Wisgrow Math A - Plus

Session 119 |Polynomial 1 | Worksheet 119

1) The sides of a rectangle are $(x-3)$ and $(x+1)$

   a) Find the area $a(x)$
   b) If $x = 4$then what is its area ?
   c) If $x = 0$ is it possible to get a rectangle ? Why?
   d) What is the condition for x to get a rectangle?

Answers

   a) $a(x) = (x-3)(x+1) = x(x+1) - 3(x+1) = x^2 - 2x - 3$
   b) If $x = 4$ then $a(4) = 4^2 - 2 \times 4 - 3 = 16 - 8 - 3 = 5$
   c) If $x = 0$ side becomes a negative number. Square cannot be drawn
   d) $x > 3$.

2) Sides of a rectangular box are $x-1, x+2$ and $x+3$.

   a) Calculate the volume $v(x)$.
   b) If $x = 2$ then what is its volume?
   c) Is it possible to make the box for $x = 1$
   d) What is the condition for x to make the recctangular box?

Answers

   a) $v(x) = (x-1)(x+2)(x+3)$
   $v(x) = (x^2 + x - 2)(x+3) = x^3 + 4x^2 + x - 6$
   b) $v(2) = 2^3 + 4 \times 2^2 + 2 - 6 = 8 + 16 + 2 - 6 = 20$
   c) If $x = 1$then side becomes 0.Box cannot be made .
   d) $x > 1$

3) The difference between breadth ,length and length of diagonal of a rectangle differ by 1

   a) If breadth is x what is its length and diagonal of the rectangle?
   b) Make an equation connecting breadth ,length and diagonal in the form $p(x) = 0$
   c) Find the solution of the equation $p(x) = 0$
   d) Find the length , breadth and diagonal.

Answers

a) If breadth is x then length is $x+1$, diagonal $x+2$

b) $(x+2)^2 = x^2 + (x+1)^2$, $x^2+4x+4 = x^2+x^2+2x+1$, $x^2-2x-3=0$

c) $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$x = \frac{-(-2)\pm\sqrt{(-2)^2-4\times1\times-3}}{2}$
$x = 3, -1$

d) If $x=3$then bradth $=3$, length $=4$,diagonal$=5$

4) Consider the polynomial $p(x) = x^2 - 7x + 12$

a) Write $p(x) = (x-a)(x-b)$.

b) Write $p(x)$as the product of two first degree polynomials.

c) Find the solution of the equation $p(x) = 0$

Answers

a) $x^2 - 7x + 12 = (x-a)(x-b) = x^2 - (a+b)x + ab$
$a+b = 7, ab = 12$

$$(a-b)^2 = (a+b)^2 - 4ab$$
$$(a-b)^2 = (7)^2 - 4\times 12 \rightarrow a-b = \pm 1$$

If $a-b=1$ then , $a-b=1, a+b=7 \rightarrow 2a=8, a=4, b=3$
(What change occur in the answer corresponding to $a-b=-1$.Try yourself)

b) $p(x) = (x-4)(x-3)$

c) $p(x) = 0 \rightarrow (x-4)(x-3) = 0$
$x = 3, 4$

5) Consider the polynomial $p(x) = x^3 - 4x^2 + 2x + k$

a) If x is a factor then find k.

b) $x-1$ is a first degree factor of $p(x)$then what is k?

c) Use k for becoming $x-1$ a factor and write the polynomial

d) Is $(x+1$ a factor of this polynomial .

Answers

a) $k = 0$

b) If $x-1=0$ then $p(1) = 0$
$1^3 - 4\times 1^2 + 2\times 1 + k = 0, k = 1$

c) $p(x) = x^3 - 4x^2 + 2x + 1$

d) $p(-1) = (-1)^3 - 4(-1)^2 + 2(-1) + 1 = -1 - 4 - 2 + 1 \neq 0$
$x+1$ is not a factor

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Wisgrow Math A - Plus

Session 120 | Polynomial 2 | Worksheet 120

1) Consider the polynomial $p(x) = x^2 - 8x + 12$

   a) If $p(x) = (x-a)(x-b)$then what is $a+b$ and ab
   b) Find a, b and write $p(x)$ as the product of two first degree factors
   c) Find the solution of the equation $p(x) = 0$

