SSLC-2021

MATHEMATICS

E A S Y
R E V I S I O N

Concepts with examples

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SSLC 2021 MATHEMATICS CONCEPTS AND FORMULAS

ARITHMETIC SEQUENCES

A sequence of numbers in which each term is obtained by adding a constant number to the previous term is called an arithmetic sequence . The difference of any two consecutive termsof an AS will be same. It is called its common difference. The cd of an AS x_1 , x_2 , x_3 , x_4 , x_5 . . . . . . is

$$d = x_2 - x_1$$

Eg: The common difference of the AS 8 , 13 , 18 . . . . . . is d = 13 – 8 = 5

If if the first term is f , common difference is d , then the AS is

f , f+d , f+2d , f+3d , f+4d , ... ,

Its nth thrm is given by

$$x_n = f + (n-1)d$$

Eg: Find the 15th term of the AS 8 , 13 , 18 . . . . . .

Ans : x_{15} = 8 + (15-1)5 = 8 + 14x5 = 8 + 70 = 78

In an AS , the difference of any two terms is always a multiple of its common difference.

$$d = \frac{x_m - x_n}{m - n}$$

Eg: Is 100 a term of the AS 8 , 13 , 18 . . . . . . ?

Ans: No , 100 – 8 = 92 is not a multiple of common difference

Eg: In an AS , first term is 8 , 15th term is 78. Find common difference.

Ans: d = $\frac{78-8}{15-1} = \frac{70}{14} = 5$

In an AS , the remainders on dividing any term by the common difference are same..

Eg: Is 100 a term of the AS 8 , 13 , 18 . . . . . . ?

Ans: No , when we divide 100 by common difference 5 , we get 0 , which is not same as in other cases.

Number of terms in the AS

x_1 , x_2 , x_3 , x_4 , x_5 . . . . . x_n is given by

$$n = \frac{x_n - x_1}{d} + 1$$

Eg: Find the number of terms in the AS 8 , 13 , 18 158

Ans: $\frac{158-8}{5} + 1 = 31$

If a,b,c are any three consecutive terms of an AS , then

$$b = \frac{a+c}{2}$$

Ans: Find the value of x if 8 , x , 18 are three consecutive terms of an AS.

Ans : $x = \frac{18+8}{2} = 13$

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In an AS of odd number of terms , the middle term is the average of first and last terms.

$$m = \frac{x_1 + x_n}{2}$$

Eg:In an AS , the first term is 8 , 15th term is 78. Find the 8th term.

Ans: x_8 = Middle term = $\frac{8+78}{2} = 43$

In an AS of odd number of terms , their sum is the product of middle term and number of terms.

$$s = m \times n$$

Eg:In an AS , the 8th term is 43. Find the sum of first 15 terms.

Ans: Here 8th term is the middle term. Sum = 43x15 = 645

In an AS the pairs of terms equidistant from each end will have the same sum. For instance , if there are 10 terms then

$$x_1 + x_{10} = x_2 + x_9 = x_3 + x_8 = x_4 + x_7 = x_5 + x_6$$

Eg: In an AS the sum of terms at the 4th and 11th positions is 100 , what is the sum of 7th and 8th terms ?

Ans: $x_4 + x_{11} = x_7 + x_8 = 100$

The sum of terms of an AS with n terms equals half the product of number of terms with sum of the first and last terms.

$$s = \frac{n}{2} \times (x_1 + x_n)$$

Eg: Find the sum of the AS 8 , 13 , 18 78

Ans: No of terms = $\frac{78-8}{5} + 1 = 15$ **, Sum =** $\frac{15}{2}(8+78) = 645$

The sum of first n terms of an AS with first term f and common difference d is

$$S_n = \frac{n}{2}(2f + (n-1)d)$$

Eg: Find the sum of first 15 terms of the AS 8 , 13 , 18 . . . . .

Ans: Sum = $\frac{15}{2}(16 + 14x5) = 645$

The nth term of an AS is called its algebraic form. Hence the algebraic form of an AS with first term f and common difference d is given by

$$x_n = f + (n-1)d$$

Eg: Find the algebraic form of the AS 8 , 13 , 18 . . . . .

Ans: 8+(n-1)5 = 8+5n-5 = 5n+3

Generally the algebraic form (nth tem) of an AS is of the form an+b.
From it , we can get the common difference as the coefficient of n = a . Also the first term is the sum of coefficients = a+b

Eg: What type of sequence does x_n = 5n+3 represent ? Also write the first term and common difference.

Ans: Ans: Arithmetic sequence , f = 5+3 = 8 , d = 5

Generally , the sum of first n terms of an AS will be of the form $an^2 + bn$
From it we get the common difference as double of the coefficient of n^2 = 2a . Also the first term is the sum of coefficients = a+b.
Eg:The expression for the sum of n terms of an AS is $5n^2 + 3n$. Find the first term and common difference.
Ans: f = 5+3 = 8 , d = 5x2=10

Few formulas for the sum of terms

The sum of first first n natural numbers = $\frac{n(n+1)}{2}$

The sum of first first n odd numbers = n^2

The sum of first first n even numbers = n^2+n

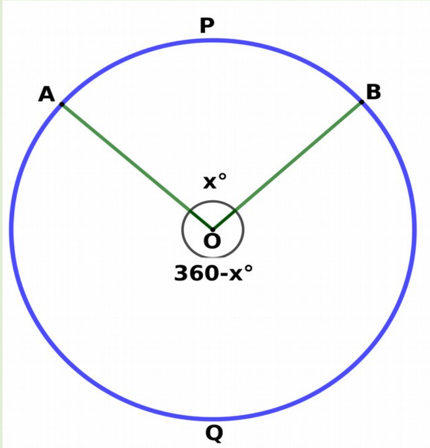
Eg:
Find the sum of first 15 natural numbers. **Ans: (15x16)½ = 120**
How many odd numbers from 1 will give a sum 400 ? **Ans: n^2 =400 , n=20**
Find the sum of first 15 even numbers. **Ans: 15 x 16 = 240**

Circles and Tangents

The sum of central angles of an arc and its alternate arc is 360^0

Eg: The central angle of an arc APB is 80^0. What is the central angle of alternate arc AQB ?

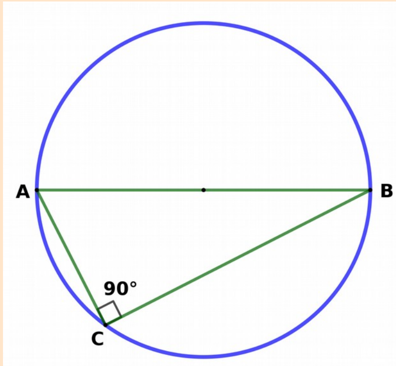
Ans : $360 – 80 = 280^0$



The angle in a semicircle is a right angle. <ACB =90^0

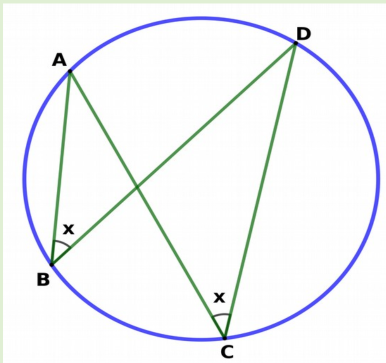
Eg: The diameter of a circle is AB. C is a point on the circle If < ABC = 28^0 find <BAC.

Ans: <C = 90^0, <BAC = 180 – (90+28) = 52^0

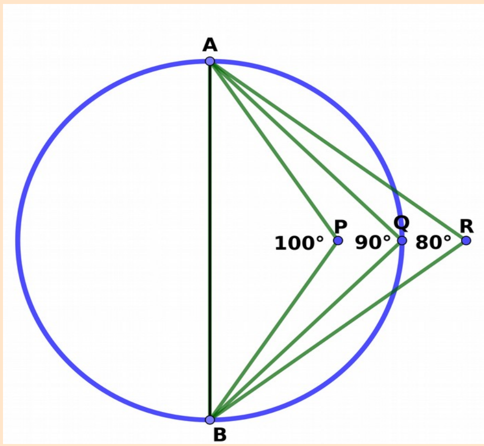


All angles inscribed in an arc are equal
< ABD = <ACD

Eg: In figure <BAC= 25^0, Find <BDC.
Ans: <BAC = <BDC = 25^0 because they are in the same arc BAC



If a circle is drawn with one side of a triangle as diameter , then it will pass through the opposite vertex if the vertex angle is 90^0. If it is less than 90^0, the vertex will fall outside the circle , if that angle is greater than 90^0 it will fall in side the circle.
Eg: In triangle ABC , <B = 32^0, <C = 40^0.Which side should be taken as the diameter so that the third vertex falls inside the circle?
Ans: Here < A = 108^0 which is more than 90 , hence take BC as diameter.



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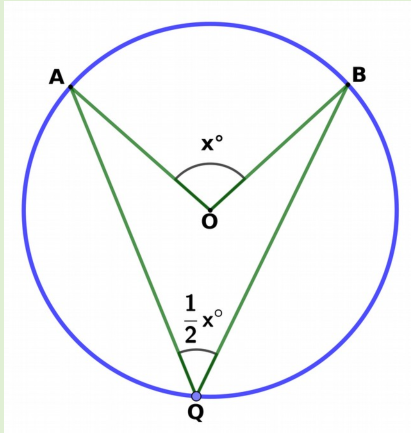
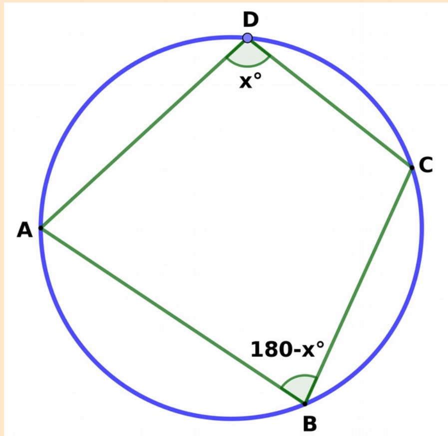


The angle made by an arc at its alternate arc is half the central angle of the arc.

$\angle AQB = \frac{1}{2}\angle AOB$

Eg: If $\angle AQB = 42^0$,find cenral angle $\angle AOB$.

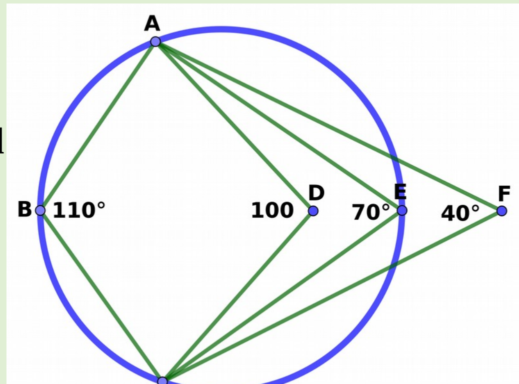
Ans: $\angle AOB = 2\text{x}42=84^0$



In a circle the sum of angles in an arc and in its alternate arc is 180^0

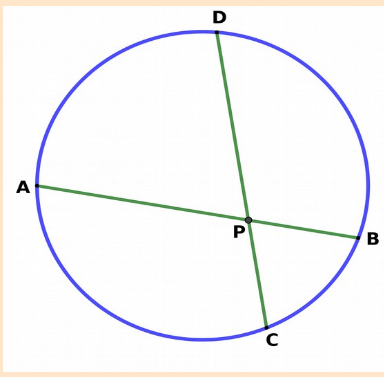
(A quadrilateral whose vertices are on a circle is called cyclic quadrilateral. The opposite angles of a cyclic quadrilateral are supplementary (sum 180).

Eg: In a cyclic quadrilateral ABCD , $\angle A = 57^0$, $\angle B= 104^0$
Find $\angle C$, $\angle D$
Ans: $\angle C = 180 - 57 = 123^0$, $\angle D = 180 -104 = 76^0$



A circle drawn through the three vertices of a quadrilateral will pass through the fourth vertex , If the sum of angles at that vertex and its opposite vertex is 180^0 . If the sum is less than 180 , the vertex will fall out side , if the sum is greater than 180 , the vertex will fall inside the circle.

