07.01.22 [FRIDAY]

FIRST REVISION EXAMINATION MODEL QUESTION PAPER - 2022

1. If $n(A \times B) = 6$ and $A = \{1, 3\}$ then $n(B)$ is
a) 1 b) 2 c) 3 d) 6.

2. If the HCF of 65 and 117 is expressible in the form of $65m - 117$, then the value of m is
a) 4 b) 2 c) 1 d) 3

3. The next sequence $3/16, 1/8, 1/12, 1/18 \ldots$ is
a) $1/24$ b) $1/27$ c) $2/3$ d) $1/8$

4. The range of the relation $R = \{(x, x^2) / x$ is a prime number less than 13 is
a) $(2,3,5,7)$ b) $\{2,3,5,7,4\}$ c) $\{4,9,25,49,121\}$
d) $\{1,4,25,49,121\}$

5. Using Euclid's division lemma if the cube of any positive integer is divided by 9 then the possible remainders are, a) 0, 1, 8 b) 1, 4, 8 c) 0, 1, 3 d) 1, 3, 5

6. Given $F_1 = 1, F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is
a) 3 b) 5 c) 8 d) 11

7. The solution of the system $x + y - 3z = -6$, $-7y + 7z = 7$, $3z = 9$ is
a) $x = 1; y = 2; z = 3$ b) $x = -1, y = 2$ $z = 3$ c) $x = -1$ $y = -2$ $z = 3$
d) $x = 1, y = -2, z = 3$

8. $\frac{3y-3}{y} \div \frac{7y-7}{3y^2}$ is
a) $9y/7$ b) $9y^3/21y-21$ c) $\frac{21y^2-42y+21}{3y^2}$ d) y^2-2y+1/y^2

9. Graph of a linear equation is a ______
a) straight line b) Circle c) Parabola d) Hyperbola

10. A system of three linear equation in three variables is inconsistent if their plane.
a) intersect only at a point b) intersect in a line.
c) coincides with each other d) do not intersect

11. The roots of quadratic equation $x^2-x-1=0$ are

a) 1,1 b) -1,1 c) $\frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$

12. The solution of $(2x-1)^2=9$ is equal to

a) -1 b) 2 c) -1,2 d) None of these.

13. Which of the following should be added to make x^4+64 a perfect square

a) $4x^2$ b) $16x^2$ c) $8x^2$ d) $-8x^2$

14. The number of points of intersection of the quadratic polynomials x^2+4x+4 with x-axis is

a) 0 b) 1 c) 0 or 1 d) 2

II ANSWER ANY 10. Q.NO.29 COMPULSORY $2 \times 10 = 20$

15. $B \times A = \{(-2,3)(-2,4)(0,3)(3,3)(3,4)\}$ find A and B

Sol:

$A = \{3,4\}$

$B = \{-2,0,3\}$

16. Let $A = \{1,2,3,4,\dots 45\}$ and R be the relation defined as "is a square of" on A. Write R as a subset of $A \times A$. Also find the domain and range of R.

Sol:

$R = \{(1,1)(2,4)(3,9)(4,16)(5,25)(6,36)\}$

Domain of R $= \{1,2,3,4,5,6\}$

Range of R $= \{1,4,9,16,25,36\}$

17. If $A = \{1,3,5\}$ $B = \{2,3\}$ then find $A \times B$ and $B \times A$

Sol:

$A \times B = \{1,3,5\} \times \{2,3\}$

$A \times B = \{(1,2)(1,3)(3,2)(3,3)(5,2)(5,3)\}$

$B \times A = \{2,3\} \times \{1,3,5\}$

$B \times A = \{(2,1)(2,3)(2,5)(3,1)(3,3)(3,5)\}$

18. A Relation R is given by the set $\{x, y / y = x+3, \in \{0,1,2,3,4\}$. Find the domain and range.

Sol:

