2003

PH : Physics

(Questions 1–30) Carry one mark each.

1. The two vectors $p = i$, $q = (i + j)/\sqrt{2}$ are—
   (A) Related by a rotation
   (B) Related by a reflection through the xy-plane
   (C) Related by an inversion
   (D) Not linearly independent

2. A 3×3 matrix has eigen values 0, $2 + i$ and $2 - i$. Which one of the following statements is correct ?
   (A) The matrix is hermitian
   (B) The matrix is unitary
   (C) The inverse of the matrix exists
   (D) The determinant of the matrix is zero.

3. The value of the integral $\int_C \frac{dz}{z^2}$, where z is a complex variable and C is the unit circle with the origin as its centre, is—
   (A) 0 (B) $2\pi i$
   (C) $4\pi i$ (D) $-4\pi i$

4. A particle with an initial velocity $v_0 i$ enters a region with an electric field $E_0 j$ and a magnetic field $B_0 k$. The trajectory of the particle will—
   (A) Be an ellipse
   (B) Be a cycloid
   (C) Be a helix with constant pitch
   (D) Not be confined to any plane

5. An object of mass m rests on a surface with coefficient of static friction μ. Which of the following is **NOT** correct ?
   (A) The force of friction is exactly μmg
   (B) The maximum force of friction is μmg
   (C) The force of friction is along the surface
   (D) The force of friction opposes any effort to move the object

6. The lagrangian of a particle of mass m moving in a plane is given by $L = \frac{1}{2}m[v_x^2 + v_y^2] + a(xv_y - yv_x)$, where v_x and v_y are velocity components and a is a constant. The canonical momenta of the particle are given by—
   (A) $p_x = mv_x$ and $p_y = mv_y$
   (B) $p_x = mv_x + ay$ and $p_y = mv_y + ax$
   (C) $p_x = mv_x - ay$ and $p_y = mv_y + ax$
   (D) $p_x = mv_x - ay$ and $p_y = mv_y + ax$

7. Two events are separated by a distance of 6×10^5 km and the first event occurs 1's before the second event. The interval between the two events—
   (A) is time-like
   (B) is light-like (null)
   (C) is space-like
   (D) Cannot be determined from the information given.

8. An electric charge, + Q is placed on the surface of a solid, conduction sphere of radius a. This distance measured from the centre of the sphere is denoted as r, then—
   (A) The charge gets distributed uniformly through the volume of the sphere
   (B) The electrostatic potential has the same value of $r < a$
   (C) An equal and opposite charge gets included in the bottom half of the sphere
   (D) The electric field is given by $\frac{1}{4\pi\epsilon_0 r^2}$ for $r < a$

9. An electric field applied along the length of a long cylinder produces a polarization P. The depolarization field produced in this configuration is—
   (A) $\frac{4\pi P}{3}$ (B) $\frac{-4\pi P}{3}$
   (C) $2\pi P$ (D) 0

10. Which one of the following Maxwell's equations implies the absence of magnetic monopoles ?

(A) $\nabla . \overline{E} = \frac{\pi}{\epsilon_0}$

(B) $\nabla . \overline{B} = 0$

(C) $\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$

(D) $\nabla \times \overline{B} = \frac{1}{c^2}\frac{\partial \overline{B}}{\partial t} + \mu_0 \overline{J}$

11. An electromagnetic wave is propagating in free space in the z-direction. If the electric field is given by $\overline{E} = \cos(\omega t - kz)i$, where $\omega t = ck$, then the magnetic field is given by—

(A) $\overline{B} = \left(\frac{1}{c}\right)\cos(\omega t - kz)\, i$

(B) $\overline{B} = \left(\frac{1}{c}\right)\sin(\omega t - kz)j$

(C) $\overline{B} = \left(\frac{1}{c}\right)\cos(\omega t - kz)\, j$

(D) $\overline{B} = \left(\frac{1}{c}\right)\cos(\omega t - kz)\, ji$

12. Given a wave with the dispersion relation $\omega = ck + m$, for $k > 0$ and $m > 0$, which one of the following is true ?

(A) The group velocity is greater than the phase velocity

(B) The group velocity is less than the phase velocity

(C) The group velocity and the phase velocity are equal

(D) There is no definite relation between the group velocity and the phase velocity

13. Which of the following is a valid normalized wave function for a particle in a one dimensional infinite potential well of width L centered at $x = 0$?

