

Final touch
Answer Key

1. 12, 22, 32, ...

a) $d = 10$

b) $x_9 = 92$

c) 200 is not a term of this sequence. because all terms of this sequence ends with the digit 2. Last term of last digit of 200 is not 2.

d) Three digit term: 102

2) 10, 14, 18, ...

a) $d = 4$

b) $x_{21} = x_1 + 20d$
 $= 10 + 20 \times 4$
 $= 10 + 80 = 90$

c) $x_n = dn + f - d$
 $= 4n + 10 - 4 = 4n + 6$

d) $4n + 6 = 66$
 $4n = 66 - 6 = 60$
 $n = \frac{60}{4} = 15$
 $x_{15} = 66$

3) $x_n = 3n + 5$

a) $x_1 = 3 + 5 = 8$, $d = 3$

b) $x_{15} = 3 \times 15 + 5$
 $= 45 + 5 = 50$

c) 200 is a multiple of the common difference 3. \therefore 200 is not the difference of any two terms.

4. 11, 14, 17, ...

a) $x_8 = 11 + 7 \times 3 = 11 + 21 = 32$

b) $d = \frac{14}{8} - \frac{11}{8} = \frac{3}{8}$

c) Integer term: $\frac{32}{8} = 4$

5. $x_8 = 30$, $x_{11} = 42$

a) $d = \frac{x_{11} - x_8}{11 - 8} = \frac{42 - 30}{3} = \frac{12}{3} = 4$

b) $x_1 = x_8 - 7d$
 $= 30 - 7 \times 4$
 $= 30 - 28 = 2$

c) $x_n = dn + f - d$
 $= 4n + 2 - 4$
 $= 4n - 2$

6. $x_9 = 16$, $x_{16} = 9$

a) $d = -1$

b) $x_{25} = x_{9+16} = 0$

c) $S_{49} = 0 \times 49 = 0$

7. $S_9 = 225$

a) $x_5 = \frac{S_9}{9} = \frac{225}{9} = 25$

b) $x_4 + x_6 = 2 \times x_5 = 2 \times 25 = 50$

c) $x_1 + x_9 = 50$
 $6 + x_9 = 50$
 $x_9 = 50 - 6 = 44$

d) $d = \frac{44 - 6}{9 - 1} = \frac{38}{8} = \frac{19}{4}$

8) $S_6 = 120$

Sequence: 20, 20, 20, 20, 20, 20

b) $x_1 = 10$, $x_6 = \frac{120}{3} - 10 = 30$

$d = \frac{30 - 10}{6 - 1} = \frac{20}{5} = 4$

Sequence: 10, 14, 18, ...

$$9. a) 1 + 2 + 3 + \dots + 20 = \frac{20 \times 21}{2} = 210.$$

$$b) 6 + 12 + 18 + \dots + 120 = 6 \times 210 = 1260$$

$$c) 8 + 14 + 20 + \dots + 124 = 1260 + 20 \times 2 \\ = 1260 + 40 = 1300$$

$$d) 14 + 26 + 38 + \dots + 242 = 1260 + 1300 \\ = 2560.$$

$$10. 1, 3, 5, \dots$$

$$a) x_{20} = 2 \times 20 - 1 = 39.$$

$$b) 1 + 3 + 5 + \dots + 39 = 20^2 = 400.$$

$$c) 3 + 9 + 15 + \dots + 117 = 400 \times 3 = 1200.$$

$$d) 7 + 13 + 19 + \dots$$

$$S_{20} = 1200 + 20 \times 4 \\ = 1200 + 80 = 1280.$$

$$11. 9, 15, 21, \dots$$

$$a) d = 6.$$

$$b) x_{11} = x_1 + 10d \\ = 9 + 10 \times 6 = 69.$$

$$c) S_n = \text{Mid term} \times \text{no. of terms}$$

$$S_{21} = x_{11} \times 21 = 69 \times 21 \\ = 1449.$$

$$d) \text{difference of Sums: } 10^2 \times 6 = 600.$$

$$12. 8, 11, 14, \dots$$

$$a) d = 3.$$

$$b) x_{25} = x_1 + 24d \\ = 8 + 24 \times 3 \\ = 8 + 72 = 80$$

$$c) S_n = \frac{n}{2} (x_1 + x_n)$$

$$S_{25} = \frac{25}{2} (8 + 80) = \frac{25}{2} \times 88 = 1100$$

OR

$$x_{13} = \frac{x_1 + x_{25}}{2} = \frac{8 + 80}{2} = 44$$

$$S_{25} = 44 \times 25 = 1100.$$

$$13. a) 100, 105, 110, \dots$$

$$b) 102, 107, 112, \dots$$

$$c) 997.$$

$$d) n = \frac{997 - 102}{5} + 1 = \frac{895}{5} + 1 \\ = 179 + 1 = 180.$$

$$S = \frac{180}{2} (102 + 997)$$

$$= 90 \times 1099 = 98910.$$

$$14. x_7 = 15, \quad x_{13} = 25.$$

$$a) x_7 = 15$$

$$x_{10} = \frac{15 + 25}{2} = 20.$$

$$x_{13} = 25$$

$$x_{16} = 25 + 5 = 30.$$

$$b) S_{31} = x_{16} \times 31 = 30 \times 31 = 930.$$

$$15. a) 1 + 3 + 5 + \dots + 19 = 10^2 = 100.$$

$$b) \frac{1}{2} + 1\frac{1}{2} + 2\frac{1}{2} + \dots + 9\frac{1}{2} \\ = \frac{1 + 3 + 5 + \dots + 19}{2} = \frac{100}{2} = 50$$

$$c) \frac{1}{10} + \frac{3}{10} + \frac{5}{10} + \dots + \frac{19}{10} = \frac{100}{10} = 10.$$

$$16) a) \text{Next lines:}$$

$$11, 12, 13, 14, 15$$

$$16, 17, 18, 19, 20, 21.$$

$$b) \text{last term} = \frac{20 \times 21}{2} = 210.$$

$$\text{first term: } 210 - 19 = 191.$$

$$c) S = \frac{20}{2} (191 + 210)$$

$$= 10 \times 401.$$

$$= 4010.$$

17. a) Next lines

10 11 12 13 14 15 16

17 18 19 20 21 22 23 24 25

b) 1, 4, 9, 16, ...