$x = 0 \quad y = 0+3 = 3$

$x = 1 \quad y = 1+3 = 4$

$x = 2 \quad y = 2+3 = 5$

$x = 3 \quad y = 3+3 = 6$

$x = 4 \quad y = 4+3 = 7$

Domain is $\{0,1,2,3,4\}$

Range is $\{3,4,5,6,7\}$

19. If $\overset{t_1}{3+k}$, $\overset{t_2}{18-k}$, $\overset{t_3}{5k+1}$ are in A.P, then find k.

Sol:

given, $t_2 - t_1 = t_3 - t_2$

$$18-k-(3+k) = 5k+1-(18-k)$$

$$18-k-3-k = 5k+1-18+k$$

$$-k-k-5k-k = 1-18-18+3$$

$$-8k = -36+4$$

$$-8k = -32$$

$$k = 32/8 = 4$$

$$\boxed{k=4}$$

20. Find the L.C.M of x^4-1, x^2-2x+1

Sol:

$$x^4-1^4 = (x^2)^2 - (1^2)^2 \quad \text{[this is of the form } a^2-b^2 = (a+b)(a-b)]$$

$$= (x^2+1)(x^2-1)$$

$$= (x^2+1)(x+1)(x-1)$$

$$x^2-2x+1 = (x-1)(x-1)$$

$$= (x-1)^2$$

[Common factor – least power & Uncommon factor]

L.C.M is $(x+1)(x-1)^2(x^2+1)$

21. Determine the nature of $15x^2+11x+2=0$

Sol: $a = 15 \quad b = 11 \quad c = 2$

$$\Delta = b^2 - 4ac$$

$$= 11^2 - 4 \cdot 15 \cdot 2$$

$$= 121 - 120$$

$\Delta = 1 > 0 \Rightarrow$ The roots are real and not equal.

22. Find the sum and product of the quadratic equation $x^2+8x-65=0$

$a=1 \quad b=+8 \quad c=-65$

Sol: Sum $= -b/a = -8/1 = -8$

Product $c/a = -65/1 = -65$

23. If one root of the equation $2y^2-ay+64=0$ is twice the other then find the values of a.

Sol: Let the root be α

given other root is twice $\Rightarrow 2\alpha$

$2y^2-ay+64=0$

$a=2 \quad b=-a \quad c=64$

Sum of the roots $= -b/a = +a/2$

Product of the roots $= c/a = \frac{64}{2} = 32$

Sum of the roots $\alpha+2\alpha = \frac{a}{2}$

$3\alpha = \frac{a}{2}$

$3(4) = \frac{a}{2}$

$12.2 = a$

$\boxed{a = 24}$

Product of roots

$\alpha . 2\alpha = 2\alpha^2$

$2\alpha^2 = 32$

$\alpha^2 = \frac{32}{2}$

$\alpha^2 = 16$

$\boxed{\alpha = \pm 4}$

$3(-4) = a/2$

$\boxed{a = -24}$

24. Find the L.C.M of the polynomials x^4-27a^3x, $(x-3a)^2$ whose GCD is $x-3a$

Sol: L.C.M $\times$ GCD $= f(x) \times g(x)$

L.C.M $\times$ $x-3a = (x^4-27a^3x) \times (x-3a)^2$

L.C.M $= \frac{(x^4-27a^3x) \times (x-3a)^2}{x-3a}$

$= (x^4 - 27a^3x)(x - 3a)$

25 Find the square root of $9x^2 - 24xy + 30xz - 40yz + 25z^2 + 16y^2$

Sol:

$\sqrt{(3x)^2 - 2(3x)(4y) + 2(4y)(5z) - 2(5z)(3x) + (5z)^2 + (4y)^2}$

$a = 3x$
$b = 4y$
$c = 5z$

$= \sqrt{(3x - 4y + 5z)^2}$

$= |3x - 4y + 5z|$

26 Find the value of x for which the roots of the equation $(5k-6)x^2 + 2kx + 1 = 0$ are real and equal

Sol: Given, $\Delta = 0$

$a = 5k-6 \quad b = 2k \quad c = 1$

$b^2 - 4ac = 0$

$(2k)^2 - 4.(5k-6)(1) = 0$

$4k^2 - 20k + 24 = 0$

$\div 4 \Rightarrow \quad k^2 - 5k + 6 = 0$

$(k-3)(k-2) = 0$

$\boxed{k = 3, 2}$

27 If the sum and product of the roots are $-3/2$ and -1. Find the equation.

Sol: Quadratic equation is

$x^2 - (\text{sum of the roots})x + \text{Product of roots} = 0$

$x^2 + \frac{3}{2}x - 1 = 0$ ($\times 2$)

$\boxed{2x^2 + 3x - 2 = 0}$

28 The father's age is six times his son's age. Six years hence the age of father will be four times his son's age. Find the present age of son and the father.

