GATE EC 2007

Q.1 to Q.20 carry one mark each

MCQ 1.1 If E denotes expectation, the variance of a random variable X is given by

(A) $E[X^2] - E^2[X]$ (B) $E[X^2] + E^2[X]$

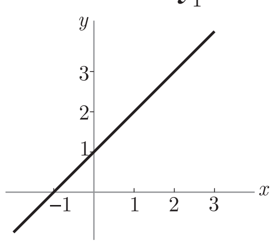
(C) $E[X^2]$ (D) $E^2[X]$

SOL 1.1 The variance of a random variable x is given by

$E[X^2] - E^2[X]$

Hence (A) is correct option.

MCQ 1.2 The following plot shows a function which varies linearly with x. The value of the integral $I = \int_1^2 y dx$ is



(A) 1.0 (B) 2.5

(C) 4.0 (D) 5.0

SOL 1.2 The given plot is straight line whose equation is

$$\frac{x}{-1} + \frac{y}{1} = 1$$

or $y = x + 1$

Now $I = \int_1^2 y dx = \int_1^2 (x+1)\, dx$

$$= \left[\frac{(x+1)^2}{2}\right]_1^2 = \frac{9}{2} - \frac{4}{2} = 2.5$$

Hence (B) is correct answer.

MCQ 1.3 For $|x| << 1$, coth (x) can be approximated as

(A) x (B) x^2

(C) $\frac{1}{x}$ (D) $\frac{1}{x^2}$

SOL 1.3 Hence (C) is correct answer.

$$\coth x = \frac{\cosh x}{\sinh x}$$

as $|x| << 1, \cosh x \approx 1$ and $\sinh x \approx x$

Thus $\coth x \approx \frac{1}{x}$

MCQ 1.4 $\lim_{\theta \to 0} \frac{\sin\left(\frac{\theta}{2}\right)}{\theta}$ is

(A) 0.5 (B) 1

(C) 2 (D) not defined

SOL 1.4 Hence (A) is correct answer.

$$\lim_{\theta \to 0} \frac{\sin(\frac{\theta}{2})}{\theta} = \lim_{\theta \to 0} \frac{\sin(\frac{\theta}{2})}{2(\frac{\theta}{2})} = \frac{1}{2}\lim_{\theta \to 0} \frac{\sin(\frac{\theta}{2})}{(\frac{\theta}{2})} = \frac{1}{2} = 0.5$$

MCQ 1.5 Which one of following functions is strictly bounded?

(A) $1/x^2$ (B) e^x

(C) x^2 (D) e^{-x^2}

SOL 1.5 Hence (D) is correct answer.

We have,
$$\lim_{x \to 0} \frac{1}{x^2} = \infty$$
$$\lim_{x \to \infty} x^2 = \infty$$
$$\lim_{x \to \infty} e^{-x} = \infty$$
$$\lim_{x \to \infty} e^{-x^2} = 0$$
$$\lim_{x \to 0} e^{-x^2} = 1$$

Thus e^{-x^2} is strictly bounded.

MCQ 1.6 For the function e^{-x}, the linear approximation around $x = 2$ is

(A) $(3 - x)e^{-2}$ (B) $1 - x$

(C) $[3 + 3\sqrt{2} - (1 - \sqrt{2})x]e^{-2}$ (D) e^{-2}

SOL 1.6 Hence (A) is correct answer.

We have $f(x) = e^{-x} = e^{-(x-2)-2} = e^{-(x-2)}e^{-2}$

$$= \left[1 - (x-2) + \frac{(x-2)^2}{2!}...\right]e^{-2}$$

$$= [1 - (x-2)]e^{-2}$$ Neglecting higher powers

$$= (3 - x)\, e^{-2}$$

MCQ 1.7 An independent voltage source in series with an impedance $Z_s = R_s + jX_s$ delivers a maximum average power to a load impedance Z_L when

(A) $Z_L = R_s + jX_s$ (B) $Z_L = R_s$

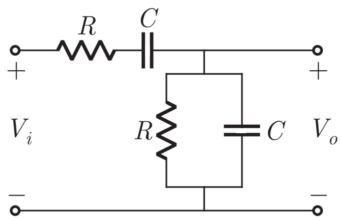
(C) $Z_L = jX_s$ (D) $Z_L = R_s - jX_s$

SOL 1.7 According to maximum Power Transform Theorem

$$Z_L = Z_s^* = (R_s - jX_s)$$

Hence (D) is correct option.

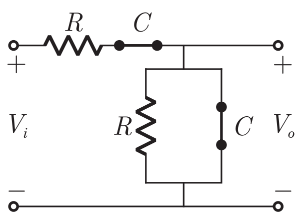
MCQ 1.8 The RC circuit shown in the figure is



(A) a low-pass filter (B) a high-pass filter

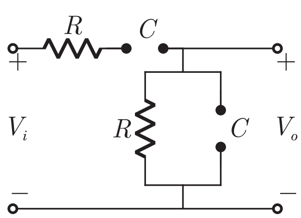
(C) a band-pass filter (D) a band-reject filter

SOL 1.8 At $\omega \to \infty$, capacitor acts as short circuited and circuit acts as shown in fig below



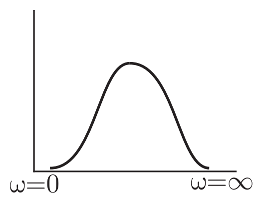
Here we get $\frac{V_0}{V_i} = 0$

At $\omega \to 0$, capacitor acts as open circuited and circuit look like as shown in fig below



Here we get also $\frac{V_0}{V_i} = 0$

So frequency response of the circuit is as shown in fig and circuit is a Band pass filter.



Hence (C) is correct option.

MCQ 1.9 The electron and hole concentrations in an intrinsic semiconductor are n_i per cm^3 at 300 K. Now, if acceptor impurities are introduced with a concentration of N_A per cm^3 (where $N_A >> n_i$, the electron concentration per cm^3 at 300 K will be)

(A) n_i (B) $n_i + N_A$

(C) $N_A - n_i$ (D) $\frac{n_i^2}{N_A}$

SOL 1.9 As per mass action law

$$np = n_i^2$$

If acceptor impurities are introduces

$$p = N_A$$

Thus $$nN_A = n_i^2$$

or $$n = \frac{n_i^2}{N_A}$$

Hence option (D) is correct.

MCQ 1.10 In a p^+n junction diode under reverse biased the magnitude of electric field is maximum at

(A) the edge of the depletion region on the p-side

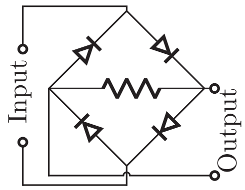
(B) the edge of the depletion region on the n-side

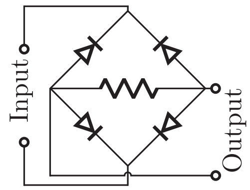
(C) the p^+n junction

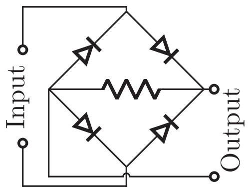
(D) the centre of the depletion region on the n-side

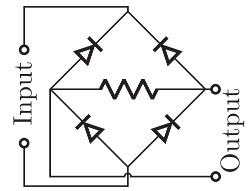
SOL 1.10 The electric field has the maximum value at the junction of p^+n.
Hence option (C) is correct.

MCQ 1.11 The correct full wave rectifier circuit is

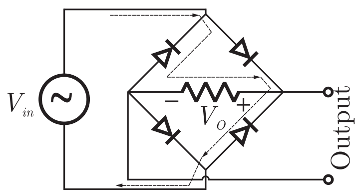
(A) 

(B) 

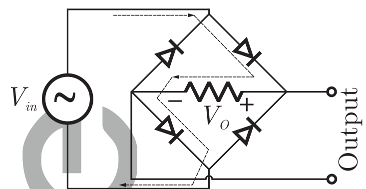
(C) 

(D) 

SOL 1.11 The circuit shown in (C) is correct full wave rectifier circuit.



During Negative Cycle



During Positive Cycle

Hence (C) is correct option.

MCQ 1.12 In a transconductance amplifier, it is desirable to have

(A) a large input resistance and a large output resistance

(B) a large input resistance and a small output resistance

(C) a small input resistance and a large output resistance

(D) a small input resistance and a small output resistance

SOL 1.12 In the transconductance amplifier it is desirable to have large input resistance and large output resistance.

Hence (A) is correct option.

MCQ 1.13 $X = 01110$ and $Y = 11001$ are two 5-bit binary numbers represented in two's complement format. The sum of X and Y represented in two's complement format using 6 bits is

(A) 100111 (B) 0010000

(C) 000111 (D) 101001

SOL 1.13 MSB of Y is 1, thus it is negative number and X is positive number

Now we have $X = 01110 = (14)_{10}$

and $Y = 11001 = (-7)_{10}$

$$X + Y = (14) + (-7) = 7$$

In signed two's complements from 7 is

$$(7)_{10} = 000111$$

Hence (C) is correct answer.

MCQ 1.14 The Boolean function $Y = AB + CD$ is to be realized using only 2 - input NAND gates. The minimum number of gates required is

(A) 2 (B) 3

(C) 4 (D) 5

SOL 1.14 Hence (B) is correct answer.

$$Y = AB + CD = \overline{\overline{AB}.\overline{CD}}$$

This is SOP form and we require only 3 NAND gate

MCQ 1.15 If the closed-loop transfer function of a control system is given as $T(s)\frac{s-5}{(s+2)(s+3)}$, then It is

(A) an unstable system (B) an uncontrollable system

(C) a minimum phase system (D) a non-minimum phase system

SOL 1.15 In a minimum phase system, all the poles as well as zeros are on the left half of the $s-$plane. In given system as there is right half zero ($s = 5$), the system is a non-minimum phase system.

Hence (D) is correct option.

MCQ 1.16 If the Laplace transform of a signal $Y(s) = \frac{1}{s(s-1)}$, then its final value is

(A) -1 (B) 0

(C) 1 (D) Unbounded

SOL 1.16 Hence (D) is correct answer.

$$Y(s) = \frac{1}{s(s-1)}$$

Final value theorem is applicable only when all poles of system lies in left half of S -plane. Here $s = 1$ is right $s-$plane pole. Thus it is unbounded.

MCQ 1.17 If $R(\tau)$ is the auto correlation function of a real, wide-sense stationary random process, then which of the following is NOT true

(A) $R(\tau) = R(-\tau)$

(B) $|R(\tau)| \leq R(0)$

(C) $R(\tau) =- R(-\tau)$

(D) The mean square value of the process is $R(0)$

SOL 1.17 Autocorrelation is even function.

Hence (C) is correct option

MCQ 1.18 If $S(f)$ is the power spectral density of a real, wide-sense stationary random process, then which of the following is ALWAYS true?

(A) $S(0) \leq S(f)$ (B) $S(f) \geq 0$

(C) $S(-f) = -S(f)$ (D) $\int_{-\infty}^{\infty} S(f)\,df = 0$

SOL 1.18 Power spectral density is non negative. Thus it is always zero or greater than zero. Hence (B) is correct option.