Eg: In quadrilateral ABCD , $\angle A = 100^0$, $\angle B = 80^0$, $\angle C = 50^0$. If we draw a circle passing through A , B , C where will be the position of vertex D?
Ans: $\angle D = 360 -230 = 130^0$. $\angle B + \angle D = 210^0$. it is greater than 180, D is inside the circle.



If two chords AB and CD of a circle cross each other at a point P inside the circle then , the product of parts of each chords are equal.   PAxPB = PCxPD

Eg: The chords AB and CD intersect at P , AB = 35cm , PA = 20cm, PD = 60, Find PC.

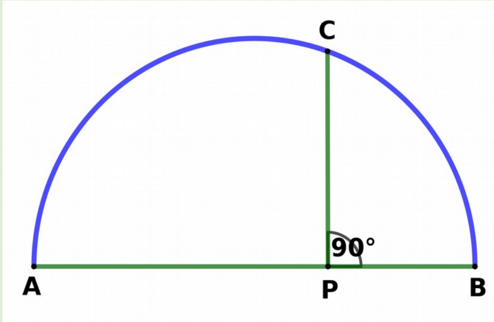
Ans: PB = 35-20=15. 20x15= PC x 60 .   PC = 5cm

AB is the diameter of a semicircle and PC is perpendicular to AB.

Then , $PA \times PB = PC^2$

Eg: PA = 18cm , PB = 8cm , Find PC.

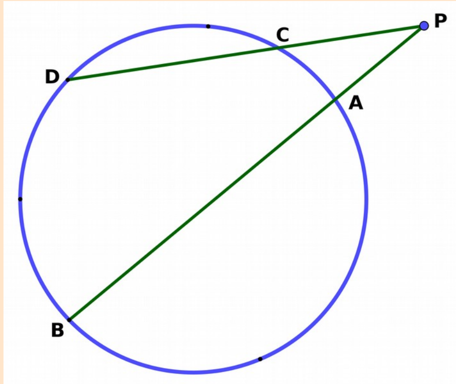
Ans: $18 \times 8 = PC^2$, $PC = \sqrt{144} = 12$cm



In a circle , the two chords AB and CD extended meet at a point P out side the circle. Then PA x PB = PC x PD.

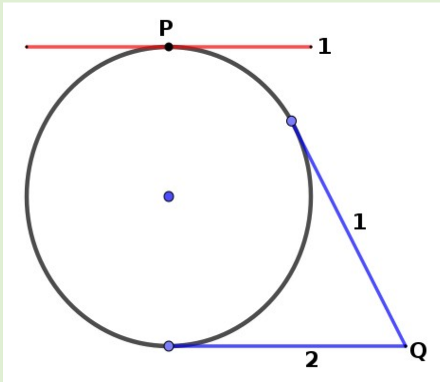
Eg: PA = 8cm , AB = 12cm , PC = 10 ,Find CD.

Ans: 8 x (8+12) = 10 x PD . PD = 16 , CD = 16-10 = 6cm.



Tangents

A line touching a circle at one point is called a Tangent to the circle.
Through a point on a circle there exists only one tangent , where as two tangents are possible from a point out side.



a) The tangent at a point on circle is perpendicular to the radius through that point.

$OA \perp PA$, <OAP = <OBP = 90^0

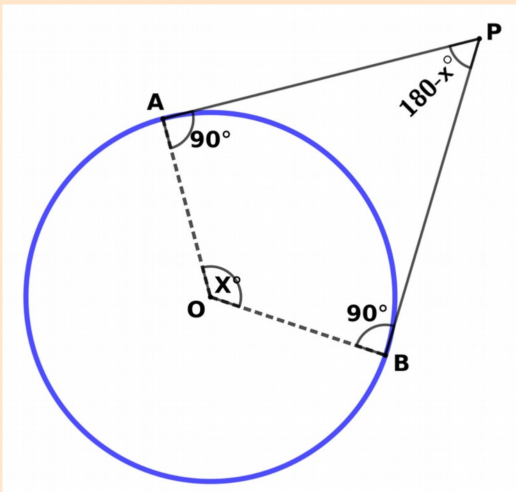
b) The two tangents drawn to a circle from a point out side the circle are equal in length.

PA = PB

c) The angle between the two tangents , and the angle between the two corresponding radii are supplementary (Sum 180)

<AOB + <APB = 180^0

d) PAOB is a cyclicquadrilateral.



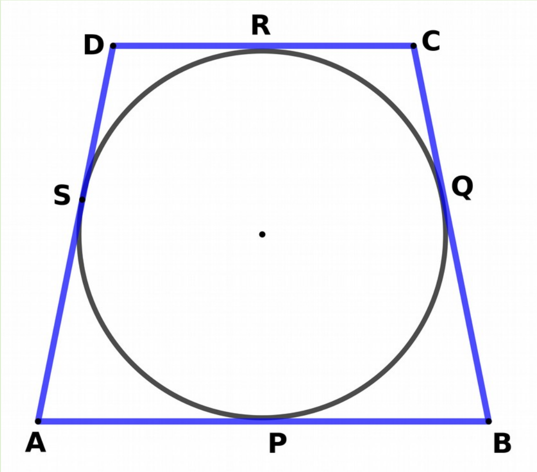
Eg: PA and PB are tangents to a circle of centre O. If PA = 4cm , AO = 3cm , < APB = 40^0 Find PB and <AOB. Also Find OP.

Ans : PB = PA = 4cm ,
<AOB = 180 – 40 = 140^0 ,

Since AOP is a right triangle , by Pythagoras theorem , hypotenuse OP = 5cm

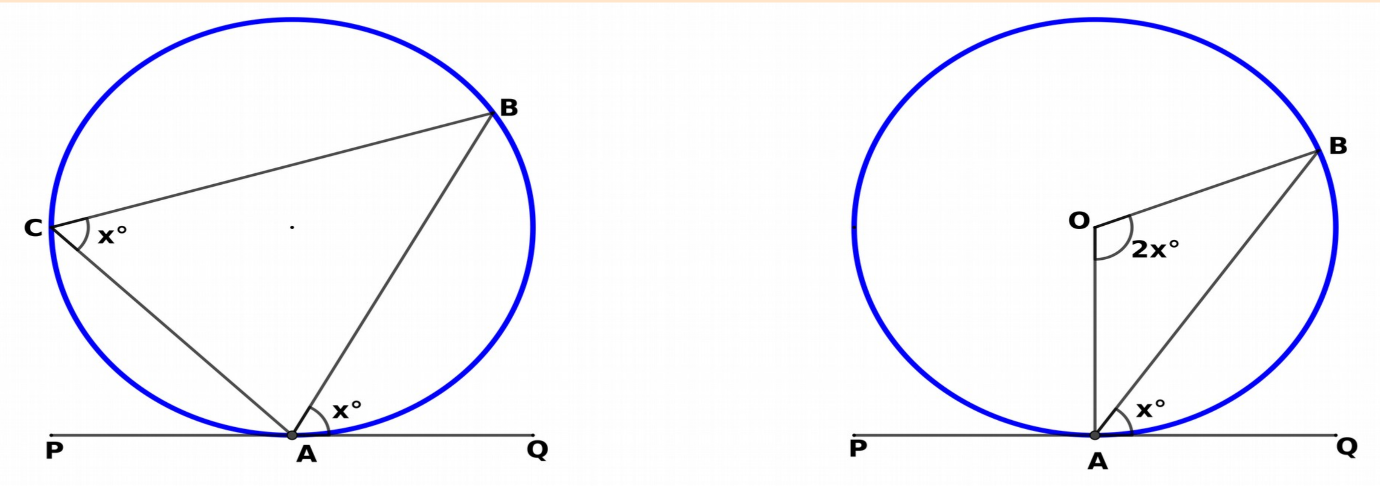
If a quadrilateral is formed by drawing tangents through 4 points of a circle , then the sum of pairs of its opposite sides are equal.

AB + CD = AD + BC



Eg1: In figure , AP = 5cm , BQ = 6cm , CR = 4cm , DS = 3cm , Find perimeter of ABCD.
Ans: 30cm ,
(AP=AS , BQ=BP , CQ=CR , DR=DS)

Eg2: In a quadrilateral ABCD , AB = 10cm , BC = 12cm , CD = 13cm , AD = 11cm. Is it possible to draw a quadrilateral with all sides touching the circle ?
Ans : Yes , because AB + CD = BC + AD = 23cm .



The angle made by a tangent and a chord at a point on a circle is equal to the angle made in the segment on other side of the chord.
It is also equal to half the central angle of the chord.

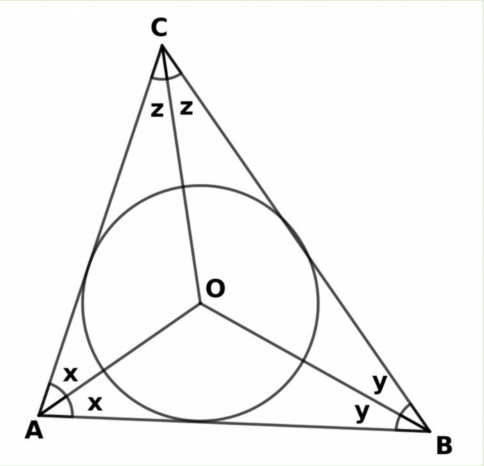
<BAQ = <BCA , <BAQ = ½<BOA

Eg: From above figures <PAC = 40^0 , < QAB = 50^0 , Find <B and <C.

Ans: <B =<PAC=40^0 , <C=<QAB=50^0 , <AOB = 2 x <QAB=100^0.

A circle touching the sides of a polygon is called its in-circle. The centre of the incircle of a triangle is the point of intersection of its angle bisectors. The radius of in-circle of a triangle is equal the " semi perimeter divided by area"

$$r = A/s$$

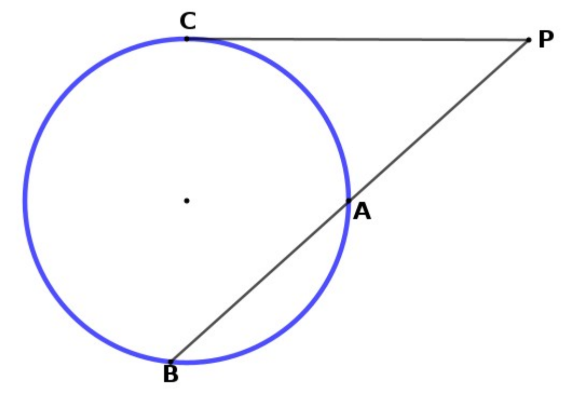


Eg: Find the radius of in-circle of a right triangle with perpendicular sides 3cm , 4cm .

Ans : A = $6cm^2$, s = ½(3+4+5) = 6cm . r = 6/6 = 1cm

In a circle , the tangent at a point C on the circle and the chord AB extended meet at a point P out side the circle. Then

$$PA \times PB = PC^2$$



Eg: PA = 4cm , AB = 5cm , Find PC

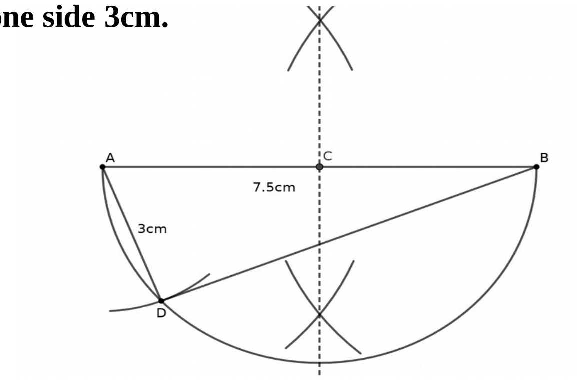
Ans: 4 x (4+5) = PC^2

PC = 6cm

GEOMETRIC CONSTRUCTIONS STD 10

1.Draw a right triangle of hypotenuse 7.5cm and one side 3cm.

a) Draw 7.5cm long line AB.
b) Draw a semi circle with diameter AB.
c) From A cut an arc of radius 3cm at D.
d) Join AD, BD.