(A) $\left(\frac{2}{L}\right)[\cos(2\pi x/L) + \sin(2\pi x/L)]$

(B) $\left(\frac{2}{L}\right)^{1/2} \sin\left[\frac{n\pi x}{L}\right]$ for odd n

(C) $\left(\frac{2}{L}\right)^{1/2} \cos\left(\frac{n\pi x}{L}\right)$ for odd n

(D) $\left(\frac{2}{L}\right)\cos\left(\frac{\pi x}{L}\right)$

14. The commutator $[x, p^2]$, where x and p are position and momentum operators respectively is—

(A) $2i\hbar p$ (B) $-i\hbar p$

(C) $2i\hbar x p$ (D) $-2i\hbar x p$

15. A spin half particle is in the state $S_z = \frac{\hbar}{2}$. The expectation values of S_x, S_x^2, S_y, S_y^2 are given by—

(A) $0, 0, \frac{\hbar^2}{4}, \frac{\hbar^2}{4}$ (B) $0, \frac{\hbar^2}{4}, \frac{\hbar^2}{4}, 0$

(C) $0, \frac{\hbar^2}{4}, 0, \frac{\hbar^2}{4}$ (D) $\frac{\hbar^2}{4}, \frac{\hbar^2}{4}, 0, 0$

16. The spectral term for the atom with 70% filled subshell and only S = 3/2 is—

(A) $^3P_0$ (B) $^4F_{9/2}$

(C) $^3F_{1/2}$ (D) $^4P_{1/2}$

17. The hyperfine splitting of the spectral lines of an atom is due to—

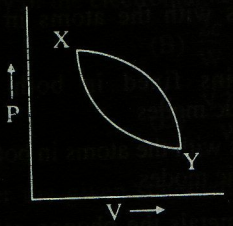
(A) The coupling between the spins of two or more electrons

(B) The coupling between the spins and the orbital angular momenta of the electrons

(C) The coupling between the electron spins and the nuclear spin

(D) The effect of external electromagnetic fields.

18. A piston containing an ideal gas is originally in the state X(see figure). The gas is taken through a thermal cycle X → Y → X, as shown. The work done by the gas is positive if the direction of the thermal cycle is—



(A) Clockwise

(B) Counter clockwise

(C) Neither clockwise nor counter clockwise

(D) Clockwise from X → Y and counter-clockwise from Y → X

19. A second order phase transition is one is which—
    (A) The plot of entropy as a function of temperature shows a discontinuity
    (B) The plot of specific heat as a function of temperature shows a discontinuity
    (C) The plot of volume as a function of pressure shows a discountinuity
    (D) The plot of comprehensibility as a function of temperature is continuous

20. Consider the Fermi-Dirac distribution function $f(E)$ at room temperature (300K), where E refers to energy. If E_F is the fermi energy, which of the following is true ?
    (A) $f(E)$ is a step function
    (B) $f(E_F)$ has a value of 1/2
    (C) States with $E < E_F$ are filled completely
    (D) $f(E)$ is large and tends to infinity as E decreases much below E_F

21. If the ionic radii of Mn and S are 0·080 and 0·184 nm respectively, the structure of MnS will be—
    (A) Cubic closed packed
    (B) Body centered cubic
    (C) NaCl type
    (D) Primitive cubic cell

22. A cubic cell consists of two atoms of masses m_1 and m_2 ($m_1 > m_2$) with m_1 and m_2 atoms situated on alternate planes. Assuming only nearest neighbour interactions, the centre of mass of the two atoms—
    (A) Moves with the atoms in the optical mode and remains fixed in the acoustic mode
    (B) Remains fixed in the optical mode and moves with the atoms in the acoustic mode
    (C) Remains fixed in both optical and acoustic modes
    (D) Moves with the atoms in both optical and acoustic modes.