MCQ 1.19 A plane wave of wavelength λ is traveling in a direction making an angle 30° with positive $x-$ axis and 90° with positive $y-$ axis. The $\vec{E}$ field of the plane wave can be represented as (E_0 is constant)

(A) $\vec{E} = \hat{y}E_0 e^{j\left(\omega t - \frac{\sqrt{3}\pi}{\lambda}x - \frac{\pi}{\lambda}z\right)}$ (B) $\vec{E} = \hat{y}E_0 e^{j\left(\omega t - \frac{\pi}{\lambda}x - \frac{\sqrt{3}\pi}{\lambda}z\right)}$

(C) $\vec{E} = \hat{y}E_0 e^{j\left(\omega t + \frac{\sqrt{3}\pi}{\lambda}x + \frac{\pi}{\lambda}z\right)}$ (D) $\vec{E} = \hat{y}E_0 e^{j\left(\omega t - \frac{\pi}{\lambda}x + \frac{\sqrt{3}\pi}{\lambda}z\right)}$

SOL 1.19 Hence (A) is correct option.

$$\gamma = \beta\cos 30^\circ x \pm \beta \sin 30^\circ y$$
$$= \frac{2\pi}{\lambda}\frac{\sqrt{3}}{2}x \pm \frac{2\pi}{\lambda}\frac{1}{2}y$$
$$= \frac{\pi\sqrt{3}}{\lambda}x \pm \frac{\pi}{\lambda}y$$
$$E = a_y E_0 e^{j(\omega t - \gamma)} = a_y E_0 e^{j\left[\omega t - \left(\frac{\pi\sqrt{3}}{\lambda}x \pm \frac{\pi}{\lambda}y\right)\right]}$$

MCQ 1.20 If C is code curve enclosing a surface S, then magnetic field intensity $\vec{H}$, the current density $\vec{j}$ and the electric flux density $\vec{D}$ are related by

(A) $\iint_S \vec{H}\cdot d\vec{s} = \oiint_c \left(\vec{j} + \frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{t}$ (B) $\int_S \vec{H}\cdot d\vec{l} = \oiint_S \left(\vec{j} + \frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{S}$

(C) $\oiint_S \vec{H}\cdot d\vec{S} = \int_C \left(\vec{j} + \frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{t}$ (D) $\oint_C \vec{H}\cdot d\vec{l} \oint_c = \iint_S \left(\vec{j} + \frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{s}$

SOL 1.20 Hence (D) is correct option.

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad \text{Maxwell Equations}$$
$$\iint_s \nabla \times H\cdot ds = \iint_s \left(J + \frac{\partial D}{\partial t}\right).ds \qquad \text{Integral form}$$
$$\oint H\cdot dl = \iint_s \left(J + \frac{\partial D}{\partial t}\right).ds \qquad \text{Stokes Theorem}$$

Q.21 to Q.75 carry two marks each.

MCQ 1.21 It is given that $X_1, X_2 \dots X_M$ at M non-zero, orthogonal vectors. The dimension of the vector space spanned by the $2M$ vectors $X_1, X_2, \dots X_M, -X_1, -X_2, \dots -X_M$ is

(A) $2M$ (B) $M+1$

(C) M

(D) dependent on the choice of $X_1, X_2, \dots X_M$

SOL 1.21 For two orthogonal vectors, we require two dimensions to define them and similarly for three orthogonal vector we require three dimensions to define them. $2M$ vectors are basically M orthogonal vector and we require M dimensions to define them. Hence (C) is correct answer.

MCQ 1.22 Consider the function $f(x) = x^2 - x - 2$. The maximum value of $f(x)$ in the closed interval $[-4, 4]$ is

(A) 18 (B) 10

(C) -225 (D) indeterminate

SOL 1.22 We have

$$f(x) = x^2 - x + 2$$
$$f'(x) = 2x - 1 = 0 \rightarrow x = \frac{1}{2}$$
$$f''(x) = 2$$

Since $f''(x) = 2 > 0$, thus $x = \frac{1}{2}$ is minimum point. The maximum value in closed interval $[-4, 4]$ will be at $x = -4$ or $x = 4$

Now maximum value

$$= \max[f(-4), f(4)]$$
$$= \max(18, 10)$$
$$= 18$$

Hence (A) is correct answer.

MCQ 1.23 An examination consists of two papers, Paper 1 and Paper 2. The probability of failing in Paper 1 is 0.3 and that in Paper 2 is 0.2. Given that a student has failed in Paper 2, the probability of failing in Paper 1 is 0.6. The probability of a student failing in both the papers is

(A) 0.5 (B) 0.18

(C) 0.12 (D) 0.06

SOL 1.23 Hence (C) is correct answer.

Probability of failing in paper 1 is $P(A) = 0.3$

Possibility of failing in Paper 2 is $P(B) = 0.2$

Probability of failing in paper 1, when student has failed in paper 2 is $P(\frac{A}{B}) = 0.6$

We know that

$$P\left(\frac{A}{B}\right) = \frac{(P \cap B)}{P(B)}$$

or $P(A \cap B) = P(B)\, P\left(\frac{A}{B}\right) = 0.6 \times 0.2 = 0.12$

MCQ 1.24 The solution of the differential equation $k^2 \frac{d^2 y}{dx^2} = y - y_2$ under the boundary conditions

(i) $y = y_1$ at $x = 0$ and

(ii) $y = y_2$ at $x = \infty$, where k, y_1 and y_2 are constants, is

(A) $y = (y_1 - y_2)\exp\left(-\frac{x}{k^2}\right) + y_2$

(B) $y = (y_2 - y_1)\exp\left(-\frac{x}{k}\right) + y_1$

(C) $y = (y_1 - y_2)\sinh\left(\frac{x}{k}\right) + y_1$

(D) $y = (y_1 - y_2)\exp\left(-\frac{x}{k}\right) + y_2$

SOL 1.24 Hence (D) is correct answer.

We have $$k^2 \frac{d^2y}{dx^2} = y - y_2$$

or $$\frac{d^2y}{dx^2} - \frac{y}{k^2} = -\frac{y_2}{k^2}$$

A.E. $$D^2 - \frac{1}{k^2} = 0$$

or $$D = \pm\frac{1}{k}$$

$$\text{C.F.} = C_1 e^{-\frac{x}{k}} + C_2 e^{\frac{x}{k}}$$

$$\text{P.I.} = \frac{1}{D^2 - \frac{1}{k^2}}\left(\frac{-y_2}{k^2}\right) = y_2$$

Thus solution is

$$y = C_1 e^{-\frac{x}{k}} + C_2 e^{\frac{x}{k}} + y_2$$

From $y(0) = y_1$ we get

$$C_1 + C_2 = y_1 - y_2$$

From $y(\infty) = y_2$ we get that C_1 must be zero.

Thus $$C_2 = y_1 - y_2$$

$$y = (y_1 - y_2)\, e^{-\frac{x}{k}} + y_2$$

MCQ 1.25 The equation $x^3 - x^2 + 4x - 4 = 0$ is to be solved using the Newton - Raphson method. If $x = 2$ is taken as the initial approximation of the solution, then next approximation using this method will be

(A) 2/3

(B) 4/3

(C) 1

(D) 3/2

SOL 1.25 We have

$$f(x) = x^3 - x^2 + 4x - 4$$

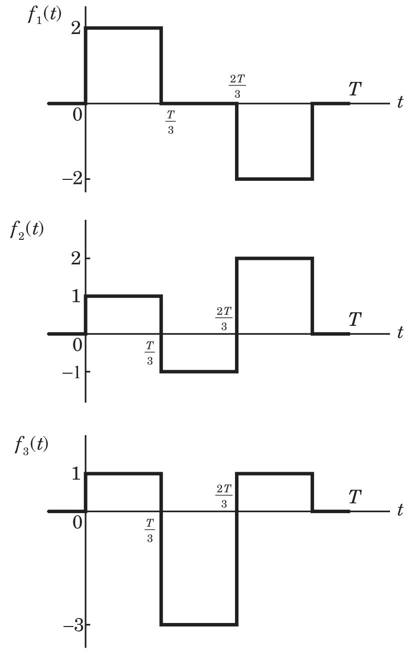
$$f'(x) = 3x^2 - 2x + 4$$

Taking $x_0 = 2$ in Newton-Raphosn method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{2^3 - 2^2 + 4(2) - 4}{3(2)^2 - 2(2) + 4} = \frac{4}{3}$$

Hence (B) is correct answer.

MCQ 1.26 Three functions $f_1(t), f_2(t)$ and $f_3(t)$ which are zero outside the interval $[0, T]$ are shown in the figure. Which of the following statements is correct?



(A) $f_1(t)$ and $f_2(t)$ are orthogonal

(B) $f_1(t)$ and $f_3(t)$ are orthogonal

(C) $f_2(t)$ and $f_3(t)$ are orthogonal

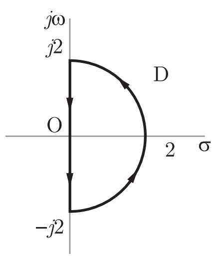
(D) $f_1(t)$ and $f_2(t)$ are orthonormal

SOL 1.26 For two orthogonal signal $f(x)$ and $g(x)$

$$\int_{-\infty}^{+\infty} f(x)\,g(x)\,dx = 0$$

i.e. common area between $f(x)$ and $g(x)$ is zero.
Hence (C) is correct options.

MCQ 1.27 If the semi-circular control D of radius 2 is as shown in the figure, then the value of the integral $\oint_D \frac{1}{(s^2-1)}ds$ is



(A) $j\pi$

(B) $-j\pi$

(C) $-\pi$

(D) π

SOL 1.27 We know that

$$\oint_D \frac{1}{s^2-1}ds = 2\pi j \qquad \text{[sum of residues]}$$

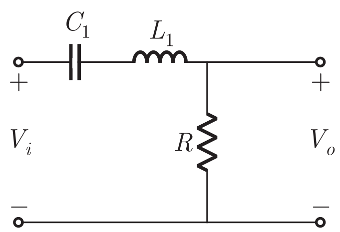
Singular points are at $s = \pm 1$ but only $s = +1$ lies inside the given contour, Thus Residue at $s = +1$ is

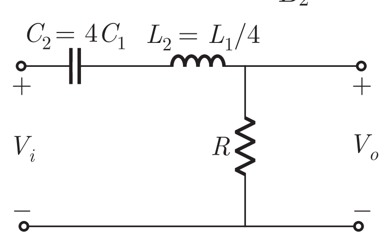
$$\lim_{s \to 1}(s-1)f(s) = \lim_{s \to 1}(s-1)\frac{1}{s^2-1} = \frac{1}{2}$$

$$\oint_D \frac{1}{s^2-1}\,ds = 2\pi j\left(\frac{1}{2}\right) = \pi j$$

Hence (A) is correct answer.

MCQ 1.28 Two series resonant filters are as shown in the figure. Let the 3-dB bandwidth of Filter 1 be B_1 and that of Filter 2 be B_2. the value $\frac{B_1}{B_2}$ is





(A) 4 (B) 1

(C) 1/2 (D) 1/4

SOL 1.28 We know that bandwidth of series RLC circuit is $\frac{R}{L}$. Therefore

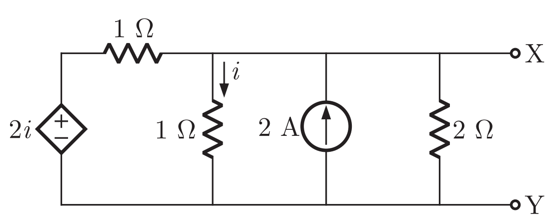
Bandwidth of filter 1 is $B_1 = \frac{R}{L_1}$

Bandwidth of filter 2 is $B_2 = \frac{R}{L_2} = \frac{R}{L_1/4} = \frac{4R}{L_1}$

Dividing above equation $\frac{B_1}{B_2} = \frac{1}{4}$

Hence (D) is correct option.

