + AQ) + (PB + BR)$
$(PA + AC) + (PB + BC)$
$(PA + PB + AC + BC)$
$PA + PB + AB$
Perimetre of the triangle.

4) In the figure PA, QB are the parallel tangents. PQ touches the circle at R
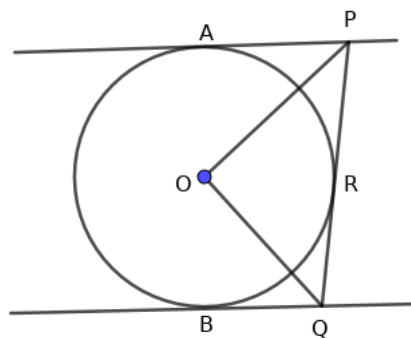
a) Prove that $\triangle PAO$ and $\triangle PRO$ are equal triangles

b) Prove that $\triangle QBO$ and $\triangle QRO$ are equal triangles

c) Find $\angle POQ$
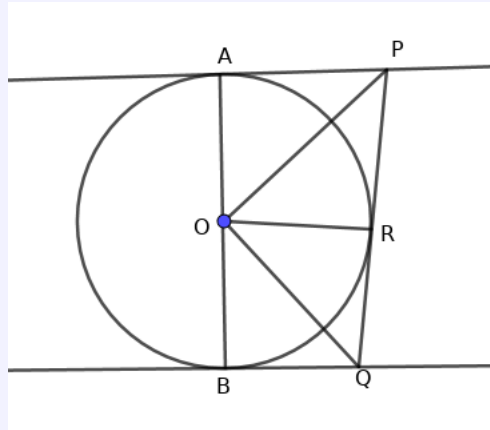
a) $OA = OR, PA = PR, OP$ is common. $\triangle PAO$ and $\triangle PRO$ are equal triangles

b) $OB = OR, QB = QR, OQ$ is common. $\triangle QBO$ and $\triangle QRO$ are equal triangles

c) $\angle AOP = \angle ROP, \angle QOR = \angle QOB$
$\angle AOP + \angle ROP + \angle QOR + \angle QOB = 180°$
$2 \times \angle ROP + 2 \times \angle QOR = 180°$
$\angle ROP + \angle QOR = 90°, \angle POQ = 90°$

5) XP, XQ are the tangents to the circle from X outside the circle. The line AB touches the circle at R
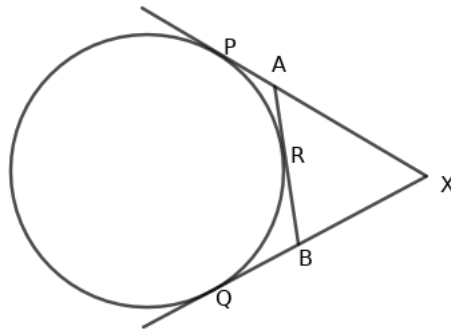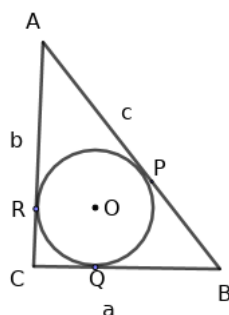

Prove that $XA + AR = XB + BR$

Answers

⋆ $XP = XQ$ Tangents from outer point to the circle are equal.

⋆ $XA + AP = XB + BQ$

⋆ Since $AP = AR$ and $BQ = BR$, $XA + AR = XB + BR$

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Session 81 │Tangents 7 │ Worksheet 81

1) In the right triangle $\triangle ABC$, $\angle C = 90°$. a, b, c are the sides opposite to A, B and C. A circle touches sides of the triangle.

   a) If the radius of the circle is r write the lengths PB and AP

   b) Prove that $r = \frac{a+b-c}{2}$

   c) If the perpendicular sides are 6cm and 8 cm then find the length of the hypotenuse

   d) If the perpendicular sides are 6 and 8 cm mthen find the radius of the circle .
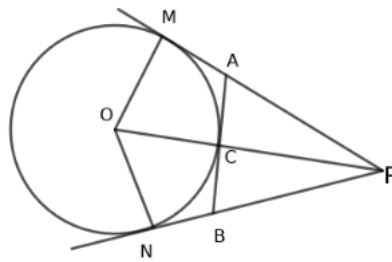

Answers

   a) See diagram
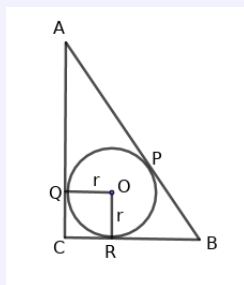
   $ORCQ$ is a square. $OR = OQ = r$. $\therefore CR = CQ = r$
   $BR = a - r, BP = a - r$
   $AQ = AP = b - r$

  b) $c = PA + PB = b - r + a - r$
    $c = a + b - 2r, 2r = a + b - c, r = \frac{a+b-c}{2}$

  c) $AB = \sqrt{6^2 + 8^2} = 10$

  d) $r = \frac{a+b-c}{2} = \frac{6+8-10}{2} = 2$cm

2) In the figure PM, PN are the tangents to the circle.The distance from P to the centre of the circle is 13cm, radius of the circle is 5cm . The line AB touches the circle at C
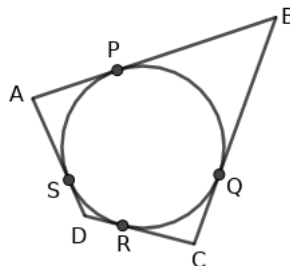

a) Find the length of PM and PN

b) If $AM = x$ then find AC and AP

c) Find x

d) What is the length of AB

Answers

a) $PM = PN = \sqrt{13^2 - 5^2} = 12$cm

b) If $AM = x$ then $AC = x, AP = 12 - x$

c) Since OC is perpendicular to AB, $\triangle ACP$ is a right triangle.
$(12 - x)^2 = x^2 + (13 - 5)^2$
$12^2 + x^2 - 24x = x^2 + 8^2, 24x = 80, x = \frac{80}{24}$ cm

d) $AB = 2 \times \frac{80}{24} = \frac{20}{3}$ cm

3) The sides of $ABCD$ touches the circle at P, Q, R, S

a) Prove that $AB + CD = AD + BC$

b) If $AB = 12$cm $CD = 8$cm , $AD = 14$cm then find BC.

Answers

a)

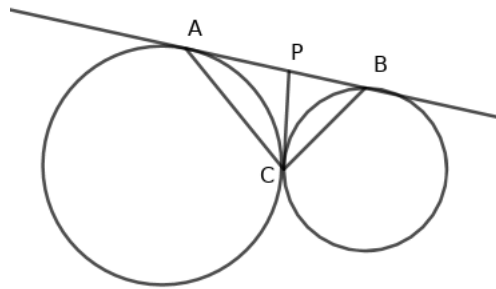$$AP = AS \tag{1}$$

$$BP = BQ \tag{2}$$

$$DR = DS \tag{3}$$

$$CR = CQ \tag{4}$$

Adding these equations , $AP + BP + DR + CR = AS + BQ + DS + CQ$
$(AP + BP) + (DR + CR) = (AS + DS) + (BQ + CQ)$
$AB + CD = AD + BC$

b) $12 + 8 = 14 + BC, BC = 20 - 14 = 6$cm

4) In the figure ,the line AB touches a circle. CP is the common tangent .

a) Prove that $PA = PB$

b) Prove that $\triangle ABC$ is a right triangle.