c) Last term: $9^2 = 81$

d) first term: 82,
last term = $10^2 = 100$

18

18. a) $\angle C = 90^\circ$ angle in semicircle.

b) $\angle B = 90 - 10 = 80$

$\angle A = 90 + 10 = 100$

19. a) $\angle A + \angle C = 180^\circ$

$\therefore ABCD$ is Cyclic

b) $\angle B = \frac{180}{3} = 60^\circ$, $\angle D = 120^\circ$

20. a) $\angle D = \angle CBP = 80^\circ$

b) $\angle ABC = 180 - 80 = 100$

c) $\angle A + \angle C = 180^\circ$

21) a) $\angle B = \angle A = 70^\circ$

$\angle C = 180 - 70 = 110^\circ$

b) $\angle A + \angle C = 180^\circ \therefore ABCD$ is cyclic.

c) $\angle C$ is greater than 110°

\therefore point C is inside.

22) a) $\angle OBA = \angle OAB = 20^\circ$

b) $\angle ACB = \frac{180 - 40}{2} = 70^\circ$

c) $\angle ADB = 180 - 70 = 110^\circ$

d) If $\angle OAB = x$, then

$\angle ACB = 90 - x$.

$\angle ACB + \angle ADB = 180$

$\Rightarrow 90 - x + y = 180$

$y - x = 90$

23) a) $\angle OCA = \angle OAC = 30^\circ$

~~$\angle ABC$~~ $\angle ACB = 90^\circ$

b) $\angle BOC = 30 \times 2 = 60^\circ$

$\therefore \angle OBC = \angle OCB = 60^\circ$

24) a) $\angle A = 30 + 20 = 50^\circ$

b) $\angle BOC = 50 \times 2 = 100^\circ$

$\angle BDC = 180 - 50 = 130^\circ$

25) a) $\angle BOD = 2 \times 80 = 160^\circ$

b) $\angle A = \frac{160}{2} = 80^\circ$

$\angle BCD = 180 - 80 = 100^\circ$

26) a) $\angle ACP = 90 - 40 = 50^\circ$

b) $\angle AOD = 50 \times 2 = 100^\circ$

c) Central angle of arc -

$BMC = 2 \times 40 = 80^\circ$

27) a) $\angle ACB = 40^\circ = \angle ADB$.

b) $\angle ACP = 180 - 40 = 140^\circ$

c) $\angle P + \angle COD = 360 - (140 + 140)$

$= 360 - 280 = 80 = \angle AOB$.

28) $\angle D = 360 - (100 + 70 + 70) = 120$

$\angle A + \angle C = 100 + 70 \neq 180$, not cyclic.

- 29) a) $\angle BAC = \angle BDC = 60^\circ$
 $\angle BAD = 60 + 45 = 105$
 b) $\angle ABC = 180 - (60 + 30) = 90^\circ$
 $\angle ADC = 90^\circ$
 c) $\angle BPC = 60 + 45 = 105$

- 30) a) $\angle A = 80, \angle QPC = 80$
 $\angle D = 100$
 $\angle A + \angle C \neq 180 \therefore$ not cyclic
 b) $\angle A = \angle B = 80$
 then $\angle B + \angle D = 180$
 $\therefore ABCD$ is cyclic.

- 31) a) $PB = 14 - 10 = 4 \text{ cm.}$
 b) $PD = \frac{PA \times PB}{PC} = \frac{10 \times 4}{5} = 8 \text{ cm.}$
 $CD = 8 + 5 = 13 \text{ cm}$

- 32) a) $\angle A = \angle PCB$
 $\angle APC = \angle BPC = 90^\circ$
 $\therefore \frac{PA}{PC} = \frac{PC}{PB} \Rightarrow PA \times PB = PC^2$
 b) $18 \times PB = 12^2$
 $PB = \frac{12 \times 12}{18} = 8 \text{ cm.}$

33, 34 figures.

- 35) a) $\angle BAO = 30^\circ$
 $\angle BAC = 30 + 20 = 50^\circ$
 b) $\angle AOD = 30 + 30 = 60^\circ$
 c) $\angle COD = 140 - 60 = 80^\circ$
 d) $\angle AOD = 60^\circ$
 $\therefore \angle OAD = \angle ODA = 60^\circ$
 (Since $OA = OD$ radius)
 $\therefore \triangle OAD$ is equilateral

- 36) Probability = $\frac{31-28}{7} = \frac{3}{7}$
 37) Probability of white = $1 - \frac{2}{3} = \frac{1}{3}$

- 38) Probability of getting Red ball = $\frac{10}{18} = \frac{5}{9}$
 b) " blue ball = $\frac{8}{18} = \frac{4}{9}$
 c) Probability of Red = $\frac{11}{20}$
 $\frac{11}{20} < \frac{5}{9} \therefore$ Probability decrease.

- 39) No. of prime numbers is 3
 \therefore Probability = $\frac{3}{6} = \frac{1}{2}$
 40) a) Probability of odd = $\frac{15}{30} = \frac{1}{2}$
 b) " even = $\frac{1}{2}$
 c) " prime = $\frac{10}{30} = \frac{1}{3}$

- 41) Probability of digits equal = $\frac{9}{90} = \frac{1}{10}$
 " Product a square = $\frac{15}{90} = \frac{1}{6}$
 (11, 14, 41, 19, 91, 22, 28, 33, 44, 49, 55, 66, 77, 88, 99).

42) a) Probability of inside = $\frac{1}{2}$
b) Probability of outside = $\frac{1}{2}$

43) Probability of that point
is inside = $\frac{1}{4}$
b) Outside : $\frac{3}{4}$

44) a) Probability of the dot is
inside the small rectangle = $\frac{2}{5}$
b) ... inside the large rectangle = $\frac{3}{5}$

45) $n^2 = 400$, $n = 20$.