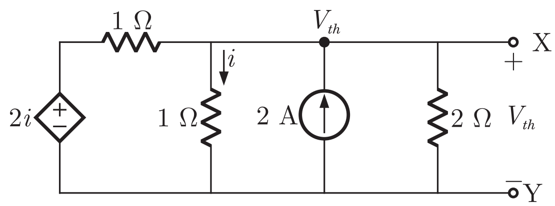
MCQ 1.29 For the circuit shown in the figure, the Thevenin voltage and resistance looking into $X - Y$ are



(A) $\frac{4}{3}$ V, 2 Ω (B) 4 V, $\frac{2}{3}$ Ω

(C) $\frac{4}{3}$ V, $\frac{2}{3}$ Ω (D) 4 V, 2 Ω

SOL 1.29 Here V_{th} is voltage across node also. Applying nodal analysis we get



$$\frac{V_{th}}{2} + \frac{V_{th}}{1} + \frac{V_{th} - 2i}{1} = 2$$

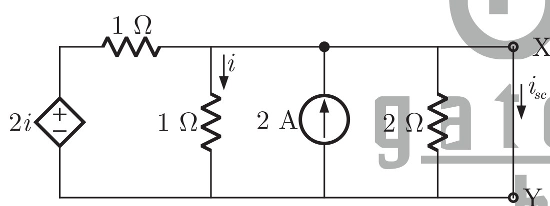
But from circuit $i = \frac{V_{th}}{1} = V_{th}$

Therefore

$$\frac{V_{th}}{2} + \frac{V_{th}}{1} + \frac{V_{th} - 2V_{th}}{1} = 2$$

or $V_{th} = 4$ volt

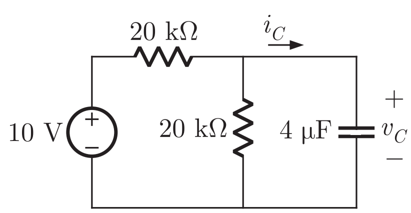
From the figure shown below it may be easily seen that the short circuit current at terminal XY is $i_{sc} = 2$ A because $i = 0$ due to short circuit of 1 Ω resistor and all current will pass through short circuit.



Therefore $R_{th} = \frac{V_{th}}{i_{sc}} = \frac{4}{2} = 2\,\Omega$

Hence (D) is correct option.

MCQ 1.30 In the circuit shown, v_C is 0 volts at $t = 0$ sec. For $t > 0$, the capacitor current $i_C(t)$, where t is in seconds is given by



(A) $0.50\exp(-25t)$ mA (B) $0.25\exp(-25t)$ mA

(C) $0.50\exp(-12.5t)$ mA (D) $0.25\exp(-6.25t)$ mA

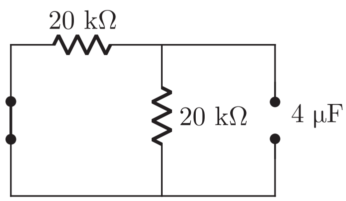
SOL 1.30 The voltage across capacitor is

At $t = 0^+$, $V_c(0^+) = 0$

At $t = \infty$, $V_C(\infty) = 5$ V

The equivalent resistance seen by capacitor as shown in fig is

$$R_{eq} = 20 \| 20 = 10k\Omega$$



Time constant of the circuit is

$$\tau = R_{eq} C = 10k \times 4\mu = 0.04 \text{ s}$$

Using direct formula

$$V_c(t) = V_C(\infty) - [V_c(\infty) - V_c(0)]\, e^{-t/\tau}$$

$$= V_C(\infty)(1 - e^{-t/\tau}) + V_C(0)\, e^{-t/\tau} = 5(1 - e^{-t/0.04})$$

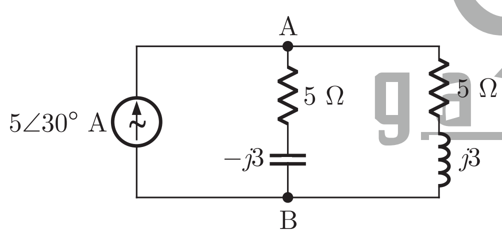
or $V_c(t) = 5(1 - e^{-25t})$

Now $I_C(t) = C\dfrac{dV_C(t)}{dt}$

$$= 4 \times 10^{-6} \times (-5 \times 25 e^{-25t}) = 0.5 e^{-25t} \text{ mA}$$

Hence (A) is correct option.

MCQ 1.31 In the ac network shown in the figure, the phasor voltage V_{AB} (in Volts) is



(A) 0 (B) 5∠30°

(C) 12.5∠30° (D) 17∠30°

SOL 1.31 Hence (D) is correct option.

Impedance $= (5 - 3j) \| (5 + 3j) = \dfrac{(5 - 3j) \times (5 + 3j)}{5 - 3j + 5 + 3j}$

$$= \frac{(5)^2 - (3j)^2}{10} = \frac{25 + 9}{10} = 3.4$$

$$V_{AB} = \text{Current} \times \text{Impedance} = 5\angle 30^\circ \times 34 = 17\angle 30^\circ$$

MCQ 1.32 A p^+n junction has a built-in potential of 0.8 V. The depletion layer width a reverse bias of 1.2 V is 2 µm. For a reverse bias of 7.2 V, the depletion layer width will be

(A) 4 µm (B) 4.9 µm

(C) 8 µm (D) 12 µm

SOL 1.32 Hence option (A) is correct.

$$W = K\sqrt{V + V_R}$$

Now $2\mu = K\sqrt{0.8 + 1.2}$

From above two equation we get

$$\frac{W}{2\mu} = \frac{\sqrt{0.8+7.2}}{\sqrt{0.8+1.2}} = \frac{\sqrt{8}}{\sqrt{2}} = 2$$

or $\qquad W_2 = 4\,\mu\text{ m}$

MCQ 1.33 Group I lists four types of $p-n$ junction diodes. Match each device in Group I with one of the option in Group II to indicate the bias condition of the device in its normal mode of operation.

Group - I	Group-II
(P) Zener Diode	(1) Forward bias
(Q) Solar cell	(2) Reverse bias
(R) LASER diode	
(S) Avalanche Photodiode	

(A) P - 1, Q - 2, R - 1, S - 2 (B) P - 2, Q - 1, R - 1, S - 2

(C) P - 2, Q - 2, R - 1, S- -2 (D) P - 2, Q - 1, R - 2, S - 2

SOL 1.33 Zener diode and Avalanche diode works in the reverse bias and laser diode works in forward bias.

In solar cell diode works in forward bias but photo current is in reverse direction. Thus

Zener diode : Reverse Bias

Solar Cell : Forward Bias

Laser Diode : Forward Bias

Avalanche Photo diode : Reverse Bias

Hence option (B) is correct.

MCQ 1.34 The DC current gain (β) of a BJT is 50. Assuming that the emitter injection efficiency is 0.995, the base transport factor is

(A) 0.980 (B) 0.985

(C) 0.990 (D) 0.995

SOL 1.34 Hence option (B) is correct.

$$\alpha = \frac{\beta}{\beta+1} = \frac{50}{50+1} = \frac{50}{51}$$

Current Gain = Base Transport Factor $\times$ Emitter injection Efficiency

$$\alpha = \beta_1 \times \beta_2$$

or $\qquad \beta_1 = \frac{\alpha}{\beta_2} = \frac{50}{51\times 0.995} = 0.985$

MCQ 1.35 Group I lists four different semiconductor devices. match each device in Group I with its charactecteristic property in Group II

Group-I	Group-II
(P) BJT	(1) Population iniversion
(Q) MOS capacitor	(2) Pinch-off voltage

(R) LASER diode (3) Early effect
(S) JFET (4) Flat-band voltage

(A) P - 3, Q - 1, R - 4, S - 2
(B) P - 1, Q - 4, R - 3, S - 2
(C) P - 3, Q - 4, R - 1, S - 2
(D) P - 3, Q - 2, R - 1, S - 4

SOL 1.35 In BJT as the B-C reverse bias voltage increases, the B-C space charge region width increases which x_B (i.e. neutral base width) $> A$ change in neutral base width will change the collector current. A reduction in base width will causes the gradient in minority carrier concentration to increase, which in turn causes an increased in the diffusion current. This effect si known as base modulation as early effect.

In JFET the gate to source voltage that must be applied to achieve pinch off voltage is described as pinch off voltage and is also called as turn voltage or threshold voltage.

In LASER population inversion occurs on the condition when concentration of electrons in one energy state is greater than that in lower energy state, i.e. a non equilibrium condition.

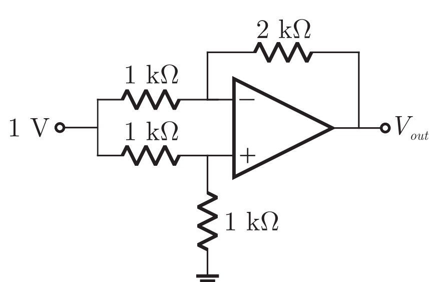
In MOS capacitor, flat band voltage is the gate voltage that must be applied to create flat ban condition in which there is no space charge region in semiconductor under oxide.

Therefore

BJT : Early effect
MOS capacitor: Flat-band voltage
LASER diode : Population inversion
JFET : Pinch-off voltage

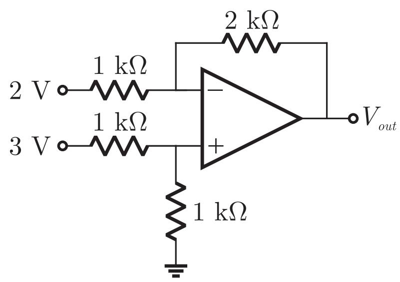
Hence option (C) is correct.

MCQ 1.36 For the Op-Amp circuit shown in the figure, V_0 is



(A) -2 V
(B) -1 V
(C) -0.5 V
(D) 0.5 V

SOL 1.36 We redraw the circuit as shown in fig.



Applying voltage division rule

$$v_+ = 0.5 \text{ V}$$

We know that $v_+ = v_-$

Thus $v_- = 0.5$ V

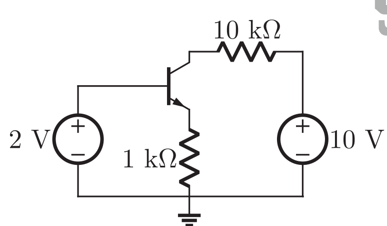
Now $$i = \frac{1-0.5}{1k} = 0.5 \text{ mA}$$

and $$i = \frac{0.5 - v_0}{2k} = 0.5 \text{ mA}$$

or $$v_0 = 0.5 - 1 = -0.5 \text{ V}$$

Hence (C) is correct option.

MCQ 1.37 For the BJT circuit shown, assume that the β of the transistor is very large and $V_{BE} = 0.7$ V. The mode of operation of the BJT is



(A) cut-off (B) saturation

(C) normal active (D) reverse active

SOL 1.37 If we assume β very large, then $I_B = 0$ and $I_E = I_C$; $V_{BE} = 0.7$ V. We assume that BJT is in active, so applying KVL in Base-emitter loop

$$I_E = \frac{2 - V_{BE}}{R_E} = \frac{2-0.7}{1k} = 1.3 \text{ mA}$$

Since β is very large, we have $I_E = I_C$, thus

$$I_C = 1.3 \text{ mA}$$

Now applying KVL in collector-emitter loop

$$10 - 10I_C - V_{CE} - I_C = 0$$

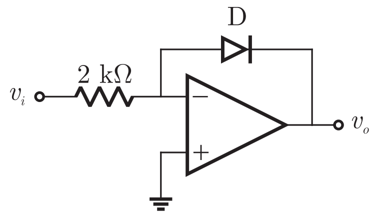
or $$V_{CE} = -4.3 \text{ V}$$

Now $$V_{BC} = V_{BE} - V_{CE}$$
$$= 0.7 - (-4.3) = 5 \text{ V}$$

Since $V_{BC} > 0.7$ V, thus transistor in saturation.

Hence (B) is correct option

MCQ 1.38 In the Op-Amp circuit shown, assume that the diode current follows the equation $I = I_s \exp(V/V_T)$. For $V_i = 2\,V$, $V_0 = V_{01}$, and for $V_i = 4\,V$, $V_0 = V_{02}$.
The relationship between V_{01} and V_{02} is



(A) $V_{02} = \sqrt{2}\, V_{o1}$ (B) $V_{o2} = e^2 V_{o1}$

(C) $V_{o2} = V_{o1} 1n2$ (D) $V_{o1} - V_{o2} = V_T 1n2$

SOL 1.38 Here the inverting terminal is at virtual ground and the current in resistor and diode current is equal i.e.

$$I_R = I_D$$

or $$\frac{V_i}{R} = I_s e^{V_D/V_T}$$

or $$V_D = V_T 1n \frac{V_i}{I_s R}$$

For the first condition

$$V_D = 0 - V_{o1} = V_T 1n \frac{2}{I_s R}$$

For the first condition

$$V_D = 0 - V_{o1} = V_T 1n \frac{4}{I_s R}$$

Subtracting above equation

$$V_{o1} - V_{o2} = V_T 1n \frac{4}{I_s R} - V_T 1n \frac{2}{I_s R}$$

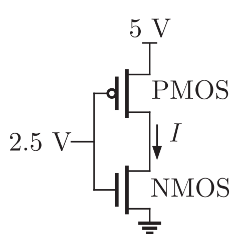
or $$V_{o1} - V_{o2} = V_T 1n \frac{4}{2} = V_T 1n2$$

Hence (D) is correct option.