c) If $AC = BC = 10$cm then find the length AB

Answers

a) $PA = PC, PB = PC$(Tangents from outer point to a circle are equal)
$PA = PB$

b) In $\triangle APC$, two sides $PA = PC$. $\therefore$ the angles opposite to equal sides are equal. $\angle A = \angle C = x$
In $\triangle BPC$, two sides $PB = PC$. $\therefore$ the angles opposite to equal sides are equal. $\angle B = \angle C = y$
Consider $\triangle ABC$, $\angle A + \angle B + \angle C = 180°$, $x + x + y + y = 180, 2x + 2y = 180, x + y = 90°$. $\triangle ABC$ is a right triangle.

c) $\triangle ABC$ is a $45^circ - 45° - 90°$ triangle. $AB = 10\sqrt{2}$cm

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Wisgrow Math A – Plus

Session 82 │Tangents 7 │ Worksheet 82

1) In the figure $ABCD$ is a parallelogram .The circle touches the sides at P, Q, R, S
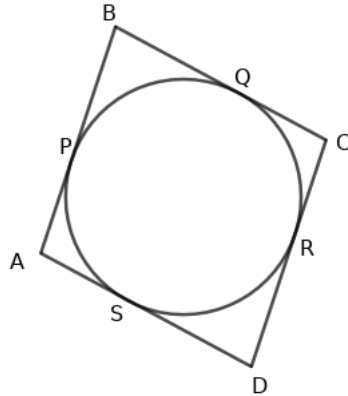
a) Prove that $AD + BC = AB + CD$

b) Prove that $ABCD$ is a rhombus

Answers

a) $BQ = BP, CQ = CR, AS = AP, DS = DR$
Adding these equations,
$(BQ + CQ) + (AS + DS) = (BP + CR) + (AP + DR)$
$BC + AD = AB + CD$

b) Since $ABCD$ is a parallelogram , $AD = BC, AB = CD$
$BC + AD = AB + CD$ becomes $2AD = 2AB, AD = AB$
That is ,
$AB = BC = CD = AD$
$ABCD$ is a rhombus.

---

2) In the quadrilateral $ABCD, \angle D = 90°$
The sides AB, BC, CD, DA touches the circle at P, Q, R, S.
$BC = 38$ cm , $CD = 25$ cm , $BP = 27$cm
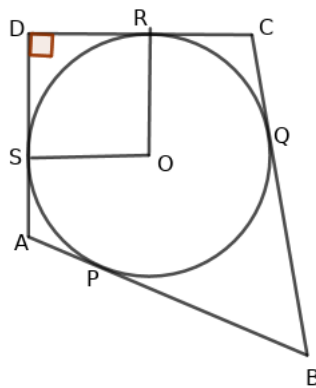
a) Prove that $ORDS$ is a square

b) Find the length of CQ

c) What is the side of $ORDS$?

d) What is the radius of the circle which touches the sides?

Answers

a) CD is tangent and OR is radius . Therefore CD is perpendicular to OR.
   Similarly AD is perpendicular to OS, $\angle D = 90°$. In $ORDS$, $\angle O$ will be $90°$
   Also, $DR = DS$. All sides of $ORDS$ are equal, all angles are $90°$. That is $ORDS$ is a square.

b) $BP = BQ = 27$, $BC = 38$,$QC = 38 - 27 = 11$cm

c) $CQ = CR = 11$cm, $DR = CD - 11 = 25 - 11 = 14$cm
   Side of $ORDS$ is 14 cm

d) Since $ORDS$ is a square radius is its side. Radius $= 14$cm

3) In the figure PQ is the common tangent to the circles . Radius of the big circle is 6cm , radius of the small circle is 3cm . The distance between the centres is 15cm.
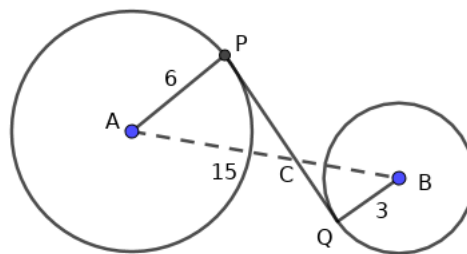
a) Are the tringles APC and BQC similar ?

b) What is the length AC and BC?

c) What is the length of PQ?
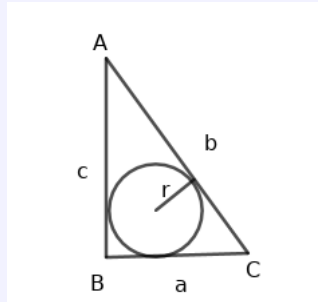
Answers

a) Consider $\triangle APC$ and $\triangle BQC$
   $\angle P = \angle Q$, $\angle ACP = \angle BCQ$. So $\triangle APC$ and $\triangle BQC$ are similar

b) $\frac{AP}{BQ} = \frac{AC}{BC}$
   If $AC = x$ then $BC = 15 - x$
   $\frac{6}{3} = \frac{x}{15-x}$,$6(15 - x) = 3x$, $90 - 6x = 3x$, $90 = 9x$, $x = 10$cm
   $AC = 10$cm , $BC = 5$cm

c) $PC = \sqrt{10^2 - 6^2} = 8$cm, $CQ = \sqrt{5^2 - 3^2} = 4$.
   $PQ = 8 + 4 = 12$cm

4) In the traingle ABC, $\angle B = 90°$, area of the triangle 30cm, sum of the perpendicular sides is 17cm

a) What is the length of AC?

b) What is the radius of the circle ?

Answers

a) $a + c = 17, \frac{1}{2} \times a \times c = 30, ac = 60$
   $(a + c)^2 = a^2 + c^2 + 2ac, 17^2 = a^2 + c^2 + 120, a^2 + c^2 = 169$
   Since $a^2 + c^2 = b^2, b^2 = 169, b = 13$

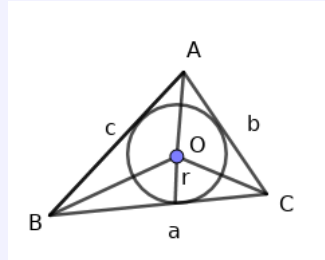b) $r = \frac{a+c-b}{2} = \frac{17-13}{2} = 2$cm

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Session 83 │Tangents 8 │ Worksheet 83

1) In $\triangle ABC$, a, b, c are the sides opposite to A, B and C .
   r is the radius of the circle touches the sides, area of the triangle is A,half of its perimetre is s
   Prove that $A = rs$

Answers

a) see diagarm

In the diagram
Area of $\triangle ABC$= area $\triangle BOC$+area of $\triangle OAC$+area of $\triangle OAB$
$A = \frac{1}{2}a \times r + \frac{1}{2}b \times r + \frac{1}{2}c \times r$
$A = r(\frac{a+b+c}{2}) = rs$

---

2) Sides of a triangular metal sheets are 26cm, 24cm and 10cm

   a) What kind of triangle is this ?

   b) What is the perimetre of this triangle?

   c) What is the area of this triangle?

   d) Can this metal sheet is used to cover the upper open face of a cyclindrical vessel of radius 5cm?

Answers

a) $24^2 + 10^2 = 576 + 100 = 676 = 26^2$.
   This is a right triangle

b) Perimeter$26 + 24 + 10 = 60$cm

c) Area $\frac{1}{2} \times 24 \times 10 = 120$ sq.cm

d) Radius of incircle $r = \frac{A}{s}$
   $s = \frac{26+24+10}{2} = 30$
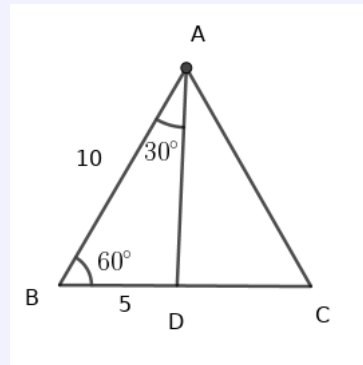 $r = \frac{120}{30} = 4$cm
   Radius of the upper end of the cylinder is 5cm .It is more than radius of incircle. Not possible to cover.