46) a) ~~Area~~ ^{Side} of the small square:
 $\sqrt{625} = 25 \text{ cm}$
b) Side of the large square = $25 + 5$
 $= 30 \text{ cm}$
c) Area of the large square = $30^2 = 900 \text{ cm}^2$

47) a) Area = x^2 , Perimeter = $4x$.
b) $x^2 + 4x + 4 = 900$
 $(x+2)^2 = 900$
 $x+2 = 30$, $x = 28$.
Side of the square = 28 cm .

48) a) $(\frac{6}{2})^2 = 9$ should add.
b) $x^2 + 6x + 9 = 135 + 9$
 $(x+3)^2 = 144$, $x+3 = \pm 12$
 $x = 12 - 3 = 9$, $x = -12 - 3 = -15$
Numbers: 9 or -15.

49) $x(x+8) = 384$
 $x^2 + 8x = 384$
 $x^2 + 8x + 4^2 = 384 + 16$
 $(x+4)^2 = 400$
 $x+4 = 20$
 $x = 16$
Number: 16 and 24.

50) Perimeter = 40 cm
 $l+b = \frac{40}{2} = 20 \text{ cm}$
 $l = x$, $b = 20 - x$.
Then: $x(20-x) = 64$
 $20x - x^2 = 64$
 $\therefore x^2 - 20x = -64$
Completing the square
 $x^2 - 20x + 10^2 = 100 - 64$
 $(x-10)^2 = 36$
 $x-10 = 6$, $x = 6+10 = 16 \text{ cm}$
 $l = 16 \text{ cm}$, $b = 20 - 16 = 4 \text{ cm}$
OR

$l = 10 + x$, $b = 10 - x$.
Then Area = $(10+x)(10-x) = 64$
 $10^2 - x^2 = 64$
 $100 - x^2 = 64 \Rightarrow x^2 = 36$
 $x = \sqrt{36} = 6$.
 $l = 10 + 6 = 16 \text{ cm}$, $b = 10 - 6 = 4 \text{ cm}$.

51. a) $PA = x$, $PB = x + 10$.
b) $x(x+10) = 12^2$
 $x^2 + 10x = 144$
 $x^2 + 10x + 5^2 = 144 + 25$
 $(x+5)^2 = 169$, $x+5 = 13$, $x = 13 - 5 = 8$
 $PA = 8 \text{ cm}$, $PB = 18 \text{ cm}$

52) a) $OP = x$, $AP = 13 + x$, $PB = 13 - x$

$PA \times PB = PC^2 \Rightarrow (13+x)(13-x) = 12^2$

$169 - x^2 = 144$

$169 - x^2 = 144$

$x^2 = 25$, $x = \sqrt{25} = 5$

$PA = 13 + 5 = 18 \text{ cm}$

$PB = 13 - 5 = 8 \text{ cm}$

53) a) $\angle C = 45^\circ$

b) $AB = BC = 6 \text{ cm}$

c) $AC = 6\sqrt{2} \text{ cm}$

54) a) $\angle P = 45^\circ$ b) $PR = 5\sqrt{2} \text{ cm}$

55) a) $\angle C = 60^\circ$

b) $AB = 3\sqrt{3} \text{ cm}$, $AC = 6 \text{ cm}$

56) a) $AP = \frac{10}{2} = 5 \text{ cm}$

$PC = 5\sqrt{3} \text{ cm}$

b) $BP = AP = 5 \text{ cm}$

$AB = 5\sqrt{2} \text{ cm}$

57) a) $PS = \frac{12}{2} = 6 \text{ cm}$

b) $\text{Area} = \frac{1}{2}bh = \frac{1}{2} \times 18 \times 6 = 54 \text{ cm}^2$

c) $\angle Q = 150^\circ$ then $\text{Area} = 54 \text{ cm}^2$

58) a) Perpendicular distance = $\frac{9\sqrt{2}}{2} = 4.5\sqrt{2} \text{ cm}$

b) $\text{Area} = bh = 20 \times \frac{9\sqrt{2}}{2} = 90\sqrt{2} \text{ cm}^2$

59) a) $\angle CBD = 180 - 120 = 60^\circ$

b) $BC = \frac{10}{2} = 5 \text{ cm}$, $CD = 5\sqrt{3} \text{ cm}$

c) $\text{Area} = \frac{1}{2}bh = \frac{1}{2} \times 15 \times 5\sqrt{3} = \frac{75\sqrt{3}}{2} \text{ cm}^2$

Perimeter = $15 + 5\sqrt{3} + 10\sqrt{3} = (15 + 15\sqrt{3}) \text{ cm}$

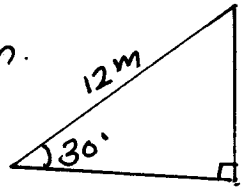
60) a) $\angle ADB = \angle ACB = 60^\circ$

b) $\angle ABD = 90^\circ$

c) $BD = \frac{6}{\sqrt{3}} \text{ cm}$, $\therefore \text{radius} = \frac{6}{\sqrt{3}} \text{ cm}$

61) a) Height of well = $\frac{12}{2} = 6 \text{ m}$

b) Distance = $6\sqrt{3} \text{ m}$



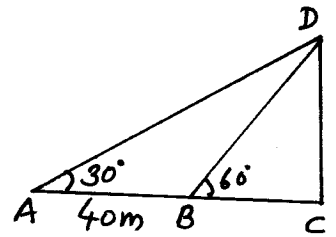
62) $\angle A = \angle ADB = 30^\circ$

$\therefore AB = BD = 40 \text{ m}$

$\therefore BC = \frac{40}{2} = 20 \text{ m}$

$CD = 20\sqrt{3} \text{ m}$

Height of the building = $20\sqrt{3} \text{ m}$



63) a) $BC = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12 \text{ cm}$

b) $\sin A = \frac{BC}{AC} = \frac{12}{13}$

$\cos A = \frac{AB}{AC} = \frac{5}{13}$

c) $\cos C = \frac{BC}{AC} = \frac{12}{13} = \sin A$

64) ~~PA~~ $QR = 4 \times 2 = 8 \text{ cm}$

a) $PQ = \sqrt{10^2 - 8^2} = \sqrt{100 - 64} = \sqrt{36} = 6 \text{ cm}$

b) $\cos P = \frac{6}{10} = \frac{3}{5}$

$\cos R = \frac{4}{5}$

c) $\frac{\sin P}{\cos R} = \frac{4/5}{4/5} = 1$

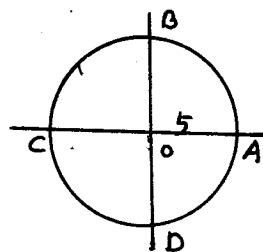
65) a) $AP = 20 \times \sin 40 = 20 \times 0.64 = 12.8 \text{ cm}$

b) $\text{Area} = \frac{1}{2} \times 25 \times 12.8 = 160 \text{ cm}^2$

c) $\angle B = 140$ then $\text{Area} = 160 \text{ cm}^2$

66). $(0, 3)$, $-y$ axis
 $(4, 0)$, $(-5, 0) \rightarrow x$ axis.