Sol: Let son's age be x.

given, fathers age is $6x$

After six years son's age is $x+6$

fathers age is $6x+6$

given, $6x+6 = 4(x+6)$

$6x+6 = 4x+24$

$6x-4x = 24-6$

$2x = 18$

$x = 9$

Sons age is 9 & Father's age is 54

29 Simplify: $\frac{x^3}{x-y} + \frac{y^3}{y-x}$

Sol:

$\frac{x^3}{x-y} - \frac{y^3}{x-y}$

$\frac{x^3-y^3}{x-y} = \frac{\cancel{(x-y)}(x^2+xy+y^2)}{\cancel{(x-y)}}$

$= x^2+xy+y^2$

III ANSWER THE FOLLOWING: Q:NO:43 Compulsory:

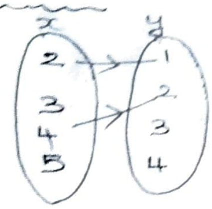
30. Represent each of the given relation by
a) an arrow diagram b) a graph c) a set in Roster form, whenever possible $\{(x,y) / x = 2y \ x \in (2,3,4,5), y \in (1,2,3,4$

Sol: when,

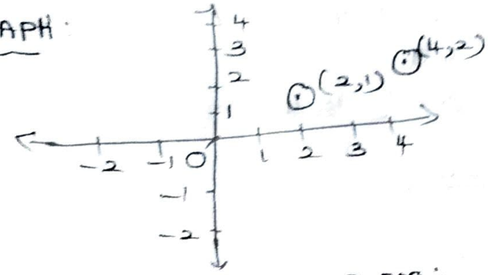
$y = 1 \quad x = 2(1) = 2$

$y = 2 \quad x = 2(2) = 4$

a) ARROW DIAGRAM:



b) GRAPH:



c) A SET IN ROSTER FORM:

$R = \{(2,1) \ (4,2)\}$

31. Find HCF of 252525 and 363636

Sol: Using Euclid's div. algorithm,

$363636 = 252525 \times 1 + 111111$

Remainder $\neq 0$

$252525 = 111111 \times 2 + 30303$

Re $\neq 0$

$111111 = 30303 \times 3 + 20202$

Rem $\neq 0$

$30303 = 20202 \times 1 + 10101$

Rem $\neq 0$

$20202 = 10101 \times 2 + 0$

$R \neq 0$

$\therefore$ HCF is 10101

32. The ratio of 6^{th} and 8^{th} term of an AP is 7:9. Find the ratio of 9^{th} term to 13^{th} term.

Sol: Given, $t_6 : t_8 = 7:9$

$$\frac{t_6}{t_8} = \frac{7}{9}$$

we know $t_n = a + (n-1)d$.

$$\frac{a+5d}{a+7d} = \frac{7}{9}$$

$$9(a+5d) = 7(a+7d)$$

$$9a + 45d = 7a + 49d$$

$$9a - 7a = 49d - 45d$$

$$2a = 4d$$

$$a = \frac{4d}{2} = 2d$$

$$\boxed{a = 2d}$$

To find

$$\frac{t_9}{t_{13}} = \frac{a+8d}{a+12d}$$

$$= \frac{2d+8d}{2d+12d} \quad [\text{sub } a = 2d]$$

$$= \frac{\cancel{10d}}{\cancel{14d}} = \frac{5}{7}$$

$$\therefore \boxed{t_9 : t_{13} = 5 : 7}$$

33 Let $A = \{x \in N / 1 < x < 4\}$ $B = \{x \in W / 0 \leq x < 2\}$
$C = \{x \in N / x < 3\}$, Then verify $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Sol: $A = \{2, 3\}$ $B = \{0, 1\}$ $C = \{1, 2\}$

$B \cup C = \{0, 1, 2\}$

LHS

$A \times (B \cup C) = \{2, 3\} \times \{0, 1, 2\}$

$= \{(2,0)(2,1)(2,2)(3,0)(3,1)(3,2)\}$ —(1)

$A \times B = \{2, 3\} \times \{0, 1\}$

$= \{(2,0)(2,1)(3,0)(3,1)\}$

$A \times C = \{2, 3\} \times \{1, 2\}$

$= \{(2,1)(2,2)(3,1)(3,2)\}$

RHS

$(A \times B) \cup (A \times C) = \{(2,0)(2,1)(2,2)(3,0)(3,1)(3,2)\}$ —(2)

from (1) & (2) Verified.