---

3) Side of an equilateral triangle is 10cm

   a) What is the altitude of this triangle?

   b) Find the perimetre and area of the triangle

   c) Find the radius of the incircle of this triangle.

a) Look at the picture

$\triangle ABD$ is a $30° - 60° - 90°$ right triangle . $AD = 5\sqrt{3}$cm

b) Perimetre 30cm , area $\frac{1}{2} \times 10 \times 5\sqrt{3} = 25\sqrt{3}$sq.cm

c) $A = rs \rightarrow r = \frac{A}{s} = \frac{25\sqrt{3}}{15} = \frac{5\sqrt{3}}{3}$ cm

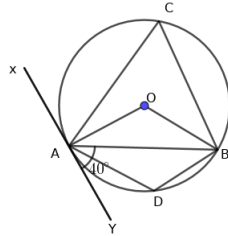**4)** Draw an angle and construct a circle which touches the arms of the angle.

Answers

a) Draw the angle and bisect it

b) Mark a point O on the bisector. Draw perpendicular from the point to the arm.

c) Draw a circle with O as the center and perpendicular distance as the radius

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Session 84 │Tangents 9 │ Worksheet 84

1) In the figure AB is a chord , line XY is a tangent at A. If $\angle YAB = 40°$ then
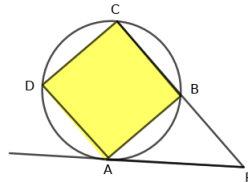

    a) Find $\angle ACB$

    b) Find $\angle AOB$?

    c) Find $\angle ADB$?
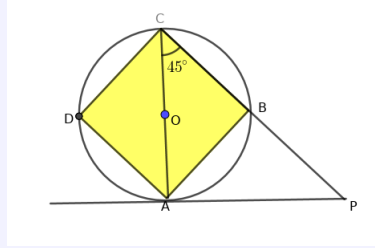
Answers

    a) $\angle ACB = 40°$

    b) $\angle AOB = 2 \times 40 = 80°$

    c) $\angle ADB = 180 - 40 = 140°$

**2)** $ABCD$ is a square .The vertices of the square are on the circle. Tangent at A meet CB produced at P.

    a) What is $\angle BAP$?

    b) What is $\angle ABP$?

    c) What is $\angle APB$?

    d) If $AP = 20$cm then what is the area of the square ?

a) Draw diagonal AC of the square
   $\angle ACB = 45°, \angle BAP = 45°$

b) Since $ABCD$ is a square $\angle ABC = 90°, \therefore \angle ABP = 90°$

c) $\angle APB = 180 - (90 + 45) = 45°$

d) Since $\triangle ABP$ is a $45° - 45° - 90°$ triangle and $AP = 20$cm, $AB = \frac{20}{\sqrt{2}} = 10\sqrt{2}$cm
   Area $= (10\sqrt{2})^2 = 200$ sq.cm

3) AP is the tangent of a circle with centre O. The angle between AB and tangent is $140°$
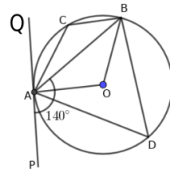
a) What is the measure of $\angle ACB$

b) What is the central angle of arc ADB?

c) What is the measure of $\angle ADB$

d) Name an angle in the figure equal to $\angle ADB$

Answers

a) $\angle ACB = 140°$

b) The central angle of ADB is $2 \times 140 = 280°$

c) $\angle ADB = 40°$

d) $\angle ADB = \angle QAB$

4) Draw an equilateral triangle of side 4cm.Construct a circle touches its sides (Incircle)
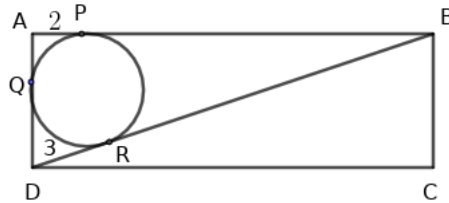
Answers

a) Draw the triangle

b) Draw the bisector of the angles. (bisectors of two angles are necessary)

c) Mark the intersecting point of the bisectors.Draw perpendicular from this point to the side

d) Draw a circle with this point as the center and perpendicular distance as the radius .

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Session 85 │ Tangents 10 │ Worksheet 85

1) In the figure $ABCD$ is a rectangle .A circle touches the triangle formed by two sides and diagonal at P, Q, R. If $AP = 2$cm, $DQ = 3$cm then


   a) What is AD?

   b) What is the length of the side AB?

   c) What is the length of the diagonal of the rectangle?

   d) What is the radius of the circle?

Answers

   a) $DR = DQ = 3$cm , $AP = AQ = 2$cm
      $AD = 3 + 2 = 5$ cm

   b) Let $BP = BR = x$, $AB = x + 2, BD = x + 3$
      $(x + 3)^2 = (x + 2)^2 + 5^2$, $x^2 + 6x + 9 = x^2 + 4x + 4 + 25$
      $2x = 20, x = 10$cm
      $AB = 10 + 2 = 12$cm

   c) $BD = 13$cm

   d) Mark the centre of the circle O.
      $OQAP$ is a square. $OP = OQ = 2$cm , Radius of the circle is 2cm

2) ABC is an equilateral triangle.Tangents are drawn at the vertices to the circumcircle.These tangents form another triangle PQR.
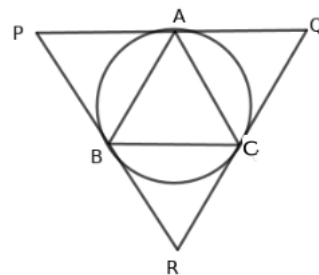

   a) Prove that PQR is an equilateral triangle.

   b) If the perimetre of ABC is 12cm then what is the perimetre of $\triangle PQR$.

   c) How many times the area of PQR is that of ABC?

a) $\triangle ABC$ is an equilateral triangle.
$\angle A = \angle CBR = \angle BCR = 60°$, $\angle R = 60°$
$\angle B = \angle CAQ = \angle ACQ = 60°$, $\angle Q = 60°$,$\angle P = 60°$. $\triangle PQR$ is an equilateral triangle.

b) $PACB$ is a parallelogram.$BC = PA$
$QABC$ is a parallelogram $BC = AQ$ $\therefore PQ = 2 \times BC$,
Similarly $PR = 2 \times AC, QR = 2 \times AB$
Perimetre of $\triangle PQR = 2 \times 12 = 24$cm

c) $PACB, QABC, RBAC$ are equal parallelograms. Each one can be divided into two equal triangles.We can see four equal triangles in the picture.
Area of $\triangle PQR = 4$ times the area of $\triangle ABC$

3) In the figure AP is the diametre of the circle.$AB = 6\sqrt{3}$cm $PB = 6$ cm
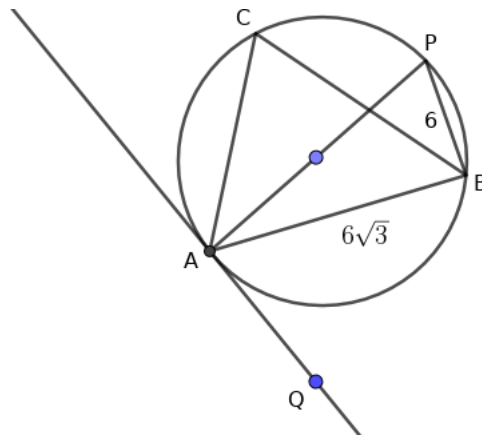
a) What is the radius of the circle?