MCQ 1.39 In the CMOS inverter circuit shown, if the trans conductance parameters of the NMOS and PMOS transistors are

$$k_n = k_p = \mu_n C_{ox} \frac{W_n}{L_n} = \mu C_{ox} \frac{W_p}{L_p} = 40 \mu A/V^2$$

and their threshold voltages ae $V_{THn} = |V_{THp}| = 1$ V the current I is



(A) 0 A (B) 25 μA

(C) 45 μA (D) 90 μA

SOL 1.39 Hence (D) is correct option

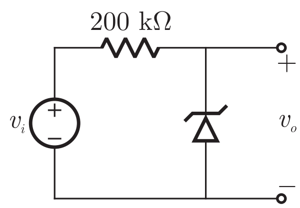
We have $V_{thp} = V_{thp} = 1$ V

and $\frac{W_P}{L_P} = \frac{W_N}{L_N} = 40\mu\text{A/V}^2$

From figure it may be easily seen that V_{as} for each NMOS and PMOS is 2.5 V

Thus $I_D = K(V_{as} - V_T)^2 = 40\frac{\mu\text{A}}{\text{V}^2}(2.5-1)^2 = 90\,\mu\text{A}$

MCQ 1.40 For the Zener diode shown in the figure, the Zener voltage at knee is 7 V, the knee current is negligible and the Zener dynamic resistance is 10 Ω. If the input voltage (V_i) range is from 10 to 16 V, the output voltage (V_0) ranges from

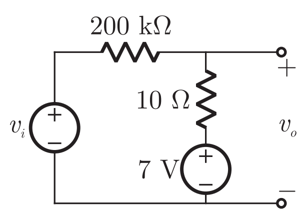


(A) 7.00 to 7.29 V (B) 7.14 to 7.29 V

(C) 7.14 to 7.43 V (D) 7.29 to 7.43 V

SOL 1.40 We have $V_Z = 7$ volt, $V_K = 0, R_Z = 10\Omega$

Circuit can be modeled as shown in fig below



Since V_i is lies between 10 to 16 V, the range of voltage across 200 kΩ

$$V_{200} = V_i - V_Z = 3 \text{ to } 9 \text{ volt}$$

The range of current through 200 kΩ is

$$\frac{3}{200k} = 15 \text{ mA to } \frac{9}{200k} = 45 \text{ mA}$$

The range of variation in output voltage

$$15\text{m} \times R_Z = 0.15 \text{ V to } 45\text{m} \times R_Z = 0.45$$

Thus the range of output voltage is 7.15 Volt to 7.45 Volt

Hence (C) is correct option.

MCQ 1.41 The Boolean expression $Y = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}BC\overline{D} + A\overline{B}\,\overline{C}D + AB\overline{C}\,\overline{D}$ can be minimized to

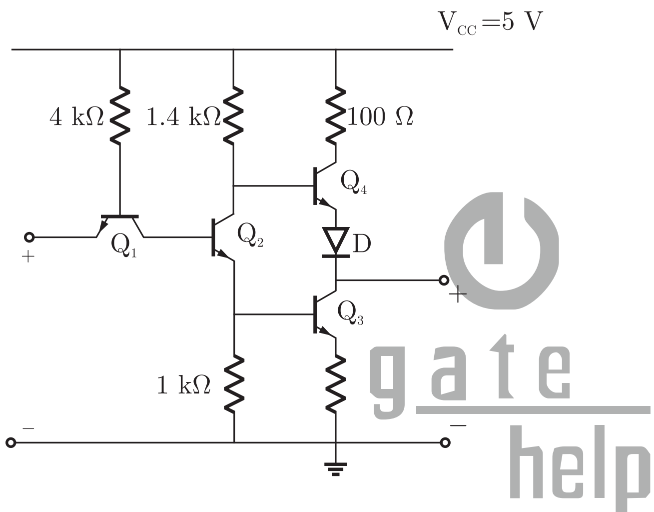
(A) $Y = \overline{A}\overline{B}\overline{C}D + \overline{A}B\overline{C} + A\overline{C}D$ (B) $Y = \overline{A}\overline{B}\overline{C}D + BC\overline{D} + A\overline{B}\overline{C}D$

(C) $Y = \overline{A}BC\overline{D} + \overline{B}\overline{C}D + A\overline{B}\overline{C}D$ (D) $Y = \overline{A}BC\overline{D} + \overline{B}\overline{C}D + AB\overline{C}\overline{D}$

SOL 1.41 Hence (D) is correct answer.

$$\begin{aligned} Y &= \overline{A}BC\overline{D} + \overline{A}BC\overline{D} + A\overline{B}\overline{C}D + AB\overline{C}\overline{D} \\ &= \overline{A}BC\overline{D} + AB\overline{C}\overline{D} + A\overline{B}\overline{C}D + \overline{A}\overline{B}\overline{C}D \\ &= \overline{A}BC\overline{D} + AB\overline{C}\overline{D} + \overline{B}\overline{C}D(A + \overline{A}) \\ &= \overline{A}BC\overline{D} + AB\overline{C}\overline{D} + \overline{B}\overline{C}D \qquad A + \overline{A} = 1 \end{aligned}$$

MCQ 1.42 The circuit diagram of a standard TTL NOT gate is shown in the figure. $V_i = 25$ V, the modes of operation of the transistors will be



(A) Q_1: revere active; Q_2: normal active; Q_3: saturation; Q_4: cut-off

(B) Q_1: revere active; Q_2: saturation; Q_3: saturation; Q_4: cut-off

(C) Q_1: normal active; Q_2: cut-off; Q_3: cut-off; Q_4: saturation

(D) Q_1: saturation; Q_2: saturation; Q_3: saturation; Q_4: normal active

SOL 1.42 In given TTL NOT gate when $V_i = 2.5$ (HIGH), then

$Q_1 \rightarrow$ Reverse active

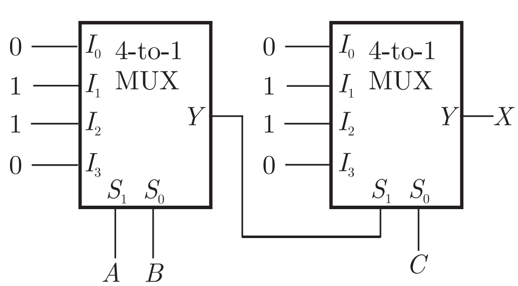
$Q_2 \rightarrow$ Saturation

$Q_3 \rightarrow$ Saturation

$Q_4 \rightarrow$ cut - off region

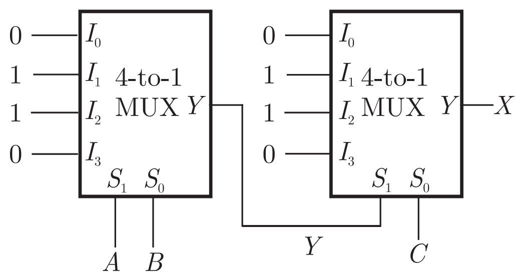
Hence (B) is correct answer.

MCQ 1.43 In the following circuit, X is given by



(A) $X = A\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{AB}C + ABC$
(B) $X = \overline{A}BC + A\overline{B}C + AB\overline{C} + \overline{AB}\,\overline{C}$
(C) $X = AB + BC + AC$
(D) $X = \overline{AB} + \overline{B}\,\overline{C} + \overline{A}\,\overline{C}$

SOL 1.43 The circuit is as shown below

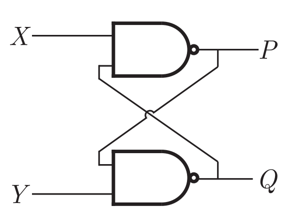


$$Y = \overline{A}B + A\overline{B}$$

and
$$X = \overline{Y}C + Y\overline{C}$$
$$= \overline{(\overline{A}B + A\overline{B})}\,C + (\overline{A}B + A\overline{B})\,\overline{C}$$
$$= (\overline{A}\,\overline{B} + AB)\,C + (\overline{A}B + A\overline{B})\,\overline{C}$$
$$= \overline{A}\,\overline{B}C + ABC + \overline{A}B\overline{C} + A\overline{B}\,\overline{C}$$

Hence (A) is correct answer.

MCQ 1.44 The following binary values were applied to the X and Y inputs of NAND latch shown in the figure in the sequence indicated below :
$X = 0, Y = 1; X = 0,\ Y = 0;\ X = 1;\ Y = 1$
The corresponding stable P, Q output will be.

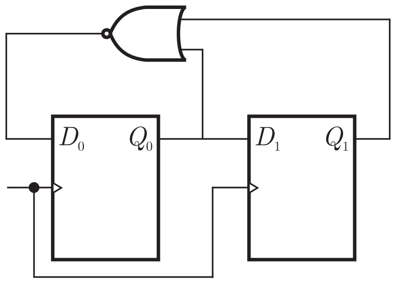


(A) $P = 1, Q = 0;\ P = 1, Q = 0;\ P = 1, Q = 0$ or $P = 0, Q = 1$
(B) $P = 1, Q = 0;\ P = 0, Q = 1;$ or $P = 0, Q = 1;\ P = 0, Q = 1$
(C) $P = 1, Q = 0;\ P = 1, Q = 1;\ P = 1, Q = 0$ or $P = 0, Q = 1$
(D) $P = 1, Q = 0;\ P = 1, Q = 1;\ P = 1, Q = 1$

SOL 1.44 Hence (C) is correct answer.

For $X = 0, Y = 1$ $P = 1, Q = 0$
For $X = 0, Y = 0$ $P = 1, Q = 1$
For $X = 1, Y = 1$ $P = 1, Q = 0$ or $P = 0, Q = 1$

MCQ 1.45 For the circuit shown, the counter state $(Q_1 Q_0)$ follows the sequence



(A) 00,01,10,11,00 (B) 00,01,10,00,01

(C) 00,01,11,00,01 (D) 00,10,11,00,10

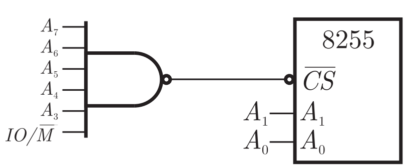
SOL 1.45 For this circuit the counter state (Q_1, Q_0) follows the sequence 00, 01, 10, 00 ... as shown below

Clock	$D_1 D_0$	$Q_1 Q_0$	Q_1 NOR Q_0
		00	1
1st	01	10	0
2nd	10	01	0
3rd	00	00	0

1	0	0	1
0	d	0	0
0	0	d	1
1	0	0	1

Hence (A) is correct answer.

MCQ 1.46 An 8255 chip is interfaced to an 8085 microprocessor system as an I/O mapped I/O as show in the figure. The address lines A_0 and A_1 of the 8085 are used by the 8255 chip to decode internally its thee ports and the Control register. The address lines A_3 to A_7 as well as the $IO/\overline{M}$ signal are used for address decoding. The range of addresses for which the 8255 chip would get selected is



(A) F8H - FBH (B) F8GH - FCH

(C) F8H - FFH (D) F0H - F7H

SOL 1.46 Chip 8255 will be selected if bits A_3 to A_7 are 1. Bit A_0 to A_2 can be 0 or. 1. Thus address range is

1 1 1 1 1 0 0 0	F8H
1 1 1 1 1 1 1 1	FFH

Hence (C) is correct answer.

