

WANDOOR GANITHAM – S.S.L.C STUDY MATERIAL 2022

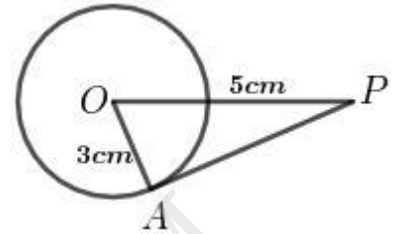
REVISION – TANGENTS – PART 1 - ANSWERS

1 In the figure ,O is the centre of the circle and AP is a tangent

OA = 3 centimetres , OP = 5 centimetres .

a) What is the measure of $\angle OAP$?

b) What is the length of the tangent PA ?



Answer

a) $\angle OAP = 90^\circ$

b) $PA^2 + 3^2 = 5^2 \implies PA^2 + 9 = 25 \implies PA^2 = 25 - 9 = 16$

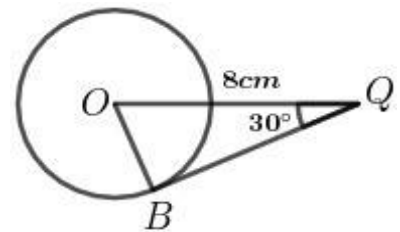
$$PA = \sqrt{16} = 4 \text{ cm}$$

2 In the figure , O is the centre of the circle and QB is a tangent . OQ = 8 cm , $\angle OQB = 30^\circ$

a) What is the measure of $\angle OBQ$?

b) What is the radius of the circle ?

c) What is the length of the tangent ?



Answer

a) $\angle OBQ = 90^\circ$

b) 4 cm

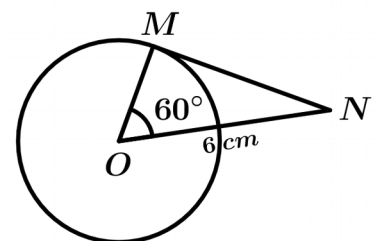
c) $4\sqrt{3}$ cm

3 In the figure , O is the centre of the circle and MN is a tangent . ON = 6 cm , $\angle MON = 60^\circ$

a) What is the measure of $\angle OMN$?

b) What is the radius of the circle ?

c) What is the length of the tangent ?



Answer

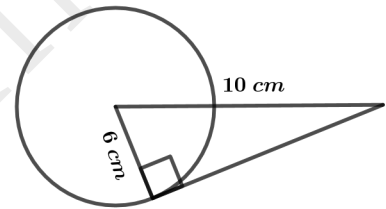
- a) $\angle OMN = 90^\circ$
- b) 3 cm
- c) $3\sqrt{3} \text{ cm}$

4 There is a point 10 cm away from the centre of a circle of radius 6 cm . A tangent is drawn through that point .

- a) What is the angle between a tangent at a point and the radius through that point ?
- b) What is the length of the tangent ?

Answer

- a) 90°
- b) $\sqrt{10^2 - 6^2} = \sqrt{100 - 36} = \sqrt{64} = 8 \text{ cm}$



5 There is a point 13 cm away from the centre of a circle of radius 5 cm . A tangent is drawn through that point .

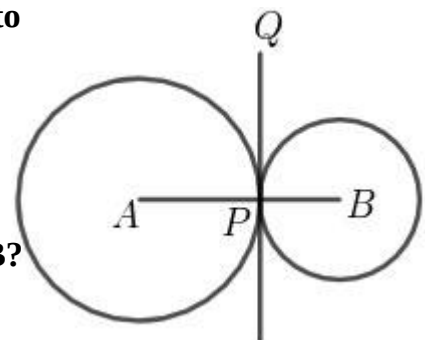
- a) What is the angle between a tangent at a point and the radius through that point ?
- b) What is the length of the tangent ?

Answer

- a) 90°
- b) $\sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12 \text{ cm}$

6 In the figure ,two circles intersect at P . PQ is a tangent to the circle with centre A .

- a) What is the measure of $\angle APQ$?
- b) Prove that PQ is a tangent to the circle with centre B?



Answer

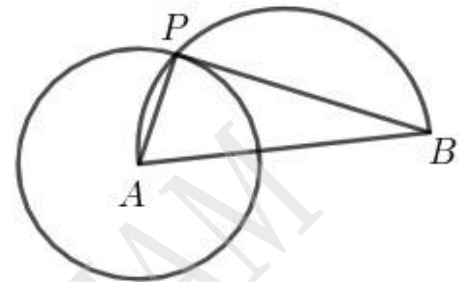
a) $\angle APQ = 90^\circ$

b) $\angle BPQ = 90^\circ \implies PQ$ is a tangent to the circle with centre B .