23. In simple metals the phonon contribution to the electrical resistivity at temperature T is—
    (A) Directly proportional to T above Debye temperature and to T^3 below it
    (B) Inversely proportional to T for all temperature
    (C) Independent to T for all temperatures
    (D) Directly proportional to T above Debye temperature and to T^5 below it

24. The effective mass of an electron in a semiconductor can be—
    (A) Negative near the bottom of the band
    (B) A scalar quantity with a small magnitude
    (C) Zero at the centre of the band
    (D) Negative near the top of the band

25. The dielectric constant of water is 80. However its refractive index is 1·75 invalidating the expression $n = \epsilon^{1/2}$. This is because—
    (A) The water molecule has a permanent dipole moment
    (B) The boiling point of water is 100°C
    (C) The two quantities are measured in different experiments
    (D) Water is transparent to visible light

26. The nucleus of the atom $^9Be_4$ consists of—
    (A) 13 up quarks and 13 down quarks
    (B) 13 up quarks and 14 down quarks
    (C) 14 up quarks and 13 down quarks
    (D) 14 up quarks and 14 down quarks

27. Which one of the following nuclear reactions is possible ?
    (A) $^{14}N_7 \rightarrow {}^{13}C_6 + \beta^+ + \upsilon_e$
    (B) $^{13}N_7 \rightarrow {}^{13}C_6 + \beta^+ + \upsilon_e$
    (C) $^{13}N_7 \rightarrow {}^{13}C_6 + \beta^+$
    (D) $^{13}N_7 \rightarrow {}^{13}C_6 + \beta^+ + \upsilon_e$

28. Suppose that a neutron at rest in free space decays into a proton and an electron. This process would violate—
    (A) Conservation of charge
    (B) Conservation of energy
    (C) Conservation of linear momentum
    (D) Conservation of angular momentum

29. Which one of the following is TRUE for a semiconductor p–n junction with no external bias ?
    (A) The total charge in the junction is not conserved
    (B) The p side of the junction is positively charged

(C) The p side of the junction is negatively charged

(D) No charge develops anywhere in the function

30. Which one of the set of values given below does NOT satisfy the Boolean relation R = PQ' (where Q' denotes NOT Q) ?

(A) P = 1, Q = 1, R = 0

(B) P = 1, Q = 1, R = 1

(C) P = 0, Q = 0, R = 0

(D) P = 0, Q = 1, R = 1

(Questions—31–90) Carry two marks each.

31. The curl of the vector $\overline{A} = zi + xj + yk$ is given by—

(A) $i+j+k$ (B) $i-j+k$

(C) $i+j-k$ (D) $-i-j-k$

32. Consider the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$. At time $t = 0$, it is given that $x = 1$ and $\frac{dx}{dt} = 0$. At $t = 1$, the value of x is given by—

(A) $\frac{1}{e}$ (B) $\frac{2}{e}$

(C) 1 (D) $\frac{3}{e}$

33. S_{ij} and A_{ij} represent a symmetric and an anti-symmetric real-valued tensor respectively in three dimensions. The number of independent components of S_{ij} and A_{ij} are—

(A) 3 and 6 respectively

(B) 6 and 3 respectively

(C) 6 and 6 respectively

(D) 9 and 6 respectively

34. Consider the four statements given below about the function $f(x) = x^4 - x^2$ in the range $-\infty < x < +\infty$. Which one of the following statements is correct ?

P. The plot of $f(x)$ versus x has two maxima and two minima.

Q. The plot of $f(x)$ versus x cuts the x-axis at four points.

R. The plot of $f(x)$ versus x has three extrema

S. No part of the plot of $f(x)$ versus x lies in the fourth quadrant.

Pick the right combination of correct choices from those given below—

(A) P and R (B) R only

(C) R and S (D) P and Q

35. The Fourier transform of the function $f(x)$ is $F(k) = \int e^{ikx} f(x)\, dx$. The fourier transform of $\frac{df(x)}{dx}$ is—

(A) $\frac{dF(k)}{dk}$ (B) $\int \frac{F(k)}{dk}$

(C) $-ikF(k)$ (D) $ikF(k)$

36. A particle of mass m is moving in a potential of the form $V(x, y, z) = \frac{1}{2} m\omega^2 (3x^2 + 3y^2 + 2z^2 + 2xy)$. The oscillation frequencies of the three normal modes of the particle are given by—

(A) $\omega, \sqrt{3}\omega$ and $\sqrt{3}\omega$

(B) $\sqrt{2}\omega, \sqrt{3}\omega$ and $\sqrt{3}\omega$

(C) $\sqrt{2}\omega, \sqrt{2}\omega$ and 2ω

(D) $\sqrt{2}\omega, 2\omega$ and 2ω

37. The speed of a particle whose kinetic energy is equal to its rest mass energy is given by (c is the speed of light in vacuum)—