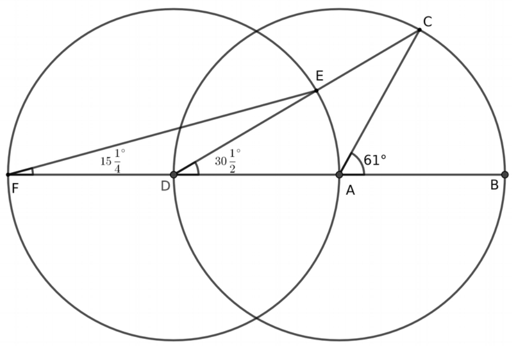


Fact: Angle in a semicircle is a right angle

$<ADB = 90^0$

2) Construct an angle of measure 15¼^0.

a)Draw a circle.
b)Taking 61^0 at centre , draw two radii AB , AC
c) Extend AB to D.
d) Join CD.
e) Draw a circle with D as centre and DA as radius.
f) Extend AD to F.
g) Join EF.

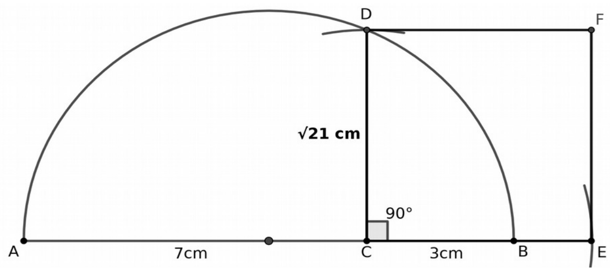


Fact: Angle made by an arc in its alternate arc is half its central angle

<EFA = ½<CDB = ½<CAB

3) Draw a line segment of length √21cm.Also draw a square of area 21cm^2

a) draw 7+3 = 10 cm long line AB.
b) Draw semicircle with AB as diameter.
c) From C draw a perpendicular to D.
d) With CD as a side complete the square.

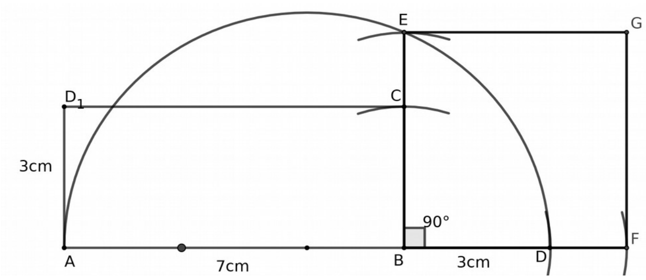


Fact: If AB is the diameter of a semi circle and DC perpendicular to AB , then AP x PB = PC^2.

AC x CB = CD^2

4)Draw a rectangle of sides 7cm , 3cm. Draw a square of same area.

a)Draw the given rectangle.
b)Extend AB to D where BD = BC.
c)Draw a semicircle with AD as diameter.
d) From B draw a perpendicular to E.
e)With BE as a side complete the square.

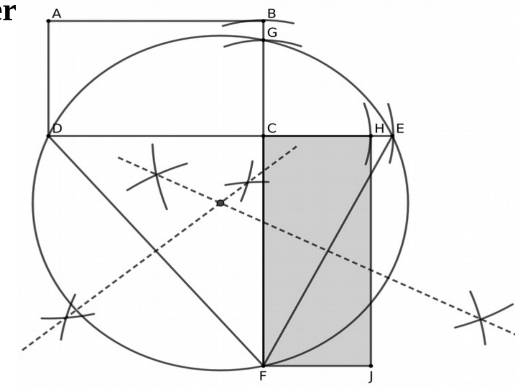


Fact: If AB is the diameter of a semi circle and DC perpendicular to AB , then AP x PB = PC^2.

AC x CB = CD^2

5)Draw a rectangle of sides 6cm , 3cm. Draw another rectangle of same area but one side 7cm.

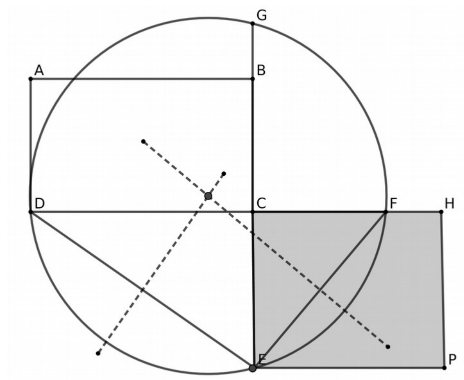
a)Draw the given rectangle.
b)Extend DC to E where CB = CE.
c)Extend BC to F by 7cm.
d)Join DF , EF.
e) Draw perpendicular bisectors of sides DF , EF.
f) Draw circum circle of triangle DEF.
g)Draw CH = CG
g)Draw rectangle with CF and CH as sides.



Fact: If two chords AB , CD intersect at P , then AP x PB = PC x PD. DC x CE = CG x CF

6) Draw a rectangle of area 18cm^2. Draw another rectangle of same area but one side 5cm.

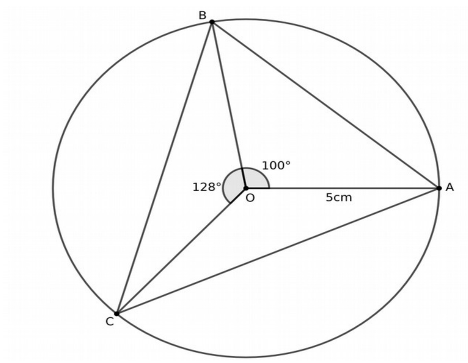
a)Draw the given rectangle.
b)Extend DC to F where CF = BC.
c)Extend BC to E by 5 cm.
d)Join DE , FE.
e) Draw perpendicular bisectors of sides DE ,FE.
f) Draw circum circle of triangle DEF.
g)Draw CH = CG
g)Draw rectangle with CE and CH as sides.



Fact: If two chords AB , CD intersect at P , then AP x PB = PC x PD. DC x CF = CG x CE

7) Draw a circle of radius 5cm. Draw a triangle of angles 50^0 , 64^0 with vertices on the circle.

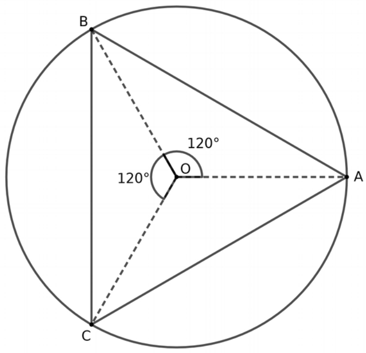
a) Draw a circle of radius 5cm .
b)Taking twice the given angles at centre draw 3 radii OA , OB , OC.
c)Join end points of radii.



Fact: Angle made by an arc in its alternate arc is half its central angle <BCA = ½<BOA

8) Draw an equilateral triangle of Circum-radius 4cm.

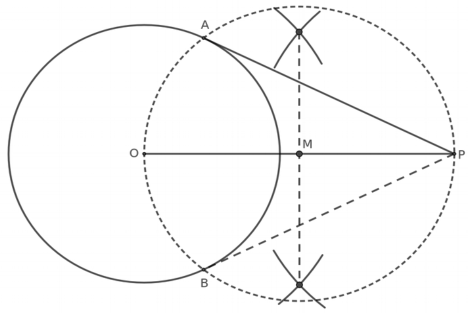
a) Draw a circle of radius 4cm.
b)Taking twice the required angles at centre draw 3 radii OA , OB , OC.
c)Join end points of radii.



Fact: Angle made by an arc in its alternate arc is half its central angle <BCA = ½<BOA

9) Draw a circle of radius 3cm. Mark a point 7cm from centre. Draw tangents from there on to the circle.

a) Draw a circle of radius 3cm.
b)Mark a point P , 7cm away from centre
c) Join OP.
d) Draw a circle with OP as diameter cutting at A , B
e) Join PA , PB.

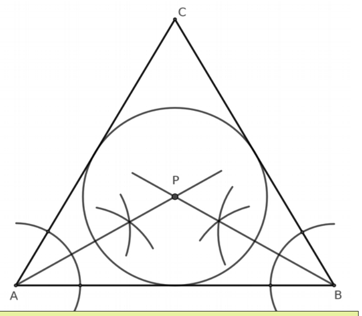


Fact: If PA and PB are tangents from a point P to a circle with centre O , then PAOB is a cyclic quadrilateral

$<AOB + <APB = 180^0$

10) Draw a triangle of radius 4cm. Draw its In-circle.

a)Draw the given triangle.
b)Draw angle bisectors of any two angles to meet at a point.
c) Draw a proper circle touching the sides of triangle.

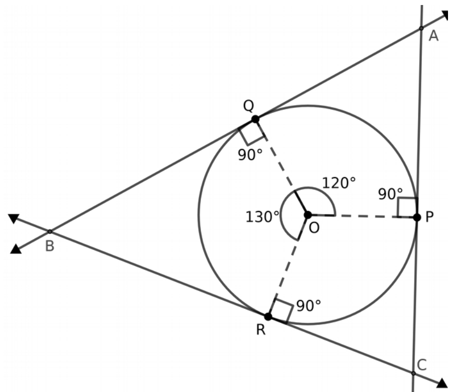


Fact: Angle bisectors of a triangle meet at the centre of the In-circle

AP and BP meet at P

11) Draw a triangle of In-radius 3cm and angles 60^0 , 50^0

a) Draw a circle of radius 3cm.
b)subtract the given angles from 180. (will get 120^0 , 130^0) and use them to draw 3 radii OP ,OQ,OR .
c)Draw tangents at their end points.

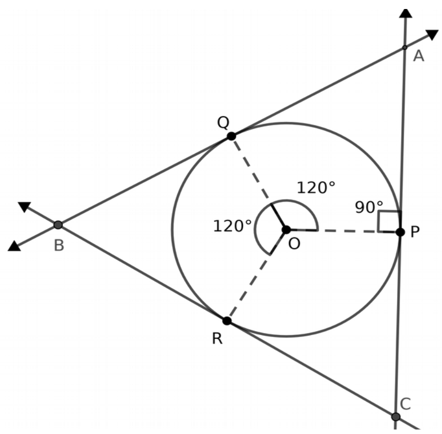


Fact: Angle between two tangents from a point out side and angle between two radii to that tangents are supplementary.

$<QBR + <QOR = 180^0$

12)Draw a circle of radius 3 cm. Draw an equilateral triangle with its sides touching the circle.

a) Draw a circle of radius 3cm.
b)subtract the given angles from 180. (will get 120^0 each) and use them to draw 3 radii OP ,OQ,OR.
c)Draw tangents at their end points.

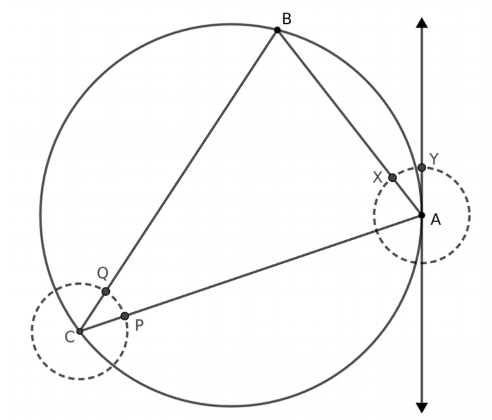


Fact: Angle between two tangents from a point out side and angle between two radii to that tangents are supplementary.

$<QBR + <QOR = 180^0$

13) Draw a circle. Mark a point on it . Draw a tangent through it without using radius.

a)Draw a circle and mark a point A on it.
b)Draw a chord AB through it.
c)Mark another point C on the alternate arc
and join BC , AC.
d)Measure this angle <QCP and reproduce at <XAY.
e) Draw tangent through AY
(Draw a small circle with centre C and small radius.
Measure QP. Draw another circle with centre A
and same radius . Mark X , Y such that XY= PQ.)

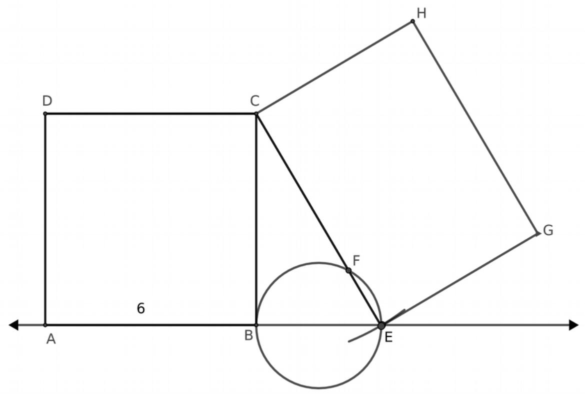


Fact: Angle between a tangent and a chord at a point , on one side equals the angle on the part of circle on the other side of chord

<BAY = <BCA

14)Draw a square of area 36 cm^2. Draw a rectangle of one side 7cm , using the concept of tangents.

a)Draw a square of side 6cm.
b)Extend side AB
c) Cut a point E such that CE =7cm.
d) Draw a circle with diameter BE.
e) Draw rectangle with sides CE , CF.