(The tangent at a point on the circle is perpendicular to the radius through that point

7 In the figure , a circle and a semicircle intersect at P .

A is the centre of the circle and AB is the diameter of the semicircle .



a) What is the measure of $\angle APB$?

b) Prove that PB is a tangent to the circle with centre A ?

Answer

a) $\angle APB = 90^\circ$

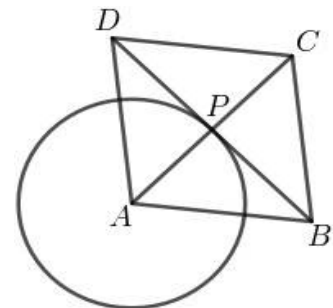
b) The tangent at a point on the circle is perpendicular to the radius through that point . So PB is a tangent to the circle with centre A .

8 In the figure , diagonals of a rhombus intersect at a point

P on the circle with centre A .

a) What is the measure of $\angle APD$?

b) Prove that PD is a tangent to the circle with centre A ?



Answer

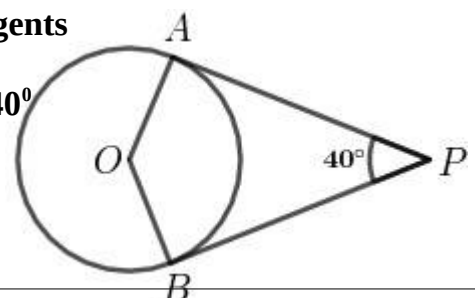
a) $\angle APD = 90^\circ$

b) The tangent at a point on the circle is perpendicular to the radius through that point . So PD is a tangent to the circle with centre A .

9 In the figure , O is the centre of the circle and the tangents through the points A and B intersect at P . $\angle APB = 40^\circ$

a) What is the measure of $\angle OAP$?

b) What is the measure of $\angle AOB$?



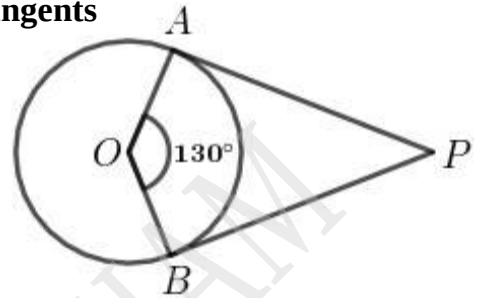
Answer

- a) $\angle OAP = 90^\circ$
- b) $\angle AOB = 40^\circ$ ($PA = PB$)

10 In the figure , O is the centre of the circle and the tangents through the points A and B intersect at P .

$\angle AOB = 130^\circ$

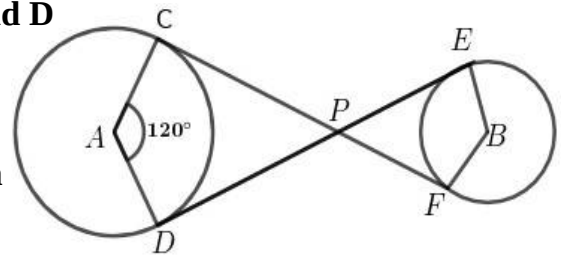
- a) What is the measure of $\angle OAP$?
- b) What is the measure of $\angle APB$?



Answer

- a) $\angle OAP = 90^\circ$
- b) $\angle APB = 180^\circ - 130^\circ = 50^\circ$

11 In the figure , tangents through the points C and D of a circle with centre A meet at P . The lines CP and DP are extended to touch the circle with centre at the points E and F . $\angle CAD = 120^\circ$



- a) What is the measure of $\angle ACP$?
- b) What is the measure of $\angle CPD$?
- c) What is the measure of $\angle EBF$?

Answer

- a) $\angle ACP = 90^\circ$
- b) $\angle CPD = 180^\circ - 120^\circ = 60^\circ$
- c) $\angle EPF = 60^\circ$
 $\angle EBF = 180^\circ - 60^\circ = 120^\circ$

12 In the figure circle touches the sides of the triangle at the points

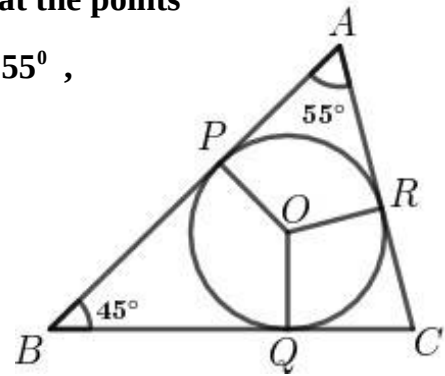
P, Q and R . O is the centre of the circle . $\angle BAC = 55^\circ$,

$\angle ABC = 45^\circ$

a) What is the measure of $\angle BPO$?

b) What is the measure of $\angle POQ$?

c) What is the measure of $\angle QOR$?