(A) $\frac{c}{3}$ (B) $\frac{\sqrt{2}c}{3}$

(C) $\frac{c}{2}$ (D) $\frac{\sqrt{3}c}{2}$

38. Electromagnetic waves are propagating along a hollow, metallic waveguide whose cross section is a square of side W. The minimum frequency of the electromagnetic waves is—

(A) $\frac{c}{W}$ (B) $\frac{2c}{W}$

(C) $\frac{\pi c}{W}$ (D) $\frac{\sqrt{2}\pi c}{W}$

39. Consider the given statements about $E(\bar{r}, t)$ and $B(\bar{r}, t)$, the electric and magnetic vectors respectively in a region of free space

P. Both $\overline{E}$ and $\overline{B}$ are conservative vector fields.

Q. Both $\overline{E}$ and $\overline{B}$ are central force fields.

R. $\overline{E}$ and $\overline{B}$ are mutually perpendicular in the region.

S. Work done by $\overline{B}$ on a moving charge in the region is zero.

Choose the right combination of correct Statements from the following—

(A) P and R (B) R and S

(C) S only (D) P and Q

40. An infinite conducting sheet in the x-y plane carries a surface current density K along the y-axis. The magnetic field $\overline{B}$ for $z > 0$ is—

(A) $\overline{B} = 0$

(B) $\overline{B} = \mu_0 K\, k/z$

(C) $\overline{B} = \mu_0 k\, i/2$

(D) $\overline{B} = \mu_0 Kj/(x^2 + z^2)^{0.5}$

41. A parallel beam of infrared radiation of wavelength of $1{\cdot}01 \times 10^{-6}$ m is incident normally on a screen with two slits 5×10^{-6}m apart and the resulting interference pattern is observed on a distant screen. What is the largest number of maxima that can be observed on the screen ?

(A) 4 (B) 9

(C) 13 (D) Infinitely many

42. A parallel beam of electrons of a given by momentum pass through a screen S_1 containing a slit and then produces a diffraction pattern on a screen S_2 placed behind it. The width of the central maximum observed on the screen S_2 can be increased by—

(A) Decreasing the distance between the screens S_1 and S_2

(B) Increasing the width of the slit in screen S_1

(C) Decreasing the momentum of the electrons

(D) Increasing the momentum of the electrons

43. An electron in a time independent potential is in a state which is the superposition of the ground state (E_0 = 11eV) and the first excited state (E_1 = 1eV). The wave function of the electron will repeat itself with a period of—

(A) $3{\cdot}1 \times 10^{-18}$ s (B) $2{\cdot}1 \times 10^{-15}$ s

(C) $1{\cdot}2 \times 10^{-12}$ s (D) $1{\cdot}0 \times 10^{-9}$ s

44. A particle has the wavefunction $\psi(x, t) = A \exp(i\omega t) \cos(kx)$. Which one of the following is correct ?

(A) This is an eigen state of both energy and momentum

(B) This is an eigen state of momentum and not energy

(C) This is an eigen state of energy and not momentum

(D) This is not an eigen state of energy or momentum

45. A free particle with energy E whose wave function is a plane wave with wavelength λ enters a region of constant potential $V > 0$ where the wavelength of the particle is 2λ. The ratio (V/E) is—

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$

(C) $\frac{3}{4}$ (D) $\frac{4}{5}$

46. The vibrational spectrum of a molecule exhibits a strong line with P and R branches at a frequency υ_1 and a weaker line at a frequency υ_2. The frequency υ_3 is not shown up. Its vibrational Raman spectrum shows a strongly polarized line at frequency υ_3 and no feature at υ_1 and υ_2—

(A) The molecule could be linear

(B) The molecule lacks a center of inversion

(C) υ_1 arises from a symmetric stretching mode

(D) υ_3 arises from a bending mode

47. Three values of rotational energies of molecules are given below in different units—

P. $10\ cm^{-1}$

Q. 10^{-23} J

R. 10^{14} MHz

Choose the correct arrangement in the increasing order of energy—

(A) P,Q,R (B) R,Q,P

(C) R,P,Q (D) Q,R,P

48. The short wavelength cut off of the continuous X-ray spectrum from a nickel target is 0·0825. The voltage required to be applied to an X-ray tube is—