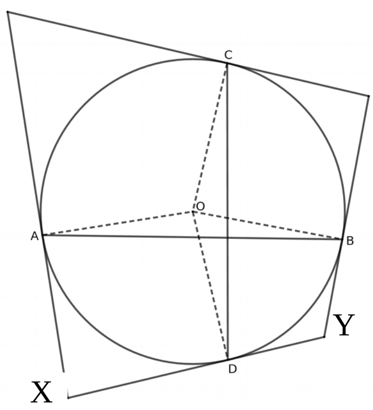


Fact: Consider a tangent PC and a line from P which cut the circle at A and B then PA x PB = PC^2.

CF x CE = CB^2

15. Draw a circle and two perpendicular chords. Draw tangents at their end points to form a quadrilateral.

a) Draw the circle
b) Draw two perpendicular chords
c) Draw radii to their end points
d) Draw tangents at their end points



Fact: Tangent at a point on a circle is perpendicular to the radius through towards that point.

<ODX = 90^0

Mathematics of chances

In an experiment with well defined results which can be counted , The probability of getting a particular result is represented by the fraction " number of favourable result / Total number of results "

When a coin is tossed up , we get two results : Head , Tail

Probability of a head = $\frac{1}{2}$

Probability of a Tail = $\frac{1}{2}$

They have the same probability = $\frac{1}{2}$

Eg: If a die with 6 faces , with faces denoting digits from 1 to 6 is tossed up ...

The possible out comes (Results) = 1,2,3,4,5,6 (6 numbers)

Odd number out comes : 1,3,5 (3 numbers)

Even number outcomes : 2,4,6 (3 numbers)

Hence probability of odd numbers coming = $\frac{3}{6}$

Hence probability of even numbers coming= $\frac{3}{6}$

They have same probability of happening = $\frac{1}{2}$

Eg: Two cubes in which faces denote first 6 natural numbers are tossed up together.

a) Possible pairs of outcomes : [(1,1) , (1,2) , (1,3) , (1,4) , (1,5) , (1,6) , (2,1)................(6,5) , (6,6)] (Total 6x6 = 36 outcomes)

b) Probability of getting same numbers in both dice = $\frac{6}{36}=\frac{1}{6}$

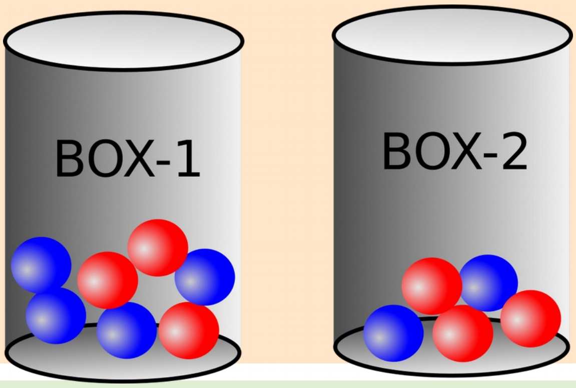
(because (1,1) , (2,2) , (3,3) , (4,4) , (5,5) , (6,6) (6 cases)

c) Probability of getting sum of the two numbers 8 = $\frac{5}{36}$

(because (2,6) , (3,5) , (4,4) , (5,3) , (6,2) (5 cases)

Eg: One box contains 7 balls of which 4 are blue , 3 are read. Another box contain 2 blue and 3 read balls.

One ball is taken at random from the box 1 and then box 2



a)Write the probabilities of getting read balls from each boxes separately ($\frac{3}{7},\frac{3}{5}$)

b)Which box has greater probability ?

Ans: Box 2 ($\frac{15}{35},\frac{21}{35}$)

If one ball each are taken from the two boxes at a time.

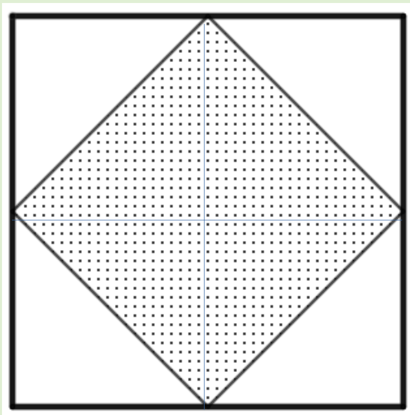
How many pairs of balls are possible ? Ans: 7x5=35

Probability getting both balls red ? Ans : $\frac{3x3}{35}=\frac{9}{35}$

Probability getting different colours ?Ans : $\frac{(3x2)+(4x3)}{35}=\frac{18}{35}$

Probability getting same colours ? Ans: $\frac{(3x3)+(4x2)}{35}=\frac{17}{35}$

Mid points of a square are joined to form another square and is shaded. Without looking to it a dot is put in the big square. What is the chance that it falls inside the shaded square ?



Let the length of side of the bigger square be x unit. Then the diagonal of the smaller square will be x unit.

Probability = Shaded area / Total area

$$= \frac{\frac{1}{2}x^2}{x^2}=\frac{1}{2}$$

(OR)

probability = no of shaded parts / Total parts. $\frac{4}{8}=\frac{1}{2}$

SSLC 2021 MATHEMATICS CONCEPTS AND FORMULAS

Second degree equations.

Some Identities

$(x+a)(x+b) = x^2+(a+b)x+ab$

$(x+a)(x-a) = x^2 – a^2$

$(x+a)^2 = x^2+2ax+b^2$

$(x+a)^2 = x^2-2ax+b^2$

Solutions of some equations

$x^2 = k$, $x = \pm\sqrt{k}$

$(x-a)^2 = k$, $x-a = \pm\sqrt{k}$, $x = a\pm\sqrt{k}$

$(x+a)^2 = k$, $x+a = \pm\sqrt{k}$, $x = -a\pm\sqrt{k}$

$$ax^2+bx+c=0, x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Examples:

1. $x^2 = 16$, $x = \pm 4$

2. $x^2 = 7$, $x = \pm\sqrt{7}$

3. $(x-1)^2 = 25$, $x-1 = \pm 5$, $x = 1\pm 5 = 1+5 , 1-5 = 6 , -4$

4. $(x+2)^2 = 36$, $x+2 = \pm 6$, $x = -2\pm 6 = -2+6 , -2 -6 = 4 , -8$

5. $x^2 -3x+2 =0$,

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4x1x2}}{2x1}=\frac{3\pm\sqrt{9-8}}{2}=\frac{3\pm\sqrt{1}}{2}=\frac{3\pm 1}{2}=\frac{4}{2},\frac{2}{2}=2,1$$

6. $3x^2 + x -2 =0$,

$$x=\frac{-1\pm\sqrt{(1)^2-4x3x-2}}{2x3}=\frac{-1\pm\sqrt{1+24}}{2}=\frac{-1\pm\sqrt{25}}{6}=\frac{-1\pm 5}{6}=\frac{4}{6},\frac{-6}{6}=\frac{2}{3},-1$$

Completing the square method

What should be added to $x^2 +bx$ to make it a perfect square?

$x^2 +bx + ? = (x+........)^2$

adding $\left(\frac{b}{2}\right)^2$ **with** $x^2 +bx$ **gives a perfect square.**

$x^2 +ax + \left(\frac{b}{2}\right)^2 = (x+ \frac{b}{2})^2$

$x^2 -bx + \left(\frac{b}{2}\right)^2 = (x- \frac{b}{2})^2$

Add square of half of the Coefficient of x

Eg: x^2 -6x = 40 Solve this equation using the above method.

Ans: $x^2 -6x + \left(\frac{-6}{2}\right)^2 = 40 + \left(\frac{-6}{2}\right)^2$, $(x-3)^2 = 49$, $x-3 = \pm 7$, $x = 7+3 , -7+3 = 10,-4$

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Write the following concepts into second degree equations.

a)The product of two consecutive natural numbers is 306
Ans : $x(x+1) = 306$

b)The product of two consecutive odd numbers is 143
Ans: $x(x+2) = 143$

c)The product of two consecutive multiples of 3 is 180
Ans : $x(x+3) = 180$

d) The product of two consecutive terms of an AS with common difference 4 is 117
Ans: $x(x+4) = 117$

e) The length of a rectangle is 3cm more the breadth. Its area is 108 cm^2
Ans : $x(x+3) = 108$

f) one of the shorter side of a right triangle is 2cm more than twice the other. Its area is 30 cm^2
Ans : $\frac{1}{2} x(2x+2) = 30$

g) The product of first n natural numbers is 171

Ans : $\frac{1}{2}n(n+1) = 171$

h)The sum of two natural numbers is 17 , while their product is 66
Ans : $x(17-x) = 66$

i) The difference of two numbers is 6 , their product is 55

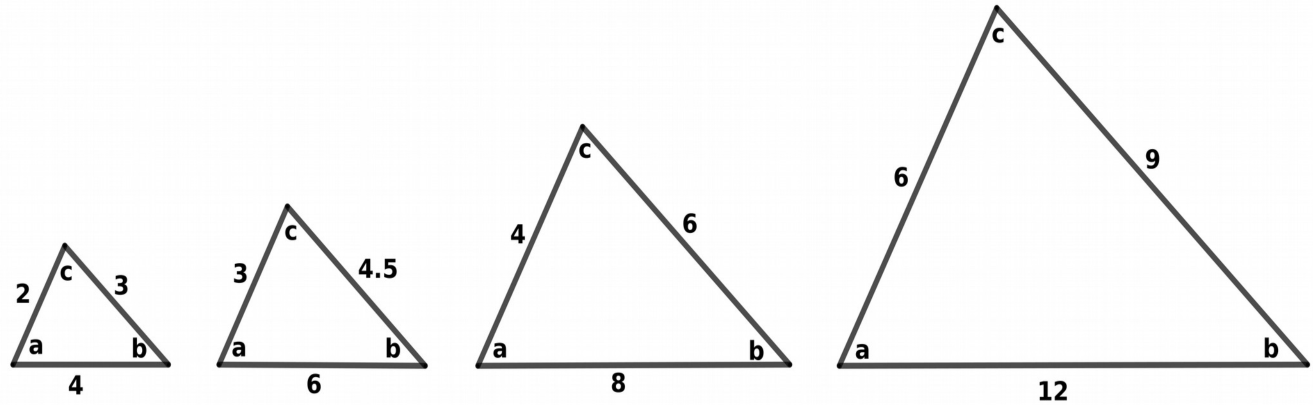
Ans : $x(x+6) = 55$

Try to do all these using formula or completing square

SSLC 2021 MATHEMATICS CONCEPTS AND FORMULAS

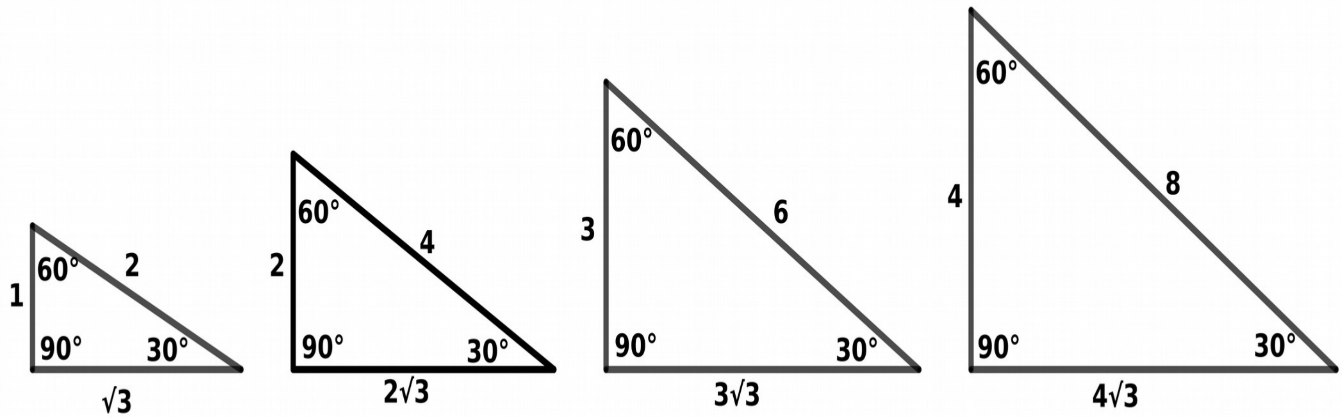
Trigonometry

Triangles with same set of angles are called similar triangles..
Their sides will be in same ratio