Answer

a) $\angle BPO = 90^\circ$

b) $\angle POQ = 180^\circ - 45^\circ = 135^\circ$

c) $\angle ACB = 180^\circ - (45^\circ + 55^\circ) = 180^\circ - 100^\circ = 80^\circ$

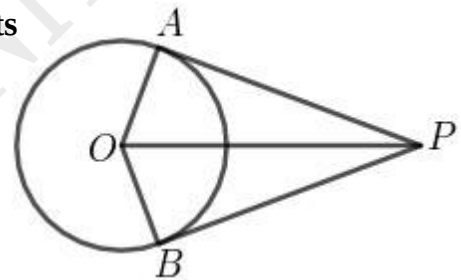
$\angle QOR = 180^\circ - 80^\circ = 80^\circ$

13 In the figure , O is the centre of circle and the tangents

through the points A and B intersect at P .

a) What is the measure of $\angle OAP$?

b) Prove that the tangents have the same length .



Answer

a) $\angle OAP = 90^\circ$

b) $\angle OBP = 90^\circ$

In right triangle OAP , $PA = \sqrt{OP^2 - OA^2}$

In right triangle OBP , $PB = \sqrt{OP^2 - OB^2}$

$$PA = PB$$

$$(OA = OB)$$

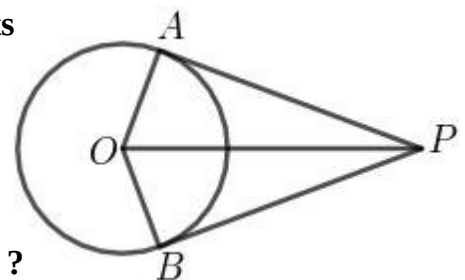
14 In the figure , O is the centre of circle and the tangents

through the points A and B intersect at P .

a) What is the measure of $\angle OAP$?

b) Prove that the triangles AOP and BOP are equal ?

c) Prove that OP is the bisector of $\angle APB$?



Answer

a) $\angle OAP = 90^\circ$

b) $PA = PB$ (The tangents to a circle from a point are of the same length)

$OA = OB$ (Radii of a circle are equal)

$OP = OP$ (Common side)

Since the sides of the triangle AOP are equal to the sides of the triangle BOP ,
they are equal triangles .

c) $\angle APO = \angle BPO$ (Angles opposite to equal sides of equal triangles are equal)

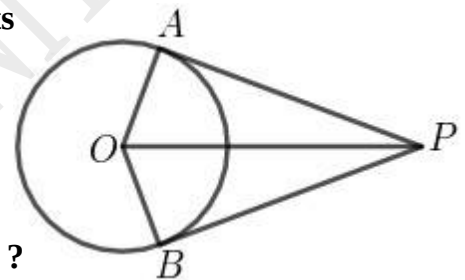
$\therefore OP$ is the bisector of $\angle APB$.

15 In the figure , O is the centre of circle and the tangents through the points A and B intersect at P .

a) What is the measure of $\angle OAP$?

b) Prove that the triangles AOP and BOP are equal ?

c) Prove that OP is the bisector of $\angle AOB$?



Answer

a) $\angle OAP = 90^\circ$

b) $PA = PB$ (The tangents to a circle from a point are of the same length)

$OA = OB$ (Radii of a circle are equal)

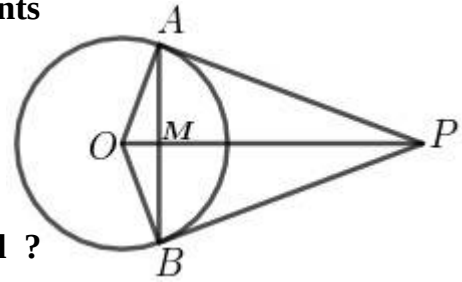
$OP = OP$ (Common side)

Since the sides of the triangle AOP are equal to the sides of the triangle BOP ,
they are equal triangles .

c) $\angle AOP = \angle BOP$ (Angles opposite to equal sides of equal triangles are equal)

$\therefore OP$ is the bisector of $\angle AOB$.