73). $A(5, 0)$, $B(0, 5)$
 $C(-5, 0)$, $D(0, -5)$



67) a) $O(0, 0)$.
 b) $C(4, 0)$, $A(4, 3)$, $B(0, 3)$.

74) a) Radius = $16 - 6 = 10$.

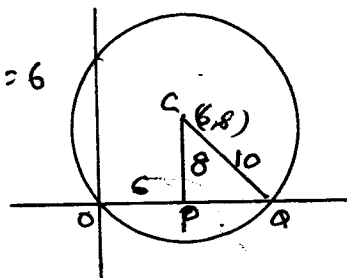
$$PQ = \sqrt{10^2 - 8^2}$$

$$\sqrt{100 - 64} = \sqrt{36} = 6$$

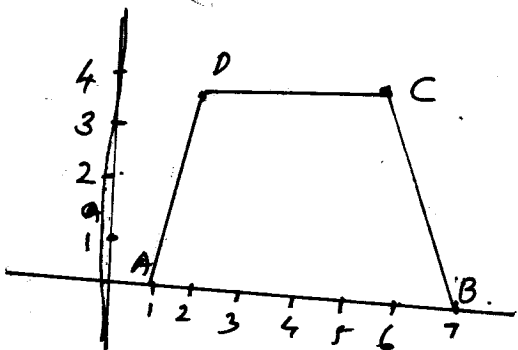
$$OQ = 6 + 6 = 12$$

$$\therefore Q(12, 0)$$

$$O(0, 0)$$



68)



b) isosceles trapezium.

c) $AB = 6$, $CD = 4$, $h = 4$.

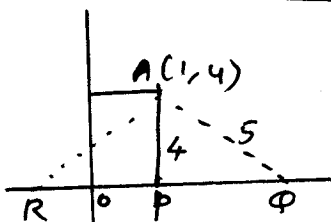
$$A_{\text{area}} = \frac{1}{2} \times 4(6 + 4) = 20 \text{ sq. unit}$$

69) $A(-5, -2)$, $C(5, 1)$.

70) $P(3\sqrt{3}, 3)$, $Q(-3, 3\sqrt{3})$.

71) a) $(1, 0)$

b) $(0, 4)$



$$c) PQ = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3$$

$$OQ = 1 + 3 = 4$$

$$Q(4, 0)$$

$$OR = 3 - 1 = 2, R(-2, 0)$$

72) distance between $(6, 8)$ and $(10, 8)$
 is $10 - 6 = 4 \therefore (10, 8)$ on the circle.

distance between $(6, 8)$, $(6, 13)$ is
 $13 - 8 = 5 > 4 \therefore$ outside.

$(0, 0)$, $(6, 8)$ distance: 10, outside

$(6, 8)$, $(-1, 9)$ distance: $\sqrt{(6+1)^2 + (8-9)^2}$
 $= \sqrt{7^2 + 1^2} = \sqrt{50} > 4 \therefore$ outside

75)

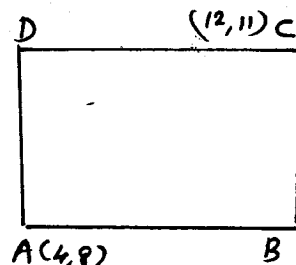
$B(12, 8)$

$D(4, 11)$.

$AB = 12 - 4 = 8$ unit

$BC = 11 - 8 = 3$ unit

Perimeter = $2(8 + 3) = 22$ unit.



76) $B(7, 3)$, $C(7, 8)$, $D(2, 8)$

$$77) AB = 1 - 3 = 4$$

$$BC = \sqrt{1^2 + 2^2} = \sqrt{1 + 4} = \sqrt{5}$$

$$CD = \sqrt{-3^2 + 1^2} = \sqrt{9 + 1} = \sqrt{10}$$

$$AD = 1 - 2 = 3$$

$$AC = \sqrt{-3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13}$$

$$BD = \sqrt{(1+3)^2 + (1+2)^2} = \sqrt{4^2 + 3^2}$$

$$= \sqrt{16 + 9} = \sqrt{25} = 5$$

80) a) $\angle P = 90^\circ$

b) $\angle Q = 90 - 42 = 48^\circ$

81) a) $\angle POQ = 90^\circ$

$$b) PQ^2 = 13^2 - 5^2 = 169 - 25 = 144$$

$$PQ = \sqrt{144} = 12 \text{ cm}$$

82) a) $\angle OAB = 90^\circ$
 b) $OB = 2 \times 6 = 12 \text{ cm}$
 $AB = 6\sqrt{3} \text{ cm}$

83) ~~a) $PA^2 = PC^2 = AC^2$~~

a) $PB = PA = 8 \text{ cm}$
 b) $PC^2 = PA^2 + AC^2$
 $= 8^2 + 6^2 = 64 + 36 = 100$
 $PC = \sqrt{100} = 10 \text{ cm}$

84) $\angle C = \angle PAB = 50^\circ$
 b) $\angle PBA = \angle PAB = 50^\circ$
 c) $\angle P = 180 - (50 + 50) = 80^\circ$