b) What are the angles of $\triangle APB$?

c) What is the measure of $\angle ACB$?

d) What is the measure of $\angle BAQ$?

a) $AP = \sqrt{(6\sqrt{3})^2 + 6^2} = 12$. Radius of the circle is 6 cm

b) Since AP is the diametre $\angle B = 90°$. Sides of $\triangle APB$ are in the ratio $1 : \sqrt{3} : 2$. It is a $30° - 60° - 90°$ triangle.
$\angle A = 30°, \angle P = 60°, \angle B = 90°$

c) $\angle ACB = 60°$(angle in the same arc)

d) $60°$

5) In the figure $QR$is the diametre of the circle, PA is the tangent ,$\angle RPA = 30°$.
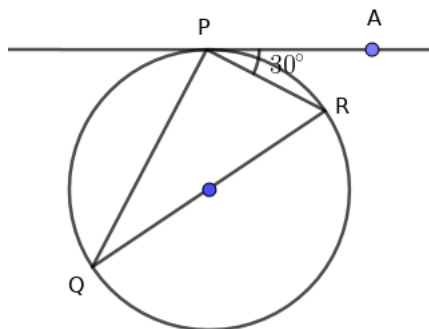
a) What is the measure of $\angle PQR$?

b) What is the measure of $\angle PRQ$?

c) What is the acute angle formed by PA with PQ?

Answers
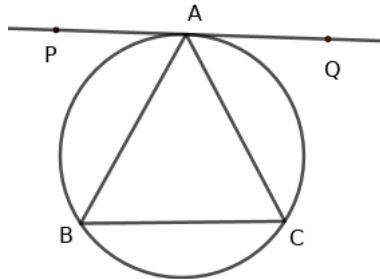
a) $\angle PQR = 30°$

b) $\angle PRQ = 60°$

c) Acute angle is $60°$

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Session 86 │Tangents 11│ Worksheet 86

1) In $\triangle ABC$ $AB = AC$, a tangent PQ is drawn through A to its circumcircle.Prove that PQ is parallel to BC.

Answers

★ Since $AB = AC$ opposite angles are equal. $\angle B = \angle C$

★ $\angle PAB = \angle C$(In a circle the angle between a chord and tangent at its end is equal to the angle in the other side of the chord on the circle.)

★ Since $\angle B = \angle C$, $\angle PAB = \angle B$.The equality of altrenate angles shows that BC is parallel to the tangent at A.
PQ is parallel BC

2) In $\triangle ABC$ a tangentPQ is drawn through A to the circumcircle of the triangle.If BC is parallel to PQ then prove that $AB = AC$
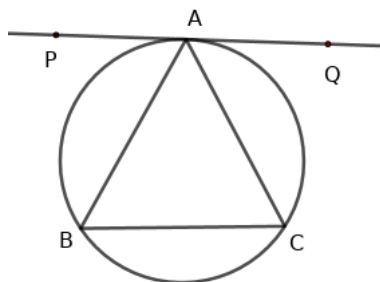
Answers

★ $\angle PAB = \angle C$(In a circle the angle between a chord and tangent at its end is equal to the angle in the other side of the chord on the circle.)

★ $\angle PAB = \angle B$ (Line PQ is parallel to BC, alternate angles are equal)

★ $\angle PAB = \angle B = \angle C$
$\angle B = \angle C$
Sides opposite to equal angles are equal. $AB = AC$

3) In the figure PQ is the diametre of the circle , MN is the tangent to the circle at P.
If $\angle RPN = 50°$
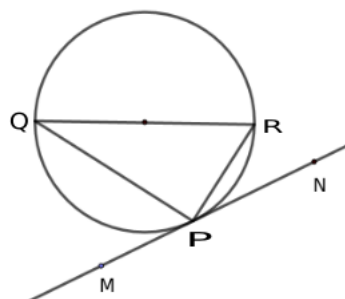
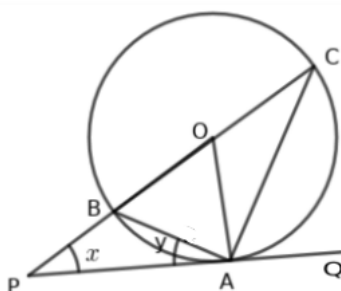a) What is the measure of $\angle PQR$?

b) What is the measure of $\angle PRQ$?

c) What is the measure of $\angle QPM$?

Answers

a) $\angle PQR = 50°$
(In a circle the angle between a chord and tangent at its end is equal to the angle in the other side of the chord on the circle.)

b) $\angle QPR = 90°, \angle PRQ = 90 - 50 = 40°$

c) $\angle QPM = \angle PRQ = 40°$

4) In the figure BC is the diametre of the circle, PA is a tangent .If $\angle APB = x, \angle PAB = y$ then

a) What is the measure of $\angle BCA$ and $\angle CAQ$

b) Whatn is the measure of $\angle ABC$?

c) Find $x + 2y$

Answers

a) $\angle BCA = y$ (In a circle the angle between a chord and tangent at its end is equal to the angle in the other side of the chord on the circle.)
Since $OC = OA$ opposite angles are equal .$\angle OAC = y$.
Radius is perpendicular to the tangent $\angle CAQ = 90 - y$

b) $\angle ABC = 90 - y$

c) $\angle PBA = 180 - (90 - y) = 90 + y$
In triangle $PBA, 90 + y + x + y = 180, x + 2y = 90°$

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Wisgrow Math A – Plus

Session 87 │Tangents 12 │ Worksheet 87

1) $ABCD$ is a cyclic quadrilaeral . PQ is a tangent at C. BD is the diametre of the circle.
$\angle DCP = 40°, \angle ABD = 60°$
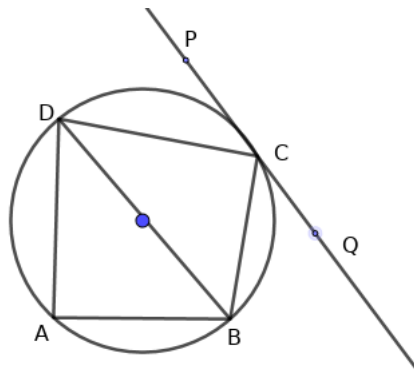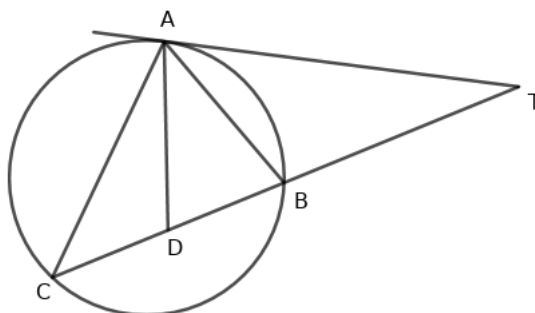

   a) What is the measure of angle DBC?

   b) What is the measure of angle BCQ?

   c) What is the measure of angle BDC?

   d) What is the measure of ADB?