(A) 0·15 kV (B) 1·5 kV

(C) 15 kV (D) 150 kV

49. The spin-orbit coupling constant for the upper state of sodium atom which emits D lines of wave numbers 16956·2 and 16973·4 cm^{-1} is—

(A) 15 cm^{-1} (B) 11·4 cm^{-1}

(C) 12·5 cm^{-1} (D) 15·1 cm^{-1}

50. Consider the following statements about molecular spectra—

P. CH_4 does not give pure rotational Raman lines

Q. SF_6 could be studied by rotational Raman spectroscopy

R. N_2 shows infrared absorption spectrum

S. CH_3CH_3 shows vibrational Raman and infrared absorption lines

T. H_2O_2 shows pure rotational spectrum

Choose the right combination of correct statements—

(A) P and Q (B) P, R and T

(C) P, S and T (D) Q and R

51. The temperature of a cavity of fixed volume is doubled. Which of the following is true for black-body radiation inside the cavity ?

(A) Its energy and the number of photons both increase 8 times

(B) Its energy increases 8 times and the number of photons increases 16 times

(C) Its energy increases 16 times and the number of photons increases 8 times

(D) Its energy and the number of photons both increase 16 itmes

52. A sample of ideal gas with initial pressure P and volume V is taken through an isothermal expansion proceed during which the change in entropy is found to be ΔS. The universal gas constent is R. Then the work done by the gas in given by—

(A) $\frac{PV\Delta S}{nR}$ (B) $nR\Delta S$

(C) PV (D) $\frac{P\Delta S}{nRV}$

53. Hydrogen molecule (mass m) are in thermal equilibrium at a temperature T. Assuming classical distribution of velocity, the most probable speed at room temperature is—

(A) $\frac{(k_B T)}{m}$ (B) $\frac{2k_B T}{m}$

(C) $\frac{\sqrt{2}k_B T}{m}$ (D) $\frac{m}{\sqrt{2}k_B T}$

54. Consider the energy E in the first Brillouin zone as a function of the magnitude of the wave vector k for a crystal lattice constant a, then—

(A) The slope of E versus k is proportional to the group velocity

(B) The slope of E versus k has its maximum value at $|k| = \frac{\pi}{a}$

(C) The plot of E versus k will be parabolic in the interval $\left(\frac{-\pi}{a}\right) < k < \left(\frac{\pi}{a}\right)$

(D) The slope of E versus k is non-zero for all k in the interval $\left(\frac{-\pi}{a}\right) < k < \left(\frac{\pi}{a}\right)$

55. An external magnetic field of magnitude H is applied to a type-I superconductor at a temperature below the transition point. Then which one of the following statements is NOT true for H less than the critical field H_C ?

(A) The sample is diamagnetic

(B) Its magnetization varies linearly with H

(C) The lines of magnetic induction are pushed out from the sample

(D) The sample exhibits mixed states of magnetization near H_C.

56. A ferromagnetic material has a curie temperature 100K, then—

(A) Its susceptibility is doubled when it is cooled from 300k to 200k

(B) All the atomic magnets in it get oriented in the same direction above 100k

(C) The plot of inverse susceptibility versus temperature is linear with a slope T_C

(D) The plot of its susceptibility versus temperature is linear with an intercept T_C

57. The point group symmetries of the three molecules shown in figures 1—3 are respectively—

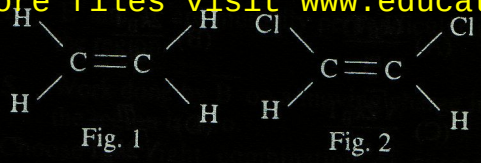


Fig. 1 Fig. 2

[notation: $C_{2v} = 2mm$; $C_{2h} = 2/m$;

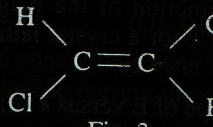


Fig. 3

$D_{2h} = mmm$]

(A) C_{2h}, C_{2v}, C_{2h} (B) C_{2v}, C_{2h}, C_{2h}
(C) D_{2h}, C_{2v}, C_{2h} (D) C_{2v}, D_{2h}, C_{2h}

58. The energy density of states of an electron in a one dimensional potential well of infinitely high walls is (the symbols have their usual meaning)—

