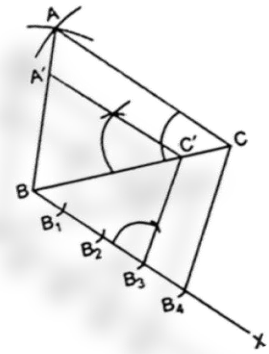
Government of Karnataka

Department of Public Instruction

OFFICE OF THE D.D.P.I.

KOLAR DISTRICT , KOLAR



2021-22

$Mean = \frac{\sum fx}{N}$

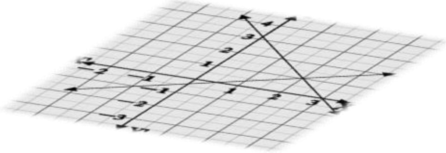
GLANCE ME ONCE

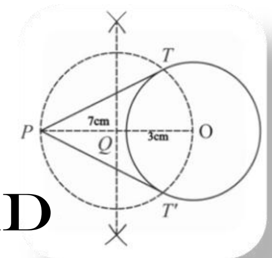
π

$ax^2 + bx + c = 0$

Subject : MATHEMATICS

$tan\theta = \frac{Opp}{Adj}$





CLASS: 10TH STANDARD

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(As per the reduction of **20%** of the Syllabus, **Unit-8,9 and 14** are not considered for the year 2021-22.)

Unit-1: ARITHMETIC PROGRESSION

Multiple Choice Questions

1. The n[th] term of an arithmetic progression with first term 'a' and common difference 'd', is
   (A) $a_n=a+(n-1)d$ (B) $a_n=a-(n-1)d$ (C) $a_n=a-(n+1)d$ (D) $a_n=a+(n+1)d$

2. In an arithmetic progression, if the first term is 'a' and the common difference is 'd', then the sum of its first 'n' terms is
   (A) $S_n=\frac{2}{n}[a+(n-1)d]$ (B) $S_n=2[a+(n-1)d]$
   (C) $S_n=\frac{n}{2}[a+(n-1)d]$ **(D) $S_n=\frac{n}{2}[2a+(n-1)d]$**

3. If a_1, a_2, a_3, a_4, ... are in arithmetic progression, then the common difference is
   (A) a_2-a_1 (B) a_1-a_2 (C) a_2-a_3 (D) a_3-a_4

4. The common difference of the arithmetic progression, 3, 7, 11, 15, …. . is
   (A) -4 (B) 3 **(C) 4** (D) 5

5. An arithmetic progression among the following is
   (A) 3, 5, 7, 10, . . (B) 3,5,6,9, . . (C) -2,-1, 0, 3, . . **(D) 4, 7, 10, 13, . .**

6. If the n[th] term of an arithmetic progression is 3n-2, then its 9[th] term is
   (A) 15 **(B) 25** (C) 29 (D) 11

7. If the terms $4, x, 10$ are in arithmetic progression then the value of 'x' is
   (A) 6 **(B) 7** (C) 8 (D) 9

8. The 25[th] term of an arithmetic progression, 3, 8, 13, 18, …… is
   (A) 25 **(B) 123** (C) 128 (D) 80

9. The sum of the first 30 odd natural numbers is
   (A) 300 (B) 600 (C) 150 **(D) 900**

10. The sum of 5+10+15+20+……………………… to 10 terms is
    (A) 50 (B) 75 (C) 100 **(D) 275**

One Mark Questions

1. Write the formula to find the sum of first 'n' terms of an arithmetic progression with the first term 'a' and the last term a_n.

$$S_n = \frac{n}{2}(a + a_n)$$

2. Write the formula to find the sum of first 'n' terms of an arithmetic progression whose the first term is 'a' and the common difference is 'd'.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

3. If the common difference of an arithmetic progression is 3, then find the value of $a_7 - a_2$.

$$a_7 - a_2 = a + 6d - (a + d)$$
$$= a + 6d - a - d = 5d = 5(3) = 15$$
$$\therefore a_7 - a_2 = \mathbf{15}$$

Two Marks Questions

1. If the first and the last term of an A.P are 4 and 40 respectively. Find the sum of first 20 terms. a=4, l=40, n=20 $S_n=\frac{n}{2}(a+l)$ $S_{20}=\frac{20}{2}(4+40) = 10x44=440$ $\therefore \mathbf{S_{20} = 440}$	2. Find the 12th term of an A.P, 2, 5, 8, 11, . . using formula a=2, d=5–2=3, n=12 $a_n=a+(n-1)d$ $a_{12}=2+(12-1)3$ $= 2+33 = 35$ $\therefore \mathbf{a_{12} = 35}$
3. Find the sum of first 20 terms of the arithmetic series 2+7+12+……………… using the formula. a =2, d=5, n=20 $S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{20}=\frac{20}{2}[2(2)+(20-1)(5)] = 10[4+95]$ $= 10[99] = 990$ $\therefore \mathbf{S_{20} = 990}$	4. Find the 10th term from last (towards the first term) of the A.P, 4, 7, 10, 13, . . . 64. From last term, the A.P becomes 64, . . . 13,10,7,4. a=64 d= 10 – 13= -3 , n=10 $a_n=a+(n-1)d$ $a_{10}=64+(10-1)(-3)$ $= 64 - 27 = 37$ $\therefore \mathbf{a_{10} = 37}$

5. Examine, whether 92 is a term of the A.P., 2, 5, 8, 11, . .

a=2 d= 5 – 2= 3

Let a_n =92

$a_n = a + (n-1)d$

92 = 2 + (n-1)3 = 2+ 3n -3

3n= 93 n= 31

Since n is an whole number, 92 is a term of the A.P 2,5,8,11, . .

Three Marks questions

1. The interior angles of a quadrilateral are in A.P. The smallest among them is 15^0. Find the measure of remaining angles. Let the angles be $a-3d, a-d, a+d, a+3d$. By the angle sum property of quadrilateral $(a-3d)+(a-d)+(a+d)+(a+3d)=360°$ $4a = 3600 \quad => \quad a = 900$ Substituting the value of "a" in a-3d =15 $90-3d = 15 \quad => \quad d = 25$ **The measure of remaining angles 65^0, 115^0 and 165^0**	2. In an A.P., the 3rd term is 3 and the 5th term is −11. Find its 50th term. $a_3=3, \ a_5=-11 \ \ a_{50}=?$ *a+2d= 3* *a+4d= -11* (subtraction) *-2d= 14* => *d= -7* Substituting the value of "d" in a+2d= 3 *a+2(-7)=3* => *a=17* $a_n=a+(n-1)d$ $a_{50}=17+(50-1)(-7)$ $\mathbf{a_{50} = -326}$

Four or Five Marks Questions

1. In an A.P, the sum of 3rd and 6th term is 28 and the sum of 4th and 8th term is 34. Find the A.P.

According to the data

$a_3 + a_6 = 28$

$a+2d+a+5d=28$

$2a+7d=28$ ---------- (1)

$a_4 + a_8 = 34$

$a+3d+a+7d=34$

$2a+10d=34$ ---------- (2)

solving (1) and (2)

$2a+7d=28$

$2a+10d=34$ *(subtraction)*

$-3d = -6 \quad => d=2$

Substituting the value of *"d"* in $a + 5d = 17$

$a = 7$

A.P. is 7, 9, 11, 13, . .

2. A sum of Rs. 1600 is to be used to give ten cash prizes to the students of a school for their overall academic performances. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Here, $n = 10, d = 20.$

Let the amounts of the prizes be

$a, a-20, a-40, \dots\dots.., a-180$

$a + a-20 + a-40 + \dots\dots + a-180 = 1600$

$a = a,\ l = a-180,\ S_n=1600,\quad n=10$

$S_n = \frac{n}{2}[a+l]$

$S_{10} = \frac{10}{2}[a + a - 180]$

$1600 = 5(2a - 180)$

$2a - 180 = 320 \quad => a = 250$

Value of each prize is 250,230,210, ------------70

3. The 4th term of an A.P is 14 and 8th term is 8 less than twice the 5th term. Find the sum of first 25 terms of the A.P.

$a_4=14,\ a_8=2a_5-8,\ S_{25}=?$

$a + 3d = 14 \ ------ \ (1)$

$a + 7d = 2(a + 4d) - 8$

$a + d = 8 \ -------- \ (2)$

solving (1) and (2)

$a+3d=14$

$a+d=8$

$2d=6$

$d=3$

By substituting the value of "d" in $a + d = 8$ we get

$a = 5$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{25} = \frac{25}{2}[2x5 + (25-1)3]$

$= \frac{25}{2}[10+72]$

$\therefore \boldsymbol{S_{25} = 1025}$

4. The sum of three terms of an A.P is 18 and the sum of the squares of extremes is 104. Find the A.P and the sum of first 40 terms.