Answers

   a) $\angle DBC = 40°$

   b) $\angle BCD = 90°, \angle BDC = 90 - 40 = 50°, \angle BCQ = 50°$

   c) $\angle BDC = 50°$

   d) Since $DAB = 90°$, $\angle ADB = 90 - 60 = 30°$

2) Tangent from an outer point T to the circle is AT. B and C are the points a line from T cut the circle.In triangle ACB, AD is the bisector of $\angle A$, $\angle A = 70°$,$\angle CAD = 40°$
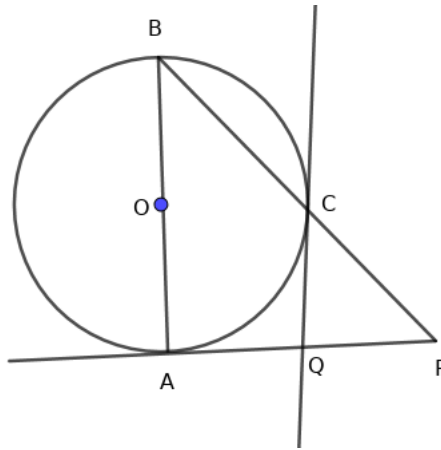

   a) What is the measure of $\angle ADB$?

   b) What is the measure of $\angle BAT$?

   c) Find the angles of $\triangle DAT$

3) AB is the diametre of the circle, PA is a tangent . The line PB cut the circle at C, also CQ is the tangent at C
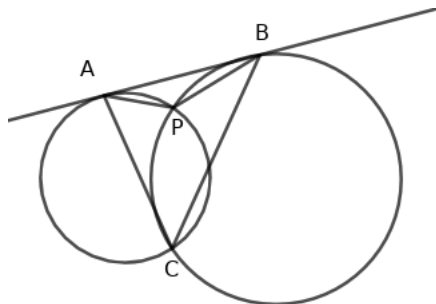

a) If AC is drawn then what is the measure of $\angle ACB$?

b) If $\angle ACQ = x$ then what are the acute angles of $\triangle ABC$?

c) Is $AQ = QC$? Why?

d) Prove that the line CQ bisects AP.

4) Two circles intersect at P, C.AB is the common tangent.

Prove that $\angle APC + \angle ACB = 180°$

5) In the figure AB is the diameter of the circle. P is a point on AB produced.The line from P touches the circle at C.If $\angle CAB = 30°$ and the radius of the circle is 6 cm
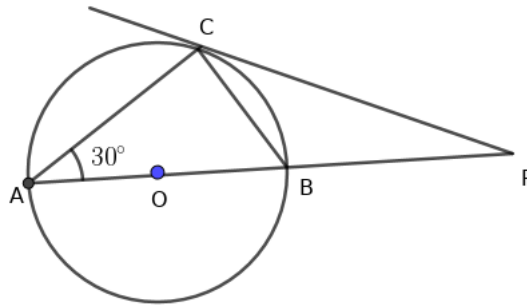
a) Find the lengths AC and BC

b) Prove that $BP = BC$.

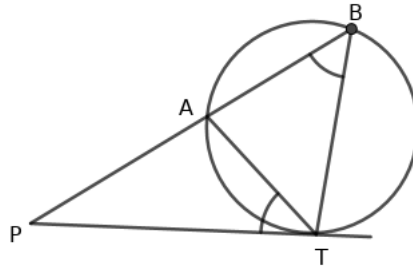**Answers**

★ $AB = 12$ cm . $\triangle ACB$ is a $30° - 60° - 90°$ right triangle. $BC = 6$ cm , $AC = 6\sqrt{3}$ cm

★ $\angle BCP = 30°, \angle CBP = 180 - 60 = 120°$,
In $\triangle PBC$, $\angle P = \angle C = 30°$.
Opposite sides are equal. $BP = BC$

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Wisgrow Math A – Plus

Session 88 │Tangents 13 │ Worksheet 88

1) Let P be a point outside the circle. PT is a tangent to the circle and another line from P cut the circle at A and B.
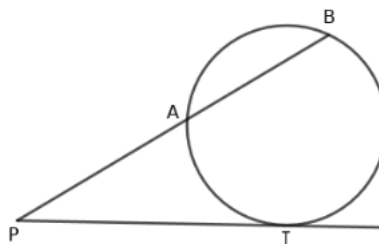

a) What is the relation between $\angle PTA, \angle PBT$?

b) Are $\triangle PTA, \triangle PBT$ similar

c) Prove that $PA \times PB = PT^2$

Answers

a) $\angle PTA = \angle PBT$(Angle between a chord of a circle and tangent at the end in one side is equal to angle in the other part of the circle)

b) $\angle PTA = \angle PBT, \angle P$ is common .$\triangle PTA \infty \triangle PTB$ are similar triangles

c) Sides opposite to the equal angles are proportional.
$\frac{PT}{PB} = \frac{PA}{PT}, PA \times PB = PT^2$

2) PT is a tangent from an outer point P to the circle.

Another line from P intersect the circle at A and B. If the length of the chord PB is 16cm and $AB = 7$ cm then
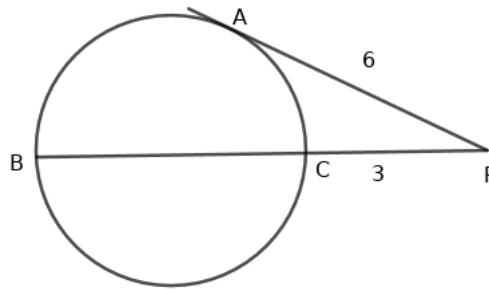

a) What is the length PA?

b) What is the relation between PA, PB, PT?

c) What is the length of the tangent PT?

d) What is the length of the other tangent from $P$to the circle.

Answers

a) $PA = 16 - 7 = 9$cm

b) $PA \times PB = PT^2$

c) $9 \times 16 = PT^2, PT = 3 \times 4 = 12$ cm

d) 12cm

3) BC is the diametre of the circle. P is a point on BC produced.
Tangent PA is drawn from P to the circle. If $PA = 6$cm and $PC = 3$cm then

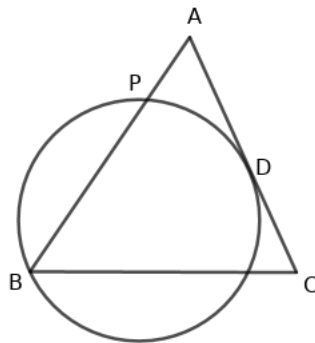   a) What is the length PB?

   b) Find the radius of the circle.

Answers

   a) $PB \times PC = PA^2$
   $PB \times 3 = 6^2, PB = 12$cm

   b) $BC = 12 - 3 = 9$cm
   radius $= \frac{9}{2} = 4.5$cm

4) In $\triangle ABC$, $AB = AC$, A circle passing through B intersect AB at P. The circle touches AC at its mid point D

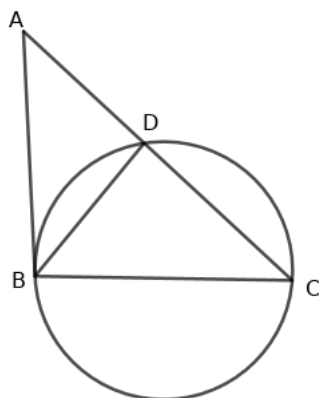Prove that $4AP = AB$

Answers

   ⋆ $AB \times AP = AD^2$

   ⋆ $AB \times AP = (\frac{AC}{2})^2$
   $AB \times AP = \frac{AC^2}{4}$

   ⋆ $AB = AC \rightarrow AB \times AP = \frac{AB^2}{4}$

   ⋆ $AP = \frac{AB}{4}$
   $AB = 4 \times AP$

5) In the figure BC is the diametre of the circle and AB is a tangent.