85) a) $\angle AQP = \angle APQ = \frac{180 - 70}{2} = 55^\circ$
 b) $\angle PRQ = \angle APQ = 55^\circ$
 $\angle PQR = \angle BPR = \frac{180 - 80}{2} = 50^\circ$
 $\angle QPR = 180 - (50 + 55) = 75^\circ$
 c) $\angle CQR = \angle CRQ = \angle QPR = 75^\circ$
 $\therefore \angle QCR = 180 - (75 + 75) = 30^\circ$

86) a) $\angle POR = 360 - (120 + 130) = 110^\circ$
 $\therefore \angle PAR = 180 - 110 = 70^\circ$
 $\therefore \angle APR = \angle ARP = \frac{180 - 70}{2} = 55^\circ$
 b) $\angle B = 180 - 120 = 60^\circ$
 $\angle A = 70^\circ$
 $\therefore \angle C = 180 - (70 + 60) = 50^\circ$

87) a) $\angle APO = \angle OAP = 32^\circ$
 $\angle AOP = 180 - (32 + 32) = 116^\circ$
 b) $\angle OPT = 90^\circ$
 $\angle POT = 32 + 32 = 64^\circ$
 $\angle PTO = 90 - 64 = 26^\circ$

88) $AR = AC = 11 - 7 = 4 \text{ cm}$
 $PC = QC = 7 \text{ cm}$
 $BP = BR = 8 - 4 = 4 \text{ cm}$

89) $AP = AS, BP = BQ, CQ = CR$
 and $DR = DS$
 $AB + CD = AP + PB + CR + DR$
 $= AS + BQ + CQ + DS$
 $= (BQ + CQ) + (AS + DS)$
 $= BC + AD$

b) Perimeter $= 2(2 + 5 + 3 + 4)$
 $= 2 \times 14 = 28 \text{ cm}$
 c) $AD + BC = AB + CD = 20 \text{ cm}$
 Perimeter $= 20 \times 2 = 40 \text{ cm}$

90) a) $PC = 3 + 9 = 12 \text{ cm}$
 b) $PB \times PC = PA^2$
 $PA^2 = 3 \times 12 = 36$
 $PA = \sqrt{36} = 6 \text{ cm}$

91) a) $\angle OAP = 90^\circ$
 b) $\angle OQA = \angle PQA$
 $\angle AOQ = \angle PAQ$
 c) $OQ \times QP = AO^2$
 $OQ \times QP = r^2 = OQ^2$
 $OQ \times QR + OQ^2 = r^2$
 $OQ(PQ + OQ) = r^2$
 $OQ \times OP = r^2$

92, 93, 94 figures.

95) $l = 10 \text{ cm}$.

$$r = \frac{l}{2} = \frac{10}{2} = 5 \text{ cm.}$$

(because semicircle)

b) $CSA = \hat{\pi} r l = \hat{\pi} \times 5 \times 10 = 50\hat{\pi} \text{ cm}$

c) $TSA = 50\hat{\pi} + 25\hat{\pi} = 75\hat{\pi} \text{ cm}^2$.

96) $R = 12 \text{ cm}$, $x = 120^\circ$

a) $l = R = 12 \text{ cm}$.

b) $\frac{r}{l} = \frac{x}{360} \Rightarrow \frac{r}{12} = \frac{120}{360}$

$$\frac{r}{12} = \frac{1}{3} \Rightarrow r = \frac{12}{3} = 4 \text{ cm.}$$

97) $x = 216^\circ$ $R = 10 \text{ cm}$

a) $l = R = 10 \text{ cm}$.

b) $\frac{r}{l} = \frac{x}{360} \Rightarrow \frac{r}{10} = \frac{216}{360}$

$$r = \frac{10 \times 216}{360} = 6 \text{ cm.}$$

c) $h^2 = l^2 - r^2 = 10^2 - 6^2$
 $= 100 - 36 = 64$

$$h = \sqrt{64} = 8 \text{ cm.}$$

d) $V = \frac{1}{3} \hat{\pi} r^2 h = \frac{1}{3} \hat{\pi} \times 6^2 \times 8$
 $= 96\hat{\pi} \text{ cm}^3$.

98) $\frac{x}{360} = \frac{r}{R} \Rightarrow x = \frac{18 \times 360}{30}$

$$l^2 = \sqrt{24^2 + 18^2} = 216 //$$

$$= \sqrt{576 + 324} = \sqrt{900} = 30 \text{ cm}$$

99) $l = 2r$.

b) $l = 10 \text{ cm}$

$$r = 5 \text{ cm.}$$

100) $r = 12 \text{ cm}$, $h = 16 \text{ cm}$.

a) $l^2 = 16^2 + 12^2 = 256 + 144 = 400$

$l = \sqrt{400} = 20 \text{ cm}$.

b) $CSA = \hat{\pi} r l = \hat{\pi} \times 12 \times 20 = 240\hat{\pi} \text{ cm}$

c) $TSA = 240\hat{\pi} + 144\hat{\pi} = 384\hat{\pi} \text{ cm}^2$

d) $V = \frac{1}{3} \hat{\pi} r^2 h$

$$= \frac{1}{3} \hat{\pi} \times 12 \times 12 \times 16$$

$$= 768\hat{\pi} \text{ cm}^3$$

100A.

a) $\hat{\pi} r^2 = 64\hat{\pi}$, $r = \sqrt{64} = 8 \text{ cm}$

$$l = 20 \text{ cm.}$$

$$h = \sqrt{20^2 - 8^2} = \sqrt{400 - 64}$$

$$= \sqrt{336} \text{ cm.}$$

b) $CSA = \hat{\pi} r l = \hat{\pi} \times 8 \times 20 = 160\hat{\pi} \text{ cm}$

$$TSA = 160\hat{\pi} + 64\hat{\pi} = 224\hat{\pi} \text{ cm}^2$$

c) $V = \frac{1}{3} \hat{\pi} \times 8 \times 8 \times \sqrt{336}$.