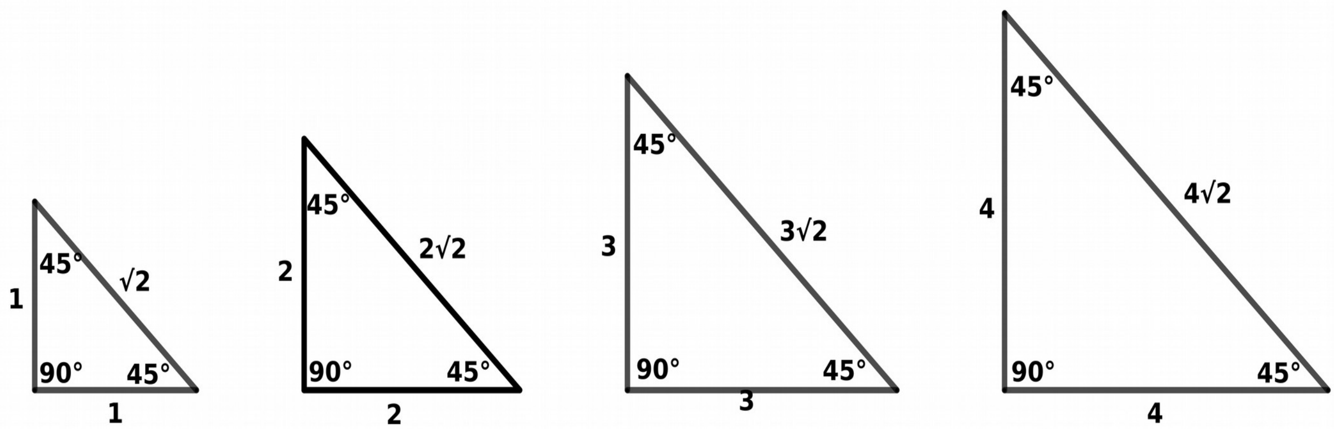
$2 : 3 : 4 = 3 : 4.5 : 6 = 4 : 6 : 8 = 6 : 9 : 12$

The ratio of sides of a triangle with angles 30^0 , 60^0 , 90^0 is $1 : \sqrt{3} : 2$



$1 : \sqrt{3} : 2 = 2 : 2\sqrt{3} : 4 = 3 : 3\sqrt{3} : 6 = 4 : 4\sqrt{3} : 8$

The ratio of sides of a triangle with angles 45^0 , 45^0 , 90^0 is $1 : 1 : \sqrt{2}$



$1 : 1 : \sqrt{2} = 2 : 2 : 2\sqrt{2} = 3 : 3 : 3\sqrt{2} = 4 : 4 : 4\sqrt{2}$

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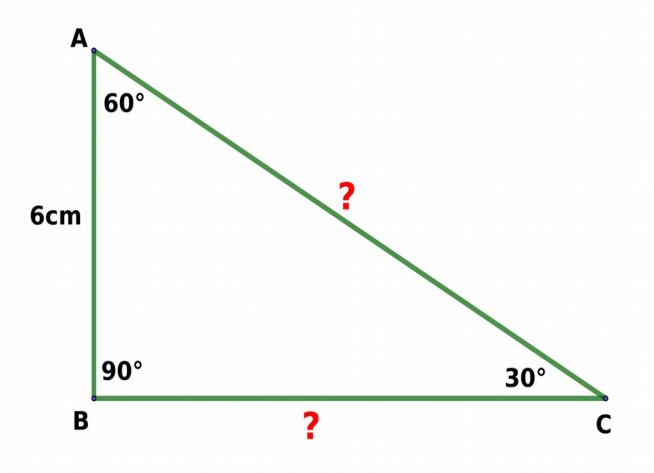
Eg:Find the missing sides of the following triangles



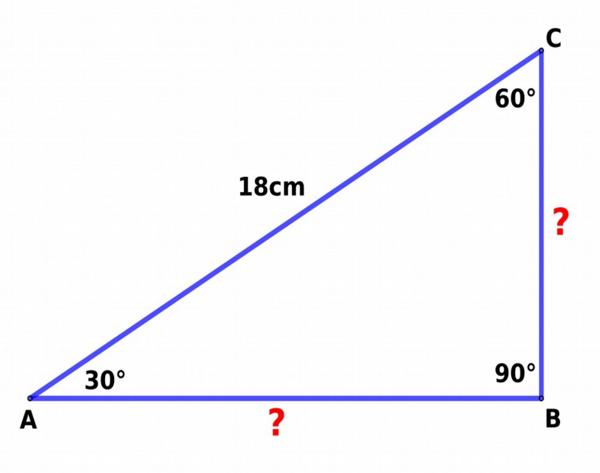


Figure 1

Method-1

$AB : BC : AC = 1 : \sqrt{3} : 2$

$6 : BC : AC = 6 : 6\sqrt{3} : 12$

$BC = 6\sqrt{3}$ cm , $AC = 12$cm

OR

Figure 2

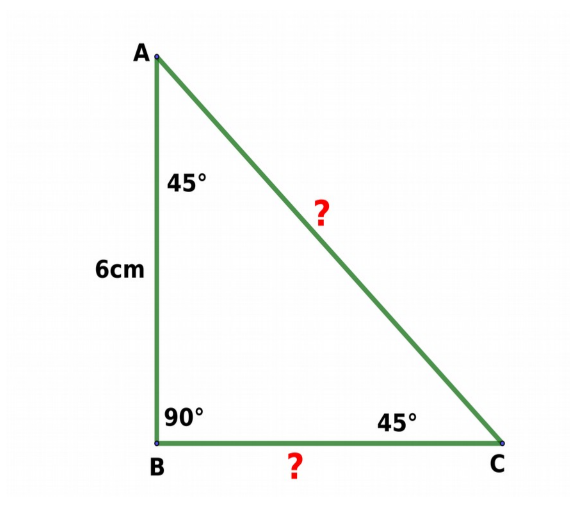
Method-2

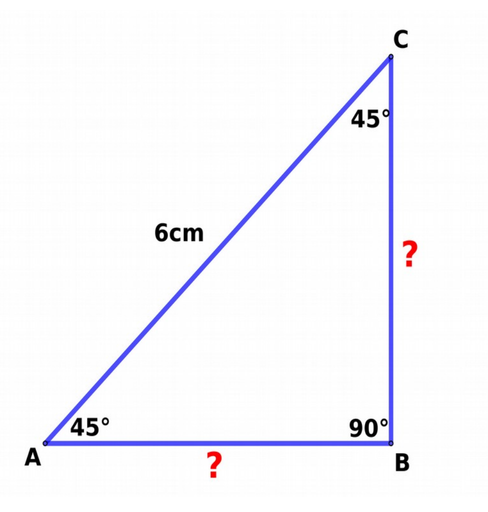
$AB : BC : AC = 1 : \sqrt{3} : 2$

$AB = x$, $BC = \sqrt{3}x$, $AC = 2x$

$AC=18 \Rightarrow 2x = 18$, $x=9$

$AB = x= 9$, $BC = \sqrt{3}x = 9\sqrt{3}$,





ചോദ്യം 3

Method-1

$AB : BC : AC = 1 : 1 : \sqrt{2}$

$6 : BC : AC = 6 : 6 : 6\sqrt{2}$

$BC = 6$cm , $AC = 6\sqrt{2}$cm

OR

ചോദ്യം 4

Method-2

$AB : BC : AC = 1 : 1 : \sqrt{2}$

$AB = x$, $BC = x$, $AC = \sqrt{2}x$

$AC = 6 \Rightarrow \sqrt{2}x = 6$, $x = 3\sqrt{2}$

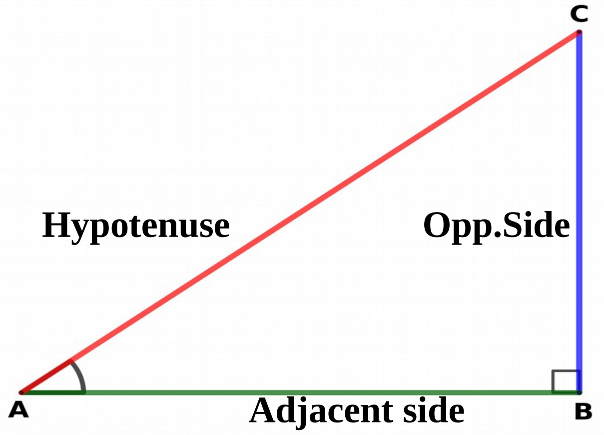
$AB = x = 3\sqrt{2}$, $BC = x = 3\sqrt{2}$

New measures

$$\text{Sin } A = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

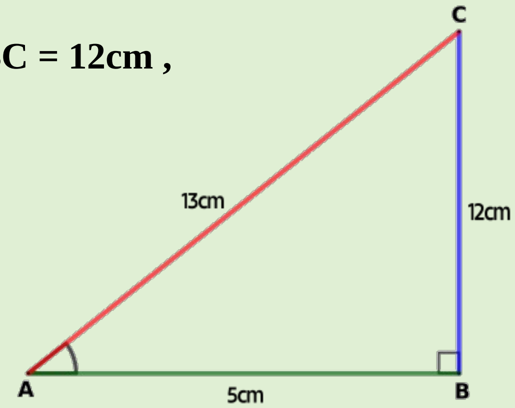
$$\text{Cos } A = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\text{Tan } A = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{BC}{AB}$$

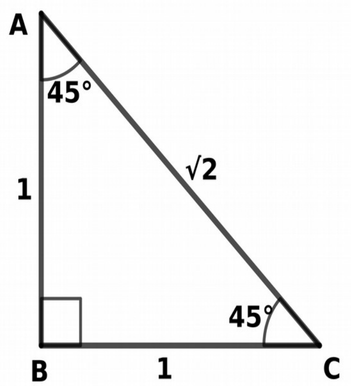


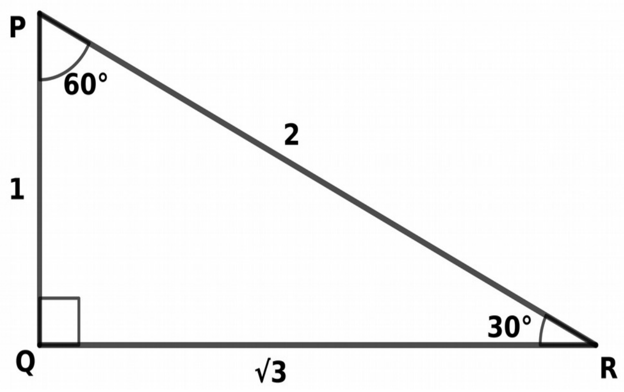
Eg : In a Right triangle ABC , <B = 90⁰ , AB = 5cm , BC = 12cm ,
Find SinA , CosA , TanA
Ans: Using pythagorus theorem , $AC=\sqrt{5^2+12^2}=13$

$$SinA=\frac{12}{13}, CosA=\frac{5}{13}, TanA=\frac{12}{5}$$



Sin , Cos , Tan Values of some angles.