16 In the figure , O is the centre of circle and the tangents through the points A and B intersect at P .



- What is the measure of $\angle OAP$?
- Prove that the triangles AOP and BOP are equal ?
- Prove that the angles of the triangles AOM and BOM are equal ?
- Prove that OP is the bisector of AB ?
- What is the measure of $\angle AMO$?

Answer

a) $\angle OAP = 90^\circ$

b) $PA = PB$ (The tangents to a circle from a point are of the same length)

$OA = OB$ (Radii of a circle are equal)

$OP = OP$ (Common side)

Since the sides of the triangle AOP are equal to the sides of the triangle BOP , they are equal triangles .

c) $OA = OB$ (Radii of a circle are equal)

$OM = OM$ (Common side)

$\angle AOM = \angle BOM$ ($\angle AOP = \angle BOP$)

Since two sides of the triangle AOM and the angle made by them are equal to two side of the triangle BMO , they are equal triangles .

d) $AM = BM$ (Sides opposite to equal angles of equal triangles are equal)

\therefore OP is the bisector of AB .

e) $\angle AMO = \angle AMO$

$\angle AMO + \angle AMO = 180^\circ$ (linear pair)

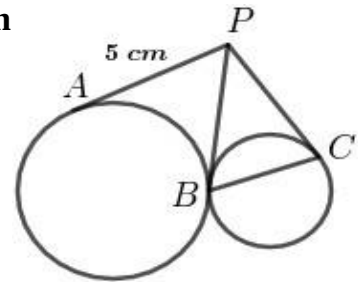
$\therefore \angle AMO = \angle AMO = 90^\circ$

17 In the figure two circle intersect at B . The tangents through

A , B , C meet at P . PA = 5 cm .

a) What is the length of PB ?

b) Prove that PBC is an isosceles triangle ?



Answer

a) PB = 5cm . (The tangents to a circle from a point are of the same length)

b) PB = PC

∴ PBC is an isosceles triangle .

18 In the figure two circle intersect at B . The tangents through

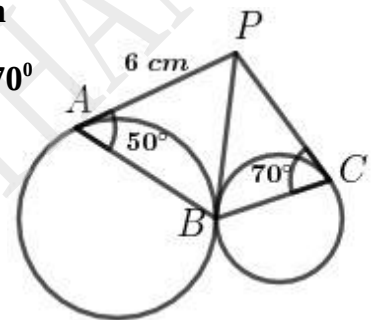
A , B , C meet at P . PA = 6 cm , $\angle BAP = 50^\circ$, $\angle BCP = 70^\circ$

a) What is the length of PB ?

b) What is the measure of $\angle APB$?

c) What is the measure of $\angle BPC$?

c) What is the measure of $\angle APC$?



Answer

a) PB = 6 cm (The tangents to a circle from a point are of the same length)

b) $\angle ABP = \angle BAP = 50^\circ$ (PA = PB)

c) $\angle BCP = \angle CBP = 70^\circ$ (PB = PC)

$$\angle BPC = 180^\circ - (70^\circ + 70^\circ) = 180^\circ - 140^\circ = 40^\circ$$

$$d) \angle APB = 180^\circ - (50^\circ + 50^\circ) = 180^\circ - 100^\circ = 80^\circ$$

$$\angle APC = \angle APB + \angle BPC = 80^\circ + 40^\circ = 120^\circ$$

19 In the figure , tangents through the points C and D

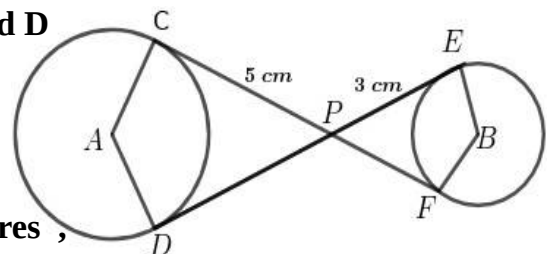
of a circle with centre A meet at P . The lines CP and DP are extended to touch the circle with

centre at the points E and F . PC = 5 centimetres ,

PE = 3 centimetres

a) What is the length of PD ?

b) What is the length of CF ?



Answer

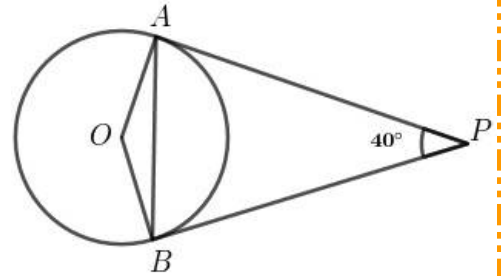
a) $PD = PC = 5 \text{ cm.}$

b) $PE = PF = 3 \text{ cm.}$

$CF = 5 + 3 = 8 \text{ cm.}$

20 In the figure , O is the centre of the circle and tangents through the points A and B intersect at P .

$\angle APB = 40^\circ$



a) What is the measure of $\angle OAP$?

b) What is the measure of $\angle AOB$?

c) What is the measure of $\angle ABP$?

d) What is the measure of $\angle OAB$?