a) Write the relation between AC, AD and AB
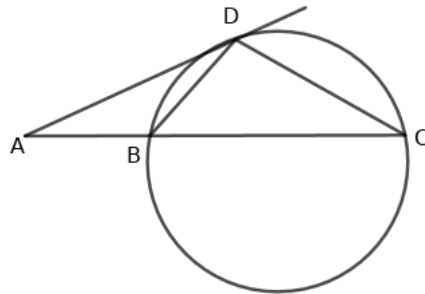
b) Prove that $AC \times CD = BC^2$

Answers

a) $AC \times AD = AB^2$

b) Consider$\triangle ABC$ and $\triangle BDC$.
$\angle ABC = 90°$(Angle between diameter and tangent)
$\angle BDC = 90°$(angle in the semicircle)
$\angle ABC = \angle BDC, \angle C$ is common
$\triangle ABC$ are similar $\triangle BDC$
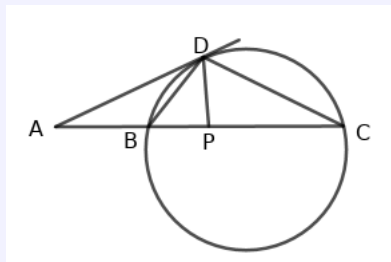$\frac{BC}{CD} = \frac{AC}{BC}$
$AC \times CD = BC^2$

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[1]John P A, jpavpz@gmail.com, sjpuzzles@gmail.com ,9847307721

Session 89 │ Tangents 14 │ Worksheet 89

1) In the figure $AB = BD$, also the line AD is a tangent from A.


   a) What is the relation between AB, AC and AD

   b) Prove that $AB \times AC = CD^2$

   c) What kind of triangle is $\triangle ACD$?

   d) If $\angle BAD = 30°$ and perpendicular distance from D to AC is 12 then what is the length of tangent AD?

Answers

   a) $AB \times AC = AD^2$

   b) Consider $\triangle ABD$ and $\triangle ACD$.
   $\angle ADB = \angle ACD$(Angle between chord and the tangent at the end is equal to angle in the other side of the chord)
   Since $AB = BD$, opposite angles are equal. .$\angle BAD = \angle ADB$
   Therefore $\angle ADB = \angle ACD \rightarrow AD = CD$ $AB \times AC = AD^2 \rightarrow AB \times AC = CD^2$

   c) In $\triangle ACD$, $\angle A = \angle C$,So opposite sides are equal . This is an isosceles triangle.

   d) $\triangle APD$is a $30° - 60° - 90°$triangle. Side opposite to $30°$ is 12cm .Therefore $AD = 24$cm . Length of tangent is24cm

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2) In the figure $AC$is the diametre and BA is a tangent to the circle. The line BC intersect the circle at P If the radius of the circle is 2.5cm and the length of tangent is 12cm
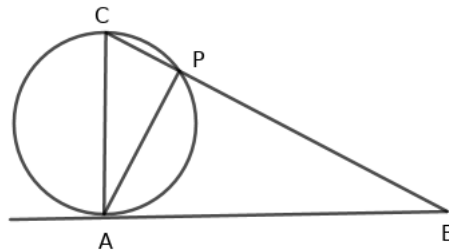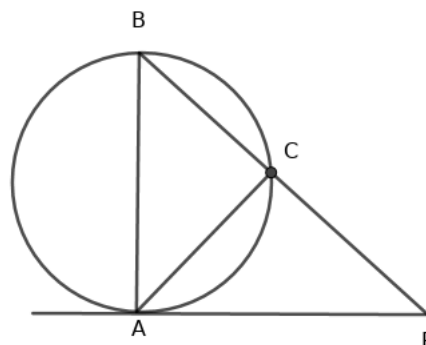
a) What is the length BC?

b) What is the length PC?

c) What is the length AP?

Answers

a) $\triangle CAB$ is a right triangle . (Diameter is perpendicular to the tangent)
$BC^2 = AC^2 + AB^2 = 5^2 + 12^2 = 169, BC = 13$cm

b) $BP \times BC = BA^2, BP \times 13 = 12^2, BP = \frac{144}{13} = 11.08$ cm
$PC = 13 - 11.08 = 1.92$cm

c) $AP^2 = AC^2 - PC^2, AP^2 = 5^2 - 1.92^2 = 25 - 3.68 = 21.32$cm, $AP = 4.6$cm

3) PA is a tangent from the outer point to a circle of diametre AB.The line PB intersect the circle at C.If the radius of the circle is 5cm and $AC = 6$cm then
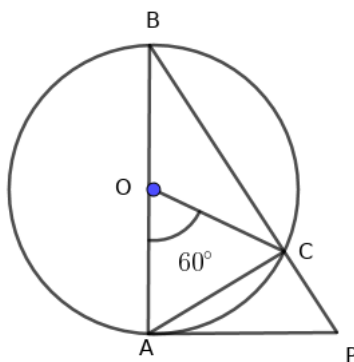

a) What is the length BC?

b) Find PC

Answers

a) $\triangle ACB$ is a right triangle . $BC = \sqrt{AB^2 - AC^2} = \sqrt{10^2 - 6^2} = 8$cm

b) $\triangle BAP$ is a right triangle .Take $PC = x, PA = y$
$(8 + x)^2 = y^2 + 10^2$
$\triangle ACP$ is a right triangle . $y^2 = 6^2 + x^2$
$10^2 + y^2 = (8 + x)^2 \rightarrow 10^2 + 6^2 + x^2 = (8 + x)^2, 136 + x^2 = 64 + x^2 + 16x, 16x = 72, x = \frac{9}{2} = 4.5$cm . $PC = 4.5$cm

c) $y = \sqrt{6^2 + 4.5^2} = 7.5$cm . Length of tangent 7.5 cm

4) O is the centre of a circle of diametre AB. PA is a tangent from P to the circle, line PB intersect the circle at C.If $\angle AOC = 60°, AC = 6$ cm then

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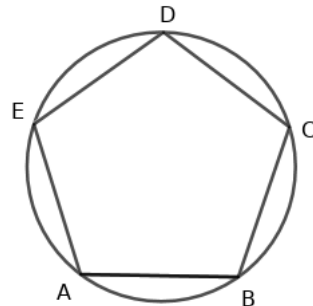a) What is the measure of $\angle ABC$?

b) What is the diametre of the circle?

c) What is the length BC?

d) What is the length PC?

<div style="background:#002fa7;color:#fff;padding:4px 8px;font-weight:bold">Answers</div>

a) $\angle ABC = 30°$

b) $\triangle OAC$ is an equilateral triangle . $OA = OC = AC = 6$cm . Diametre 12cm

c) $\triangle ACB$ is a right triangle . $AB^2 = AC^2 + BC^2$, $BC^2 = 144 - 36 = 108$, $BC = \sqrt{108} = 6\sqrt{3}$cm

d) Consider $\triangle APC$. $\angle A = 90 - 60 = 30°$, $\angle C = 90°$, $\angle P = 60°$.This is a $30° - 60° - 90°$right triangle .
Side opposite to $60°$ is 6cm . $PC = \frac{6}{\sqrt{3}} = 2\sqrt{3}$cm . $PC = 2\sqrt{3}$cm note : Length of tangent is $4\sqrt{3}$cm
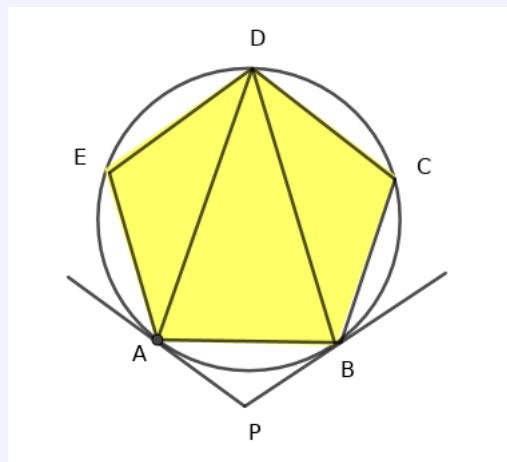
Wisgrow Math A – Plus

Session 90 │Tangents 15 │ Worksheet 90

1) $ABCDE$ is a regular pentagon and its circumcircle. The tangents to the circumcircle at A and B intersect at P.


   a) Draw AD, what are the angles of $\triangle ADE$?

   b) What is the measure of $\angle ADB$?

   c) Tangents at A, B intersect at P. What is the measure of $\angle BAP$?

   d) What is the measure of $\angle APB$?