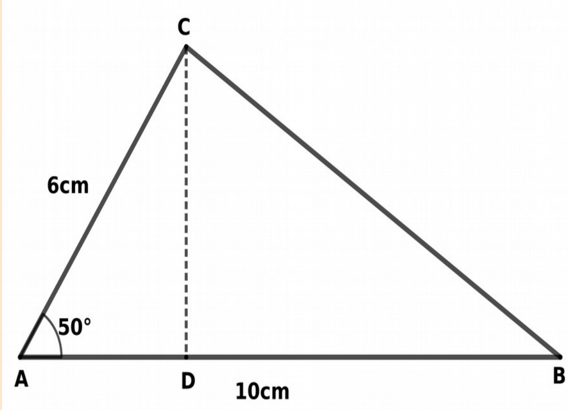
$$Sin30=\frac{1}{2}, Cos30=\frac{\sqrt{3}}{2}, Tan30=\frac{1}{\sqrt{3}}$$

$$Sin45=\frac{1}{\sqrt{2}}, Cos45=\frac{1}{\sqrt{2}}, Tan45=\frac{1}{1}=1$$

$$Sin60=\frac{\sqrt{3}}{2}, Cos60=\frac{1}{2}, Tan60=\frac{\sqrt{3}}{1}=\sqrt{3}$$

Eg: In triangle ABC , AB =10cm , AC = 6cm , <A = 50^0 . Find the altitude from C, hence the area of the triangle.

Ans: Draw altitude from C, to form a right triangle ADC. CD is the opposite side of <A , hence we may use Sine value to find CD.



$$\sin 50 = \frac{CD}{AC} = \frac{CD}{6}$$

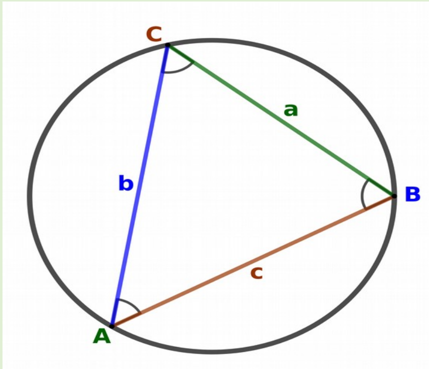
$$CD = 6 x \sin 50 = 6 x 0.766 = 4.6 cm$$

$$A = \frac{1}{2} x AB x CD = \frac{1}{2} x 10 x 4.6 = 23 cm^2$$

Sin values , Circum diameter , chord length relation

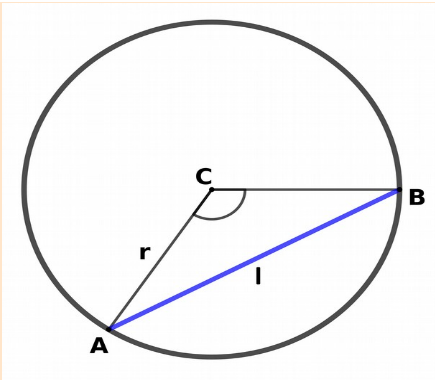
If one angle and opposite side of a triangle are given,then the diameter of cir-cum circle is obtained by dividing the side by the sin of the opposite angle.

$$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC} = 2r$$

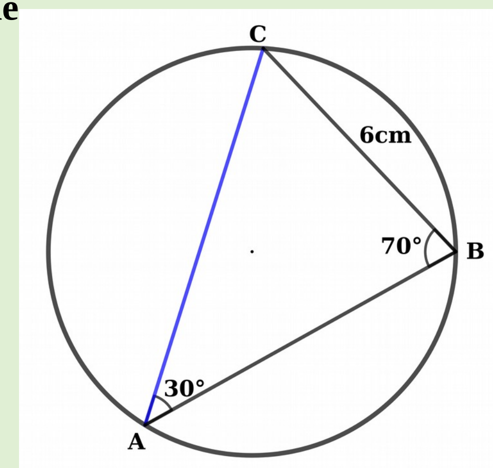


If the radius and central angle of a chord is known,then the length of the chord is obtained by multiplying the diameter by the sin of half the central angle.

$$l = 2rSin(\frac{C}{2})$$



Eg: The two angles of a triangle are 30^0 , 70^0 . If the side opposite to 30^0 angle is 6cm long , what is the length of side opposite to 70^0 angle ? What is the circum diameter of cir-cum circle ?

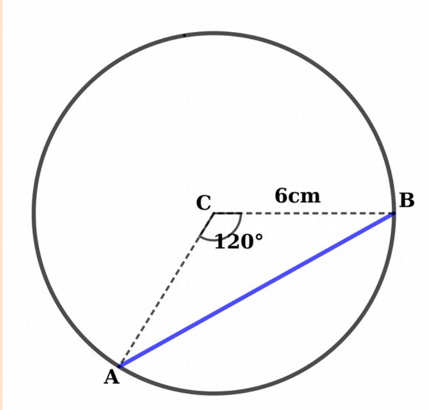


Ans: $d = \frac{BC}{SinA} = \frac{6}{\sin 30} = \frac{6}{1/2} = 12 cm$

Same time $d = \frac{AC}{\sin 70}$

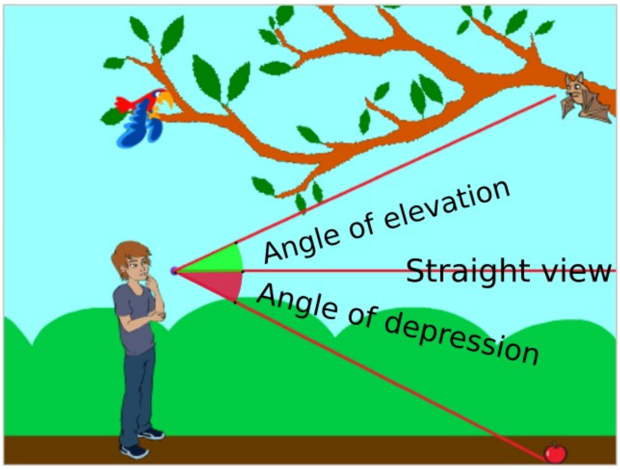
$$AC = d x \sin 70 = 12 x 0.94 = 11.3 cm$$

Eg: If a chord of central 120^0 is drawn in a circle of radius 6cm , what will be its length?

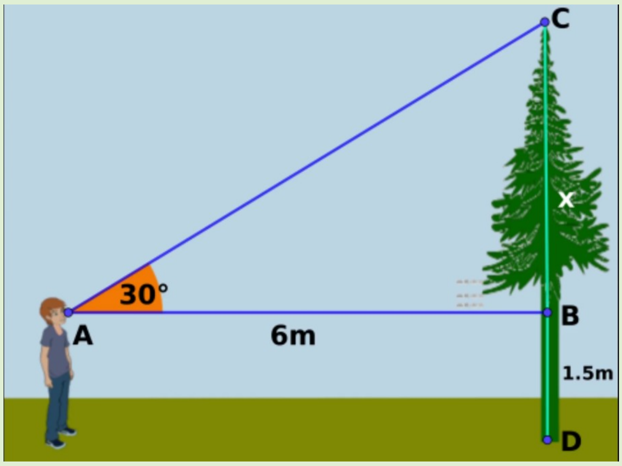


$$l=2r\,x\sin\left(\frac{C}{2}\right)=12\,xSin60=12\,x\frac{\sqrt{3}}{2}=6\sqrt{3}\,cm^2$$

Angle of elevation and depression



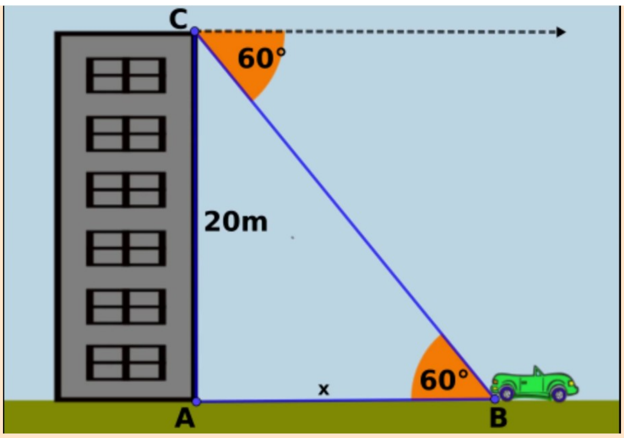
Eg: A boy standing 6m away from the foot of a tree observe its tip at an angle of elevation 30^0. If the boy is 1.5 m high , find the height of the tree.
Ans:



$$\tan 30=\frac{BC}{6},\ BC=6\,x\tan 30=6\,x\,1.73=10.38\,m$$

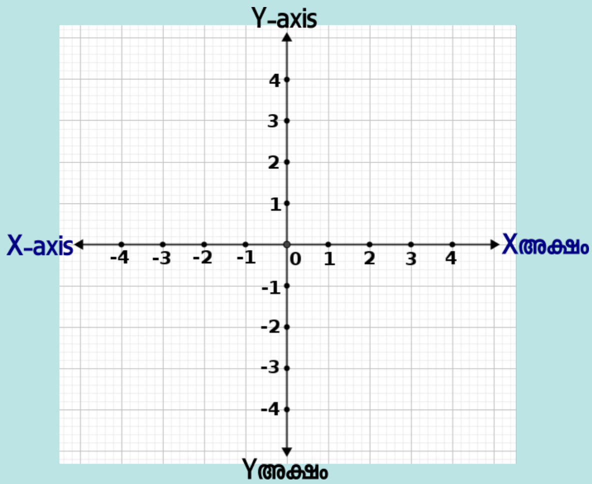
CD = 10.38+1.50 = 11.88m

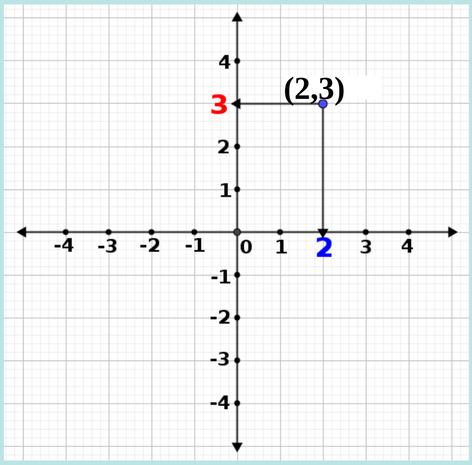
A boy standing on the top of a building 20m high observe a car on the floor , at an angle of depression 60^0. How far is the car from building ?



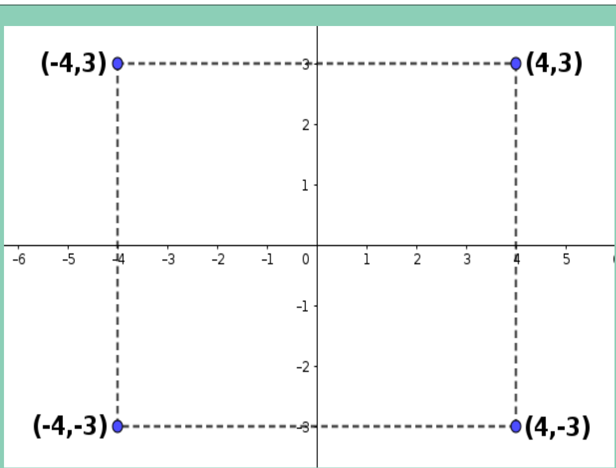
$$\tan 60=\frac{20}{AB},\ AB=20\div\tan 60=\frac{20}{\sqrt{3}}\,m$$

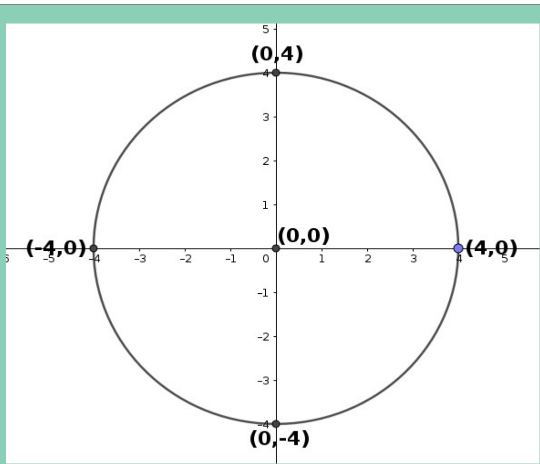
Coordinates





If (3,2) is the coordinate of a point, then 3 is called the x-coordinate and 2 is called the y-coordinate. Generally coordinate of any point is represented by (x,y).
The point of intersection of the X and Y axes is called Origin , represented by(0,0)



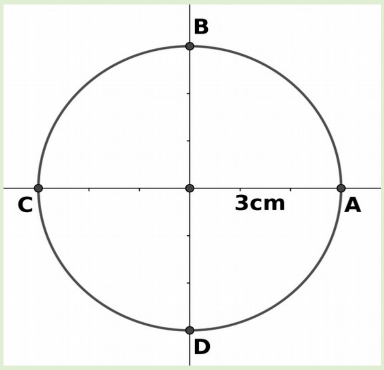


The y-coordinate of all points on X-axis is 0
Eg: (-4,0) , (0,0) , (4,0)
The x-coordinate of all points on Y-axis is 0
Eg: (0,4) , (0,0) , (0,-4)
The y-coordinate of all points on a line parallel to the X-axis are same.
Eg: (-4,3) , (0,3) , (4,3)
The x-coordinate of all points on a line parallel to the Y-axis are same.
Eg: (4,3) , (4,0) , (4,-3)
Actually the x- coordinates are the distance of the points from Y-axis

Actually the y- coordinates are the distance of the points from X-axis

Eg: Find the coordinates of the points where a circle with centre origin and radius 3 cm cut the coordinate axes.