Answers

a) $x^2 - 8x + 12 = (x-a)(x-b) = x^2 - (a+b)x + ab$,$a + b = 8, ab = 12$

b) $(a-b)^2 = (a+b)^2 - 4ab$
$(a-b)^2 = 8^2 - 4 \times 12 = 16, a - b = 4.$
$a + b = 8, a - b = 4 \rightarrow 2a = 12, a = 6, b = 2$
$p(x) = (x-6)(x-2)$

c) $p(x) = 0 \rightarrow (x-6)(x-2) = 0, x = 6, x = 2$

2) If $p(x) = x^3 - 4x^2 + 6x - k$then

   a) Find k such that $x - 1$ a factor of $p(x)$
   b) Write the polynomial . Is $(x+1)$ a factor of $p(x)$
   c) What is the speciality of the coefficients of $p(x)$ having $x - 1$ a factor
   d) Write three polynomials having $x - 1$ a factor

Answers

a) Since $(x-1)$a factor $p(1) = 0.$
$1^3 - 4 \times 1^2 + 6 \times 1 - k = 0, 1 - 4 + 6 - k = 0, k = 3$

b) $p(x) = x^3 - 4x^2 + 6x - 3$
$p(-1) = (-1)^3 - 4 \times (-1)^2 + 6 \times (-1) - 3 = -1 - 4 - 6 - 3 = -14 \neq 0$
$p(-1) \neq 0$. Therefore $(x+1)$not a factor.

c) Sum of the coefficients will be zero $(x-1)$

d) It can be ant polynomial with sum of the coefficients is zero .
$x^3 - x^2 + x - 1$,$2x^3 - 4x^2 + 5x - 3, x^3 - 4x^2 + 2x + 1$

3) Consider the polynomials $p(x) = x^3 + 1$, $q(x) = x^3 + x^2 + x + 1$

   a) Find $p(-1)$ and $q(-1)$
   b) What is the factor common to both the polynomials
   c) Find $r(x) = p(x) + q(x)$
   d) what is the first degree factor of $r(x)$

Answers

a) $p(-1) = (-1)^3 + 1 = -1 + 1 = 0$
$q(-1) = (-1)^3 + (-1)^2 + (-1) + 1 = -1 + 1 - 1 + 1 = 0$

b) $p(-1) = 0, q(-1) = 0$ implies $(x - 1)$ is a factor of both .$(x - 1)$is the common factor

c) $r(x) = (x^3 + 1) + (x^3 + x^2 + x + 1) = 2x^3 + x^2 + x + 2$

d) $r(-1) = 2(-1)^3 + (-1)^2 + (-1) + 2 = -2 + 1 - 1 + 2 = 0$
$x + 1$ is the factor of $r(x)$

4) $x^2 - 1$ is the factor of $p(x) = a^3 + bx^2 + cx + d$

a) Find $p(1), p(-1)$
b) Show that $a = -c, b = -d$
c) Write a polynomial having $x^2 - 1$ a factor

Answers

a) $x^2 - 1 = (x - 1)(x + 1)$
$(x - 1), (x + 1)$ are the factors of $p(x)$
$p(-1) = 0, p(1) = 0$

b) $p(1) = 0 \rightarrow a + b + c + d = 0$
$p(-1) = 0 \rightarrow a - b + c - d = 0, a + c = b + d$
$a + b + c + d = 0 \rightarrow 2(a + c) = 0, a + c = 0, a = -c, b = -d$

c) The condition $a = -c, b = -d$ should be satisfied. Example $3x^3 - 4x^2 - 3x + 4$

5) If $p(x) = x^3 - 8$ then

a) Check whether $x - 2$ a factor of $p(x)$
b) Write a first degree factor of $x^3 - 27$
c) What is the second degree factor of $x^3 - 27$

Answers

a) $p(2) = 2^3 - 8 = 8 - 8 = 0$
$x - 2$ is a factor of $p(x)$

b) $q(x) = x^3 - 27$ implies $q(3) = 3^3 - 27 = 27 - 27 = 0$
$x - 3$ is a factor of $x^3 - 27$

c) $x^3 - 27 = x^3 - 3^3 = (x - 3)(ax^2 + bx + c)$
.$ax^2 + bx + c$ is the second degree factor .