Let the three terms be $a-d, a, a+d$

$(a-d) + (a) + (a+d) = 18$

$3a = 18$

$\boldsymbol{a = 6}$

$(a-d)^2 + (a+d)^2 = 104$

$a^2+d^2-2ad+a^2+d^2+2ad = 104$

$2a^2+2d^2 = 104$

$a^2+d^2 = 54$

$6^2+d^2=52$

$\boldsymbol{d = \pm 4}$

Let $d = 4$, then t**he A.P is 2, 6, 10, . . .**

Sum of 40 terms is $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{40} = \frac{40}{2}[2x6 + (40-1)4]$

$\therefore \boldsymbol{S_{40} = 3360}$

Unit-2: TRIANGLES

Multiple Choice Questions

1. If two triangles are congruent, then the ratio of their areas is

(A) 1:1 (B) 1:2 (C) 2:1 (D) 2:3

2. In two similar triangles, if the corresponding sides are in the ratio 4:9, then the ratio of their areas is

(A) 81:16 **(B) 16:81** (C) 9:4 (D) 2:3

3. In a ΔABC, if $\angle B = 90^0$ then $AB^2 =$

(A) $AB^2 + BC^2$ **(B) $AC^2 - BC^2$** (C) $\sqrt{AC^2 - BC^2}$ (D) $AC^2 + BC^2$

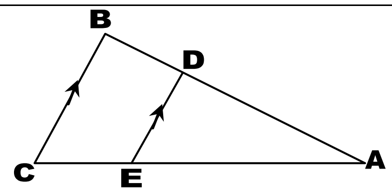
4. In a right angled triangle, if lengths of the perpendicular sides are 3cm and 4cm, then the length of the hypotenuse is

(A) 5cm (B) 9cm (C) 16cm (D) 7cm

5. A pole of height 10m casts a shadow of length 4m on the ground. At the same time the length of the shadow cast by a building of height 50m is

(A) 20m (B) 10m (C) 25m (D) 30m

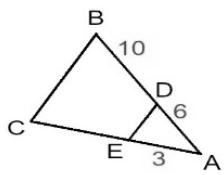
6. In the given figure, DE || BC, then $\frac{AD}{DB} =$



(A) $\frac{BD}{AD}$ (B) $\frac{BC}{DE}$

(C) $\frac{CE}{AE}$ **(D) $\frac{AE}{EC}$**

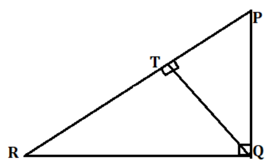
7. In the adjoining figure, in ΔABC, DE||BC, if AD = 6cm, BD = 10cm and AE=3cm then CE is



(A) 5 (B) 3

(C) 6 (D) 10

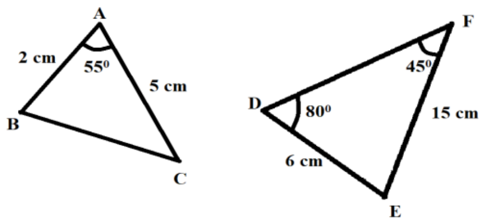
8. In the figure, in ΔPQR, $\angle Q = 90^0$, QT⊥PR then $QT^2 =$



(A) PT.PR (B) QR.TR

(C) PR.TR **(D) PT.RT**

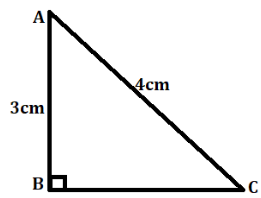
9. In the adjoining figure, similarity criterion used to say that, the triangles are similar is



(A) S.S.S. **(B) S.A.S.**

(C) A.A.A. (D) A.S.A.

10. In triangle ABC $\angle B = 90^0$, AC= 4cm, AB= 3cm, measure of BC is.



(A) 5cm (B) 7cm

(C) $\sqrt{7}$cm (D) 1cm

One Mark questions

1. Write the statement of Pythagoras theorem.

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

2. Write the statement of Basic proportionality (Thales) theorem.

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

3. Each side of a square is 12cm. Find its diagonal.

Diagonal of a square = $\sqrt{2}$ (side of a square)

Diagonal of a square = $\mathbf{12\sqrt{2}\ cm}$.

Two Marks questions

1. $\Delta ABC \sim \Delta DEF$ and their areas be $64cm^2$ and $121cm^2$. If EF=15.4cm then find BC.

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\frac{64}{121} = \frac{BC^2}{15.4^2}$$

$$\frac{8^2}{11^2} = \frac{BC^2}{15.4^2}$$

$$\frac{8}{11} = \frac{BC}{15.4}$$

$$BC = \frac{123.2}{11}$$

$\therefore BC = 11.2cm.$

2. ABC is an isosceles triangle right angled at B. Prove that $AC^2 = 2AB^2$.

In ΔABC, $\angle B = 90^0$

$\therefore \angle A = \angle C$ and $AB = BC$ [Given]

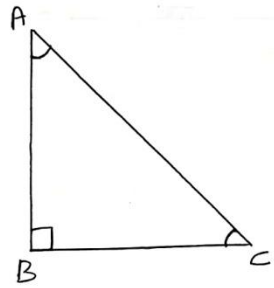
From Pythagoras Theorem, we have,

$AC^2 = AB^2 + BC^2$

$AC^2 = AB^2 + AB^2$

$AC^2 = 2AB^2$

$\therefore$ **Hence the proof.**



3. In the adjoining figure, in ΔABC, $\angle B = 90^0$ and BD⊥AC. Show that BC^2=AC.CD

In ΔBDC and ΔABC

$\angle BDC = \angle ABC$ [BD⊥AC]

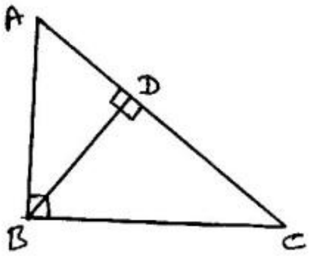
$\angle BCD = \angle ACB$ [Common angle]

$\therefore \Delta BDC \sim \Delta ABC$ [By AA-criterion]

$$\frac{BD}{AB} = \frac{DC}{BC} = \frac{BC}{AC}$$

$BC^2 = AC \times DC$

Hence the proof.



4. Given ΔABC~ΔPQR, such that $\angle A = 40^0$ and $\angle Q = 60^0$. Find the measure of $\angle C$.

$\angle B = \angle Q = 60^0$ [ΔABC~ΔPQR]

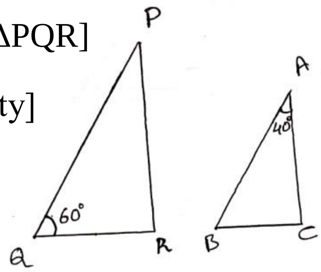
In ΔABC, [angle sum property]

$\angle A + \angle B + \angle C = 180^0$

$40° + 60° + \angle C = 180°$

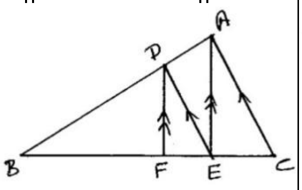
$\angle C = 180° - 100° = 80°$

$\therefore$ **$\angle C = 80^0$**



Three marks questions

1. In the adjoining figure, DE||AC and DF||AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$



In ΔBEA, DF||AE

$\frac{BF}{FE} = \frac{BD}{DA}$ ------(1)

In ΔBCA, DE||AC

$\frac{BE}{EC} = \frac{BD}{DA}$ ------(2)

From equation (1) and (2)

$\frac{BF}{FE} = \frac{BE}{EC}$

∴ **Hence the proof.**

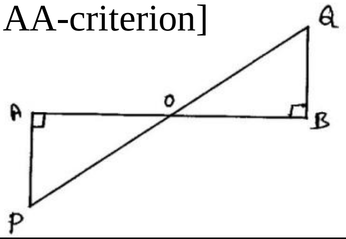
2. In the adjoining figure, AP and BQ are perpendiculars on AB. prove that $\frac{AO}{PO} = \frac{BO}{QO}$.

In ΔAOP and ΔBOQ

$\angle OAP = \angle OBQ = 90^0$ [Given]

$\angle AOP = \angle BOQ$ [Vertically opposite angles]

∴ ΔAOP~ΔBOQ [By AA-criterion]



$\frac{AO}{BO} = \frac{PO}{QO}$

∴ $\frac{AO}{PO} = \frac{BO}{QO}$

3. In the adjoining figure, in a trapezium ABCD, AB||CD and AB=2CD. Find the ratio of the areas of ΔAOB and ΔCOD.

In ΔAOB *and* ΔCOD we have,

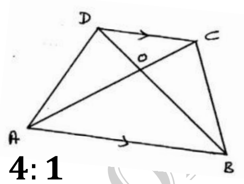
$\angle AOB = \angle COD$ [Vertically opposite angles]

$\angle OAB = \angle OCD$ [Alternate angles, AB||DC]

$\Delta AOB \sim \Delta COD$ [By AA-criterion]

$\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{DC^2}$

$\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{(2DC)^2}{(DC)^2} = \frac{4}{1}$



∴ $\boldsymbol{ar(\Delta AOB) : ar(\Delta COD) = 4 : 1}$

4. A ladder of 15m long reaches a window of a building 12m above the ground. Find the distance of the foot of the ladder from the base of the wall.

Let AC be the ladder and AB be the wall with the window at A.

Also, AC=15m and AB=12m

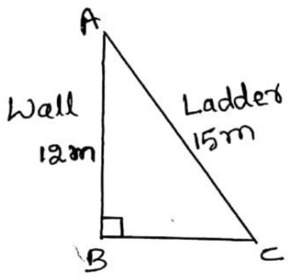
From Pythagoras Theorem,

$BC^2 = AC^2 - AB^2$

$BC^2 = 15^2 - 12^2$

$BC^2 = 81$ => $\boldsymbol{BC = 9}$

Thus, the distance of the foot of the ladder from the base of the wall is 9m.



5. A vertical pole of height 12m casts a shadow of length 8m on the plane ground. At the same time a tower casts a shadow of length 40m on the plane ground. Find the height of the tower.