Ans : A(3,0) , B(0,3) , C(–3,0) , D(0,–3)



Eg: The sides of a rectangle are parallel to the Axes. If the coordinate of a pair of opposite vertices are given, Find the coordinates of other vertices.

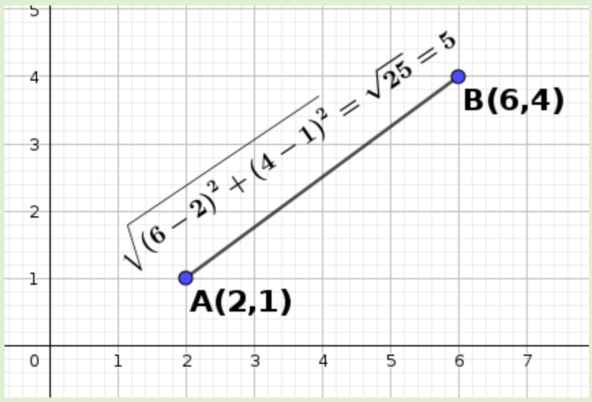
Ans : Q(7,1) , S(3,4)

The distance between the points

$(x_1, y_1), (x_2, y_2)$

is

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$



Eg: Are the vertices with coordinates A(-1,2) , B(1,0) , C(-1,-2) , D(-3,0) is of a square ?

Ans: $AB=\sqrt{(1--1)^2+(0-2)^2}=\sqrt{2^2+(-2)^2}=\sqrt{8}$

$BC=\sqrt{(-1-1)^2+(-2-0)^2}=\sqrt{(-2)^2+(-2)^2}=\sqrt{8}$

$CD=\sqrt{(-3--1)^2+(0--2)^2}=\sqrt{(-2)^2+2^2}=\sqrt{8}$

$AD=\sqrt{(-3--1)^2+(0-2)^2}=\sqrt{(-2)^2+(-2)^2}=\sqrt{8}$

$AC=\sqrt{(-1--1)^2+(-2-2)^2}=\sqrt{0^2+(-4)^2}=\sqrt{16}=4$

$BD=\sqrt{(-3-1)^2+(0-0)^2}=\sqrt{(-4)^2+0^2}=\sqrt{16}=4$

Since Sides are equal
Diagonals are equal
It is a square

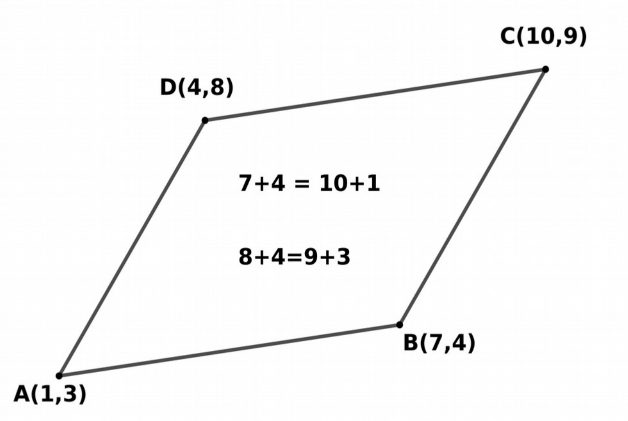
SSLC 2021 MATHEMATICS CONCEPTS AND FORMULAS

Coordinates-part2

If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$, $D(x_4, y_4)$ are the vertices of a parallelogram then

$x_1 + x_3 = x_2 + x_4$, $y_1 + y_3 = y_2 + y_4$
Hence $x_1 + x_3 - x_2$, $y_4 = y_1 + y_3 - y_2$.
(D = A + C - B)

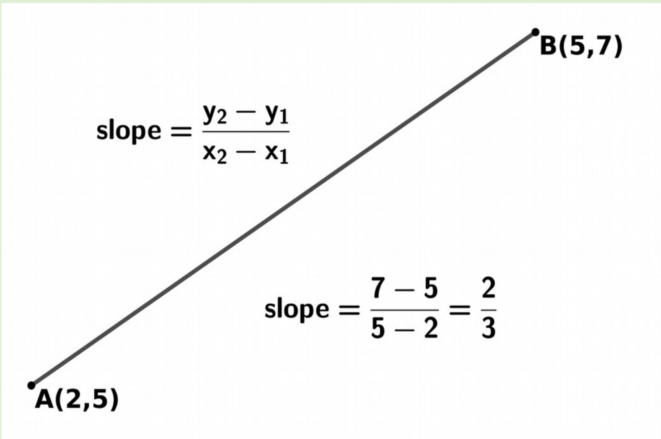
Eg: The vertices of a parallelogram are P(1,3) , Q(6,4) , S(4,9) , Find the fourth vertex.
Ans : R = Q+S-P = (6+4-1 , 9+4-3) = R(9,10)



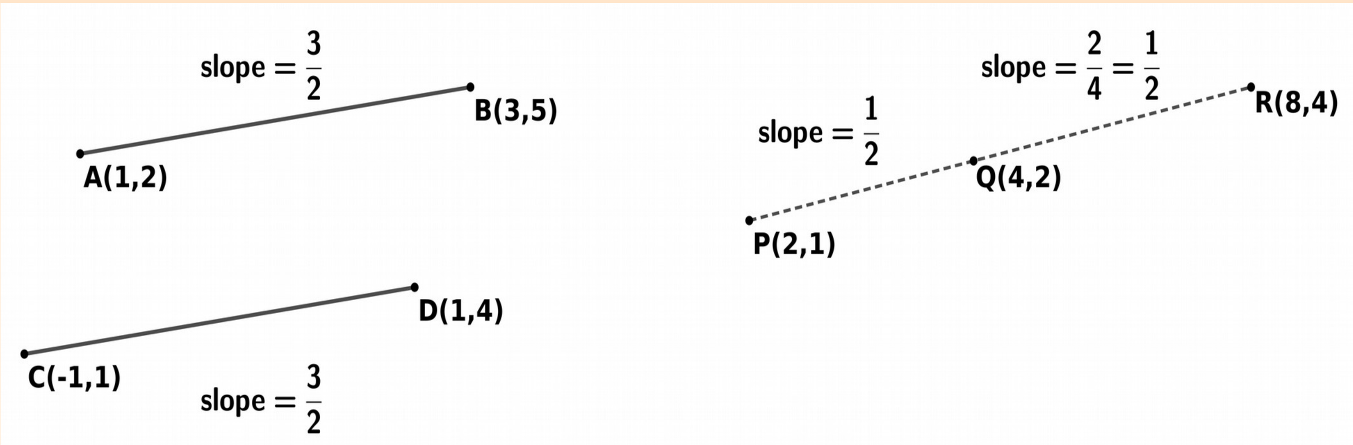
The slope of a line joining two points is the the difference of y-coordinates divided by the difference of x-coordinates of the points.

If $A(x_1, y_1)$, $B(x_2, y_2)$ are two points, then

Slope of AB = $\dfrac{y_2 - y_1}{x_2 - x_1}$



If two different lines have same slope , they are parallel. If two consecutive lines have same slope then they are part of same line. In other words if Lines AB and BC have same slope , then A , B , C are three points of the same line.



In figure slope of AB = slope of CD. Hence AB and BC are parallel lines.
In figure slope of PQ = slope of QR. Hence P , Q , R are on the same line.

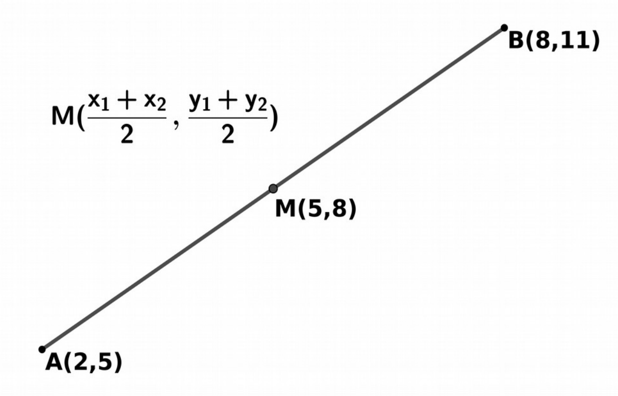
Eg: Show that A(1,3) , B(7,4) , C(10,9) , D(4,8) are the vertices of a parallelogram .
Ans: Slope of AB = Slope of CD = 1/6 , Slope of AD = Slope of BC = 5/3
Since opposite sides have same slope , they are parallel , Hence ABCD is a parallelogram.

If (x,y) is a point on a line with slope $\frac{p}{q}$, then (x+q , y+p) , (x-q , y-p) are also points on the line.

Eg: (2 , -3) , (5 , 1) are two points of a line . Find the slope of the line and write one more point of it.
Ans: Slope of the line = 4/3 . Another point on it = (5+3 , 1+4) = (8,5)

The mid point of a line joining the points $A(x_1, y_1)$, $B(x_2, y_2)$ is given by

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$



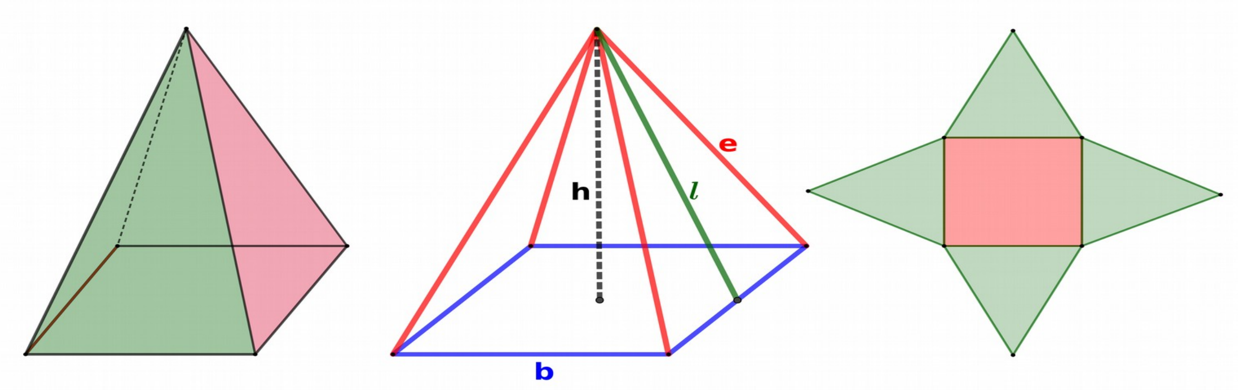
If M is the mid point of two points A and B then we can see that

A+B = 2M and B = 2M- A

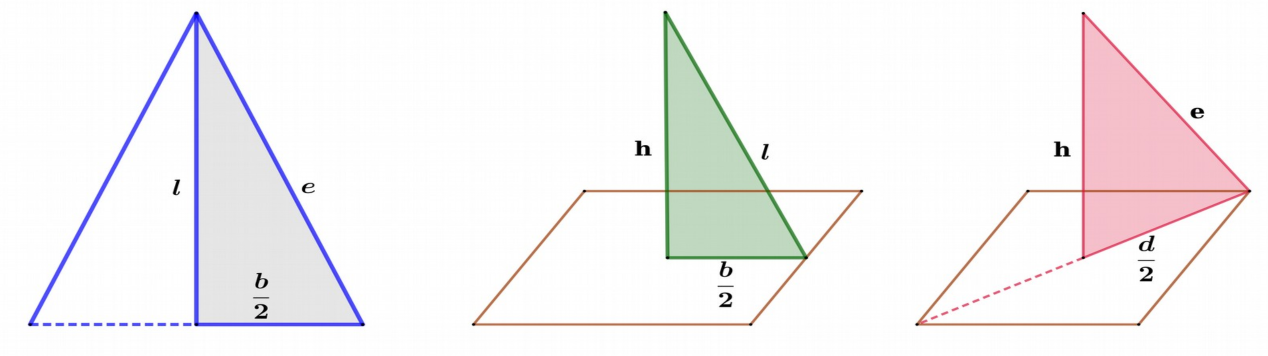
Eg: Show that A(1,3) , B(7,4) , C(10,9) , D(4,8) are the vertices of a parallelogram
Ans: Mid point of AC = (11/2 , 6). Mid point of BD = (11/2 , 6)
Since diagonals have same mid point , they bisect each other , hence ABCD is a parallelogram.