MCQ 1.47 The 3-dB bandwidth of the low-pass signal $e^{-t}u(t)$, where $u(t)$ is the unit step function, is given by

(A) $\frac{1}{2\pi}$ Hz

(B) $\frac{1}{2\pi}\sqrt{\sqrt{2}-1}$ Hz

(C) ∞

(D) 1 Hz

SOL 1.47 Hence (A) is correct answer.

$$x(t) = e^{-t}u(t)$$

Taking Fourier transform

$$X(j\omega) = \frac{1}{1+j\omega}$$

$$|X(j\omega)| = \frac{1}{1+\omega^2}$$

Magnitude at 3dB frequency is $\frac{1}{\sqrt{2}}$

Thus $$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+\omega^2}}$$

or $$\omega = 1 \text{ rad}$$

or $$f = \frac{1}{2\pi} \text{ Hz}$$

MCQ 1.48 A Hilbert transformer is a

(A) non-linear system

(B) non-causal system

(C) time-varying system

(D) low-pass system

SOL 1.48 A Hilbert transformer is a non-linear system.
Hence (A) is correct answer.

MCQ 1.49 The frequency response of a linear, time-invariant system is given by $H(f) = \frac{5}{1+j10\pi f}$. The step response of the system is

(A) $5(1-e^{-5t})\,u(t)$

(B) $5[1-e^{-\frac{t}{5}}]u(t)$

(C) $\frac{1}{2}(1-e^{-5t})\,u(t)$

(D) $\frac{1}{5}(1-e^{-\frac{t}{5}})u(t)$

SOL 1.49 Hence (B) is correct answer.

$$H(f) = \frac{5}{1+j10\pi f}$$

$$H(s) = \frac{5}{1+5s} = \frac{5}{5(s+\frac{1}{5})} = \frac{1}{s+\frac{1}{5}}$$

Step response $$Y(s) = \frac{1}{s}\frac{a}{(s+\frac{1}{5})}$$

or $$Y(s) = \frac{1}{s}\frac{1}{(s+\frac{1}{5})} = \frac{5}{s} - \frac{5}{s+\frac{1}{5}}$$

or $$y(t) = 5(1 - e^{-t/5})\,u(t)$$

MCQ 1.50 A 5-point sequence $x[n]$ is given as $x[-3] = 1,\ x[-2] = 1,\ x[-1] = 0,\ x[0] = 5$ and $x[1] = 1$. Let $X(e^{j\omega})$ denoted the discrete-time Fourier transform of $x[n]$. The value of $\int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega$ is

(A) 5 (B) 10π

(C) 16π (D) $5 + j10\pi$

SOL 1.50 For discrete time Fourier transform (DTFT) when $N \to \infty$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\,e^{j\omega n}\,d\omega$$

Putting $n = 0$ we get

$$x[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\,e^{j\omega 0}\,d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega$$

or $$\int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega = 2\pi x[0] = 2\pi \times 5 = 10\pi$$

Hence (B) is correct answer.

MCQ 1.51 The $z-$transform $X(z)$ of a sequence $x[n]$ is given by $X[z] = \frac{0.5}{1-2z^{-1}}$. It is given that the region of convergence of $X(z)$ includes the unit circle. The value of $x[0]$ is

(A) -0.5 (B) 0

(C) 0.25 (D) 05

SOL 1.51 Hence (B) is correct answer.

$$X(z) = \frac{0.5}{1 - 2z^{-1}}$$

Since ROC includes unit circle, it is left handed system

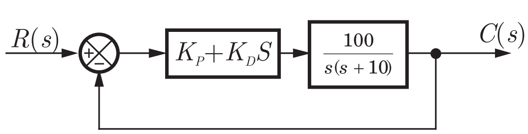
$$x(n) = -(0.5)(2)^{-n}u(-n-1)$$
$$x(0) = 0$$

If we apply initial value theorem

$$x(0) = \lim_{z \to \infty} X(z) = \lim_{z \to \infty}\frac{0.5}{1 - 2z^{-1}} = 0.5$$

That is wrong because here initial value theorem is not applicable because signal $x(n)$ is defined for $n < 0$.

MCQ 1.52 A control system with PD controller is shown in the figure. If the velocity error constant $K_V = 1000$ and the damping ratio $\zeta = 0.5$, then the value of K_P and K_D are



(A) $K_P = 100,\ K_D = 0.09$ (B) $K_P = 100,\ K_D = 0.9$
(C) $K_P = 10,\ K_D = 0.09$ (D) $K_P = 10,\ K_D = 0.9$

SOL 1.52 Hence (B) is correct option

We have $$K_v = \lim_{s \to 0} sG(s)H(s)$$

or $$1000 = \lim_{s \to 0} s\frac{(K_p + K_D s)100}{s(s+100)} = K_p$$

Now characteristics equations is

$$1 + G(s)H(s) = 0$$
$$1000 = \lim_{s \to 0} s\frac{(K_p + K_D s)100}{s(s+100)} = K_p$$

Now characteristics equation is

$$1 + G(s)H(s) = 0$$

or $$1 + \frac{(100 + K_D s)100}{s(s+10)} = 0$$ $K_p = 100$

or $s^2 + (10 + 100K_D)s + 10^4 = 0$

Comparing with $s^2 + 2\xi\omega_n + \omega_n^2 = 0$ we get

$$2\xi\omega_n = 10 + 100K_D$$

or $$K_D = 0.9$$

MCQ 1.53 The transfer function of a plant is

$$T(s) = \frac{5}{(s+5)(s^2+s+1)}$$

The second-order approximation of $T(s)$ using dominant pole concept is

(A) $\frac{1}{(s+5)(s+1)}$ (B) $\frac{5}{(s+5)(s+1)}$

(C) $\frac{5}{s^2+s+1}$ (D) $\frac{1}{s^2+s+1}$

SOL 1.53 Hence (D) is correct option.

We have $$T(s) = \frac{5}{(s+5)(s^2+s+1)}$$
$$= \frac{5}{5\left(1+\frac{s}{5}\right)(s^2+s+1)} = \frac{1}{s^2+s+1}$$

In given transfer function denominator is $(s+5)[(s+0.5)^2 + \frac{3}{4}]$. We can see easily that pole at $s = -0.5 \pm j\frac{\sqrt{3}}{2}$ is dominant then pole at $s = -5$. Thus we have approximated it.

MCQ 1.54 The open-loop transfer function of a plant is given as $G(s) = \frac{1}{s^2-1}$. If the plant is operated in a unity feedback configuration, then the lead compensator that an stabilize this control system is

(A) $\frac{10(s-1)}{s+2}$ (B) $\frac{10(s+4)}{s+2}$

(C) $\frac{10(s+2)}{s+10}$ (D) $\frac{2(s+2)}{s+10}$

SOL 1.54 Hence (A) is correct option.

$$G(s) = \frac{1}{s^2-1} = \frac{1}{(s+1)(s-1)}$$

The lead compensator $C(s)$ should first stabilize the plant i.e. remove $\frac{1}{(s-1)}$ term. From only options (A), $C(s)$ can remove this term

$$\text{Thus} G(s)\,C(s) = \frac{1}{(s+1)(s-1)} \times \frac{10(s-1)}{(s+2)}$$

$$= \frac{10}{(s+1)(s+2)}$$ Only option (A) satisfies.

MCQ 1.55 A unity feedback control system has an open-loop transfer function

$$G(s) = \frac{K}{s(s^2+7s+12)}$$

The gain K for which $s = 1 + j1$ will lie on the root locus of this system is

(A) 4 (B) 5.5

(C) 6.5 (D) 10

SOL 1.55 For ufb system the characteristics equation is

$$1 + G(s) = 0$$

or $$1 + \frac{K}{s(s^2+7s+12)} = 0$$

or $$s(s^2+7s+12) + K = 0$$

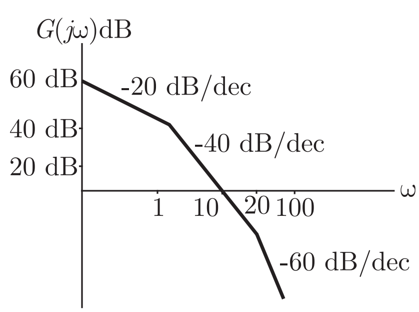
Point $s = -1 + j$ lie on root locus if it satisfy above equation i.e

$$(-1+j)[(-1+j)^2 + 7(-1+j) + 12) + K] = 0$$

or $$K = +10$$

Hence (D) is correct option.

MCQ 1.56 The asymptotic Bode plot of a transfer function is as shown in the figure. The transfer function $G(s)$ corresponding to this Bode plot is



(A) $\frac{1}{(s+1)(s+20)}$ (B) $\frac{1}{s(s+1)(s+20)}$

(C) $\dfrac{100}{s(s+1)(s+20)}$ (D) $\dfrac{100}{s(s+1)(1+0.05s)}$

SOL 1.56 At every corner frequency there is change of -20 db/decade in slope which indicate pole at every corner frequency. Thus

$$G(s) = \frac{K}{s(1+s)\left(1+\frac{s}{20}\right)}$$

Bode plot is in $(1+sT)$ form

$$20\log\frac{K}{\omega}\Big|_{\omega=0.1} = 60 \text{ dB} = 1000$$

Thus $K = 5$

Hence $G(s) = \dfrac{100}{s(s+1)(1+.05s)}$

Hence (D) is correct option.

MCQ 1.57 The state space representation of a separately excited DC servo motor dynamics is given as

$$\begin{bmatrix} \frac{d\omega}{dt} \\ \frac{di_o}{dt} \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & -10 \end{bmatrix}\begin{bmatrix} \omega \\ i_a \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix}u$$

where ω is the speed of the motor, i_a is the armature current and u is the armature voltage. The transfer function $\dfrac{\omega(s)}{U(s)}$ of the motor is

(A) $\dfrac{10}{s^2+11s+11}$ (B) $\dfrac{1}{s^2+11s+11}$

(C) $\dfrac{10s+10}{s^2+11s+11}$ (D) $\dfrac{1}{s^2+s+11}$

SOL 1.57 Hence (A) is correct option.

We have $\begin{bmatrix} \frac{d\omega}{dt} \\ \frac{di_a}{dt} \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & -10 \end{bmatrix}\begin{bmatrix} \omega \\ i_n \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix}u$

or $\dfrac{d\omega}{dt} = -\omega + i_n$ ...(1)

and $\dfrac{di_a}{dt} = -\omega - 10i_a + 10u$ ...(2)

Taking laplace transform (i) we get

$$s\omega(s) = -\omega(s) = I_a(s)$$

or $(s+1)\omega(s) = I_a(s)$ ...(3)

Taking laplace transform (ii) we get

$$sI_a(s) = -\omega(s) - 10I_a(s) + 10U(s)$$

or $\omega(s) = (-10-s)I_a(s) + 10U(s)$

$= (-10-s)(s+1)\omega(s) + 10U(s)$ From (3)

or $\omega(s) = -[s^2+11s+10]\omega(s) + 10U(s)$

$$\text{or}(s^2+11s+11)\omega(s) = 10U(s)$$

$$\text{or} \qquad \frac{\omega(s)}{U(s)} = \frac{10}{(s^2+11s+11)}$$

MCQ 1.58 In delta modulation, the slope overload distortion can be reduced by

(A) decreasing the step size
(B) decreasing the granular noise
(C) decreasing the sampling rate
(D) increasing the step size

SOL 1.58 Slope overload distortion can be reduced by increasing the step size

$$\frac{\Delta}{T_s} \geq \text{slope of } x(t)$$

Hence (D) is correct option.

MCQ 1.59 The raised cosine pulse $p(t)$ is used for zero ISI in digital communications. The expression for $p(t)$ with unity roll-off factor is given by

$$p(t) = \frac{\sin 4\pi Wt}{4\pi Wt(1-16W^2t^2)}$$

The value of $p(t)$ at $t = \frac{1}{4W}$ is

(A) -0.5
(B) 0
(C) 0.5
(D) ∞

SOL 1.59 Hence (C) is correct option.