Length of the vertical pole = AB = 12m

Length of the shadow casts by the pole = BC = 8m

Length of the shadow casts by the tower = EF = 40m

Let the height of the tower = h m

In ΔABC *and* ΔDEF

$\angle B = \angle E = 90^0$

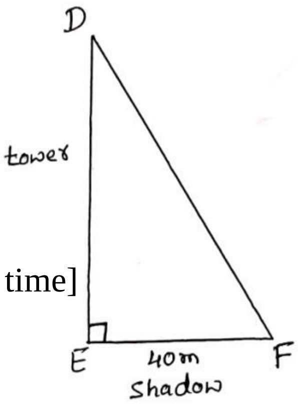
$\angle C = \angle F$ [The angles made by sun at the same time]

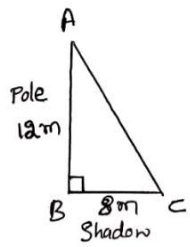
∴ $\Delta ABC \sim \Delta DEF$ [By AA-criterion of similarity]

$\frac{AB}{DE} = \frac{BC}{EF}$

$\frac{12}{h} = \frac{8}{40}$ $\frac{12 \times 40}{8} = h$

$h = 60$ ∴ **Height of the tower = 60m.**





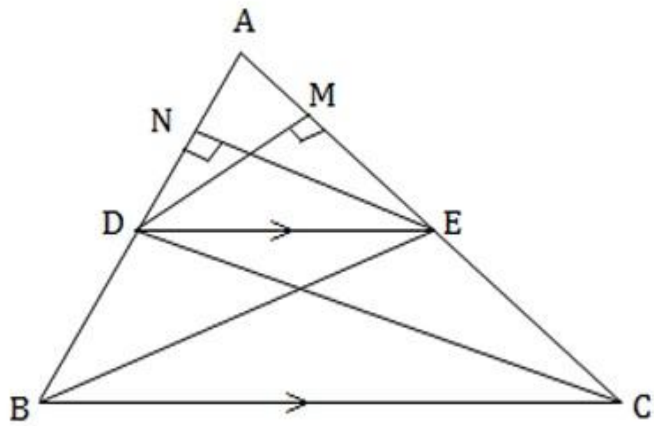
Four or Five marks questions:-

1. State and prove the Basic proportionality (Thales') theorem.

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Data : In ΔABC $DE \parallel BC$.

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$



Construction : Draw $DM \perp AC$ and $EN \perp AB$. Join BE and CD.

Proof :

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} \quad (\because \text{Area of } \Delta = \frac{1}{2} \times \text{base} \times \text{height})$$

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{DB} \quad \text{---------->}(1)$$

$$\frac{ar(\Delta ADE)}{ar(\Delta CED)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} \quad (\because \text{Area of } \Delta = \frac{1}{2} \times \text{base} \times \text{height})$$

$$\frac{ar(\Delta ADE)}{ar(\Delta CED)} = \frac{AE}{EC} \quad \text{---------->}(2)$$

But ΔBDE and ΔCED are standing on the same base DE and between $DE \parallel BC$.

$ar(\Delta BDE) = ar(\Delta CED)$ ----------> (3)

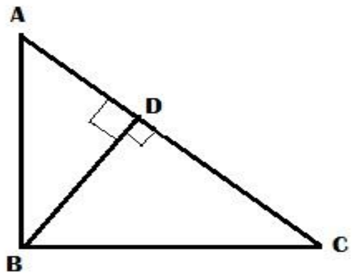
$\therefore$ from equations (1), (2) and (3)

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence the proof.

2. State and prove the Pythagoras theorem.

" In a right angled triangle , the square on the hypotenuse is equal to the sum of the squares on other two sides ".



Data : ΔABC is a right triangle and $\angle B = 90^\circ$

To Prove : $AC^2 = AB^2 + BC^2$

Construction : Draw $BD \perp AC$

Proof : In ΔADB and ΔABC

$\angle D = \angle B = 90^\circ$ ($\because$ Data and Construction)

$\angle A = \angle A$ ($\because$ Common angle)

$\Delta ADB \sim \Delta ABC$ ($\because$ AAA Similarity Criterion)

$\therefore \frac{AD}{AB} = \frac{AB}{AC}$ ($\because$ Proportional sides)

$AC.AD = AB^2$ ------> (1)

Similarly

In ΔBDC and ΔABC

$\angle D = \angle B = 90^\circ$ ($\because$ Data and Construction)

$\angle C = \angle C$ ($\because$ Common angle)

$\Delta BDC \sim \Delta ABC$ ($\because$AAA Similarity Criterion)

$\therefore \frac{DC}{BC} = \frac{BC}{AC}$ ($\because$ Proportional sides)

$AC.DC = BC^2$ ------> (2)

$AC.AD + AC.DC = AB^2 + BC^2$ [$\because$ By adding (1) and (2)]

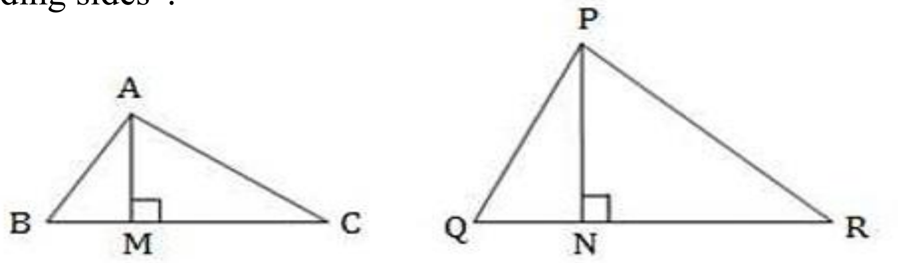
$AC(AD + DC) = AB^2 + BC^2$

$AC \times AC = AB^2 + BC^2$ ($\because$ from fig. $AD + DC = AC$)

$AC^2 = AB^2 + BC^2$

Hence the proof.

3. Prove that "The ratio of the areas of two similar triangles is equal to the ratio of their corresponding sides".



Data : $\Delta ABC \sim \Delta PQR$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

To Prove : $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

Construction : Draw $AM \perp BC$ and $PN \perp QR$.

Proof : $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN}$ ($\because$ Area of $\Delta = \frac{1}{2} \times$ base $\times$ height)

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC \times AM}{QR \times PN}$ ----------> (1)

In ΔABM and ΔPQN

$\angle B = \angle Q$ ($\because \Delta ABC \sim \Delta PQR$)

$\angle M = \angle N = 90^\circ$ ($\because$Construction)

$\therefore \Delta ABM \sim \Delta PQN$ ($\because$AA Similarity criterion)

$\therefore \frac{AM}{PN} = \frac{AB}{PQ}$ --------> (2)

But $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ------> (3) ($\because$Data)

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ}$ ($\because$substituting eqs.(2) and (3) in (1))

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2$

Now from eq.(3)

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

Hence the proof.

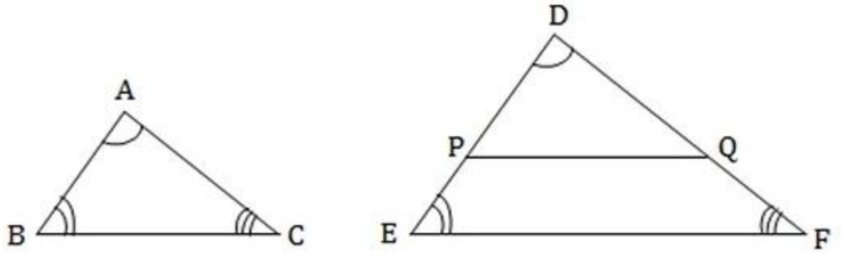
4. Prove that "If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (proportion) and hence the two triangles are similar".

Data: $In\ \Delta ABC\ and\ \Delta DEF$

$\angle A = \angle D$

$\angle B = \angle E$

$\angle C = \angle F$



To prove: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Construction: Mark points P and Q on DE and DF such that DP=AB and DQ=AC. Join PQ.

Proof: $In\ \Delta ABC\ and\ \Delta DPQ$

$\angle A = \angle D$ [Data]

AB=DP [Construction]

AC=DQ [Construction]

$\therefore \Delta ABC \cong \Delta DPQ$ [SAS postulate]

BC=PQ [By CPCT] -----(1)

$\angle B = \angle P$ [By CPCT]

$\angle B = \angle E$ [Data]

$\angle P = \angle E$ [Axiom 1]

$PQ || EF$

$\frac{DP}{DE} = \frac{PQ}{EF} = \frac{DQ}{DF}$ [Corollary of BPT]

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ [From (1) and construction]

$\therefore$ **Hence the proof.**

Unit-3:PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Multiple Choice Questions

1. A pair of linear equations $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ is said to be inconsistent if

(A) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ **(B)** $\boldsymbol{\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}}$ (C) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (D) $\frac{a_1}{a_2} = \frac{c_2}{c_1}$

2. If two lines representing the pair of linear equations $a_1x + b_1y + c_1 = 0$ *and* $a_2x + b_2y + c_2 = 0$ intersect at a point, then the correct relation among the following is

(A) $\boldsymbol{\frac{a_1}{a_2} \neq \frac{b_1}{b_2}}$ (B) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (C) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (D) $\frac{a_1}{a_2} = \frac{b_2}{b_1}$

3. The lines representing the pair of linear equations $2x + 3y - 9 = 0$ *and* $4x + 6y - 18 = 0$ are

(A) intersecting lines (B) perpendicular lines
(C) parallel lines **(D) coincident lines**

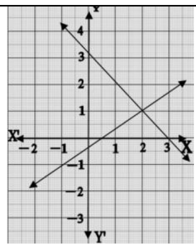
4. The Pair of linear equations $x + 2y = 6$ *and* $3x - 6y = 18$ have

(A) No solution (B) Infinitely many solutions
(C) Exactly one solution (D) Two solutions

One Mark Questions

1. The graph represents the pair of linear equations in $'x'$ and $'y'$. Write the solution for this pair of equations.

Ans : $\boldsymbol{x = 2 \text{ and } y = 1}$



2. Write the general form of pair of linear equations in two variables *'x' and 'y'*

$\boldsymbol{a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0}$, where a_1, b_1, c_1, a_2, b_2 and c_2 are all real numbers.