34 If $P_1^{x_1} \times P_2^{x_2} \times P_3^{x_3} \times P_4^{x_4} = 113400$ when P_1, P_2, P_3, P_4 are in ascending order and x_1, x_2, x_3, x_4 are integers, find the value P_1, P_2, P_3, P_4 and x_1, x_2, x_3, x_4

Sol:

$$\begin{array}{r|l} 2 & 113400 \\ \hline 2 & 56700 \\ \hline 2 & 28350 \\ \hline 5 & 14175 \\ \hline 5 & 2835 \\ \hline 3 & 567 \\ \hline 3 & 189 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

$113400 = 2^3 \times 5^2 \times 3^4 \times 7^1$

$113400 = P_1^{x_1} \times P_2^{x_2} \times P_3^{x_3} \times P_4^{x_4}$

$P_1 = 2$; $P_2 = 5$; $P_3 = 3$; $P_4 = 7$

$x_1 = 3$; $x_2 = 2$; $x_3 = 4$; $x_4 = 1$

35 The sum of 3 consecutive terms that are in AP is 27, and their product is 288. Find the 3 terms.

Sol: Let the three terms be

$a-d, a, a+d$

given, sum of three terms is 27

(ie) $a-\cancel{d}+a+a+\cancel{d} = 27$

$3a = 27$

$a = 27/3$

$\boxed{a = 9}$

product is 288

ie $(a-d)(a)(a+d) = 288$

$(9-d)\,9\,(9+d) = 288$ [sub, a = 9]

$\underset{(a-d)}{(9-d)}\ \underset{a+d}{(9+d)} = \frac{\cancel{288}^{32}}{9} = 32$ $\quad$ $(a-d)(a+d) = a^2 - d^2$

$9^2 - d^2 = 32$

$$81 - d^2 = 32$$
$$-d^2 = 32 - 81$$
$$-d^2 = -49$$
$$d^2 = 49$$
$$d = \pm 7$$

when $a = 9$
& $d = 7$

The numbers are
$a-d, a, a+d$
$9-7, 9, 9+7$
$2, 9, 16$

when $a = 9$
& $d = -7$

The numbers are
$9+7, 9, 9-7$
$16, 9, 2$

36 Find the GCD of $3x^4 + 6x^3 - 12x^2 - 24x$, $4x^4 + 14x^3 + 8x^2 - 8x$

Sol:

$$3x^4 + 6x^3 - 12x^2 - 24x = 3[x^4 + 2x^3 - 4x^2 - 8x]$$
$$4x^4 + 14x^3 + 8x^2 - 8x = 2[2x^4 + 7x^3 + 4x^2 - 4x]$$

$x^4 + 2x^3 - 4x^2 - 8x$) $2x^4 + 7x^3 + 4x^2 - 4x$ (2

$\overset{(-)}{+}2x^4 \overset{(-)}{+} 4x^3 \overset{(+)}{-} 8x^2 \overset{(+)}{-} 16x$

$3x^3 + 12x^2 + 12x$

$3(x^3 + 4x^2 + 4x)$

$\neq 0$.

$x^3 + 4x^2 + 4x$) $x^4 + 2x^3 - 4x^2 - 8x$ ($x - 2$

$\overset{(-)}{+} x^4 \overset{(-)}{+} 4x^3 \overset{(-)}{+} 4x^2$

$-2x^3 - 8x^2 - 8x$

$-2x^3 - 8x^2 - 8x$

0

$\boxed{\text{GCD is } x^3 + 4x^2 + 4x}$

$\frac{2x^4}{x^4} = $

$\frac{x^4}{x^3} = x$

$\frac{-2x^3}{x^3} = $

37. Find the squareroot of $64x^4 - 16x^3 + 17x^2 - 2x + 1$

Sol:

$\times 2 \quad \times 2$

$8x^2 - x + 1$

$8x^2 \;\Big|\; 64x^4 - 16x^3 + 17x^2 - 2x + 1$

$\cancel{64x^4}$

$16x^2 - x \;\Big|\; -16x^3 + 17x^2$

$\overset{(+)}{-\cancel{16x^3}} \overset{(-)}{+ x^2}$

$16x^2 - 2x + 1 \;\Big|\; 16x^2 - 2x + 1$

$\mp 16x^2 \pm 2x \mp 1$

0

$\dfrac{-16x^3}{x^2} = -x$

$\sqrt{64x^4 - 16x^3 + 17x^2 - 2x + 1} = |8x^2 - x + 1|$

38. Solve $pqx^2 - (p+q)^2 x + (p+q)^2 = 0$.

Sol:

$a = pq \;;\; b = -(p+q)^2 \;;\; c = (p+q)^2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-[-(p+q)^2] \pm \sqrt{[-(p+q)^2]^2 - 4pq \cdot (p+q)^2}}{2pq}$$

$$= \frac{(p+q)^2 \pm \sqrt{(p+q)^4 - 4(pq)(p+q)^2}}{2pq}$$

$$= \frac{(p+q)^2 \pm \sqrt{(p+q)^2[(p+q)^2 - 4pq]}}{2pq}$$

$$= \frac{(p+q)^2 \pm \sqrt{(p+q)^2 (p-q)^2}}{2pq} \Rightarrow (p+q) \pm (p+q)(p-q)/2pq$$

$$\Rightarrow \boxed{\frac{p+q}{q}, \quad \frac{p+q}{p}}$$

39. A bus covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more, it would have taken 30 minutes less for journey. Find the original speed of the bus.

Sol: Let the constant speed be x.

Time taken to cover 90 km $= \dfrac{90}{x}$ hrs. $\left[\text{Time} = \text{Distance}/\text{speed}\right]$

when speed increased,

Time $= \dfrac{90}{x+15}$ hrs.

given, $\frac{90}{x} - \frac{90}{x+15} = \frac{1}{2}$

$$\frac{90(x+15) - 90x}{x(x+15)} = \frac{1}{2}$$

$$\frac{\cancel{90x} + 1350 - \cancel{90x}}{x^2+15x} = \frac{1}{2}$$

$$\frac{1350}{x^2+15x} = \frac{1}{2}$$

$$x^2+15x = 2700$$

$$x^2+15x-2700=0$$

-2700 / -45 $+60$

$$(x-45)(x+60)=0$$

$x = +45$, $x = -60$

$-ve$ not possible.

$\therefore$ Original speed is $\underline{45\ km/hr}$.

40. Solve: $x+y+z=5$, $2x-y+z=9$, $x-2y+3z=16$

Sol: Let $x+y+z=5$ — (1)

$2x-y+z=9$ — (2)

$x-2y+3z=16$ — (3).

(1)+(2) $\Rightarrow$

$x+\cancel{y}+z=5$ — (1)

$2x-\cancel{y}+z=9$ — (2)

$3x+2z=14$ — (4)

(2) × 2 $\Rightarrow$ $4x-\cancel{2y}+2z=18$

(3) $\Rightarrow$ $\overset{(-)}{+}x \overset{(+)}{-}\cancel{2y} \overset{(-)}{+}3z \overset{(-)}{=}16$

(2)-(3) $\Rightarrow$ $3x - z = 2$ — (5)

Take (4) & (5)

$\cancel{3x}+2z=14$ — (4)

$\overset{-}{+}\cancel{3x} \overset{(+)}{-}z \overset{-}{=}2$ — (5)

(4)-(5) $\Rightarrow$ $3z=12$

$$\boxed{z=4}$$

Sub. $z=4$ in (4)

$3x+2z=14$

$3x+2(4)=14$

$3x+8=14$

$3x=14-8$

$3x=6$

$x=6/3=2$

$\boxed{x=2}$

Sub. $x=2, z=4$ in (1)

$x+y+z=5$

$2+y+4=5$

$y+6=5$

$y=5-6=-1$

$\boxed{y=-1}$

$\boxed{x=2;\ y=-1;\ z=4}$

41) The roots of the equation $x^2+6x-4=0$ are α, β find the quadratic equation where results are $2/\alpha$ & $\frac{2}{\beta}$