Answers

   a) Angle sum of an n sided polygon $= (n-2) \times 180°$
      $\angle AED = \frac{(5-2) \times 180}{5} = 108°$
      $ED = EA \rightarrow \angle EAD = \angle EDA = 36°$

   b) $\angle ADB = \frac{108}{3} = 36°$

   c) $\angle BAP = 36°$ (Angle between chord AB and tangent AP is equal to angle in the other side of the chord on the circle)

   d) $\angle APB = 180 - (36 + 36) = 108°$

2) In $\triangle ABC$ the sides AB, BC and AC touches a circle at D, E, F.
   If $AB = 12$cm ,$BC = 8$ cm ,$AC = 10$cm then find AD, BE and CF.
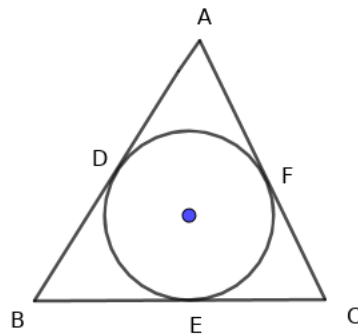
Answers

⋆ Since $AD = x$, $AF = x$.(Tangents from outer point to the circle are equal)

⋆ $BD = 12 - x$, $BE = 12 - x$, $CF = 10 - x$, $CE = 10 - x$

⋆ $BC = BE + EC$, $8 = 12 - x + 10 - x$, $8 = 22 - 2x$, $2x = 14$, $x = 7$cm

⋆ $AD = 7$cm , $BE = 12 - x = 5$cm , $CF = 10 - x = 10 - 7 = 3$cm

3) A semicircle is drawn with AB as the diametre in the square $ABCD$. DE touches the semicircle at P. If the side of the square is of length 1unit
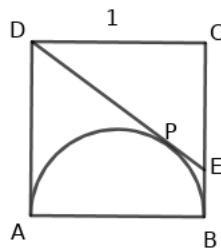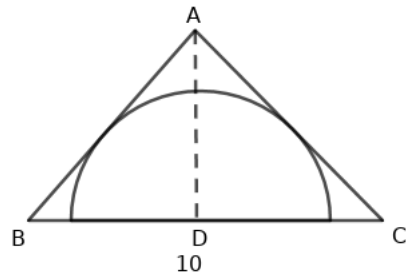

a) What is the length DP?

b) If $PE = x$ then find the equation connecting DE, CD and CE

c) Find the length of the line DE.

Answers

a) side of the square is 1. Since $DA = 1$, $DP = 1$

b) If $PE = x$ then $DE = 1 + x$. $CE = 1 - x$, $CD = 1$
$(1 + x)^2 = (1 - x)^2 + 1^2$

c) $4x = 1 \rightarrow x = \frac{1}{4} = 0.25$, $DE = 1.25$

4) In the figure $AB = AC$, $BC = 10$ cm , altitude from A to BC is 12 cm.The centre of the semicircle is on BC and the semicircle touches the sides AB and AC.
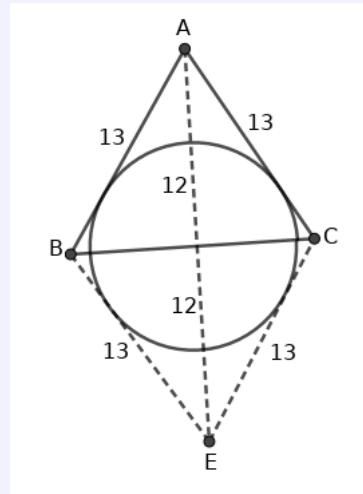
a) What is the perimetre of $\triangle ABC$?

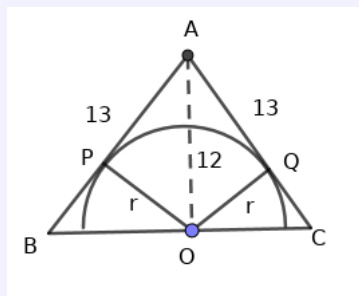b) What is the area of triangle ABC?

c) What is the radius of semicircle?

a) $AB = \sqrt{5^2 + 12^2} = 13$cm

b) Perimetre $13 + 13 + 10 = 36$cm

c) See the diagarm

$ABCE$ is a rhombus .Semicircle is completed into circle. Perimetre $= 13 \times 4 = 52$cm , Area $60 \times 2 = 120$ sq.cm

Radius of the incircle $r = \frac{A}{s} = \frac{120}{26} = 4.6$cm

Another method

Let r be the radius . It is the distance from center to the touching point$r$.

Sum of the area of ABO and area of triangle ACO is equal to area of ABC

$\frac{1}{2} \times 13 \times r + \frac{1}{2} \times 13 \times r = \frac{1}{2} \times 10 \times 12$

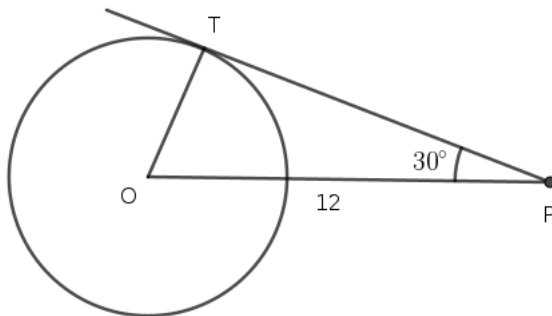$26r = 120, r = \frac{120}{26} = 4.6$സെ.മീറ്റർ

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Evaluation

Mathematics X

Scores 25

Times 1 hour

Choose the correct answer. 1 mark each

1) In the figure PT is a tangent to the circle from a point at the distance 12cm from the center of the circle. If $\angle OPT$ is $30°$ then what is the radius of the circle?

(a) 6cm   (b) 8cm   (c) 10cm   (d) 12cm

Answers

Ans :6cm

2) A quadrilateral is formed by two tangents from an exterior point and radii to the touching points. If an angle of this quadrilateral is $40°$ then what is its opposite angle?