3. In the pair of linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then write the number of solutions these equations have. **Ans: Infinitely many solutions.**

Two Marks Questions

1. Solve the following pair of linear equations by any of the algebraic method: $x + y = 8$ *and* $2x - y = 7$

$$x+y=8$$
$$2x-y=7$$
$$3x=15 \quad \text{(addition)}$$
$$x=5$$

Substituting the value of x in $x + y = 8$

$5 + y = 8$

$y = 3$

$\therefore \boldsymbol{x = 5 \text{ and } y = 3}$

2. Solve by elimination method: $x + y = 5$ and $2x + 3y = 12$

$x + y = 5$ ----- (1)

$2x + 3y = 12$ ----- (2)

Multiplying the equation (1) by 2 we get

$2x + 2y = 10$ ----- (3)

Solving equation (2) and (3)

$$2x+3y=12$$
$$2x+2y=10 \quad \text{(subtraction)}$$
$$y=2$$

Substitute the value of y in $x + y = 5$, we get $x = 3$

$\therefore \boldsymbol{x = 3 \text{ and } y = 2}$

Three or Four Marks Questions.

1. The cost of 5 oranges and 3 apples is Rs.35 and the cost of 2 oranges and 4 apples is Rs. 28. Find the cost of an orange and an apple.

Let the cost of an orange and an apple be x *and* y respectively. => $5x + 3y = 35$ and

$2x + 4y = 28$

$(5x + 3y = 35)$ x 4 => $20x + 12y = 140$

$(2x + 4y = 28)$ x 3 => $6x + 12y = 84$

Multiply the equation (1) by 4 and equation (2) by 3 we get,

$$\begin{array}{r} 20x+12y=140 \\ 6x+12y=84 \\ \hline 14x=56 \\ x=4 \end{array}$$ (subtraction)

Substituting the value of x in $5x + 3y = 35$

$$5x + 3y = 35$$
$$20 + 3y = 35$$
$$3y = 15$$
$$y = 5$$

$\therefore$ ***The cost of an orange is* Rs. 4**
***and That of an apple is* Rs. 5**.

2. Solve: $141x + 93y = 189$ and $93x + 141y = 45$.

$141x + 93y = 189$ ------ (1)

$93x + 141y = 45$ ------ (2)

By adding (1) and (2) we get

$$\begin{array}{r} 141x+93y=189 \\ 93x+141y=45 \\ \hline 234x+234y=234 \\ x+y=1 \end{array}$$ $x+y = 1$ ----- (3)

By subtracting (1) by (2) we get

$$\begin{array}{r} 141x+93y=189 \\ 93x+141y=45 \\ \hline 48x-48y=144 \\ x-y=3 \end{array}$$ $x-y = 3$ ---- (4)

Solving (3) and (4)

$$\begin{array}{r} x+y=1 \\ x-y=3 \\ \hline 2x=4 \\ x=2 \end{array}$$

By substituting the value of x in (3) or (4) we get $y = -1$

$\therefore$ $\boldsymbol{x = 2 \text{ and } y = -1}$

3. The sum of two numbers is 50 and their difference is 22, find the numbers.

Let the two numbers be x *and* y.

According to the data

$x + y = 50$ ------ (1)

$x - y = 22$ ------ (2)

Solving (1) and (2)

$$\begin{array}{r} x+y=50 \\ x-y=22 \\ \hline 2x=72 \\ x=36 \end{array}$$ (addition)

By Substituting the value of x in (1) we get

$$x + y = 50$$
$$36 + y = 50$$
$$y = 14$$

$\therefore$ ***The two numbers are* 36 *and* 14**.

4. If twice the age of the son is added to age of the father the sum is 56. But if twice the age of the father is added to the age of the son, then the sum is 82. Find the ages of the father and the son.

Let the age of son be *'x'* years and the age of father be *'y'* years

$2x + y = 56$ ------ (1)

$x + 2y = 82$ ------ (2)

Multiply the equation (2) by 2 we get

$2x + 4y = 164$ ---- (3)

Solving (1) and (3) $$\begin{array}{r} 2x+4y=164 \\ 2x+y=56 \\ \hline 3y=108 \\ y=36 \end{array}$$

By substituting the value of y in (1) we get x=10

$\therefore$ **The age of the son and the age of father ar**

10 years and 36 years respectively.

5) 4 men and 6 boys can finish a piece of work in 5 days, while 3 men and 4 boys can finish the same work in 7 days. Find the time taken by one man alone or then by 1 boy alone.

Number of days taken by 1 man = x days.
Number of days taken by 1 boy = y days.
Work done by 1 man in 1 day = $\frac{1}{x}$

Work done by 1 boy in 1 day = $\frac{1}{y}$

$$\frac{4}{x}+\frac{6}{y}=\frac{1}{5} \text{ ----- } (1)\ ;\ \frac{3}{x}+\frac{4}{y}=\frac{1}{7} \text{ ----- } (2)$$

Take $\frac{1}{x} = a$, $\frac{1}{y} = b$, then (1) and (2) becomes

$4a + 6b = \frac{1}{5}$ $\quad 20a + 30b = 1$ ----- (3)

$3a + 4b = \frac{1}{7}$ $\quad 21a + 28b = 1$ ----- (4)

By solving (3) and (4) we get $x = 35,\ y = 70$

∴ **One man will take 35 days and One boy will take 70 days to finish the work.**

6) Ritu can row, down-stream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

Let the speed of Ritu in still water be = x km/h.
Speed of current be y km/h.
The speed of downstream = $(x + y)$ km/h.
The speed of upstream = $(x - y)$ km/h

$$\text{Time} = \frac{\text{distance}}{\text{speed}}$$

$t_1 = \frac{20}{(x+y)} = 2 \quad 2x + 2y = 20$ ---(1)

$t_2 = \frac{4}{(x-y)} = 2 \Rightarrow 2x - 2y = 4$ --- (2)

$$\begin{array}{r} 2x + 2y = 20 \\ 2x - 2y = 4 \\ \hline 4x \quad = 24 \end{array}$$

$\Rightarrow x = 6$

Considering $2x + 2y = 20$
$2(6) + 2y = 20 \quad \Rightarrow \quad y = 4$

∴ **The speed of Ritu in still water = 6 km/hr and the speed of the stream = 4 km/hr.**

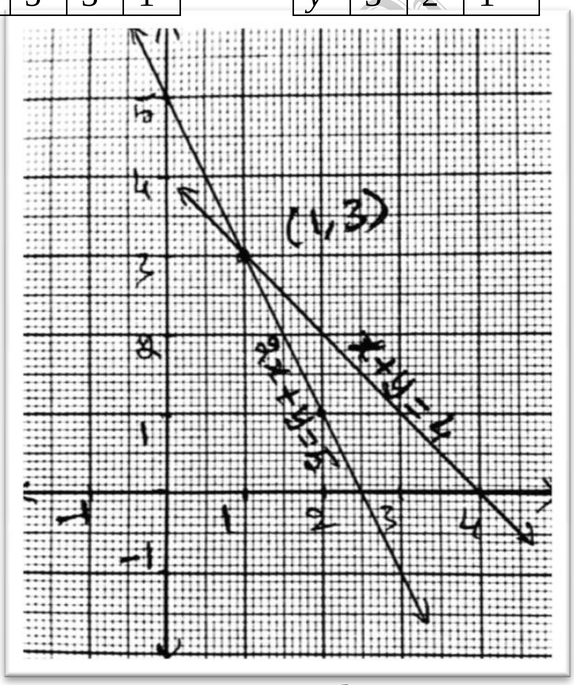
7) Solve graphically:
$2x + y = 5 \quad and \quad x + y = 4.$

$2x + y = 5$
$y = 5 - 2x$

x	0	1	2
y	5	3	1

$x + y = 4$
$y = 4 - x$

x	1	2	3
y	3	2	1



∴ **x = 1 and y = 3**

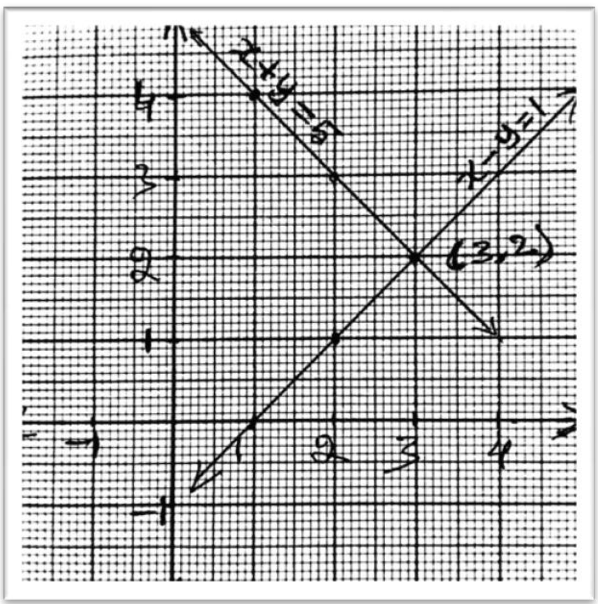
8) Solve graphically:
$x + y = 5$ and $x - y = 1$