Sol:

given, $x^2+6x-4=0$

$a=1 \quad b=6 \quad c=-4$

$\alpha+\beta = -b/a = -6/1$ | $\alpha\beta = c/a = -4/1$

$\alpha+\beta=-6$ — (1) | $\alpha\beta=-4$ — (2)

If the roots are $2/\alpha$ & $2/\beta$

Sum of roots $= \frac{2}{\alpha} + \frac{2}{\beta}$

$= \frac{2\alpha+2\beta}{\alpha\beta}$

$= \frac{2(\alpha+\beta)}{\alpha\beta} = \frac{\cancel{2}(\cancel{-6})^{3}}{\cancel{-4}\,\cancel{2}} = +3$ [from (1) & (2)]

Product of roots $= \frac{2}{\alpha} \times \frac{2}{\beta} = \frac{4}{\alpha\beta} = \frac{\cancel{4}}{\cancel{-4}} = -1$

$\therefore$ Quadratic equation is,

$x^2 - (\text{Sum of the roots})x + \text{Product of roots} = 0.$

$\boxed{x^2 - 3x - 1 = 0}$

42 Simplify: $\dfrac{1}{x^2-5x+6} + \dfrac{1}{x^2-3x+2} - \dfrac{1}{x^2-8x+15}$

Sol: $\dfrac{1}{x^2-5x+6} + \dfrac{1}{x^2-3x+2} - \dfrac{1}{x^2-8x+15}$

x^2-5x+6

6 $\quad -5$

$-3 \quad -2$

$\dfrac{-3}{x} \quad \dfrac{-2}{x}$

$(x-3)(x-2)$

x^2-3x+2

$-2 \quad 3$

$-2 \quad -1$

$\dfrac{-2}{x} \quad \dfrac{-1}{x}$

$(x-2)(x-1)$

$x^2-8x+15$

15 $\quad -8$

$-5 \quad -3$

$\dfrac{-5}{x} \quad \dfrac{-3}{x}$

$(x-5)(x-3)$

$$= \frac{1}{(x-3)(x-2)} + \frac{1}{(x-2)(x-1)} - \frac{1}{(x-5)(x-3)}$$

L.C.M is $(x-1)(x-2)(x-3)(x-5)$

$$= \frac{(x-1)(x-5)+(x-3)(x-5)-(x-1)(x-2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x^2-5x-x+5+x^2-5x-3x+15-[x^2-2x-x+2]}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x^2-5x-x+5+\cancel{x^2}-5x-3x+15-\cancel{x^2}+2x+x-2}{(x-1)(x-2)(x-3)(x-5)}$$

$$= x^2-14x+3x+18$$

$$= \frac{x^2-11x+18}{(x-1)(x-2)(x-3)(x-5)} = \frac{(x-9)\cancel{(x-2)}}{(x-1)\cancel{(x-2)}(x-3)(x-5)}$$

18

$\dfrac{-9}{x} \quad \dfrac{-2}{x}$

$$= \frac{x-9}{(x-1)(x-3)(x-5)}$$

43 If the roots of the equation $(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$ are real and equal Prove that either $a=0$ (or) $a^3+b^3+c^3=3abc$.

<u>Sol:</u> Given the roots are real and equal,

$\Delta = B^2-4AC=0$.

$(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$

$a = c^2-ab \quad b=-2(a^2-bc) \quad c=b^2-ac.$

$\Delta = 0$

$b^2-4ac=0.$

$\Rightarrow \left[-2(a^2-bc)\right]^2-4.(c^2-ab)(b^2+ac)=0.$

$4(a^2-bc)^2-4(c^2-ab)(b^2+ac)=0.$

$4\left[(a^2-bc)^2-(c^2-ab)(b^2-ac)\right]=0$

$a^4+\cancel{b^2c^2}-2a^2bc-\cancel{b^2c^2}+ac^3+ab^3-a^2bc]=0.$

$a^4-3a^2bc+ac^3+ab^3=0.$

$a\left[a^3-3abc+c^3+b^3\right]=0.$

$\Rightarrow a=0$ or $a^3-3abc+c^3+b^3=0.$

$a^3+b^3+c^3=3abc.$

Hence proved.