$x^3 - 27 = (x - 3)(ax^2 + bx + c$
$x(ax^2 + bx + c) - 3(ax^2 + bx + c) = ax^3 + bx^2 + cx - 3ax^2 - 3bx - 3c = ax^3 + (b - 3a)x^2 + (c - 3b)x - 3c$
Equating the coefficients $a = 1, (b - 3a) = 0, (c - 3b = 0), -27 = -3c, c = 9$
$c - 3b = 0 \rightarrow 9 - 3b = 0, b = 3,$
$x^3 + 3x + 9$is the second degree factor

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Wisgrow Math A - Plus

Session 121 |Polynomial 3 | Worksheet 121

1) Consider the polynomial $p(x) = 3x^2 + 4x + 1$

   a) Write $p(x)$as the product of two first degree factors

   b) Find the solution of the equation $p(x) = 0$

Answers

a) $p(x) = 3x^2 + 4x + 1 = k(x-a)(x-b) = k(x^2 - (a+b)x + ab) = kx^2 - k(a+b)x + kab$
$k = 3, a+b = -\frac{4}{3}, ab = \frac{1}{3}$
$(a-b)^2 = (a+b)^2 - 4ab \to (\frac{-4}{3})^2 - 4\frac{1}{3} = \frac{4}{9}$
$a - b = \frac{2}{3}$
$a - b = \frac{2}{3}, a + b = \frac{-4}{3} \to a = \frac{-1}{3}, b = -1$
$p(x) = k(x-a)(x-b) \to 3(x - \frac{-1}{3})(x -^- 1) = 3(\frac{3x+1}{3})(x+1) = (x+1)(3x+1)$

b) $x + 1 = 0 \to x = -1, 3x + 1 = 0 \to x = \frac{-1}{3}$

2) Consider the equation $p(x) = x^3 + 4x^2 + x - 7$

   a) Check whether$x - 1$ a factor of this polynomial or not

   b) If not what should be subtracted from $p(x)$ to get another polynomial $q(x)$ in which $x - 1$ is a factor

   c) Write $q(x)$ as the product of three first degree factors

   d) Write the solution of the equation $q(x) = 0$.

Answers

a) $p(1) = 1^3 + 4 \times 1^2 + 1 - 7 = 6 - 7 = -1 \neq 0$
$x - 1$ is not a factor

b) Since $p(1) = -1$,on subtracting -1 from $p(x)$ we get $(x-1)$ a factor .
$q(x) = x^3 + 4x^2 + x - 6$

c) $x^3 + 4x^2 + x - 6 = (x-1)(ax^2 + bx + c)$.
Equating the constant terms both sides $-6 = -c, c = 6$
Equating coefficients of x on both sides $1 = c - b \to 1 = 6 - b, b = 5$
equating coefficients of x^2on both sides $-a + b = 4, -a + 5 = 4, a = 1$
$ax^2 + bx + c = x^2 + 5x + 6 = (x+2)(x+3)$
$q(x) = (x+1)(x+2)(x+3)$

d) $q(x) : (x+1)(x+2)(x+3) = 0, x = -1, -2, -3$ are the solutions.

3) Consider the second degree polynomial $x^2 - 20x + 91$.This is the area of a rectangle with the sides the first degee polynomials.

   a) What are the sides of this rectangle.

   b) What is the condition for getting a rectangle .

   c) What is the polynomial representing the perimetre of the rectangle.