$$= \frac{64 \sqrt{336}}{3} \hat{\pi} \text{ cm}^3$$

101) $\hat{\pi} r^2 = 576\hat{\pi} \Rightarrow r = \sqrt{576} = 24 \text{ cm}$

a) $V = \frac{1}{3} \hat{\pi} r^2 h = 1920\hat{\pi}$

$$= \frac{1}{3} \hat{\pi} \times 24 \times 24 \times h = 1920\hat{\pi}$$

$$= 192h = 1920$$

$$h = \frac{1920}{192} = 10 \text{ cm}$$

b) $l = \sqrt{10^2 + 24^2} = \sqrt{100 + 576}$

$$= \sqrt{676} = 26 \text{ cm}$$

$$CSA = \hat{\pi} \times 24 \times 26 = 624\hat{\pi} \text{ cm}^2$$

101) a) Ratio of perimeters = Ratio of their radii = 2:3.

b) Ratio of their volumes.
 $= \frac{1}{3} \pi (2r)^2 \times 5h : \frac{1}{3} \pi (3r)^2 \times 4h$
 $= 5:9.$

c) Volume of Second = $9 \times 100 = 900 \text{ cm}^3$

102) $r = 12 \text{ cm}, h = 18 \text{ cm}.$

$$h = \frac{\pi \times 12 \times 12 \times 18}{\frac{1}{3} \pi \times 9 \times 9} = 96 \text{ cm}.$$

103) $r = 10 \text{ cm}, h = 24 \text{ cm}$

a) $V = \pi \times 10 \times 10 \times 24 = 2400\pi \text{ cm}^3$

b) $V = \frac{1}{3} \times 2400\pi = 800\pi \text{ cm}^3.$

c) $l = \sqrt{24^2 + 10^2} = \sqrt{576 + 100} = \sqrt{676} = 26 \text{ cm}$

$CSA = \pi \times 10 \times 26 = 260\pi \text{ cm}.$

104) a) Mid point: $\left(\frac{14+2}{2}, \frac{5+7}{2}\right) = (7, 6)$

b) Mid point: $\left(\frac{6+12}{2}, 2\right) = (9, 2)$

105) a) Centre: $\left(\frac{2+6}{2}, \frac{3+5}{2}\right) = (4, 4)$

b) Radius = $\sqrt{(6-4)^2 + (5-4)^2} = \sqrt{2^2 + 1^2} = \sqrt{5}.$

106) Midpoint of AC: $\left(\frac{8+1}{2}, \frac{6+1}{2}\right) = \left(\frac{9}{2}, \frac{7}{2}\right)$

Midpoint of BD: $\left(\frac{7+2}{2}, \frac{1+6}{2}\right) = \left(\frac{9}{2}, \frac{7}{2}\right)$

b) Midpoints of diagonals are equal: $\therefore ABCD$ is parallelogram.

107) C(9, 2) B(6, -1), D(6, 5).

108) Midpoint of AC = $\left(\frac{-1+7}{2}, 2\right) = (3, 2)$

b) B(3, -1), D(3, 5)

c) Side = ~~3~~ $\cdot \sqrt{(3+1)^2 + (2+1)^2} = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$

109) a) D(2+8-4, 4+8-3) = D(6, 9).

b) $AB = 8 - 2 = 4.$

$AD = 2.$

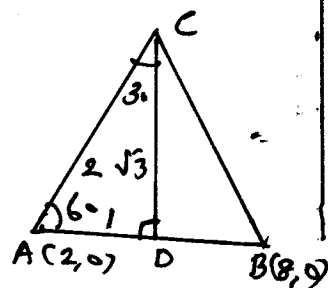
$\therefore CD = 2\sqrt{3}.$

\therefore Co-ordinates of C(4, $2\sqrt{3}$)

c) Slope of AB = $\frac{8-5}{8-4} = \frac{3}{4}$

Slope of BC = $\frac{14-8}{16-8} = \frac{6}{8} = \frac{3}{4}$

Slopes are same $\therefore A, B, C$ are on a line.



110) a) Slope = $\frac{8-4}{5-2} = \frac{4}{3}$.

b) Other points $(5+3, 8+4) = (8, 12)$
 $(8+3, 12+4) = (11, 16)$

111). a) Slope of AB = $\frac{7-4}{3-1} = \frac{3}{2}$

b) Slope of BC = $\frac{16-7}{9-3} = \frac{9}{6} = \frac{3}{2}$

Slopes are equal.

$\therefore A, B, C$ are on a line

112). Slope = $\frac{8-5}{5-1} = \frac{3}{4}$

\therefore Slope of $(5, 8), (13, 14)$ is $= \frac{6}{8} = \frac{3}{4}$

Slopes are equal \therefore Cannot draw a triangle

113) a) Other points: $(5+4, 6+3) = (9, 9)$
 $(9+4, 9+3) = (13, 12)$

b) Parallel = Slopes are equal.

\therefore Slope = $\frac{3}{4}$

114) a) Slope = $\frac{6-3}{3+1} = \frac{3}{4}$

b) Other points: $(7, 9), (11, 12)$

c) Slope = $\frac{9+3-y}{x+4-x} = \frac{3}{4}$

\therefore Slopes are equal.

\therefore This point is on this line

115) $P(x) = 2x^2 - 5x + 2$.

a) $P(2) = 2 \times 2^2 - 5 \times 2 + 2$

$2 \times 4 - 10 + 2$

$8 - 10 + 2 = 0$.

b) Factor: $P(2) = 0 \therefore x-2$

116) $5x^3 - 4x^2 + x + k$.

$x-1$ factor $\therefore 5 - 4 + 1 + k = 0$.

$6 - 4 + k = 0$

$2 + k = 0, \therefore k = -2$.

117) $P(x) = 2x^2 - 3x + 1$

$P(1) = 2 - 3 + 1 = 0$.

$P(2) = 2(2)^2 - 3(2) + 1$

$= 8 - 6 + 1 = 9 - 6 = 3$.

$P(-3) = 2(-3)^2 - 3(-3) + 1$

$2 \times 9 + 9 + 1$

$18 + 9 + 1 = 28$

118) $P(x) = 2x^2 - 5x + 1$

a) $P(3) = 2(3)^2 - 5(3) + 1$

$= 18 - 15 + 1 = 4$

b) $P(3) \neq 0 \therefore x-3$ not a factor

c) $P(x) - P(2)$.

ie 4 is subtracted.