Eg: The centre of a circle is M(4,7). The one end point of the diameter is A(3, 2) . Find the coordinate of other end.
Ans: B = 2M- A , (2x4-3 , 2x7-2) = (5,12)

SOLIDS -Square pyramids (Out of focus area)*



Base is a square, 4 lateral faces are isosceles triangles of equal size.
Base has 4 base edges (b) , Lateral faces has 4 lateral edges (e)
Height of the lateral faces is called slant height(l)
Height of the pyramid is denoted as h



$(\frac{b}{2})^2 + l^2 = e^2$ $\quad$ $(\frac{b}{2})^2 + h^2 = l^2$ $\quad$ $(\frac{d}{2})^2 + h^2 = e^2$

Base area = b^2
Area of one lateral face = ½bl
Lateral surface are = 2bl
Surface area = b^2+2bl

Volume = $\frac{1}{3}b^2h$

Eg: The base edge and lateral edge of a square pyramid are 10cm and a3cm. Find slant height and lateral surface area.

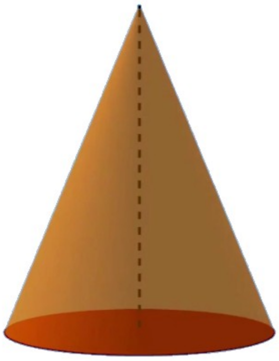
Ans : b = 10 , e = 13. Using $(\frac{b}{2})^2 + l^2 = e^2$ **, $l^2 = 13^2 - 5^2 = 144$, l = √144 = 12cm.**

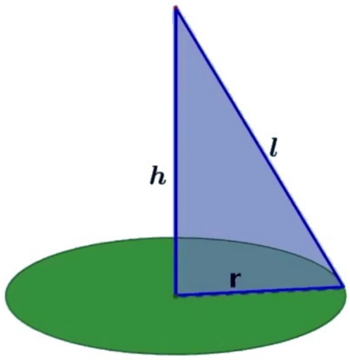
Lateral surface area = 2bl = 240cm^2

Eg: The height and slant height of a square pyramid are 8cm and 10cm.Find base edge and volume.

Ans: h = 8 , l = 10 , Using $(\frac{b}{2})^2 + h^2 = l^2$ **,** $\frac{b}{2} = \sqrt{36} = 6, b = 12\,cm$ **,**

SOLIDS - Cone

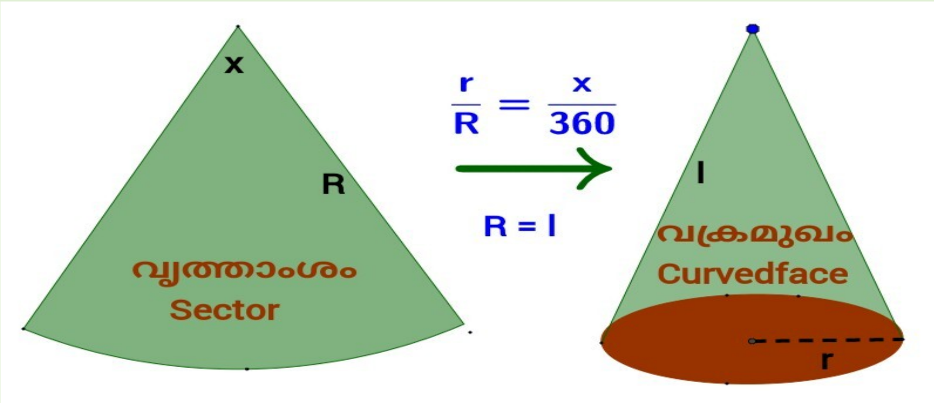




$r^2 + h^2 = l^2$

Base is a circle and also has a curved surface.
Base radius = r , slant height = l , height = h are the measures of a cone

A sector of radius R and central angle x can be rolled in to a cone. The sector becomes the curved face of the cone .The radius of the sector becomes the slant height of the cone. Also there exists a relation connecting all these measures



Base are = πr^2

Curved surface area = πrl **Volume =**

Surface area = $\pi r^2 + \pi rl$

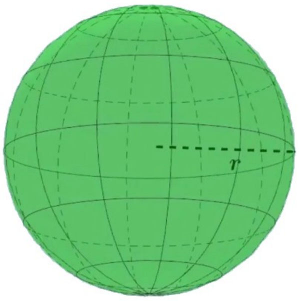
Eg:The radius of a cone is 6cm , height is 8cm. Find the slant height , curved surface area , and volume .

Ans: r = 6 , h = 8 , $r^2 + h^2 = l^2$, hence l = 10cm.
Curved surface area = $\pi rl = 60\pi\ cm^2$. Volume = = $96cm^3$.

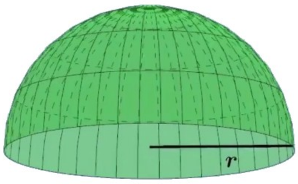
Eg: A sector of central angle 120^0 , and radius 12cm , is rolled up in to a $\frac{1}{3}\pi r^2 h$ **cone. What will be the slant height , radius and height of the cone ?**

Ans: Here R = 12 , x = 120^0. Slant height l = sector radius = 12cm , Using we get r = 4cm. $r^2 + h^2 = l^2$ gives h = $\sqrt{(144 - 16)} = \sqrt{128}$ cm

Sphere and hemisphere (Out of focus area)*



Radius = r



The surface area of a sphere of radius r = $4\pi r^2$

Volume = $\frac{4}{3}\pi r^3$

The area of plane face of a hemisphere of radius r = πr^2

Curved surface area = $2\pi r^2$

Total Surface area = $3\pi r^2$

Volume = $\frac{2}{3}\pi r^3$

Eg: Find the surface area and volume of a sphere of radius 6cm.

Ans: r = 6 , Surface area = $4\pi r^2 = 144\pi$ cm^2. Volume = $\frac{4}{3}\pi r^3$ = 288π cm^3.

Polynomials

Polynomials and Factors

$x^2+ax = x(x+a)$

$x^2+(a+b)x+ab = (x+a)(x+b)$

$x^2 – a^2 = (x+a)(x-a)$

$x^2+2ax+b^2 = (x+a)^2$

$x^2-2ax+b^2 = (x+a)^2$

Write as the product of first degree polynomials.(Factorise)

$x^2+5x = x(x+5)$

$x^2 – 16 = (x+4)(x-4)$

$x^2+6x+9 = (x+3)^2$

$x^2-12x+36 = (x-6)^2$

Write $x^2 +5x+6$ as a product of two first degree polynomials.

Let $x^2 +5x +6 = (x+a) (x+b)$

We have $x^2+(a+b)x +ab = (x+a) (x+b)$

Hence sum a+b = 5 , Product ab = 6. Guess such two numbers !

A = +3 , b = +2

Then the factors are given by $x^2 +5x+6 = (x+3) (x+2)$

Write $x^2 -5x+6$ as a product of two first degree polynomials.

Let $x^2 -5x +6 = (x+a) (x+b)$

We have $x^2+(a+b)x +ab = (x+a) (x+b)$

Hence sum a+b = -5 , Product ab = 6. Guess such two numbers !

A = -3 , b = -2

Then the factors are given by $x^2 -5x+6 = (x-3) (x-2)$

Write $x^2 +5x-6$ as a product of two first degree polynomials.

Let $x^2 +5x -6 = (x+a) (x+b)$

We have $x^2+(a+b)x +ab = (x+a) (x+b)$

Hence sum a+b = 5 , Product ab = -6. Guess such two numbers !

A = 6 , b = -1

Then the factors are given by $x^2 +5x-6 = (x+6) (x-1)$

Remainders and Factors

The remainder when p(x) is divided by x- a is P(a)
The remainder when p(x) is divided by x+a is P(-a)

If the remainder is 0 , they will be factors of p(x).

that means , If p(a) = 0 then x-a will be a factor of P(x)
If p(-a) = 0 then x+a will be a factor of P(x)

On the other hand If x-a is a factor of p(x) , then p(a) = 0.
If x+a is a factor of p(x) , then p(-a) = 0.

If p(x) is a polynomial and a is a number , then
x-a will be a factor of p(x) – p(a).
x+a will be a factor of p(x) – p(-a).

If x= a and x= -b are solutions of p(x) = 0
then x-a , x+b are the 2 factors of p(x)

Eg: When $x^3 + 2x^2 - 2x - 1$ is divided by x - 2 what is the remainder ?
Ans : $R = p(+2) = (2)^3 + 2(2)^2 - 2(2) - 1 = 11$

Eg: Is x+2 a factor of $P(x) = x^3 + x^2 - 4x - 4$?
Ans: p(-2) = 0 , x+2 is a factor.

Eg: If x-2 is a factor of p(x) , what is p(2) ?
Ans: p(2)=0

Eg: In a polynomial P(x) , If P(1) = 0 , P(- 2) = 0 , Write two factors of it.
Ans: Factors : x -1 , x +2.

Eg: p(x) = 0 has two solutions 2 , -3. Write the 2 factors of it , also write the polynomial.
Ans: Factors : x-2 , x+3 . Polynomial $p(x) = (x-2)(x+3) = x^2+x-6$

Eg: If p(x) = (x+3)(x-4). What are the solutions of p(x) = 0 ?
Ans: Solutions x = -3 and x = 4

STATISTICS

The mean or average of a set of numbers is obtained by dividing the sum of those values by the number of those values.

$$Mean = \frac{The Sum\ of\ values}{Number\ of\ values}$$

Sum of values = mean x number of values

The median of a set of numbers is that value which comes at the middle position , if the values are arranged in ascending or descending order. If there comes two values at the middle , we take the average of them

Eg: Find mean and median : 101 , 107 , 105 , 101 , 103 , 102 , 108

Ans : Sum = 727 , count = 7 , mean = 727/7 = 103.8

When values are arranged we get 101 , 101 , 102 , 103 , 105 , 107 , 108.

median = value at the middle = 103

Eg: Find median : 101 , 107 , 105 , 101 , 103 , 102 , 108 , 104

Ans: When values are arranged 101 , 101 , 102 , 103 , 104 , 105 , 107 , 108.

Values at middle = 103 , 104 . Median = (103+104) / 2 = 103.5

Eg: The mean of 30 students is 15. When teacher is also considered , the mean is 16. Find the age of teacher.

Ans: Total age of students = 30x15 = 450 .

Total age including teacher = 31x16 = 496. Age of teacher = 496 – 450 = 46

Eg: Find median :

Wage	500	550	600	650	700
Employees	8	11	16	14	10

Ans:

Wage	Employees
Up to 500	8
Up to 550	8+11=19
Up to 600	19+16=35
Up to 650	35+14=49
Up to 700	49+10=59

Total employees = 59

If arranged on wage order , person at middle = 59/2 =29.5 = 30^{th} employee.

The class in which he belongs = 600 വരെ → 35

Wage of 30^{th} employee = 600

Median wage = 600

Eg: Find the Mean and Median of first 15 natural numbers:

Ans : Sum = (15 x 16)/2 = 120 , Mean = 120 / 15 = 8

Median = middle number = 8