Answer

a) $\angle OAP = 90^\circ$

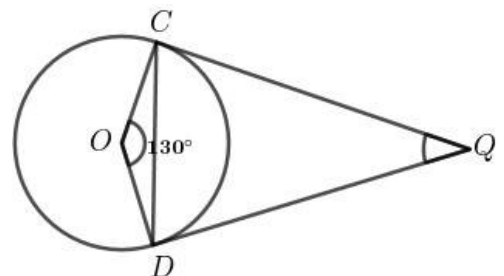
b) $\angle AOB = 140^\circ$

c) $\angle ABP = 70^\circ$

d) $\angle OAB = 20^\circ$

21 In the figure , O is the centre of the circle and tangents through the points C and D intersect at Q .

$\angle COD = 130^\circ$



a) What is the measure of $\angle OCQ$?

b) What is the measure of $\angle OCD$?

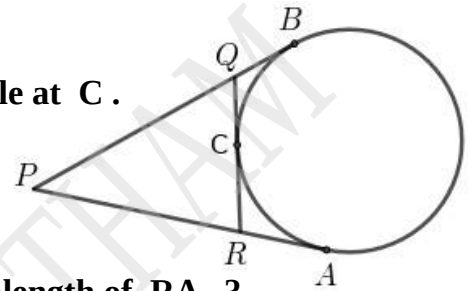
c) What is the measure of $\angle CDQ$?

c) What is the measure of $\angle CQD$?

Answer

- a) $\angle OCQ = 90^\circ$
- b) $\angle OCD = 25^\circ$
- c) $\angle CDQ = 65^\circ$
- c) $\angle CQD = 50^\circ$

22 In the figure , tangents through the points A and B of a circle intersect at P . The line QR touches the circle at C .



- a) If the length of PA is 12 centimetres , what is the length of PB ?
- b) If the length of RC is 3 centimetres , what is the length of RA ?
- c) Prove that the perimeter of the triangle PQR is double the length of PA ?

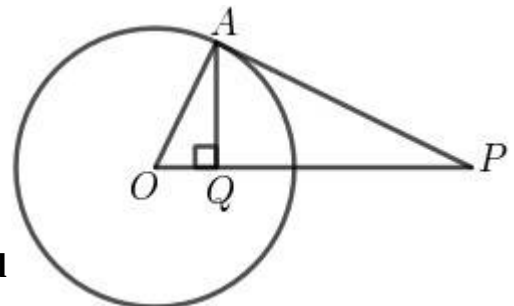
Answer

- a) $PA = PB = 12 \text{ cm}$
- c) $QC = QB$

$$\begin{aligned} \text{Perimeter of the triangle PQR} &= PQ + QR + PR \\ &= PQ + (QC + RC) + PR \\ &= PQ + (QB + RA) + PR \\ &= PB + PA \\ &= PA + PA = 2PA \end{aligned}$$

23 In the figure , O is the centre of the circle .

AP is a tangent . AQ is perpendicular to OP .



- a) What is the measure of $\angle OAP$?
- b) Prove that the angles of the triangles OAP and OAQ are same ?
- c) Prove that $OP \times OQ = OA^2$?

Answer

a) $\angle OAP = 90^\circ$

b) $\angle OAP = \angle OQA = 90^\circ$

$\angle AOP = \angle AOQ$

$\angle OPA = \angle OQA$

c) $\frac{OP}{OA} = \frac{OQ}{OA}$ (The sides of triangles with the same angles , taken in the order of size , are in the same ratio)

$OP \times OQ = OA^2$

24 In the figure , two circles intersect at P . CD is the common tangent of the circles .

Radius of the smaller circle is 4 centimetres and the radius of the larger circle is

9 centimetres . AE is perpendicular to BC .

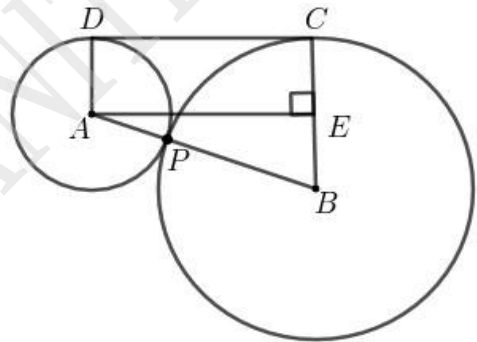
a) What is the measure of $\angle ADC$?

b) Prove that AECD is a rectangle ?

c) What is the length of BE ?

d) What is the length of AB ?

e) What is the length of the tangent CD ?