(A) $\frac{L\sqrt{m}}{\pi\hbar\sqrt{(2E)}}$ (B) $\frac{Lm}{\pi\hbar\sqrt{E}}$

(C) $\frac{Lm}{\pi\hbar\sqrt{(2E)}}$ (D) $\frac{L\sqrt{m}}{(2\pi\hbar E)}$

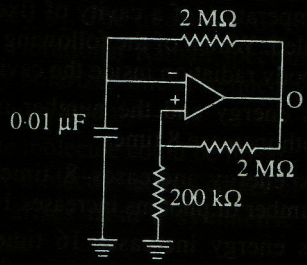
59. Which one of the following statements concerning the Compton effect is **NOT** correct ?
(A) The wavelength of the scattered photon is greater than or equal to the wavelength of the incident photon.
(B) The electron can acquire a kinetic energy equal to the energy of the incident photon
(C) The energy of the incident photon equals to the kinetic energy of the electron plus the energy of the scattered photon
(D) The kinetic energy acquired by the electron is the largest when the incident and scattered photons move in opposite directions

60. If the photons were to have a finite mass, then the coulomb potential between two stationary charges separated by a distance r would—
(A) Be strictly zero beyond some distance
(B) Fall off exponentially for large values of r
(C) Fall off as $\frac{1}{r^3}$ for large values of r
(D) Fall off as $\frac{1}{r}$ for large values of r

61. A stationary particle in free space is observed to spontaneously decay into two photons. This implies that—
(A) The particle carries electric charge
(B) The spin of the particle must be greater than or equal to 2
(C) The particle is a boson
(D) The mass of the particle must be greater than or equal to the mass of the hydrogen atom

62. The masses of a hydrogen atom, neutron and $^{238}U_{92}$ are given by 1·0078, 1·0087 and 238·0508 respectively. The binding energy of $^{238}U_{92}$ is, therefore, approximately equal to (taking 1 a.m.u. = 931·64 MeV)
(A) 120 MeV (B) 1500 MeV
(C) 1600 MeV (D) 1800 MeV

63. A bistable multivibrator with a saturation voltage ±5V is shown in the diagram. The positive and negative threshold at the inverting terminal for which the multivibrator will switch to the other state are—



(A) $\pm\frac{5}{11}$ V (B) $\pm\frac{10}{11}$ V
(C) ± 5V (D) ± 11 V

64. An avalanche effect is observed in a diode when—
(A) The forward voltage is less than the breakdown voltage
(B) The forward voltage exceeds the breakdown voltage
(C) The reverse voltage exceeds the breakdown voltage
(D) The diode is heavily doped and forward biased.

65. Which of the given relations between the Boolean variables P and Q is **NOT** correct ? (In the notation used here, P' denotes **NOT** P and Q' denotes **NOT** Q)—

(A) PQ' + PQ = P (B) (PQ') + P' + Q'
(C) PQ' + (P' + Q')' (D) PQ' + Q = P

DATA for Q. 66-67

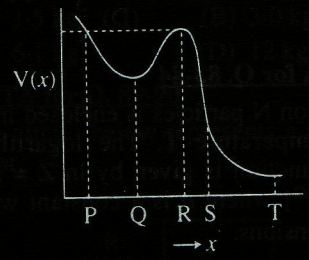
Consider the vector $\overline{V} = \frac{\overline{r}}{r^3}$

66. The surface integral of this vector over the surface of a cube of size a and centered at the origin—
(A) 0 (B) 2π
(C) $2\pi a^3$ (D) 4π

67. Which one of the following is **NOT** correct ?
(A) Value of the line integral of this vector around any closed curve is zero
(B) This vector can be written as the gradient of some scalar function
(C) The line integral of this vector from point P to point Q is independent of the path taken
(D) This vector can represent the magnetic field of some current distribution

DATA for Q. 68-69

Consider the motion of a particle in the potential V(x) shown in the figure—



68. Suppose the particle has a total energy E = V_1 in the figure. Then the speed of the particle is zero where it is at—
(A) Point P (B) Point Q
(C) Point S (D) Point T

69. Which one of the following statements is **NOT** correct about the particle ?
(A) It experiences no force when its position corresponds to the point Q on the curve
(B) It experiences no force when its position corresponds to the point R on the curve
(C) Its speed is the largest when it is at S
(D) It will be in a closed orbit between P and R if E < V_1

DATA for Q. 70-71

A particle of mass m moving with speed v collides with a stationary particle of equal mass. After the collision, both the particles move. Let θ be the angle between the two velocity vectors—