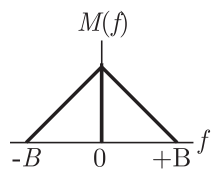
We have $p(t) = \dfrac{\sin(4\pi Wt)}{4\pi Wt(1-16W^2t^2)}$

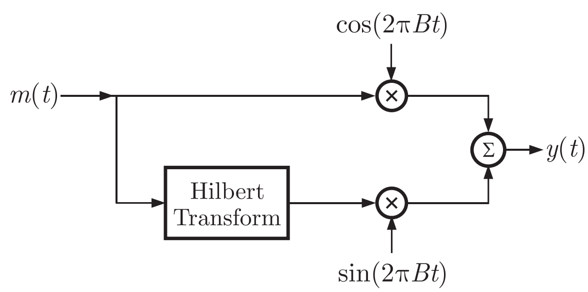
at $t = \frac{1}{4W}$ it is $\frac{0}{0}$ form. Thus applying L' Hospital rule

$$p^{(\frac{1}{4W})} = \frac{4\pi W\cos(4\pi Wt)}{4\pi W[1-48W^2t^2]}$$

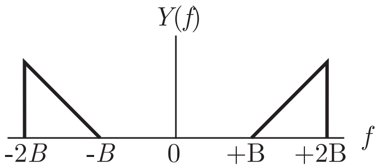
$$= \frac{\cos(4\pi Wt)}{1-48W^2t^2} = \frac{\cos\pi}{1-3} = 0.5$$

MCQ 1.60 In the following scheme, if the spectrum $M(f)$ of $m(t)$ is as shown, then the spectrum $Y(f)$ of $y(t)$ will be

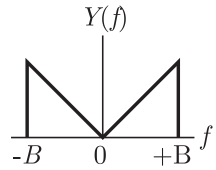




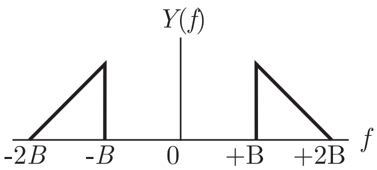
(A)



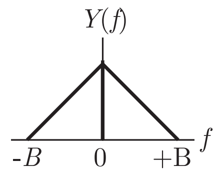
(B)



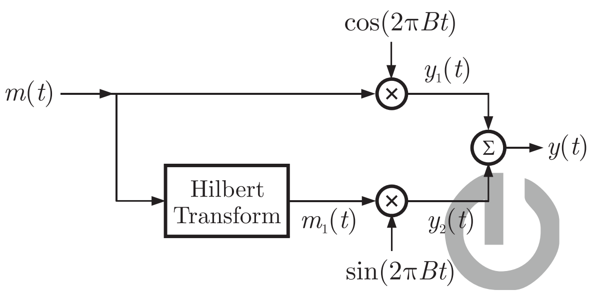
(C)



(D)



SOL 1.60 The block diagram is as shown below



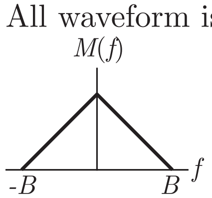
Here $M_1(f) = \hat{M}(f)$

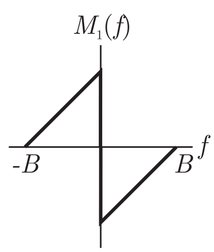
$$Y_1(f) = M(f)\left(\frac{e^{j2\pi B} + e^{-j2\pi B}}{2}\right)$$

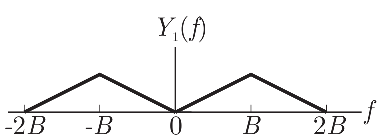
$$Y_2(f) = M_1(f)\left(\frac{e^{j2\pi B} - e^{-j2\pi B}}{2}\right)$$

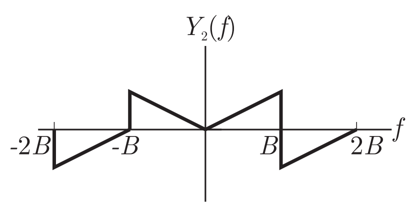
$$Y(f) = Y_1(f) + Y_2(f)$$

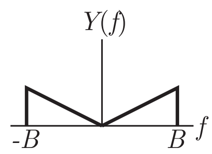
All waveform is shown below











Hence (B) is correct option.

MCQ 1.61 During transmission over a certain binary communication channel, bit errors occur independently with probability p. The probability of $AT\ MOST$ one bit in error in a block of n bits is given by

(A) p^n

(B) $1 - p^n$

(C) $np(1-p)^{n-1} + (1+p)^n$

(D) $1 - (1-p)^n$

SOL 1.61 By Binomial distribution the probability of error is

$$p_e = {}^nC_r p^r (1-p)^{n-r}$$

Probability of at most one error

= Probability of no error + Probability of one error

$$= {}^nC_0 p^0 (1-p)^{n-0} + {}^nC_1 p^1 (1-p)^{n-1}$$

$$= (1-p)^n + np(1-p)^{n-1}$$

Hence (C) is correct option.

MCQ 1.62 In a GSM system, 8 channels can co-exist in 200 kHz bandwidth using TDMA. A GSM based cellular operator is allocated 5 MHz bandwidth. Assuming a frequency reuse factor of $\frac{1}{5}$, i.e. a five-cell repeat pattern, the maximum number of simultaneous channels that can exist in one cell is

(A) 200

(B) 40

(C) 25

(D) 5

SOL 1.62 Bandwidth allocated for 1 Channel = 5 M Hz

Average bandwidth for 1 Channel $\frac{5}{5} = 1$ MHz

Total Number of Simultaneously Channel $= \frac{1\text{M} \times 8}{200k} = 40$ Channel

Hence (B) is correct option.

MCQ 1.63 In a Direct Sequence CDMA system the chip rate is 1.2288×10^6 chips per second. If the processing gain is desired to be AT LEAST 100, the data rate

(A) must be less than or equal to 12.288×10^3 bits per sec

(B) must be greater than 12.288×10^3 bits per sec

(C) must be exactly equal to 12.288×10^3 bits per sec

(D) can take any value less than 122.88×10^3 bits per sec

SOL 1.63 Hence (A) is correct option.

Chip Rate $R_C = 1.2288 \times 10^6$ chips/sec

Data Rate $R_b = \dfrac{R_C}{G}$

Since the processing gain G must be at least 100, thus for $G_{\min}$ we get

$$R_{b\max} = \frac{R_C}{G_{\min}} = \frac{1.2288 \times 10^6}{100} = 12.288 \times 10^3 \text{ bps}$$

MCQ 1.64 An air-filled rectangular waveguide has inner dimensions of 3 cm $\times$ 2 cm. The wave impedance of the TE_{20} mode of propagation in the waveguide at a frequency of 30 GHz is (free space impedance $\eta_0 = 377\,\Omega$)

(A) 308 Ω (B) 355 Ω

(C) 400 Ω (D) 461 Ω

SOL 1.64 The cut-off frequency is

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Since the mode is $TE_{20}, m = 2$ and $n = 0$

$$f_c = \frac{c}{2}\frac{m}{2} = \frac{3 \times 10^8 \times 2}{2 \times 0.03} = 10 \text{ GHz}$$

$$\eta' = \frac{\eta_o}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1 - \left(\frac{10^{10}}{3 \times 10^{10}}\right)^2}} = 400\Omega$$

Hence (C) is correct option.

MCQ 1.65 The $\vec{H}$ field (in A/m) of a plane wave propagating in free space is given by $\vec{H} = \hat{x}\frac{5\sqrt{3}}{\eta_0}\cos(\omega t - \beta z) + \hat{y}\left(\omega t - \beta z + \frac{\pi}{2}\right)$.

The time average power flow density in Watts is

(A) $\dfrac{\eta_0}{100}$ (B) $\dfrac{100}{\eta_0}$

(C) $50\eta_0^2$ (D) $\dfrac{50}{\eta_0}$

SOL 1.65 Hence (D) is correct option.

We have $$|H|^2 = H_x^2 + H_y^2 = \left(\frac{5\sqrt{3}}{\eta_o}\right)^2 + \left(\frac{5}{\eta_o}\right)^2 = \left(\frac{10}{\eta_o}\right)^2$$

For free space $$P = \frac{|E|^2}{2\eta_o} = \frac{\eta_o|H|^2}{2} = \frac{\eta_o}{2}\left(\frac{10}{\eta_o}\right)^2 = \frac{50}{\eta_o} \text{ watts}$$

MCQ 1.66 The $\vec{E}$ field in a rectangular waveguide of inner dimension $a \times b$ is given by

$$\vec{E} = \frac{\omega\mu}{h^2}\left(\frac{\lambda}{2}\right)H_0 \sin\left(\frac{2\pi x}{a}\right)^2 \sin(\omega t - \beta z)\,\hat{y}$$

Where H_0 is a constant, and a and b are the dimensions along the $x-$ axis and the $y-$ axis respectively. The mode of propagation in the waveguide is

(A) TE_{20} (B) TM_{11}

(C) TM_{20} (D) TE_{10}

SOL 1.66 Hence (A) is correct option.

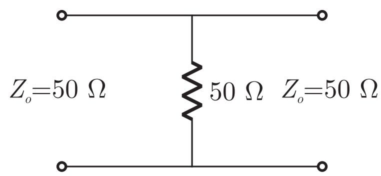
$$\vec{E} = \frac{\omega\mu}{h^2}\left(\frac{\pi}{2}\right)H_0 \sin\left(\frac{2\pi x}{a}\right)^2 \sin(\omega t - \beta z)\,\hat{y}$$

This is TE mode and we know that

$$E_y \propto \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

Thus $m = 2$ and $n = 0$ and mode is TE_{20}

MCQ 1.67 A load of $50\,\Omega$ is connected in shunt in a 2-wire transmission line of $Z_0 = 50\Omega$ as shown in the figure. The 2-port scattering parameter matrix (s-matrix) of the shunt element is



(A) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{4} & -\frac{3}{4} \\ -\frac{3}{4} & \frac{1}{4} \end{bmatrix}$

SOL 1.67 The 2-port scattering parameter matrix is

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

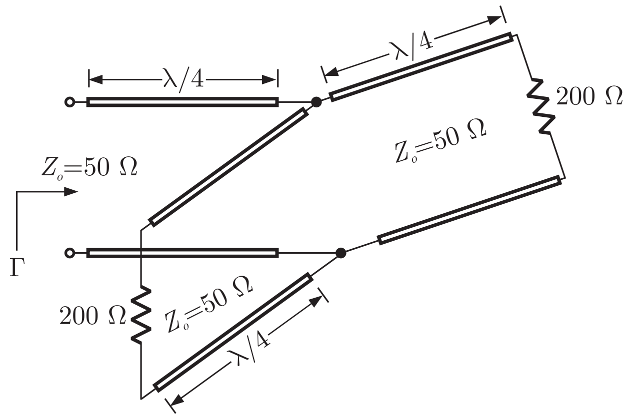
$$S_{11} = \frac{(Z_L \| Z_0) - Z_o}{(Z_L \| Z_0) + Z_o} = \frac{(50\|50) - 50}{(50\|50) + 50} = -\frac{1}{3}$$

$$S_{12} = S_{21} = \frac{2\,(Z_L \| Z_o)}{(Z_L \| Z_o) + Z_o} = \frac{2\,(50\|50)}{(50\|50) + 50} = \frac{2}{3}$$

$$S_{22} = \frac{(Z_L \| Z_o) - Z_o}{(Z_L \| Z_o) + Z_o} = \frac{(50\|50) - 50}{(50\|50) + 50} = -\frac{1}{3}$$

Hence (C) is correct option.