119) $x^2 - 1 = (x+1)(x-1)$

$x^2 - 9 = x^2 - 3^2 = (x+3)(x-3)$

$x^2 - 4 = x^2 - 2^2 = (x+2)(x-2)$

$x^2 - 100 = x^2 - 10^2 = (x+10)(x-10)$

120) $p(x) = x^2 - 5x + 7$.

a) $p(3) = 3^2 - 5 \times 3 + 7$
 $9 - 15 + 7 = 16 - 15 = 1$.

b) $p(x) - p(3) = x^2 - 5x + 7 - 1$
 $= x^2 - 5x + 6$.

c) $x^2 - 5x + 6 = (x-3)(x-2)$

121) $p(x) = x^2 - 7x + 13$

a) $p(3) = 3^2 - 7 \times 3 + 13$
 $9 - 21 + 13 = 22 - 21 = 1$

b) $p(x) - p(3) = x^2 - 7x + 13 - 1$
 $x^2 - 7x + 12$

c) $x^2 - 7x + 12 = (x-3)(x-4)$

d) Solnr: $x-3=0, x=3$
 $x-4=0, x=4$

122) Mean: $\frac{15+12+25+10+3+18+17+20+6}{9}$

$= \frac{126}{9} = 14$

3, 6, 10, 12, 15, 17, 18, 20, 25

Median = 15

123) Mean: $\frac{35+39+32+36+40+30+34+37+38+33}{10}$

$= \frac{354}{10} = 35.4$

Median:

30, 32, 33, 34, 35, 36, 37, 38, 39, 40.

$= \frac{35+36}{2} = 35.5$

124) Middle number: $\frac{1+25}{2} = 13$

Mean = Median = 13.

125) 13th family's Income: 6000

Median: 6000

126) a) Total = 21 workers.

b) Wage of 11th worker = 600.

c) Median = 600.

127) $x_6 = 40, x_9 = 58$.

a) $d = \frac{58-40}{9-6} = \frac{18}{3} = 6$

$x_1 = x_6 - 5d = 40 - 5 \times 6$
 $= 40 - 30 = 10$.

$S_n = \frac{d}{2} n^2 + (f - \frac{d}{2}) n$
 $= 3n^2 + (10 - 3)n = 3n^2 + 7n$.

b) $S_{25} = 3 \times 25^2 + 7 \times 25$
 $= 3 \times 625 + 175$
 $1875 + 175$
 $= 2050$.

128) 15, 33, 51, ...

a) $S_n = \frac{d}{2} n^2 + (f - \frac{d}{2}) n$
 $= 9n^2 + (15 - 9)n \Rightarrow 9n^2 + 6n$.

b) $S_n + 1 = 9n^2 + 6n + 1 = (3n+1)^2$
 is a perfect square.

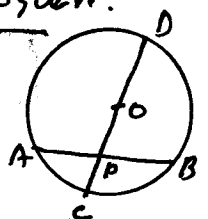
129) Let $op = x$, then

$pd = 6+x, pc = 6-x$.

$(6+x)(6-x) = 4 \times 5$

$6^2 - x^2 = 20, 36 - x^2 = 20$

$x^2 = 16, x = \sqrt{16} = 4, op = 4 \text{ cm}$



130). a) Possible pairs: $50 \times 35 = 1750$.

b) Probability of
both being Scouts } $= \frac{30 \times 20}{1750} = \frac{12}{35}$

c) both being guide = $\frac{20 \times 15}{1750} = \frac{6}{35}$

d) Probability of one
Scout and one guide } $= \frac{30 \times 15 + 20 \times 20}{1750}$
 $= \frac{450 + 400}{1750} = \frac{950}{1750} = \frac{19}{35}$

131) a) Possible pairs: $11 \times 12 = 132$.

b) Probability of both
being red } $= \frac{6 \times 8}{132} = \frac{4}{11}$

c) Probability of
both being white } $\frac{5 \times 4}{132} = \frac{5}{33}$

d) Probability of
at least one red } $= 1 - \frac{5}{33} = \frac{28}{33}$

132). a) $n^2 + 8n = 240$

b) $n^2 + 8n + 4^2 = 240 + 16$

$(n+4)^2 = 256$

$n+4 = \sqrt{256} = 16$

$n = 16 - 4 = 12$

$S_{12} = 240$

133). a) $l+b = \frac{46}{2} = 23$.

$l = 20\text{cm}, b = 3\text{cm}$.

b) Area = $20 \times 3 = 60\text{cm}^2$.

c) $l+b = \frac{62}{2} = 32$.

$lb = 60 = 30 \times 2$

$l = 30\text{cm}, b = 2\text{cm}$

134). $l+b = \frac{28}{2} = 14\text{cm}$.

a) $l = x, b = 14 - x$

c) $x^2 + (14-x)^2 = 10^2$

$x^2 + 14^2 - 2 \times 14 \times x + x^2 = 100$

$2x^2 - 28x + 196 = 100$

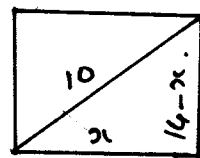
$2x^2 - 28x = 100 - 196 = -96$

$x^2 - 14x + 7^2 = 49 - 48$

$(x-7)^2 = 1$

$x-7 = 1, x = 1+7 = 8$

$l = 8\text{cm}, b = 14-8 = 6\text{cm}$.