$x + y = 5$
$y = 5 - x$

x	1	2	3
y	4	3	2

$x - y = 1$
$y = x - 1$

x	1	2	3
y	0	1	2



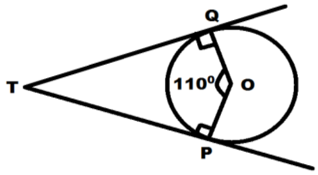
∴ **x = 3 and y = 2**

UNIT-4 : CIRCLES

Multiple Choice Questions

1 In the figure, TP and TQ are the tangents drawn to a circle with centre O. If $\angle POQ = 110^o$, then the value of $\angle PTQ$ is

A. 70^o B. 80^o C. 60^o D. 140^o



2 The tangents drawn at the ends of a diameter of a circle are

A. perpendicular to each other **B. parallel to each other** C. equal D. Not equal

3 A straight line which intersects a circle at two distinct points is

A. tangent B. chord **C. secant** D. diameter

4 If the angle between the two tangents to a circle is 40^0 , then the angle between the radii is

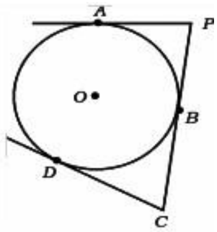
A. 90^0 B. 100^0 **C. 140^0** D. 180^0

5 Distance between two parallel tangents of a circle of radius 3.5cm is

A. 3.5cm **B. 7cm** C. 10cm D. 14cm.

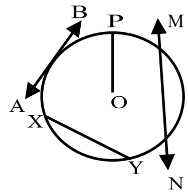
6 In the given figure PA, PC and CD are the tangents to a circle with centre O. If CD = 5 cm and AP = 3 cm, then length of the tangent PC is

A. 8 cm B. 5 cm C. 3 cm D. 2 cm



7 In the figure, Chord of the circle with centre 'O' is

A. XY B. OP C. MN D. AB



8 A tangent of length 8 cm is drawn from an external point 'A' to a circle of radius 6 cm . Then the distance between 'A' and the centre of the circle is

A. 12 cm B. 5 cm **C. 10 cm** D. 14 cm

9 Maximum number of tangents drawn to a circle from an external point is

A. 2 B. 3 C. 4 D. 5

One Mark Questions

1 What is the measure of the angle between radius and tangent at the point of contact? **Ans: 90^o**

2 Define the Secant of a Circle.

A line that intersects a circle at two points is called a Secant.

3 Define the tangent of a circle.

A line that touches a circle at only one point is called a Tangent.

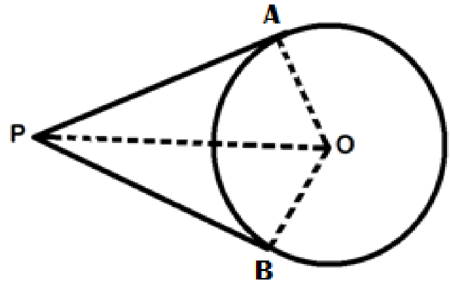
4 Define Point of contact of a circle.

The common point of the tangent and the circle is called the Point of contact.

Three Marks Questions

1 Prove that "the length of tangents drawn from an external point to a circle are equal."

Given **:** 'O' is the centre of the circle, 'P' is an external point. AP and BP are the tangents



To Prove : **A**P = BP

Construction : Join OA, OB and OP.

Proof :

In $\Delta OQP\ and\ \Delta ORP$

∠OAP = ∠OBP = 90^0 [Theorem 4.1]

OP = OP [Common side]

OA = OB [Radii of same circle]

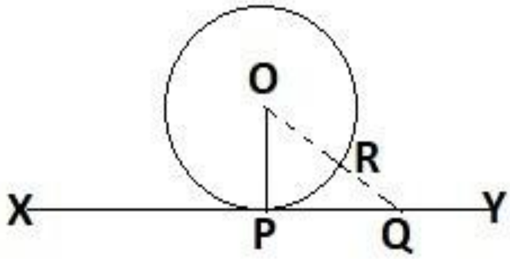
$\Delta OAP \cong \Delta OBP$ [RHS Postulate]

AP=BP [CPCT]

Hence proved.

2 Prove that " the tangent at any point of a circle is perpendicular to the radius through the point of contact."

Given **:** XY is the tangent at P to the circle with centre



To Prove : OP _|_ XY

Construction : Mark Any point 'Q' on XY, join OQ and it cuts the circle at R

Proof : OR < OQ

OR = OP (Radii of the same circle)

∴ OP < OQ

This holds good for all the points on XY

∴ OP is the least distance

=> OP _|_ XY

UNIT-5 : AREAS RELATED TO CIRCLES

Multiple Choice Questions

1 Area of Quadrant of a circle with radius *'r'* is

A. $\frac{\pi r^2}{2}$ **B.** $\boldsymbol{\frac{\pi r^2}{4}}$ C. πr D. $\frac{\pi r}{2}$

2 If the radius of a semicircle is 7cm, the length of its arc is

A. $11cm$ B. $44cm$ **C. $\boldsymbol{22cm}$** D. $14cm$

3 Length of the arc of a sector with radius 9 cm and the angle 120^0 is

A. $2\pi\ cm$ B. $3\pi\ cm$ **C. $\boldsymbol{6\pi cm}$** D. $9\pi\ cm$

4 If the angle of a sector is *'P'* (in degrees) and radius is *'R'* then its area is

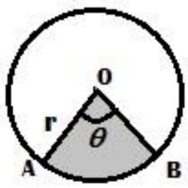
A. $\frac{P}{180}$ x $2\pi R$ B. $\frac{P}{180}\ x\pi R^2$ C. $\frac{P}{360}$ x $2\pi R$ **D. $\boldsymbol{\frac{P}{720}\ x\ 2\pi R^2}$**

5 If the ratio of circumference of two circles is 4 : 5 then the ratio of their areas is

A. 4:5 **B. 16:25** C. 64:125 D. 5:4

One Mark Questions

1 Write the formula to find the area of the shaded region in the given figure.



$\frac{\theta}{360^0}$ x πr^2

2 Define the segment of a circle.

A segment is a region covered by a chord and a corresponding arc.

3 What is meant by a sector of the circle?

The area bounded by two radii and the corresponding arc of a circle is called the Sector.

4 If the diameter of a semicircle is 14cm, then find its perimeter [use $\pi = \frac{22}{7}$]

Perimeter of the semicircle $= \pi r + d$

$= \frac{22}{7}$ x $\frac{14}{2} + 14$

∴ **Perimeter of the semicircle $=$ 36 cm**

5 If the area of a circle and the perimeter are numerically equal, then find the radius of that circle.

$\pi r^2 = 2\pi r$

$\therefore r = 2\ units$

Two Marks Questions (Use $\pi = \frac{22}{7}$ unless given)

1 In a circle of radius 21 cm an arc subtends an angle 60^0 at the centre of the circle. Find the length of the arc formed in the circle.

Length of the arc $= \frac{\theta}{360^o} \text{ x } 2\pi r$

$= \frac{60^o}{360^o} \text{ x } 2 \text{ x } \frac{22}{7} \text{ x } 21$

∴ **Length of the arc = 22 cm**

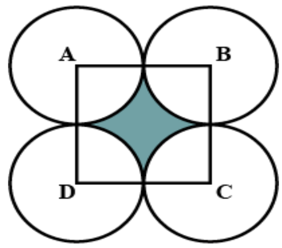
2 In a circle of radius 21 cm and arc subtends angle 60^0 at the centre of the circle, find the area of sector formed in the circle.

Area of the sector $= \frac{\theta}{360^o} \text{ X } \pi r^2$

$= \frac{60^o}{360^o} \text{ x } \frac{22}{7} \text{ x } 21 \text{ x } 21$

∴ **Area of the sector = 231 sq.cm**

3 In the figure ABCD is a square of side 14 cm . With centre A, B , C & D four circles are drawn such that each circle touch externally two of the remaining three circles. Find the Area of the shaded region.



Radius of each quadrant = $\frac{14}{2}$ = 7 cm

Area of the shaded region = Area of the square – Area of 4 Quadrants.

Area of the shaded region $= 14^2 - 4 \text{ x } \frac{\pi r^2}{4}$

$= 196 - 4 \text{ x } \frac{22}{7} \text{ x } \frac{7 \text{ x } 7}{4}$

$= 196 - 154$

∴ **Area of the shaded region = $42\ cm^2$**

4 A drain cover is made from a square metal plate of side 40 cm having 441 holes of diameter 1 cm each drilled in it. Find the area of the remaining square plate.