Answers

a) $p(x) : x^2 - 20x + 91 = 0, x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$x = 13, 7$
$p(x) = (x-7)(x-13)$
sides are $x-7, x-13$

b) Side should be greater than 0.
$x - 13 > 0 \rightarrow x > 13$

c) Perimetre $= 2(x - 13 + x - 7) = 2(2x - 20) = 4x - 40$

4) When a first degree polynomial is added to $x^3 + 2x^2$ we get a polynomial $p(x)$ such that $x^2 - 1$ is a factor.

a) Write two first degee factors of $p(x)$

b) What is the first degree factor to be added?

c) What are the solutions of the equation $p(x) = 0$

Answers

a) $x^2 - 1 = (x-1)(x+1)$ implies $x-1, x+1$ are the factors of $p(x)$

b) Term to be added is $ax + b$.
$p(x) = x^3 + 2x^2 + ax + b$
Since $(x-1)$ is a factor sum of coefficients is 0.
$1 + 2 + a + b = 0, a + b = -3$
Since $x+1$ is a factor $p(-1) = 0$
$-1 + 2 - a + b = 0, a - b = 1$
$a + b = -3, a - b = 1 \rightarrow 2a = -2, a = -1, b = -2$
കൂട്ടേണ്ട പദം $ax + b = -x - 2$
$p(x) = x^3 + 2x^2 - x - 2$

c) One factor is $x^2 - 1$. This is a second degree factor.Other factor is $px + q$.
$x^3 + 2x^2 - x - 2 = (x^2 - 1)(px + q)$
we get $q = 2, p = 1$by equating the coefficients .Third factor is $x + 2$

5) Consider the equation $p(x) = x^2 + 4x + k$

a) If $k = 0$ write the first degree factors of $p(x)$

b) If $k = 4$ what are the facors of this polynomial ?

c) What is the condition for k to get two first degree factors of $p(x)$?

Answers

a) If $k = 0$, $p(x) = x^2 + 4x$. x is a factor. $x + 4$ is also a factor.

b) If $x = 4$ then $x^2 + 4x + k = x^2 + 4x + 4 = (x+2)(x+2)$.Both factors are $x + 2$.

c) $p(x) = x^2 + 4x + k$.
$(x-a), (x-b)$ are the factors $x^2 + 4x + k = (x-a)(x-b) = x^2 - (a+b)x + ab$
$a + b = -4, ab = k$. $(a-b)^2 = 4^2 - 4k$. If k is greater than 4 , $(a-b)^2$become a negative number. It is meaningless. $k$should be less than or equal to 4.

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Session 122 |Polynomial 4 | Worksheet 122

1) Write a polynomial $p(x)$ such that $p(1) = p(-2) = p(0) = 0$

   a) What are the first degree factors of $p(x)$

   b) What should be added to $p(x)$ to get a polynomial in which $x + 1$ is a factor?

Answers

a) Since $p(1) = 0$, $x - 1$ is a factor. Since $p(-2) = 0$, $x + 2$ is a factor .Since $p(0) = 0$, x is a factor .
$p(x) = x(x-1)(x+2), p(x) = x^3 + x^2 - 2x$ First degree factors are $x - 1, x + 2, x$

b) Since $x + 1$ is a factor $p(-1) = 0$.
$p(-1) = (-1)^3 + (-1)^2 - 2 \times (-1) = -1 + 1 + 2 = 2 \neq 0$. -2should be subtracted .

2) Consider the polynomial $p(x) = 4x^2 - 16x + 15$

   a) Write $p(x)$ in the form $(x - a)(x - b)$ and find $a + b, ab$

   b) Find $a - b$

   c) Write $p(x)$as the product of two first degree factors

   d) Find the solution of the equation $p(x) = 0$

Answers

a) $4x^2 - 16x + 15 = k(x-a)(x-b) = k(x^2 - (a+b)x + ab) = kx^2 - k(a+b)x + kab$
Equating the coefficients of x^2 on both sides $k = 4$
$16 = k(a + b) \rightarrow a + b = \frac{16}{4} = 4$
$kab = 15 \rightarrow ab = \frac{15}{4}$

b) $(a - b)^2 = (a + b)^2 - 4ab = 4^2 - 4 \times \frac{15}{4} = 1, a - b = 1$

c) $a + b = 4, a - b = 1 \rightarrow 2a = 5, a = \frac{5}{2}, b = \frac{3}{2}$
$p(x) = 4(x - \frac{5}{2})(x - \frac{3}{2}) = 4(\frac{2x-5}{2})(\frac{2x-3}{2}) = (2x - 5)(2x - 3)$

d) $p(x) = 0 \rightarrow 2x - 3 = 0, x = \frac{3}{2}.2x - 5 = 0 \rightarrow x = \frac{5}{2}$

3) Consider the polynomial $p(x) = x^n + 1$

   a) What are the numbers suitable for n getting $x + 1$ a factor of $p(x)$?

   b) Can $x - 1$ a factor of $p(x)$ for any n ?

   c) Can $x^2 - 1$ a factor of $p(x)$?