135). a) $PB = x, AP = x + 10$

b) $x(x+10) = 12^2$

$x^2 + 10x = 144$

$x^2 + 10x + 5^2 = 144 + 25$

$(x+5)^2 = 169$

$x+5 = \sqrt{169} = 13$

$x = 13 - 5 = 8\text{cm}$

$PB = 8\text{cm}, AP = 8 + 10 = 18\text{cm}$

diameter = $8 + 18 = 26\text{cm}$

136) a) $\angle ADB = 65^\circ$

b) $\frac{9}{\sin 65} = 2R$

$2R = \frac{9}{0.9} = 10\text{cm}$

137) $\angle ADB = 27^\circ$

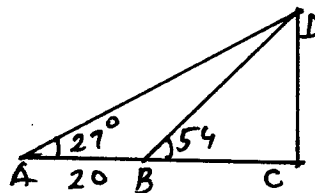
$\therefore AB = BD = 20\text{m}$

$CD = 20 \times \sin 54$

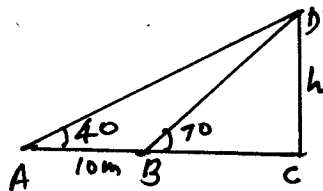
$= 20 \times 0.8$

$= 16\text{m}$

Height = 16m



138).



$$h = \frac{AB \times \tan A \times \tan B}{\tan B - \tan A}$$

$$h = \frac{10 \times \tan 40 \times \tan 70}{\tan 70 - \tan 40}$$

$$= \frac{10 \times 0.8 \times 2.8}{2.8 - 0.8} = \frac{4 \times 2.8}{2}$$

$$= 11.2 \text{ m}$$

Height = 11.2 m

139). a) $\angle ACB = 70^\circ$

$$AC = AB = 20 \text{ m}$$

$$b) CD = 20 \times \sin 40$$

$$= 20 \times 0.64 = 12.8 \text{ m}$$

$$c) AD = 20 \times \cos 40$$

$$= 20 \times 0.8 = 16 \text{ m}$$

$$BD = 20 - 16 = 4 \text{ m}$$

140) a) base edge = 10 cm

$$b) l = \frac{36 - 10}{2} = \frac{26}{2} = 13 \text{ cm}$$

$$c) \text{LSA} = 2al = 2 \times 10 \times 13$$

$$= 260 \text{ cm}^2$$

141). $a = 12 \text{ cm}$, $l = 10 \text{ cm}$.

$$a) h^2 = l^2 - (a/2)^2 = 10^2 - 6^2 = 100 - 36 = 64$$

$$h = \sqrt{64} = 8 \text{ cm}$$

$$b) v = \frac{1}{3} a^2 h = \frac{1}{3} \times 12 \times 12 \times 8$$

$$= 384 \text{ cm}^3$$

142). $a = c = 12 \text{ cm}$.

$$a) l = \frac{\sqrt{3}}{2} a = \frac{\sqrt{3} \times 12}{2} = 6\sqrt{3} \text{ cm}$$

$$b) \text{TSA} = \sqrt{3} a^2 + a^2$$

$$= \sqrt{3} \times 12^2 + 12^2$$

$$= (144\sqrt{3} + 144) \text{ cm}^2$$

143). Ratio of Volumes: $1^3 : 2^3$

$$= 1 : 8$$

Ratio of their areas: $1^2 : 2^2$

$$= 1 : 4$$

144) a) Centre = $(\frac{3+(-3)}{2}, \frac{4+(-4)}{2}) = (0, 0)$

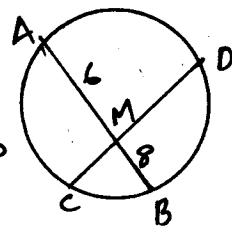
$$b) r^2 = 3^2 + 4^2 = 9 + 16 = 25$$

$$r = \sqrt{25} = 5$$

$$c) x^2 + y^2 = r^2 \Rightarrow x^2 + y^2 = 25$$

145). Let $MC = x$

$$\text{and } MD = 8 + x$$



then $(8-x)(8+x) = 6 \times 8$

$$8^2 - x^2 = 48$$

$$64 - x^2 = 48 \quad x^2 = 16$$

$$x = \sqrt{16} = 4$$

$$MC = 8 - 4 = 4 \text{ cm}$$

$$MD = 8 + 4 = 12 \text{ cm}$$

146). a) Radius = $\sqrt{25} = 5$
 b) $x^2 + y^2 = 9$

147). a) Slope = $\frac{0-2}{5-3} = \frac{-2}{2} = -1$

b) eqn: $x + y = 5$

~~147~~. b) $y = 0$

c) Point of intersection: $(5, 0)$

148). figure.

149) a) Radius = $6 - 3 = 3$ unit.

b) equation: $(x-5)^2 + (y-3)^2 = 9$

c) Distance: 3 unit.

d) Tangent = x Coordinate = 5

150) a) Point: $(4, 0)$

b) $(3, 0)$ not possible $r = 4$
 ~~$r = 1$~~

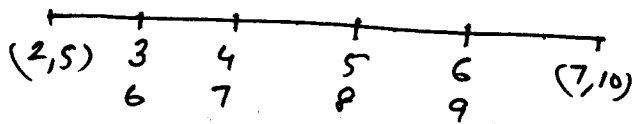
c) Inside $(3, 0)$ outside $(5, 0)$

151) a) Slope of AB = $\frac{10-2}{5-1} = \frac{8}{4} = 2$

b) equation: $y = 2x$

c) equation: $y = -2x$

152).



point: $(5, 8)$

153). a) Centre: $(\frac{8}{2}, \frac{6}{2}) = (4, 3)$

b) $r = \sqrt{(8-4)^2 + 3^2} = \sqrt{4^2 + 3^2} = \sqrt{16+9}$
 $= \sqrt{25} = 5$

equation: $(x-4)^2 + (y-3)^2 = 25$

c) Other end: $(4+4, 3+3) = (8, 6)$

154). a) Radius = $\frac{12}{2} = 6$ cm

b) TSA = $4\pi r^2 = 4\pi \times 6 \times 6 = 144\pi \text{ cm}^2$

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 6 \times 6 \times 6 = 288\pi \text{ cm}^3$

155) radius = $\sqrt{(5-1)^2 + (5-2)^2}$

$= \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$ unit.

b) Equation = $(x-1)^2 + (y-2)^2 = 25$

c) Centre: $(2, 3)$

radius = $\sqrt{81} = 9$

156)

Wages	No. Total.	
upto 300	5	1-5
" 500	12	6-12
" 700	20	13-20
" 900	30	21-30
" 1100	43	
" 1300	50	
" 1500	53	

a) Position = $\frac{53+1}{2} = 27^{\text{th}}$

b) $d = \frac{900-700}{10} = 20$

$x_{21} = 700 + \frac{20}{2} = 710$

$x_{27} = x_{21} + 6d$

$= 710 + 6 \times 20$

$710 + 120 = 830$

c) Median = 830