Area of each hole $= \pi r^2$

$= \frac{22}{7} \text{ x } \left(\frac{1}{2}\right)^2$

$= \frac{11}{14}\ cm^2$

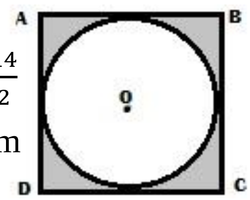
Area of 441 holes = $441 \text{ x } \frac{11}{14} = 346.5$ cm^2

Area of Square metal plate = 40^2 =1600 cm^2

Area of remaining square plate = 1600 – 346.5

= 1253.5 cm^2

5 In the figure, a circle is circumscribed in a square ABCD. If each side of the square is 14cm find the area of shaded region



Radius of the circle; $r = \frac{14}{2}$

r = 7cm

Ar(shaded region) = Ar(Square) – Ar(Circle)

$= (side)^2 - \pi r^2$

$= 14^2 - \frac{22}{7} \text{ x } 7 \text{ x } 7$

$= 196 - 154$

∴ **Area of the shaded region = 42 cm^2**

Three Marks Questions (Use $\pi = \frac{22}{7}$ unless given)

1 Find the area of a quadrant of a circle, where the circumference of circle is 44cm.

$2\pi r = \text{Circumference}$

$2\pi r = 44 \text{ cm}$

$2 \text{ x } \frac{22}{7} \text{ x r} = 44$

$r = \frac{44 \text{ x} 7}{22 \text{ x } 2} \quad => \quad r = 7 \text{ cm}$

Area of quadrant $= \frac{1}{4} \text{ x } \pi r^2$

$= \frac{1}{4} \text{ x } \frac{22}{7} \text{ x } 7 \text{ x } 7$

$= \frac{77}{2}$

$\therefore$ **Area of quadrant $= \mathbf{38.5 \; cm^2}$**

2 Area of a sector of a circle of radius 14 cm is 154 cm^2. Find the length of the corresponding arc of the sector.

Given, $r = 14 \; cm$ Area of sector $= 154 \; cm^2$

$\frac{\theta}{360^o} \text{ x } \pi r^2 = 154$

$\frac{\theta}{360^o} \text{ x } \frac{22}{7} \text{ x } 14 \text{ x } 14 = 154$

$\frac{\theta}{360^o} \text{ x } 22 \text{ x } 2 \text{ x } 14 = 154$

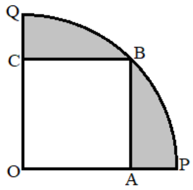
$\theta = \frac{154 \text{ x } 360}{22 \text{ x } 2 \text{ x } 14} \quad => \quad \theta = 90^0$

Length of an arc $= \frac{\theta}{360^o} \text{ x } 2\pi r$

$= \frac{90^o}{360^o} \text{ x } 2 \text{ x } \frac{22}{7} \text{ x } 14$

$\therefore$ **Length of the arc $= \mathbf{22 \; cm}$**

3 OABC is a square inscribed in a quadrant OPBQ. If OA = 20 cm. (use $\pi = 3.14$)



Ar(Square) $= 20^2$

$= 400 cm^2$

Radius of the quadrant; r= OB

$r = OB = \sqrt{OA^2 + AB^2}$

$= \sqrt{20^2 + 20^2} \; => \; r = 20\sqrt{2} \; cm$

Ar(Quadrant) $= \frac{\pi r^2}{4}$

$= \frac{3.14 \text{ x } (20\sqrt{2})^2}{4}$

$= \frac{3.14 \; x \; 400 \; x \; 4}{4}$

Ar(Quadrant) $= 628 \; cm^2$

Ar(Shaded region) = $\boldsymbol{Ar(Quadrant) - Ar(Square)}$

$= 628 - 400$

$\therefore$ ***Area of the shaded region*** $\boldsymbol{= 228 \; cm^2}$

4 The radii drawn from the end points of a chord of a circle subtend an angle of 120^0 at the centre. If the radius of the circle is 12 cm Find the area of the corresponding segment of the circle. (use $\pi = 3.14$ and $\sqrt{3} = 1.73$).

Radius $r = 12 \; cm$, $\theta = 120^o \; => \; \frac{\theta}{2} = 60^o$

$Ar(Segment) = r^2 \left(\frac{\pi\theta}{360^o} - sin\frac{\theta}{2} cos\frac{\theta}{2}\right)$

$= (12)^2 \left(\frac{3.14 \text{ x } 120^o}{360^o} - sin60^o \text{ x } cos60^o\right)$