MCQ 1.68 The parallel branches of a 2-wire transmission line re terminated in 100Ω and 200Ω resistors as shown in the figure. The characteristic impedance of the line is $Z_0 = 50\Omega$ and each section has a length of $\frac{\lambda}{4}$. The voltage reflection coefficient Γ at the input is



(A) $-j\frac{7}{5}$

(B) $\frac{-5}{7}$

(C) $j\frac{5}{7}$

(D) $\frac{5}{7}$

SOL 1.68 The input impedance is

$$Z_{in} = \frac{Z_o^2}{Z_L}; \qquad \text{if } l = \frac{\lambda}{4}$$

$$Z_{in1} = \frac{Z_{o1}^2}{Z_{L1}} = \frac{50^2}{100} = 25$$

$$Z_{in2} = \frac{Z_{o2}^2}{Z_{L2}} = \frac{50^2}{200} = 12.5$$

Now $\quad Z_L = Z_{in1} \| Z_{in2}$

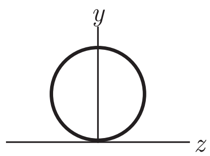
$$25 \| 12.5 = \frac{25}{3}$$

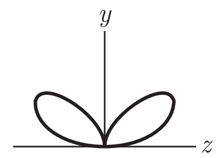
$$Z_s = \frac{(50)^2}{25/3} = 300$$

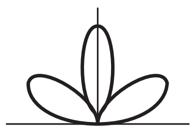
$$\Gamma = \frac{Z_S - Z_o}{Z_S + Z_o} = \frac{300-50}{300+50} = \frac{5}{7}$$

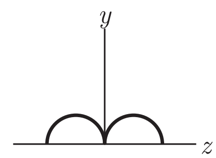
Hence (D) is correct option.

MCQ 1.69 A $\frac{\lambda}{2}$ dipole is kept horizontally at a height of $\frac{\lambda_0}{2}$ above a perfectly conducting infinite ground plane. The radiation pattern in the lane of the dipole ($\vec{E}$ plane) looks approximately as

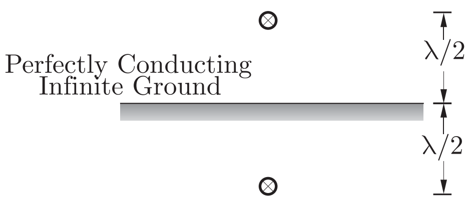
(A) 

(B) 

(C) 

(D) 

SOL 1.69 Using the method of images, the configuration is as shown below



Here $d = \lambda, \alpha = \pi$, thus $\beta d = 2\pi$

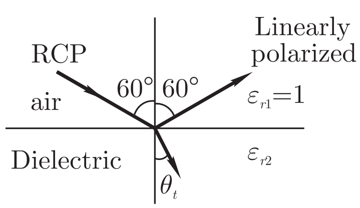
Array factor is

$$= \cos\left[\frac{\beta d \cos\psi + \alpha}{2}\right]$$

$$= \cos\left[\frac{2\pi\cos\psi + \pi}{2}\right] = \sin(\pi\cos\psi)$$

Hence (B) is correct option.

MCQ 1.70 A right circularly polarized (RCP) plane wave is incident at an angle 60° to the normal, on an air-dielectric interface. If the reflected wave is linearly polarized, the relative dielectric constant ξ_{r2} is.



(A) $\sqrt{2}$

(B) $\sqrt{3}$

(C) 2

(D) 3

SOL 1.70 The Brewster angle is

$$\tan\theta_n = \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}$$

$$\tan 60^\circ = \sqrt{\frac{\varepsilon_{r2}}{1}}$$

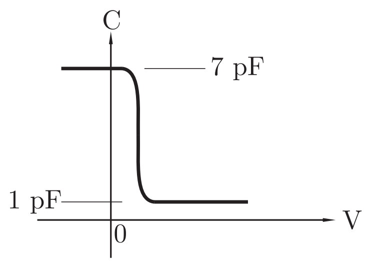
or $\varepsilon_{r2} = 3$

Hence (D) is correct option.

Common Data Questions

Common Data for Questions 71, 72, 73 :

The figure shows the high-frequency capacitance - voltage characteristics of Metal/ Sio_2/silicon (MOS) capacitor having an area of 1×10^{-4} cm^2. Assume that the permittivities ($\varepsilon_0\varepsilon_r$) of silicon and Sio_2 are 1×10^{-12} F/cm and 3.5×10^{-13} F/cm respectively.



MCQ 1.71 The gate oxide thickness in the MOS capacitor is

(A) 50 nm (B) 143 nm

(C) 350 nm (D) 1 μm

SOL 1.71 At low voltage when there is no depletion region and capacitance is decide by SiO_2 thickness only,

$$C = \frac{\varepsilon_0\varepsilon_{r1}A}{D}$$

or $$D = \frac{\varepsilon_0\varepsilon_{r1}A}{C} = \frac{3.5\times10^{-13}\times10^{-4}}{7\times10^{-12}} = 50 \text{ nm}$$

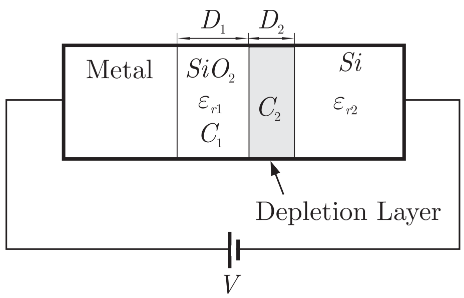
Hence option (A) is correct

MCQ 1.72 The maximum depletion layer width in silicon is

(A) 0.143 μm (B) 0.857 μm

(C) 1 μm (D) 1.143 μm

SOL 1.72 The construction of given capacitor is shown in fig below



When applied voltage is 0 volts, there will be no depletion region and we get

$$C_1 = 7 \text{ pF}$$

When applied voltage is V, a depletion region will be formed as shown in fig an total capacitance is 1 pF. Thus

$$C_T = 1 \text{ pF}$$

or $$C_T = \frac{C_1 C_2}{C_1 + C_2} = 1 \text{ pF}$$

or $$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$$

Substituting values of C_T and C_1 we get

$$C_2 = \frac{7}{6} \text{ pF}$$

Now $$D_2 = \frac{\varepsilon_0 \varepsilon_{r2} A}{C_2} = \frac{1 \times 10^{-12} \times 10^{-4}}{\frac{7}{6} \times 10^{-12}} = \frac{6}{7} \times 10^{-4} \text{ cm}$$

$$= 0.857 \ \mu\text{m}$$

Hence option (B) is correct.

MCQ 1.73 Consider the following statements about the $C - V$ characteristics plot :

S1 : The MOS capacitor has as n-type substrate

S2 : If positive charges are introduced in the oxide, the $C - V$ polt will shift to the left.

Then which of the following is true?

(A) Both S1 and S2 are true

(B) S1 is true and S2 is false

(C) S1 is false and S2 is true

(D) Both S1 and S2 are false

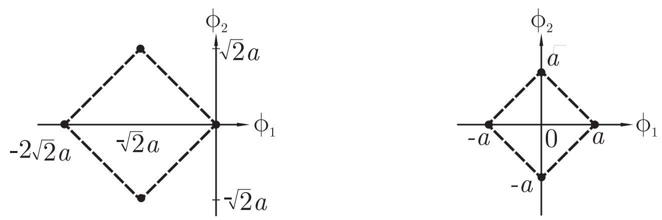
SOL 1.73 Depletion region will not be formed if the MOS capacitor has n type substrate but from C-V characteristics, C reduces if V is increased. Thus depletion region must be formed. Hence S_1 is false

If positive charges is introduced in the oxide layer, then to equalize the effect the applied voltage V must be reduced. Thus the $C - V$ plot moves to the left. Hence S_2 is true.

Hence option (C) is correct.

Common Data for Questions 74 & 75 :

Two 4-array signal constellations are shown. It is given that ϕ_1 and ϕ_2 constitute an orthonormal basis for the two constellation. Assume that the four symbols in both the constellations are equiprobable. Let $\frac{N_0}{2}$ denote the power spectral density of white Gaussian noise.



Constellation 1 Constellation 2

MCQ 1.74 The if ratio or the average energy of Constellation 1 to the average energy of Constellation 2 is

(A) $4a^2$ (B) 4

(C) 2 (D) 8

SOL 1.74 Energy of constellation 1 is

$$E_{g1} = (0)^2 + (-\sqrt{2}\,a)^2 + (-\sqrt{2}\,a)^2 + (\sqrt{2}\,a)^2 + (-2\sqrt{2}\,a)^2$$

$$= 2a^2 + 2a^2 + 2a^2 + 8a^2 = 16a^2$$

Energy of constellation 2 is

$$E_{g2} = a^2 + a^2 + a^2 + a^2 = 4a^2$$

$$\text{Ratio} = \frac{E_{g1}}{E_{g2}} = \frac{16a^2}{4a^2} = 4$$

Hence (B) is correct option.

MCQ 1.75 If these constellations are used for digital communications over an AWGN channel, then which of the following statements is true ?

(A) Probability of symbol error for Constellation 1 is lower

(B) Probability of symbol error for Constellation 1 is higher

(C) Probability of symbol error is equal for both the constellations

(D) The value of N_0 will determine which of the constellations has a lower probability of symbol error

SOL 1.75 Noise Power is same for both which is $\frac{N_0}{2}$.

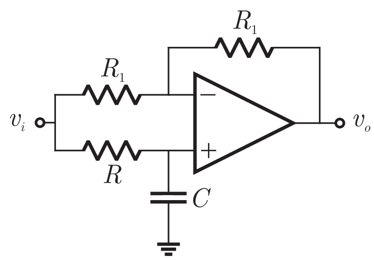
Thus probability of error will be lower for the constellation 1 as it has higher signal energy.

Hence (A) is correct option.

Linked Answer Questions : Q.76 to Q.85 carry two marks each.

Statement for Linked Answer Questions 76 & 77:

Consider the Op-Amp circuit shown in the figure.



MCQ 1.76 The transfer function $V_0(s)/V_i(s)$ is

(A) $\frac{1-sRC}{1+sRC}$ (B) $\frac{1+sRC}{1-sRC}$

(C) $\frac{1}{1-sRC}$ (D) $\frac{1}{1+sRC}$

SOL 1.76 The voltage at non-inverting terminal is

$$V_+ = \frac{\frac{1}{sC}}{R+\frac{1}{sC}}V_i = \frac{1}{1+sCR}V_i$$

Now $$V_- = V_+ = \frac{1}{1+sCR}V_i$$

Applying voltage division rule

$$V_+ = \frac{R_1}{R_1+R_1}(V_0+V_i) = \frac{(V_o+V_i)}{2}$$

or $$\frac{1}{1+sCR}V_i = \frac{(V_o+V_i)}{2}$$

or $$\frac{V_o}{V_i} = -1+\frac{2}{1+sRC}$$

$$\frac{V_0}{V_i} = \frac{1-sRC}{1+sRC}$$

Hence (A) is correct option.

MCQ 1.77 If $V_i = V_1\sin(\omega t)$ and $V_0 = V_2\sin(\omega t+\phi)$, then the minimum and maximum values of ϕ (in radians) are respectively

(A) $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ (B) 0 and $\frac{\pi}{2}$

(C) $-\pi$ and 0 (D) $-\frac{\pi}{2}$ and 0

SOL 1.77 Hence (C) is correct option.

$$\frac{V_0}{V_i} = H(s) = \frac{1-sRC}{1+sRC}$$

$$H(j\omega) = \frac{1 - j\omega RC}{1 + j\omega RC}$$

$$\angle H(j\omega) = \phi = -\tan^{-1}\omega RC - \tan^{-1}\omega RC$$
$$= -2\tan^{-2}\omega RC$$

Minimum value, $\phi_{min} = -\pi\,(\text{at } \omega \to \infty)$

Maximum value, $\phi_{max} = 0(\text{at } \omega = 0)$

Statement for Linked Answer Questions 78 & 79 :

An 8085 assembly language program is given below.

```
Line 1:  MVI A, B5H
     2:  MVI B, OEH
     3:  XRI 69H
     4:  ADD B
     5:  ANI 9BH
     6:  CPI 9FH
     7:  STA 3010H
     8:  HLT
```

MCQ 1.78 The contents of the accumulator just execution of the ADD instruction in line 4 will be

(A) C3H (B) EAH

(C) DCH (D) 69H

SOL 1.78

```
Line 1 : MVI A, B5H    ; Move B5H to A
     2 : MVI B, 0EH    ; Move 0EH to B
     3 : XRI 69H       ; [A] XOR 69H and store in A
                       ; Contents of A is CDH
     4 : ADDB          ; Add the contents of A to contents of B and
                       ; store in A, contents of A is EAH
     5 : ANI 9BH       ; [a] AND 9BH, and store in A,
                       ; Contents of A is 8 AH
     6 : CPI 9FH       ; Compare 9FH with the contents of A
                       ; Since 8 AH < 9BH, CY = 1
     7 : STA 3010 H    ; Store the contents of A to location 3010 H
     8 : HLT           ; Stop
```

Thus the contents of accumulator after execution of ADD instruction is EAH.