70. If the collision is elastic, then
(A) θ is always less than 90°
(B) θ is always equal to 90°
(C) θ is always greater than 90°
(D) θ cannot be deduced from the given data

71. If the collision is inelastic, then—
(A) θ is always less than 90°
(B) θ is always equal to 90°
(C) θ is always greater than 90°
(D) θ could assume any value in the range 0° to 180°

DATA for Q. 72-73

Consider two conducting plates of infinite extent, one plate at $z = 0$ and the other at $z = L$, both parallel to the xy plane. The vector and scalar potential in the region between the plates is given by—

$$A(\overline{r}, t) = A_0 \, i \cos(kz + \alpha) \cos(k\,ct)$$

$$\phi(\overline{r}, t) = 0$$

72. For this to represent a standing wave in the empty region between the plates—
(A) $k = \frac{\pi}{L}$ and $\alpha = 0$
(B) $k = \frac{2\pi}{L}$ and $\alpha = \frac{\pi}{2}$
(C) $k = \frac{\pi}{2L}$ and $\alpha = \frac{\pi}{2}$
(D) $k = \frac{\pi}{2L}$ and $\alpha = 0$

73. The energy density at $z = 0$ and $t = 0$ is—
(A) 0
(B) $\epsilon_0 c^2 k^2 A_0^2$
(C) $\left(\frac{1}{2}\right) \mu_0 A_0^2 k^2$
(D) $\left(\frac{1}{2}\right) \mu_0 A_0^2 k^2 + \left(\frac{1}{2}\right) \epsilon_0 c^2 k^2 A_0^2$

DATA for Q. 74-75

A particle is located in a three dimensional cubic well of width L with impenetrable walls.

74. The sum of the energies of the third and the fourth levels is—

(A) $\frac{10\pi^2\hbar^2}{mL^2}$ (B) $\frac{10\pi^2\hbar^2}{3mL^2}$

(C) $\frac{11\pi^2\hbar^2}{2mL^2}$ (D) $\frac{15\pi^2\hbar^2}{2mL^2}$

75. The degeneracy of the fourth level is given by—

(A) 1 (B) 2

(C) 3 (D) 4

DATA for Q. 76-77

The normalized wave functions ψ_1 and ψ_2 corresponds to the ground state and the first excited state of a particle in a potential. You are given the information that the operator $\hat{A}$ acts on the wave functions as $\hat{A}\psi_1 = \psi_2$ and $\hat{A}\psi_2 = \psi_1$.

76. The expectation value of A for the state $\psi = \frac{(3\psi_1 + 4\psi_2)}{5}$ is—

(A) – 0·32 (B) 0·0

(C) 0·75 (D) 0·96

77. Which of the following are eigen functions of $\hat{A}^2$?

(A) ψ_1 and ψ_2 (B) ψ_2 and not ψ_1

(C) ψ_1 and not ψ_2 (D) Neither ψ_1 and ψ_2

DATA for Q. 78-79

In the presence of an inhomogeneous weak magnetic field, spectral lines due to transition between two sets of states were observed.

(1) $^5I_5 \rightarrow {}^5H_4$ and (2) $^2D_{5/2} \rightarrow {}^2P_{3/2}$

78. The type of zeeman effect observed in (1) and (2) respectively are—

(A) Normal, normal

(B) Anomalous, anomalous

(C) Anomalous, normal

(D) Normal, anomalous

79. The number of levels into which each of the above four terms split into respectively is—

(A) 6, 4, 10, 8 (B) 4, 6, 10, 12

(C) 11, 9, 6, 4 (D) 9, 5, 12, 10

DATA for Q. 80-82

A system consists of three spin-half particles, the z-components of whose spins S_z (1), S_z (2) and S_z (3) can take value $+\frac{1}{2}$ and $-\frac{1}{2}$. The total spin of the system is $S_z = S_z(1) + S_z(2) + S_z(3)$.

80. The total number of possible micro-states of this system is—

(A) 3 (B) 6

(C) 7 (D) 8

81. The total number of micro-states with $S_z = \frac{1}{2}$ is—

(A) 3 (B) 5

(C) 6 (D) 7

82. Consider an ensemble of systems where each micro-state has equal probability. The ensemble average of S_z is—

(A) $\frac{-1}{2}$ (B) 0

(C) $\frac{1}{2}$ (D) $\frac{3}{2}$

DATA for Q. 83-84

A gas on N particles is enclosed in a volume V at a temperature T. The logarithm of the partition function is given by $\ln Z = N \ln \{(V - bN)(k_B T)^{3/2}\}$ where b is a constant with appropriate dimensions.