$= 144 \left(\frac{3.14}{3} - \frac{\sqrt{3}}{2} \; x \; \frac{1}{2}\right)$

$= 144 \left(\frac{3.14}{3} - \frac{1.73}{4}\right)$

$= 144 \left(\frac{12.56 - 5.19}{12}\right)$

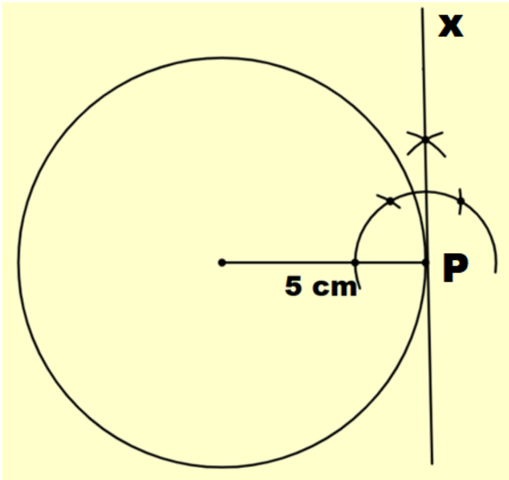
$= 12(7.37)$

$\therefore$ ***Area of the segment*** $\boldsymbol{= 88.44 \; cm^2}$

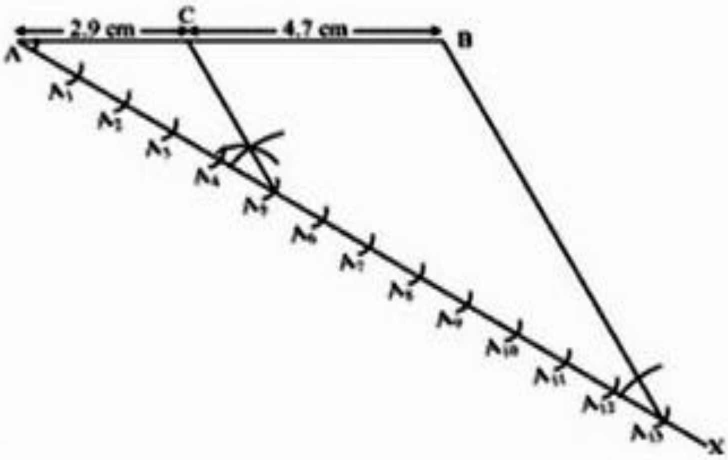
UNIT-6 : CONSTRUCTIONS

Two Marks Questions

1 Construct a tangent at any point P on a circle of radius 5 cm.

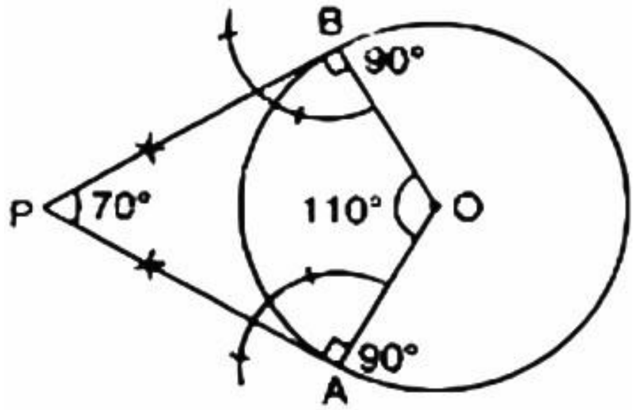


2 Draw a line segment of length 7.6 m and divide the line segment in the ratio 8 : 5.

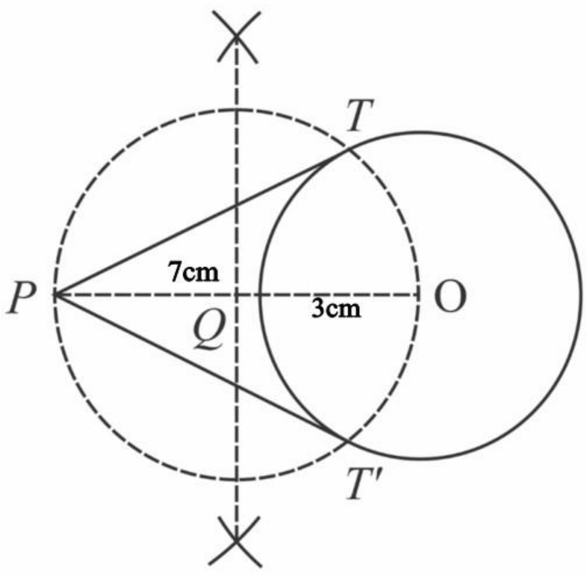


Three Marks Questions

1 Construct a pair of tangents to a circle of radius 3cm which are inclined to each other at an angle of 70^0.

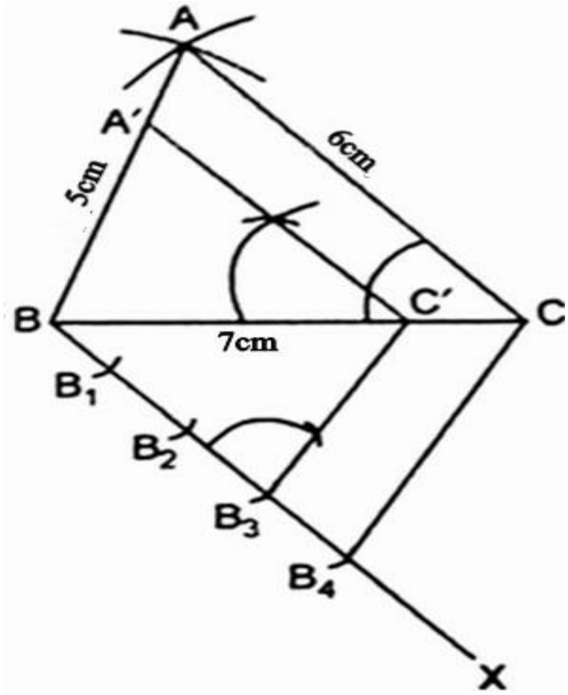


2 Draw a circle of radius 3cm and construct a pair of tangents to it from a point 7cm away from its centre.

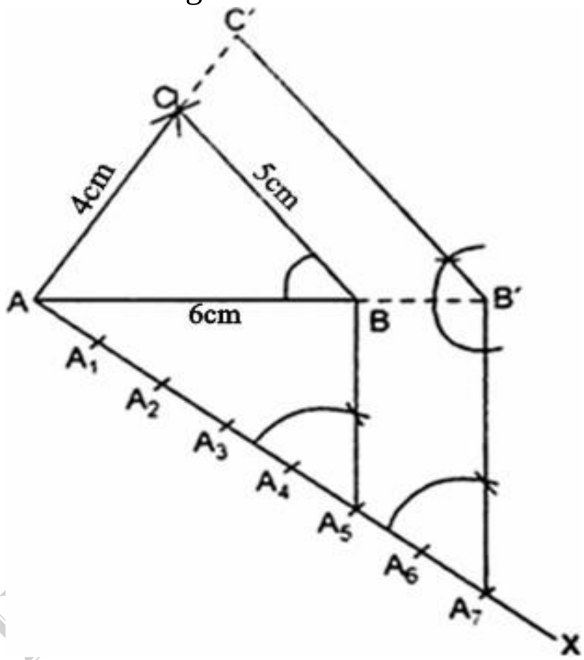


Four Marks Questions

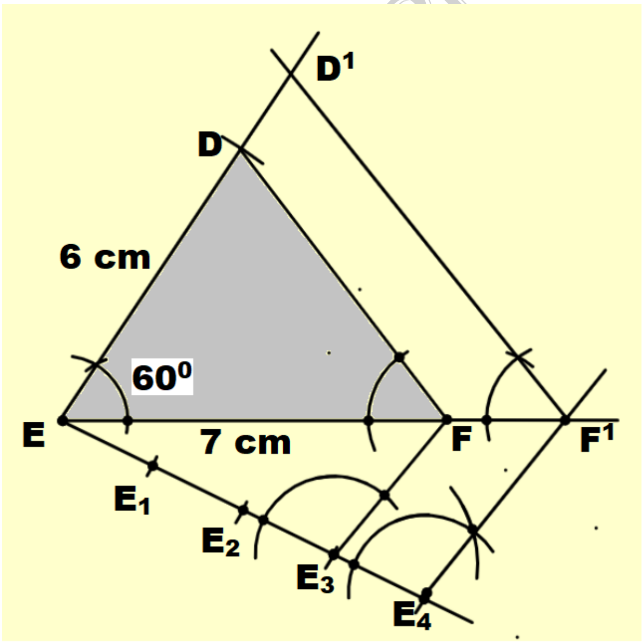
1 Construct a triangle with sides 5cm, 6cm and 7cm, then a triangle similar to it whose sides are $\frac{3}{4}$ of the corresponding sides of the first triangle.



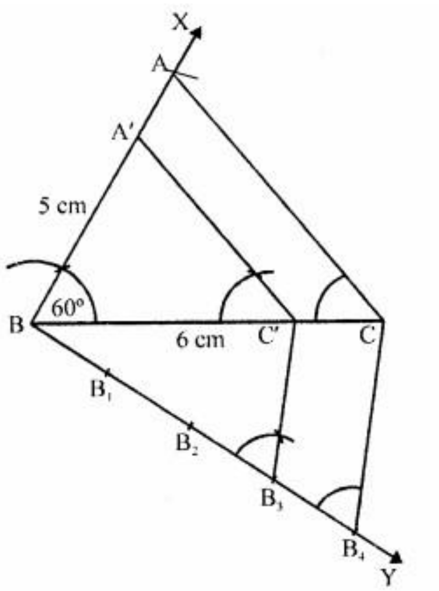
2 Construct a triangle of sides 4cm, 5cm and 6cm and then a triangle similar to it whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.



1 Draw a triangle DEF with EF=7 cm, $\angle DEF=60^0$ and DE=6 cm then construct a triangle whose sides are $\frac{4}{3}$ of the corresponding sides of the triangle DEF.



2 Construct a triangle similar to triangle ABC in which AB =4 cm, $\angle ABC=60^o$ and BC= 6 cm such that each side of the new triangle is $\frac{3}{4}$ of the corresponding sides of the triangle ABC.



UNIT-7 : COORDINATE GEOMETRY

Multiple Choice Questions

1	The co-ordinates of the mid-point of the line segment joining the points (2,0) and (6,0) is A. (2,4) B. (2,6) **C. (4,0)** D. (0,4)
2	The distance of point (4, -3) from the origin A. 4 $units$ **B. 5 $units$** C. 9 $units$ D. 16 $units$
3	The perpendicular distance of the point P (2, 3) from the x-axis is A. 1 unit B. 2 units **C. 3 units** D. 5 units
4	The Coordinates of the origin is A. (1,1) **B. (0,0)** C. (0,1) D. (1,0)
5	The coordinates of a point P on the x-axis are of the form **A. $(x, 0)$** B. $(0, y)$ C. $(y, 0)$ D. $(0, x)$
6	Area of the triangle with vertices P(0, 6), Q(0,2) and R(2, 0) is **A. 4 square units** B. 0 C. 8 square units D. 6 square units
7	If M(6, 3) is the midpoint of line joining P(-2, 5) and Q(8, y) then y = A. 4 B. 3 C. 2 **D. 1**
8	Distance of the point $P(x, y)$ from the origin is A. $\sqrt{(x-y)^2}$ B) $\sqrt{x^2 - y^2}$ **C) $\sqrt{x^2 + y^2}$** D. $\sqrt{(x+y)^2}$

One Mark Questions

1	What is the value of the *y*-coordinate of a point on *x*-axis? **Ans: 0**
2	Write the coordinates of the origin. OR Write the coordinates of the point of intersection of *x-axis* and *y-axis*. **Ans: (0,0)**
3	Write the coordinates of the midpoint of a line segment joining the points $P(x_1, y_1)$ and $Q(x_2, y_2)$. **Ans:** $P(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
4	Find the distance of the point (3, 4) from the origin. Distance from the origin $d = \sqrt{x^2 + y^2}$ $d = \sqrt{3^2 + 4^2}$ => $d = \sqrt{9 + 16}$ => $d = \sqrt{25}$ $\therefore$ $\boldsymbol{d = 5}$
5	Find the co-ordinates of the midpoint of the line segment joining the points (0, 8) and (4, 0). $P(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ $P(x, y) = \left(\frac{0+4}{2}, \frac{8+0}{2}\right)$ $P(x, y) = \left(\frac{4}{2}, \frac{8}{2}\right)$ $\boldsymbol{P(x, y) = (2, 4)}$

Two Marks Questions

1 Find the distance between the points (3,2) and (-5,6).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-5-3)^2 + (6-2)^2}$

$= \sqrt{(-8)^2 + (4)^2}$

$= \sqrt{(64 + 36}$

$= \sqrt{(100}$

$\therefore \boldsymbol{d = 10}$ **units**

2 If the distance between the points $(4, p)$ *and* $(1, 0)$ is 5 units, find the value of *'p'*

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$5 = \sqrt{(1-4)^2 + (0-p)^2}$ [Squaring on both sides]

$25 = (-3)^2 + p^2$

$25 = 9 + p^2$

$25 - 9 = p^2$

$16 = p^2$

$\therefore \boldsymbol{p = \pm 4}$

3 Find the area of a triangle with vertices $D(0, 2)$, $E(0, 6)$ *and* $F(-4, -2)$

$Area\ of\ the\ Triangle = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[0(6-(-2)) - 0(-2-2) + (-4)(2-6)]$

$= \frac{1}{2}[0 + 0 + (-4)(-4)]$

$= \frac{1}{2}(16)$

$\therefore \boldsymbol{Area\ of\ the\ Triangle = 8\ sq.units}$

4 Find the coordinates of the midpoint of the line segment joining the points $(2, 3)$ and $(4, 7)$.

$Midpoint\ P(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{2+4}{2}, \frac{3+7}{2}\right)$

$= \left(\frac{6}{2}, \frac{10}{2}\right)$

$\therefore \boldsymbol{Midpoint\ P(x, y) = (3, 5)}$

5 Find the radius of the circle whose center is $(3, 2)$ and if the circle passes through $(-5, 6)$.

Radius is the distance between center and any point on the circle.