Answer

a) $\angle ADC = 90^\circ$

b) $\angle ECD = 90^\circ$, $\angle AEC = 90^\circ$

$\therefore \angle DAE = 90^\circ \implies$ AECD is a rectangle

c) $AD = CE = 4$ cm .

$BE = BC - CE = 9 - 4 = 5$ cm .

d) $AB = AP + BP = 4 + 9 = 13$ cm.

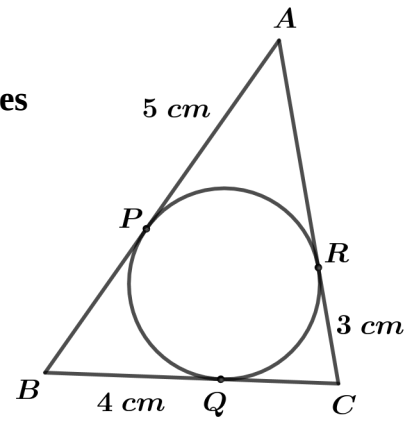
e) $BE^2 + AE^2 = AB^2 \implies 5^2 + AE^2 = 13^2 \implies 25 + AE^2 = 169$

$AE^2 = 169 - 25 = 144$

$AE = \sqrt{144} = 12$ cm.

$CD = AE = 12$ cm

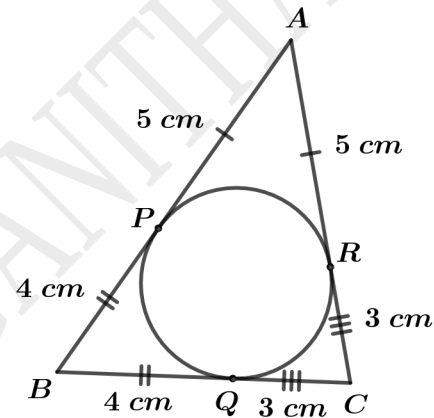
25 In the figure , the circle touches the sides of the triangle ABC at the points P, Q, R . AP = 5 centimetres
BQ = 4 centimetres , CR = 3 centimetres .



- What is the length of AR ?
- What is the length of BC ?
- What is the perimeter of the triangle ABC ?

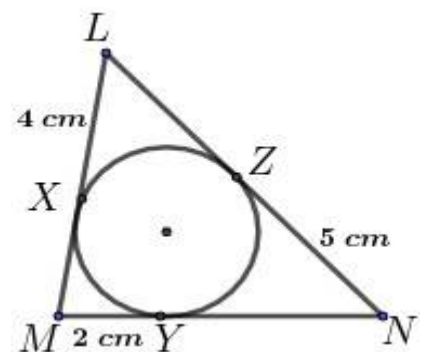
Answer

- $AR = AP = 5 \text{ cm}$
- $BP = BQ = 4 \text{ cm}$
 $CQ = CR = 3 \text{ cm}$
 $BC = 4 + 3 = 7 \text{ cm}$
- $AB = 5 + 4 = 9 \text{ cm.}$
 $AC = 5 + 3 = 8 \text{ cm.}$



Perimeter of the triangle ABC = $7 + 8 + 9 = 24 \text{ cm.}$

26 In the figure , the circle touches the sides of the triangle LMN at the points X, Y, Z . LX = 4 centimetres ,
MY = 2 centimetres , NZ = 5 centimetres.



- What is the length of LZ ?
- What is the length of MN ?
- What is the perimeter of the triangle LMN ?