83. If P is the pressure of the gas, the equation of state is given by—

(A) $P(V - bN) = Nk_BT$

(B) $P(V - bN) = k_BT$

(C) $P(V - b) = Nk_BT$

(D) $P(V - b) = Nk_BT$

84. The internal energy of the gas is given by—

(A) $U = \left(\frac{1}{2}\right) k_BT$

(B) $U = Nk_BT$

(C) $U = \left(\frac{3}{2}\right) Nk_BT$

(D) $U = 2Nk_BT$

DATA for Q. 85-86

A crystal belongs to a face centered cubic lattice with four atoms in the unit cell. The size of the crystal is 1 cm and its unit cell dimension is 1nm. f is the scattering factor of the atom.

85. The number of atoms in the crystal is—
(A) 2×10^{21} (B) 4×10^{21}
(C) 2×10^{23} (D) 4×10^{24}

86. The structure factors for (010) and (200) reflections respectively are—
(A) $2f$ and zero (B) Zero and $4f$
(C) $2f$ and $4f$ (D) Zero and zero

DATA for Q. 87-88

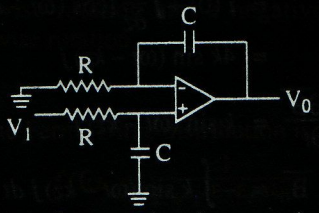
An atomic bomb consisting of ^{235}U explodes and releases an energy of 10^{14} J. It is known that each ^{235}U which undergoes fission releases 3 neutrons and about 200 MeV of energy. Further, only 20% of the ^{235}U atoms in the bomb undergo fission.

87. The total number of neutrons released is about—
(A) $4{\cdot}7 \times 10^{25}$ (B) $9{\cdot}7 \times 10^{25}$
(C) $1{\cdot}9 \times 10^{25}$ (D) $3{\cdot}7 \times 10^{25}$

88. The mass of ^{235}U in the bomb is about—
(A) 1·5 kg (B) 3·0 kg
(C) 6·1 kg (D) 12 kg

DATA for Q. 89-90

The circuit below represents a non-inverting integrator.

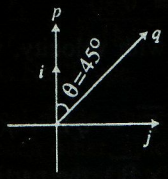


89. For high frequencies ($\omega \to \infty$) the input impendance is—
(A) 0 (B) R
(C) $\dfrac{R}{(1 + \omega RC)}$ (D) ∞

90. Which of the following is **NOT** correct ?
(A) $V_0 = \left(\dfrac{1}{RC}\right) \int V_1 dt$
(B) The voltages at the inverting and non-inverting terminals of the op-amp are nearly equal
(C) The voltage at the non-inverting terminal of the op-amp and the current in the resistor attached to it are $\dfrac{\pi}{2}$ out of phase
(D) The current in the two resistors are in phase

Answers with Hints

1. (A) $p = i$, $q = (i + j)/\sqrt{2}$, diagramatically represented as



$$q = i \sin\theta + j \cos\theta$$

After rotation $\theta = 45°$, q becomes

$$q = i \cdot \frac{1}{\sqrt{2}} + j \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}(i + j)$$

2. (D) The characteristic roots of a Hermitian matrix are all real hence statement (A) is wrong. For unitary matrix, the modulus of each characteristics is unity. Here eigen values are 0, $2 + i$ and $2 - i$

$$\text{Modulus} = |0|^2, |2 + i|^2, |2 - i|^2$$

$$= 0, \sqrt{2^2 + 1^2}, \sqrt{2^2 + 1^2}$$

$$= \neq 1$$

Statement (B) is also wrong.

For inverse of matrix, eigen values of inverse matrix will be $\infty, \dfrac{1}{2+i}, \dfrac{1}{2-i}$, statement (C) is incorrect.

Check for (D), the determinant of matrix is equal to the product of eigen values of a matrix A

$$\therefore \quad |A| = 0 \times (2 + i) \times (2 - i)$$

$$= 0$$

Hence (D) is correct.

3. (A) $\displaystyle I = \int_C \frac{dz}{z^2}$

Poles lies at $z = 0$ (centre) are of order two