(a) $140°$ (b) $120°$ (c) $100°$ (d) $110°$

Answers

Ans :$180 - 40 = 140°$

3) The perpendicular sides of a right triangle are 8 cm and 6 cm. What is the radius of the circle touches its sides.
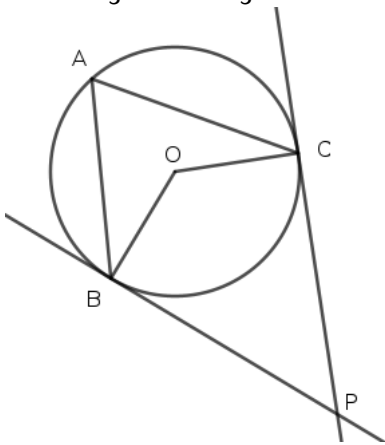
(a) 1cm   (b) 2cm   (c) 3cm   (d) 4cm

Answers

Ans : 2

$r = \frac{a+b-c}{2}$

4) In the diagram the angle between two chords AB and AC is $65°$.
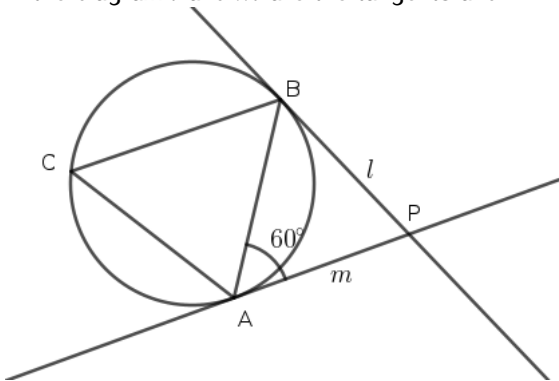

a) What is the measure of $\angle BOC$?

b) What is the angle between the tangents?

Answers

a) $130°$

b) Area $180 - 130 = 50°$

5) In the diagram l and m are the tangents and AB is a chord making an angle of $60°$ with the tangent l.

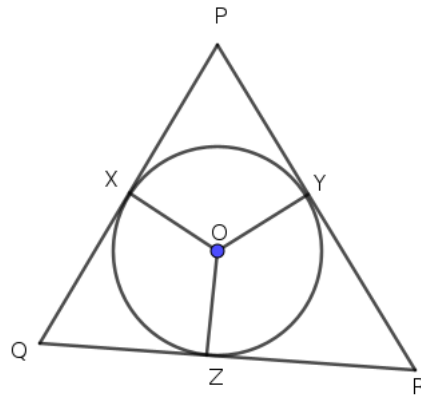a) What is the measure of $\angle ACB$?

b) What is the angle between l and m?

Answers

a) $60°$

b) $60°$
   Mark the center of the corcle as O. $\angle AOB$ will be $120°$.
   $OAPB$ is cyclic. So angle between the tangents is $180 - 120 = 60°$

6) In the figure a circle touches the sides of triangle PQR at X, Y and Z.
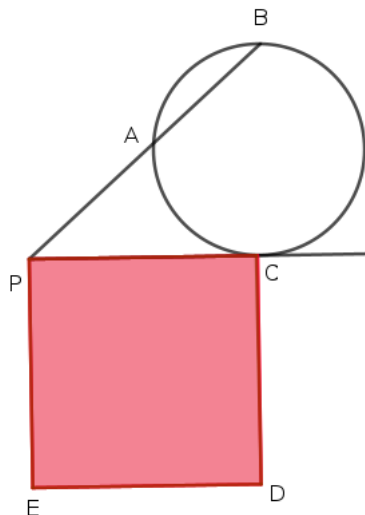

If $PX = 4, QZ = 7$ and $YR = 9$

    a) What are the lengths PY, QX and RZ?

    b) What is the perimeter of triangle PQR?

    c) If O is the center of the circle then what is $\angle XPY + XOY$?

Answers

    a) $PY = 4, QX = 7$ and $RZ = 9$

    b) Sides are $11, 13$ and 16. So the perimeter is 40

    c) $OXPY$ is cyclic. $\angle XPY + XOY = 180°$

7) The line from P cut the circle at A and B.
   PC is a tangent to the circle. $PCDE$ is a square

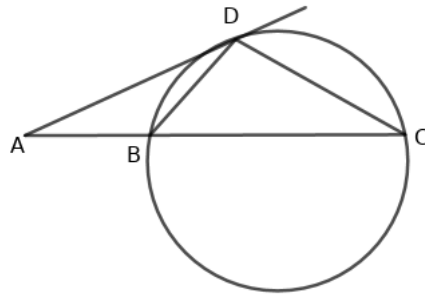If $PA = 4, AB = 5$ then

    a) What is the relation between PA, PB and PC?

    b) What is the length PC?

    c) What is the length PD?
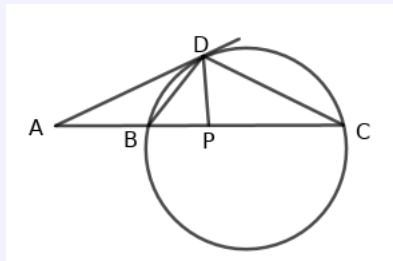
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a) $PA \times PB = PC^2$

b) $4 \times 9 = PC^2, PC = \sqrt{36} = 6$

c) $6\sqrt{2}$

8) In the figure $AB = BD$, also the line AD is a tangent from A.

a) What is the relation between AB, AC and AD

b) Prove that $AB \times AC = CD^2$

c) What kind of triangle is $\triangle ACD$?

d) If $\angle BAD = 30°$ and perpendicular distance from D to AB is 12 then what is the length of tangent AD?

Answers

a) $AB \times AC = AD^2$

b) Consider $\triangle ABD$ and $\triangle ACD$.
$\angle ADB = \angle ACD$(Angle between chord and the tangent at the end is equal to angle in the other side of the chord)
Since $AB = BD$, opposite angles are equal. $.\angle BAD = \angle ADB$
Therefore $\angle ADB = \angle ACD \rightarrow AD = CD \ AB \times AC = AD^2 \rightarrow AB \times AC = CD^2$

c) In $\triangle ACD, \angle A = \angle C$ opposite angles are equal . This is an isosceles triangle.

d) $\triangle APD$is a $30° - 60° - 90°$triangle. Side opposite to $30°$ is 12cm .Therefore $AD = 24$cm .
Length of tangent is24cm

Question 9 carries 4 score

9) In the figure $PQRS$ is a quadrilatearl in which PQ is parallel to RS.
A circle touches the sides of $PQRS$ inside at A, B, C and D.
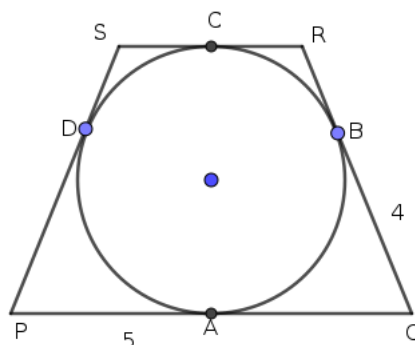
$PA = 5, QB = 4, RC = 3$ and $DS = 2$

a) Find the length of the sides PS and QR

b) Is it possible to draw a circle through the vertices of $PQRD$? How can we realize it ?

c) Calculate the perimeter of the quadrilateral $PQRS$

d) If $\angle P = x$ write the measures of other angles of $PQRS$ in terms of x

Answers

a) $PS = QR = 7$

b) Two sides are parallel and other two sides are equal .It is an isosceles trapezium. So it is cyclic. A circle can be drawn through the vertices.

c) 28

d) $\angle Q = x, \angle R = 180 - x, \angle S = 180 - x$

Question 10 carries 5 marks

```
            1
         2  3  4
      5  6  7  8  9
      ----------------------
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10) Look at the pattern carefully

a) Write the numbers in the right end of each line as a sequence

b) Using its algebraic form write the number in the right end of 30 th line

c) Which number comes just below 400 in the next line?

d) Which number comes in the left end of 21 st line ?

e) Which line ends with 1600 this pattern?

Answers

a) $1, 4, 9, 16 \cdots$

b) $30^2 = 900$

c) $21^2 - 1 = 440$

d) $20^2 + 1 = 401$

e) 1600 comes in the right end of 40 th line

[1]