Hence (B) is correct answer.

MCQ 1.79 After execution of line 7 of the program, the status of the CY and Z flags will be

(A) $CY = 0,\ Z = 0$ (B) $CY = 0,\ Z = 1$

(C) $CY = 1,\ Z = 0$ (D) $CY = 1,\ Z = 1$

SOL 1.79 The $CY = 1$ and $Z = 0$
Hence (C) is correct answer.

Statement for linked Answer Question 80 & 81 :

Consider a linear system whose state space representation is $x(t) = Ax(t)$. If the initial state vector of the system is $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then the system response is $x(t) = \begin{bmatrix} e^{-2x} \\ -2e^{-2t} \end{bmatrix}$. If the itial state vector of the system changes to $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then the system response becomes $x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$

MCQ 1.80 The eigenvalue and eigenvector pairs $(\lambda_i v_i)$ for the system are

(A) $\left(-1\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(-2\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$ (B) $\left(-1,\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(2,\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

(C) $\left(-1,\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(-2,\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$ (D) $\left(-2\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(1,\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

SOL 1.80 Hence (A) is correct option.

We have $\dot{x}(t) = Ax(t)$

Let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

For initial state vector $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ the system response is $x(t) = \begin{bmatrix} e^{-2t} \\ -2e^{-2t} \end{bmatrix}$

Thus $$\begin{bmatrix} \frac{d}{dt}e^{-2t} \\ \frac{d}{dt}(-2e^{-2t}) \end{bmatrix}_{t=0} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

or $$\begin{bmatrix} -2e^{-2(0)} \\ 4e^{-2(0)} \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} p-2q \\ r-2s \end{bmatrix}$$

We get $p - 2q = -2$ and $r - 2s = 4$ ...(i)

For initial state vector $x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ the system response is $x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$

Thus $$\begin{bmatrix} \frac{d}{dt}e^{-t} \\ \frac{d}{dt}(-e^{-t}) \end{bmatrix}_{t=0} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -e^{-(0)} \\ e^{-(0)} \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} p-q \\ r-s \end{bmatrix}$$

We get $p - q = -1$ and $r - s = 1$ ...(2)

Solving (1) and (2) set of equations we get

$$\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

The characteristic equation

$$|\lambda I - A| = 0$$

$$\begin{vmatrix} \lambda & -1 \\ 2 & \lambda + 3 \end{vmatrix} = 0$$

or $\lambda(\lambda + 3) + 2 = 0$

or $\lambda = -1, -2$

Thus Eigen values are -1 and -2

Eigen vectors for $\lambda_1 = -1$

$$(\lambda_1 I - A) X_1 = 0$$

or $\begin{bmatrix} \lambda_1 & -1 \\ 2 & \lambda_1 + 3 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = 0$

$$\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = 0$$

or $-x_{11} - x_{21} = 0$

or $x_{11} + x_{21} = 0$

We have only one independent equation $x_{11} = -x_{21}$.Let $x_{11} = K$, then $x_{21} = -K$, the Eigen vector will be

$$\begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = \begin{bmatrix} K \\ -K \end{bmatrix} = K\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Now Eigen vector for $\lambda_2 = -2$

$$(\lambda_2 I - A) X_2 = 0$$

or $\begin{bmatrix} \lambda_2 & -1 \\ 2 & \lambda_2 + 3 \end{bmatrix} \begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} = 0$

or $\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = 0$

or $-x_{11} - x_{21} = 0$

or $x_{11} + x_{21} = 0$

We have only one independent equation $x_{11} = -x_{21}$.

Let $x_{11} = K$, then $x_{21} = -K$, the Eigen vector will be

$$\begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} = \begin{bmatrix} K \\ -2K \end{bmatrix} = K\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

MCQ 1.81 The system matrix A is

(A) $\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

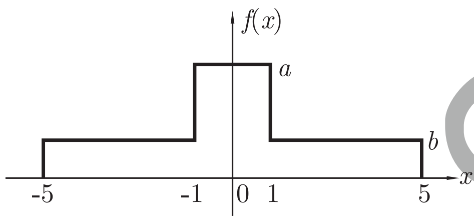
SOL 1.81 As shown in previous solution the system matrix is

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

Hence (D) is correct option.

Statement for Linked Answer Question 82 & 83 :

An input to a 6-level quantizer has the probability density function $f(x)$ as shown in the figure. Decision boundaries of the quantizer are chosen so as to maximize the entropy of the quantizer output. It is given that 3 consecutive decision boundaries are' -1'.'0' and '1'.



MCQ 1.82 The values of a and b are

(A) $a = \frac{1}{6}$ and $b = \frac{1}{12}$ (B) $a = \frac{1}{5}$ and $b = \frac{3}{40}$

(C) $a = \frac{1}{4}$ and $b = \frac{1}{16}$ (D) $a = \frac{1}{3}$ and $b = \frac{1}{24}$

SOL 1.82 Area under the pdf curve must be unity

Thus$2a + 4a + 4b \quad = 1$

$$2a + 8b = 1 \qquad ...(1)$$

For maximum entropy three region must by equivaprobable thus

$$2a = 4b = 4b \qquad ...(2)$$

From (1) and (2) we get

$$b = \frac{1}{12} \text{ and } a = \frac{1}{6}$$

Hence (A) is correct option.

MCQ 1.83 Assuming that the reconstruction levels of the quantizer are the mid-points of the decision boundaries, the ratio of signal power to quantization noise power is

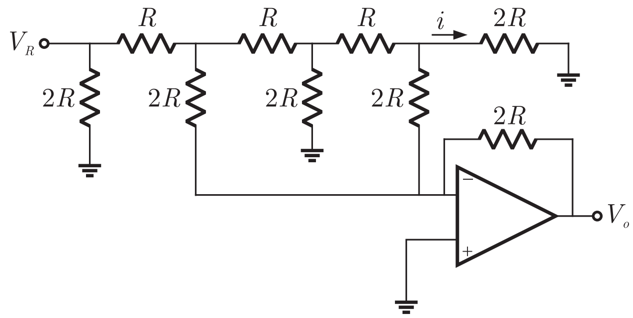
(A) $\frac{152}{9}$ (B) $\frac{64}{3}$

(C) $\frac{76}{3}$ (D) 28

SOL 1.83 Hence correct option is ( )

Statement for Linked Answer Question 84 and 85 :

In the Digital-to-Analog converter circuit shown in the figure below,
$V_R = 10\,V$ and $R = 10k\Omega$

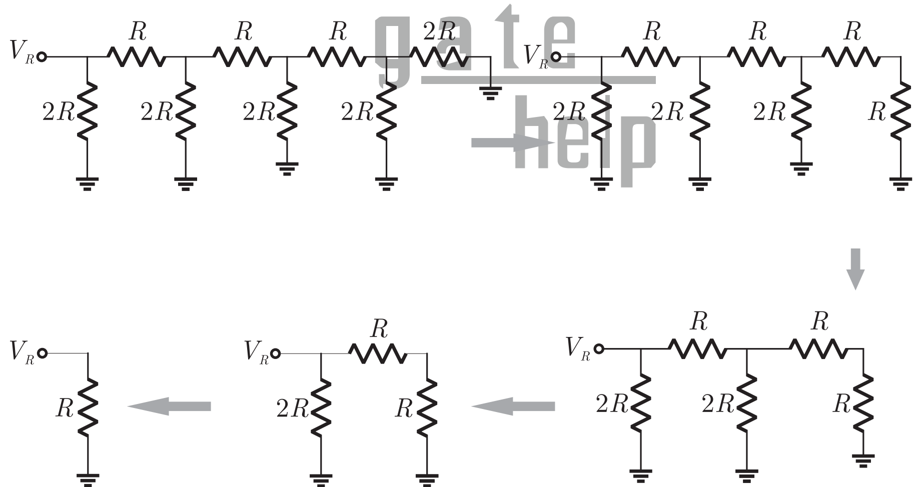


MCQ 1.84 The current i is

(A) 31.25μA (B) 62.5μA

(C) 125μA (D) 250μA

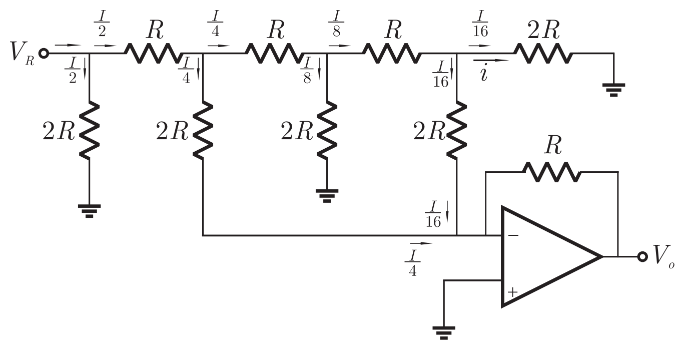
SOL 1.84 Since the inverting terminal is at virtual ground the resistor network can be reduced as follows



The current from voltage source is

$$I = \frac{V_R}{R} = \frac{10}{10k} = 1 \text{ mA}$$

This current will be divide as shown below



Now $i = \frac{I}{16} = \frac{1 \times 10^{-3}}{16} = 62.5\ \mu\text{ A}$

Hence (B) is correct answer.

MCQ 1.85 The voltage V_0 is

(A) -0.781 V (B) -1.562 V

(C) -3.125 V (D) -6.250 V

SOL 1.85 The net current in inverting terminal of OP - amp is

$$I_- = \frac{1}{4} + \frac{1}{16} = \frac{5I}{16}$$

So that $V_0 = -R \times \frac{5I}{16} = -3.125$

Hence (C) is correct answer.

Answer Sheet

1.	(A)	19.	(A)	37.	(B)	55.	(D)	73.	(C)
2.	(B)	20.	(D)	38.	(D)	56.	(D)	74.	(B)
3.	(C)	21.	(C)	39.	(D)	57.	(A)	75.	(A)
4.	(A)	22.	(A)	40.	(C)	58.	(D)	76.	(A)
5.	(D)	23.	(C)	41.	(D)	59.	(C)	77.	(C)
6.	(A)	24.	(D)	42.	(B)	60.	(B)	78.	(B)
7.	(D)	25.	(B)	43.	(A)	61.	(C)	79.	(C)
8.	(C)	26.	(C)	44.	(C)	62.	(B)	80.	(A)
9.	(D)	27.	(A)	45.	(A)	63.	(A)	81.	(D)
10.	(C)	28.	(D)	46.	(C)	64.	(C)	82.	(A)
11.	(C)	29.	(D)	47.	(A)	65.	(D)	83.	(*)
12.	(A)	30.	(A)	48.	(A)	66.	(A)	84.	(B)
13.	(C)	31.	(D)	49.	(B)	67.	(C)	85.	(C)

14.	(B)	32.	(A)	50.	(B)	68.	(D)
15.	(D)	33.	(B)	51.	(B)	69.	(B)
16.	(D)	34.	(B)	52.	(B)	70	(D)
17.	(C)	35.	(C)	53.	(D)	71	(A)
18.	(B)	36.	(C)	54.	(A)	72	(B)

****