Answer

a) $LX = LZ = 4 \text{ cm}$

b) $MY = MX = 2 \text{ cm}$

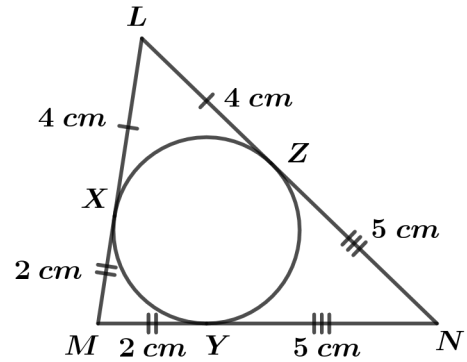
$NZ = NY = 5 \text{ cm}$

$MN = 2 + 5 = 7 \text{ cm}$

c) $LM = 4 + 2 = 6 \text{ cm}$

$LN = 4 + 5 = 9 \text{ cm.}$

Perimeter of the triangle $LMN = 7 + 9 + 6 = 22 \text{ cm.}$



27 In the figure , the circle touches the sides of the triangle

ABC at the points P,Q, R . $AB = 10 \text{ centimetres ,}$

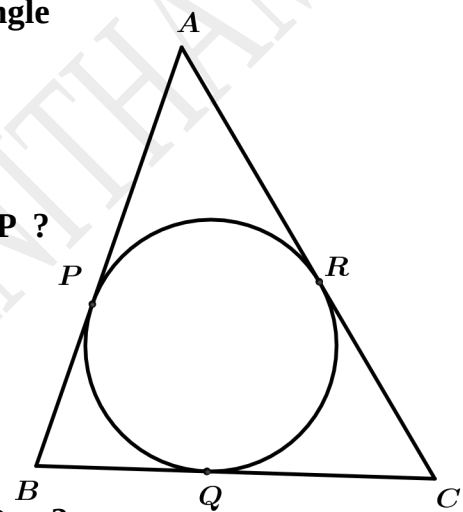
$BC = 8 \text{ centimetres , } AC = 12 \text{ centimetres .}$

a) Which other line has the same length as that of AP ?

b) If the length AR is taken as x , what is the length of BQ ?

c) What is the value of x ?

d) What are the lengths of the lines AP , BQ and CR ?



Answer

a) $AP = AR$

b) $BP = BQ = 10 - x$

c) $CQ = CR = 12 - x$

d) $10 - x + 12 - x = 8$

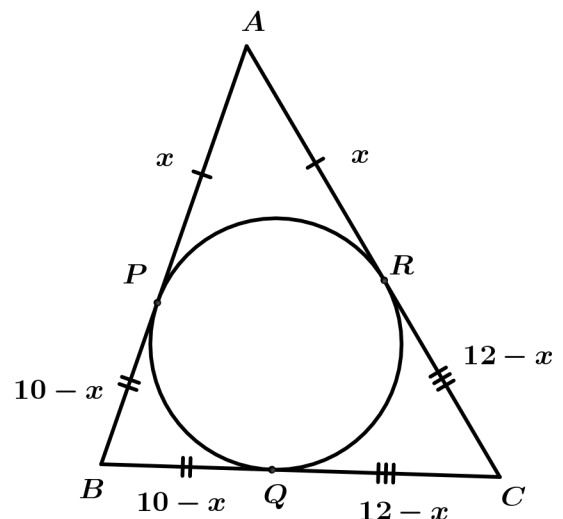
$22 - 2x = 8$

$2x = 14 \implies x = \frac{14}{2} = 7 \text{ cm}$

$AP = x = 7 \text{ cm}$

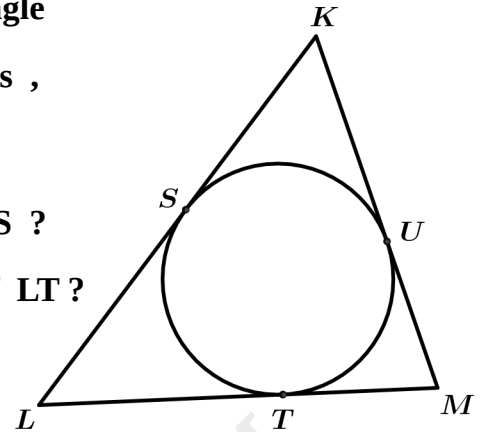
$BQ = 10 - x = 10 - 7 = 3 \text{ cm}$

$CR = 12 - x = 12 - 7 = 5 \text{ cm}$



28 In the figure , the circle touches the sides of the triangle KLM at the points S , T , U . KL = 11 centimetres , LM = 10 centimetres , KM = 7 centimetres .

- Which other line has the same length as that of KS ?
- If the length KS is taken as x , what is the length of LT ?
- What is the value of x ?
- What are the lengths of the lines KU , LS and MT ?