Answers

a) n should be odd .If n is odd $p(-1) = (-1)^n + 1 = -1 + 1 = 0$.Therefore $(x + 1)$ is a factor.

b) Whether n is odd or even $P(1) \neq 0.x - 1$ is not a factor

c) $x^2 - 1 = (x + 1)(x - 1)$.
$x + 1, x - 1$ are not factors .$x^2 - 1$ is not a factor

4) Consider the polynomial $p(x) = x^2 + 6x + k$

a) If $k = 0$then what are the first degree factors of $p(x)$?

b) What is the value of k to get two equal first degree factors ?

c) What are the values of k not for occuring a first degree factor to this polynomial?

d) If $k = 8$ what are the first degree factors of $p(x)$?

Answers

a) If $k = 0$, $p(x) = x^2 + 6x \rightarrow x(x + 6)$. First degree factors are $x, x + 6$

b) $x^2 + 6x + k = x^2 + 2 \times 3 \times x + 3^2 \rightarrow (x + 3)^2$.For this $k = 9$. $x + 3, x + 3$ are equal the factors

c) $x^2 + 6x + k = (x - a)(x - b) \rightarrow a + b = -6, ab = k$
$(a - b)^2 = (a + b)^2 - 4ab \rightarrow (a - b)^2 = (-6)^2 - 4 \times k$
$(a - b)^2 = 36 - 4k$.
If $k > 9$ then $(a - b)^2$become a negative number,it is not possible. If $k > 9$ no first degee factor exist.

d) If $k = 8$then $p(x) = x^2 + 6x + 8 = x^2 + 4x + 2x + 8 = x(x + 4) + 2(x + 4) = (x + 4)(x + 2)$
First degree factors are $(x + 4), (x + 2)$

5) Consider the polynomial $p(x) = ax^2 - 2bx + c$

a) If $x - 1$ is a factor of $p(x)$ prove that a, b, c are in an arithmetic sequence.

b) Write twp polynomials in the form $ax^2 - 2bx + c$ such that a, b, c are in an arithmetic sequence .

c) If $x^2 - 1$ is a factor of $p(x)$ then what is $a + b$?

Answers

a) If $x - 1$ is a factor then $p(1) = 0$.
$a \times 1^2 - 2b \times 1 + c = 0, a - 2b + c = 0$
$a + c = 2b, a + c = b + b \rightarrow b - a = c - b \rightarrow a, b, c$ are in an arithmetic sequence .

b) If $a = 4, b = 3, c = 2$then $4x^2 - 6x + 2$.

c) $x^2 - 1 = (x - 1)(x + 1), x - 1, x + 1$ are the factos .$p(1) = 0 \rightarrow a - 2b + c = 0$
$p(-1) = 0 \rightarrow a + 2b + c = 0$
adding these equations $2a + 2c = 0, a + c = 0$

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Wisgrow Math A - Plus

Session 123 |Polynomial 5 | Worksheet 123

1) Consider the polynomial $p(x) = x^3 + 4x^2 + x - 6$

   a) Find $p(1)$.Is $x - 1$ a factor of this polynomial ?
   b) What is the quotient when $p(x)$ is divided by $x - 1$?
   c) Write the quotient as the product of two first degree factors
   d) Find the solution of the equation $p(x) = 0$

or

Write $x^3 + 4x^2 + x - 6$ as the product of three first degree factors .Also solve the equation $x^3 + 4x^2 + x - 6 = 0$.