Answer

- $KS = KU$
- $LS = LT = 11 - x$
- $MU = MT = 7 - x$
- $11 - x + 7 - x = 10$

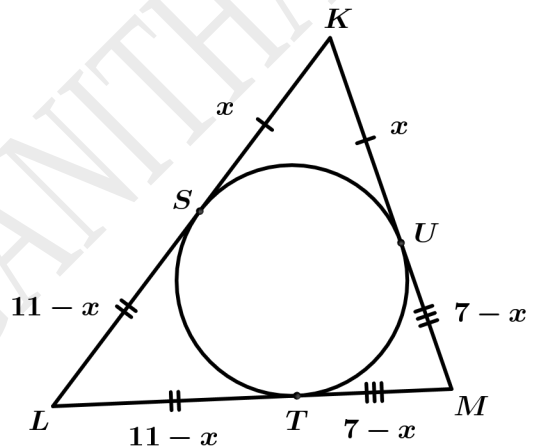
$$18 - 2x = 10$$

$$2x = 8 \implies x = \frac{8}{2} = 4 \text{ cm}$$

$$KU = x = 4 \text{ cm}$$

$$LS = 11 - x = 11 - 4 = 7 \text{ cm}$$

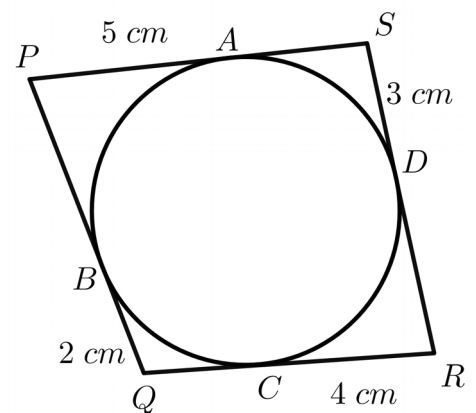
$$MT = 7 - x = 7 - 4 = 3 \text{ cm}$$



29 In the figure , the circle touches the sides of the quadrilateral PQRS at the points A , B , C , D .

PA = 5 centimetres , QB = 2 centimetres , RC = 4 centimetres , SD = 7 centimetres .

- What is the length of PB ?
- What is the length of QR ?
- What is the length of RS ?
- What is the perimeter of quadrilateral PQRS ?



Answer

a) $PB = PA = 5 \text{ cm.}$

b) $QB = QC = 2 \text{ cm.}$

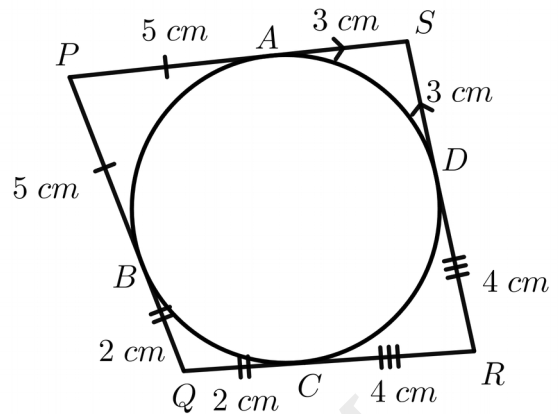
$QR = 2 + 4 = 6 \text{ cm.}$

c) $RC = RD = 4 \text{ cm.}$

$RS = 4 + 3 = 7 \text{ cm.}$

d) $SD = SA = 3 \text{ cm.}$

Perimeter of quadrilateral PQRS = $6 + 7 + 8 + 7 = 28 \text{ cm.}$



30 In the figure, the circle touches the sides of the quadrilateral ABCD at the points K, L, M, N.

$AK = 6 \text{ centimetres, } BL = 3 \text{ centimetres,}$

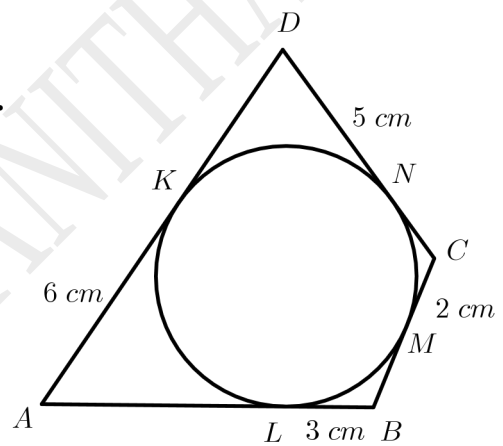
$CM = 2 \text{ centimetres, } DN = 5 \text{ centimetres.}$

a) What is the length of AL ?

b) What is the length of BC ?

c) What is the length of CD ?

d) What is the perimeter of the quadrilateral ABCD ?



Answer

a) $AK = AL = 6 \text{ cm.}$

b) $BL = BM = 3 \text{ cm.}$

$BC = 3 + 2 = 5 \text{ cm.}$

c) $CM = CN = 2 \text{ cm.}$

$CD = 2 + 5 = 7 \text{ cm.}$

d) $DN = DK = 5 \text{ cm.}$

Perimeter of the quadrilateral ABCD = $9 + 5 + 7 + 11 = 32 \text{ cm.}$