Answers

a) $p(1) = 1^3 + 4 \times 1^2 + 1 - 6 = 1 + 4 + 1 - 6 = 0$
Since $p(1) = 0$ we can write $(x - 1)$ is a factor

b) Let $ax^2 + bx + c$ is the quotient.$x^3 + 4x^2 + x - 6 = (x - 1)(ax^2 + bx + c)$
$x^3+4x^2+x-6 = x(ax^2+bx+c)-(ax^2+bx+c) = ax^3+(b-a)x^2+(c-b)x-c$
Equating the coefficients $a = 1, b - a = 4 \rightarrow b = 4 + a = 4 + 1 = 5$, $c - b = 1 \rightarrow c = 1 + b = 1 + 5 = 6$
Quotient is $x^2 + 5x + 6$

c) $x^2 + 5x + 6 = x^2 + 2x + 3x + 6 = x(x + 2) + 3(x + 2) = (x + 2)(x + 3)$

d) $p(x) = (x+1)(x+2)(x+3)$, $p(x) = 0 \rightarrow (x+1) = 0or(x+2) = 0$or $(x+3) = 0$
$x = -1, -2. - 3$

2) Consider the polynomial $p(x) = ax^4 + bx^3 + cx^2 + dx + e$

   a) Write a fourth degree polynomial having $x - 1$ a factor
   b) If $x + 1$ is a factor what is the relation between the coefficients of $p(x)$
   c) Using these conditions write a fourth degree polynomial having $x^2 - 1$ as a factor.

Answers

a) If $x - 1$ is a factor then $p(1) = 0$.That is sum of the coefficients is 0.
$p(x) = x^4 + 2x^3 + 3x^2 + 4x - 10$

b) Since $x + 1$ is a factor $p(-1) = 0$.
$a - b + c - d + e = 0$ That is $a + c + e = b + d$
Choose five numbers as coefficients . $a = 1, b = 2, c = 3, d = 4, e = 2$
$q(x) = x^4 + 2x^3 + 3x^2 + 4x + 2$

c) Combining the conditions $a + b + c + d + e = 0$ and $a + c + e = b + d$, we can write $2(b + d) = 0, b + d = 0, b = -d$
Also $a + c + e = 0$
Example $r(x) = 4x^4 + 5x^3 + 3x^2 - 5x - 7$.

3) In the second degree polynomial $p(x)$ $p(\frac{1}{2}) = 0, p(\frac{1}{3}) = 0$, constant term 4.

a) If $p(x) = ax^2 + bx + 4$ then find a, b

b) Write the polynomial .What should be added to this polynomial to get another polynomial in which $x - 1$ a factor.

Answers

a) $p(\frac{1}{2}) = 0, a(\frac{1}{2})^2 + b(\frac{1}{2}) + 4 = 0$
$\frac{a}{4} + \frac{b}{2} = -4, a + 2b = -16$
$a(\frac{1}{3})^2 + b(\frac{1}{3}) + 4 = 0, \frac{a}{9} + \frac{b}{3} = -4 \; a + 3b = -36$
Solving the equations $a + 2b = -16, a + 3b = -36$ we get $a = 24, b = -20$

b) $p(x) = 24x^2 - 20x + 4$
$p(1) = 8$, -8 should be added

4) Consider the polynomial $x^2 + kx + 6$

a) If $x - 1$ is a factor what is k?

b) Write other first degree factor.

c) Find the solution of $x^2 - 7x + 6 = 0$

Answers

a) If $x - 1$ is a factor sum of the coefficients will be 0 . $k = -7$

b) $x^2 - 7x + 6 = x^2 - x - 6x + 6 = x(x - 1) - 6(x - 1) = (x - 1)(x - 6)$. Other factor is $x - 6$

c) $x = 1, x = 6$

5) Consider the polynomial $p(x) = (x + 1)(x + 2)(x + 3) + k$

a) If $p(-1) = 10$then what is k ?

b) Write the polynomial using the number k

c) Change the constant term only to make the $x - 1$ a factor of the polynomial

Answers

a) $p(-1) = 10 \rightarrow k = 10$

b) $p(x) = (x + 1)(x + 2)(x + 3) + 10 = x^3 + 6x^2 + 11x + 6$

c) 6 should be changed to -18

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