∴ Radius of the circle = d

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-5-3)^2 + (6-2)^2}$

$= \sqrt{(-8)^2 + (4)^2}$

$= \sqrt{80}$

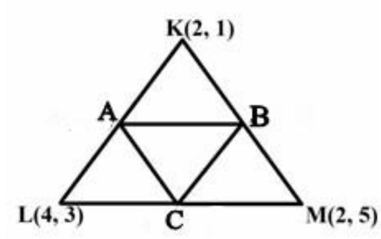
∴ **Radius of circle = $\boldsymbol{4\sqrt{5}}$ units**

Three Marks Questions	
1	Find the co-ordinates of the point which divides the line segment joining the point (1,6) and (4,3) in the ratio 1:2. $P(x,y) = \left[\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right]$ $= \left[\frac{(1)(4)+2(1)}{1+2}, \frac{(1)(3)+(2)(6)}{1+2}\right]$ $= \left[\frac{4+2}{3}, \frac{3+12}{3}\right]$ $= \left[\frac{6}{3}, \frac{15}{3}\right]$ $\therefore \boldsymbol{P(x,y) = (2,5)}$
2	If $D(1,\ 2),\ E(-5,\ 6)\ and\ F(a,\ -2)$ are collinear, then find the value of 'a.' If three points are collinear then, Area of the Triangle = 0 => $\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$ $[1(6-(-2))-5(-2-2)+a(2-6)] = 0$ x 2 $[1(6+2))-5(-4)+a(-4)] = 0$ $[8+20-4a] = 0$ $28 = 4a$ $\frac{28}{4} = a$ $\therefore \boldsymbol{a = 7}$
3	Find the area of the triangle whose vertices are (1, 2), (3, 7) and (5, 3). $Area\ of\ triangle = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$ $= \frac{1}{2}[1(7-3)+3(3-2)+5(2-7)]$ $= \frac{1}{2}[1(4)+3(1)+5(-5)]$ $= \frac{1}{2}[4+3-25]$ $= \frac{1}{2}[-18] = -9$ But Area cannot be negative **∴ Area of given triangle is 9 square units**

4	In what ratio does the point $(-4, 6)$ divide the line segment joining the points $(-6, 10)$ *and* $(3, -8)$? $\therefore$ P(x,y) = $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$ $(-4, 6) = \left(\frac{3m_1-6m_2}{m_1+m_2}, \frac{-8m_1+10m_2}{m_1+m_2}\right)$ => $-4 = \frac{3m_1-6m_2}{m_1+m_2}$ and $6 = \frac{-8m_1+10m_2}{m_1+m_2}$ Consider, $-4 = \frac{3m_1-6m_2}{m_1+m_2}$ $-4m_1 - 4m_2 = 3m_1 - 6m_2$ $2m_2 = 7m_1$ $=> \frac{m_1}{m_2} = \frac{2}{7}$ $\therefore \boldsymbol{m_1 : m_2} = \mathbf{2 : 7}$
5	Find the value of *'p'* if the point A(0, 2) is equidistant from $(3, p)$ *and* $(p, 3)$. Let $B(3, p)$ *and* $C(p, 3)$ Given AB = AC $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\sqrt{(0-3)^2 + (2-p)^2} = \sqrt{(p-0)^2 + (3-2)^2}$ $(0-3)^2 + (2-p)^2 = (p-0)^2 + (3-2)^2$ [squaring on both sides] $9 + 4 + p^2 - 4p = p^2 + 1$ $13 - 4p = 1$ $-4p = 1 - 13$ $-4p = -12$ $\therefore \boldsymbol{p} = \mathbf{3}$

Four Marks Questions

1 Find the area of the triangle formed by joining the mid-points of the triangle whose vertices are $K(2, 1), L(4, 3)\ and\ M(2, 5)$.



Midpoint = $\left(\frac{x_{1+}x_2}{2},\ \frac{y_{1+}y_2}{2}\right)$

K(2, 1), L(4, 3)

Midpoint of KL is A = $\left(\frac{2+4}{2},\ \frac{1+3}{2}\right) = \left(\frac{6}{2},\ \frac{4}{2}\right)$ = A(3, 2).

K(2, 1) , M(2, 5)

Midpoint of KM is B = $\left(\frac{2+2}{2},\ \frac{1+5}{2}\right) = \left(\frac{4}{2},\ \frac{6}{2}\right)$ = B(2, 3).

L(4, 3), M(2, 5)

Midpoint of LM is C = $\left(\frac{4+2}{2},\ \frac{3+5}{2}\right) = \left(\frac{6}{2},\ \frac{8}{2}\right)$ = C(3, 4).

A(3, 2), (B(2, 3) and (3, 4)

Area of Δ = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Area of ΔABC = $\frac{1}{2}[3(3-4) + 2(4-2) + 3(2-3)]$

$= \frac{1}{2}[3(-1) + 2(2) + 3(-1)]$

$= \frac{1}{2}[-3 + 4 - 3]$

$= -1$

But area cannot be negative, **∴Area of Triangle ABC = 1 square unit.**

2 Show that the points K(4, 5), L(7, 6), M(6, 3) and N(3, 2) are the vertices of a rhombus.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

K(4, 5), L(7, 6)

KL = $\sqrt{(7-4)^2 + (6-5)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$ units.

L(7, 6), M(6, 3)

LM = $\sqrt{(6-7)^2 + (3-6)^2} = \sqrt{(-1)^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10}$ units.

M(6, 3), N(3, 2)

MN = $\sqrt{(3-6)^2 + (2-3)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$ units.

N(3, 2), K(4, 5)

NK = $\sqrt{(3-4)^2 + (2-5)^2} = \sqrt{(-1)^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10}$ units.

KL = LM = MN = NK

Here all sides are equal.

∴ K, L, M and N are the vertices of a Rhombus.

Unit-10 : QUADRATIC EQUATIONS

Multiple Choice Questions

1. The value of the discriminant of a quadratic equation is 3. Then the nature of its roots is

   A. Real and Distinct B. Real and equal

   C. There is no any root D. Imaginary numbers

2. The standard form of quadratic equation is

   A. $ax^2 - bx + c = 0$ **B.** $\boldsymbol{ax^2 + bx + c = 0}$

   C. $ax^2 - bx - c = 0$ D. $ax^2 + bx - c = 0$

3. The quadratic equation whose roots are -1 and 2 is

   A. $\boldsymbol{x^2 - x - 2 = 0}$ B. $x^2 - x + 2 = 0$

   C. $x^2 + x - 2 = 0$ D. $x^2 + x + 2 = 0$

4. The standard form of the quadratic equation $x(x+1) = 30$ is

   A. $x^2 - x = 30$ **B.** $\boldsymbol{x^2 + x - 30 = 0}$

   C. $x^2 - x - 30 = 0$ D. $x^2 - x = 30$

5. "Sum of the squares of two consecutive odd numbers is 130. " Mathematical form of this statement is

   A. $x^2 + (x+1)^2 = 130$ B. $x^2 + (2x)^2 = 130$

   C. $\boldsymbol{x^2+(x+2)^2=130}$ D. $(x + 2x)^2 = 130$

6. If the roots of $ax^2 + bx + c = 0$ are equal, then the correct relation among the following is

   A. $\boldsymbol{\frac{b}{2a} = \frac{2c}{b}}$ B. $b^2 + 4ac = 0$ C. $\frac{b}{2a} = \frac{b}{2c}$ D. a = b

One Mark Questions

1. Write the standard form of a quadratic equation. **Ans:** $\boldsymbol{ax^2 + bx + c = 0}$ ***, where*** $\boldsymbol{a \neq 0}$

2. Find the discriminant of the quadratic equation $x^2 + 2x + 1 = 0$

   $b^2 - 4ac = 2^2 - 4(1)(1)$

   $= 4 - 4$

   $\therefore b^2 - 4ac = 0$

3. Find the roots of the quadratic equation $x^2 - 25 = 0$

   $x^2 = 25$

   $x = \sqrt{25}$

   $\therefore x = \pm 5$

4. Write the discriminant of the quadratic equation $ax^2 + bx + c = 0$

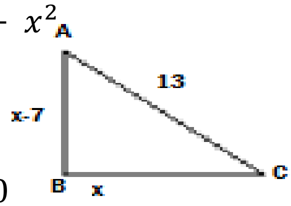
   Ans: $\boldsymbol{b^2 - 4ac}$

5. Write the formula to find the roots of the quadratic equation $ax^2 + bx + c = 0$

   Ans: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Two Marks Questions

1	Solve the quadratic equation $x^2 + 7x + 12 = 0$ by Factorization method $x^2 + 3x + 4x + 12 = 0$ $x(x + 3) + 4(x + 3) = 0$ $(x + 3)(x + 4) = 0$ $x + 3 = 0 \; or \; x + 4 = 0$ $x = -3 \; or \; x = -4$	2	Solve the quadratic equation $x^2 + x - 6 = 0$ by Factorization method $x^2 + 3x - 2x - 6 = 0$ $x(x + 3) - 2(x + 3) = 0$ $(x + 3)(x - 2) = 0$ $x + 3 = 0 \; or \; x - 2 = 0$ $x = -3 \; or \; x = 2$
3	Solve the quadratic equation $2x^2 - 15x + 18 = 0$ by Factorization method $2x^2 - 12x - 3x + 18 = 0$ $2x(x - 6) - 3(x - 6) = 0$ $(x - 6)(2x - 3) = 0$ $x - 6 = 0 \; or \; 2x - 3 = 0$ $x = 6 \; or \; x = \frac{3}{2}$	4	Solve the quadratic equation $3x^2 - x - 14 = 0$ by Factorization method $3x^2 + 6x - 7x - 14 = 0$ $3x(x + 2) - 7(x + 2) = 0$ $(x + 2)(3x - 7) = 0$ $x + 2 = 0 \; or \; 3x - 7 = 0$ $x = -2 \; or \; x = \frac{7}{3}$
5	Solve $2x^2 - 5x + 3 = 0$ by using the quadratic formula. $a = 2, \quad b = -5, \quad c = 3$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(3)}}{2(2)}$ $x = \frac{5 \pm \sqrt{25 - 24}}{4}$ $x = \frac{5 \pm 1}{4}$ $x = \frac{5+1}{4}$ or $\frac{5-1}{4}$ $\boldsymbol{x = \frac{3}{2} \; or \; x = 1}$	6	Solve $x^2 + 2x + 4 = 0$ by using the quadratic formula. $a = 1, \quad b = 2, \quad c = 4$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$ $x = \frac{-2 \pm \sqrt{4 - 16}}{2}$ $x = \frac{-2 \pm \sqrt{-12}}{2}$ $x = \frac{-2 \pm \sqrt{4(-3)}}{2}$ $x = \frac{2(-1 \pm \sqrt{-3})}{2}$ $\boldsymbol{x = (-1 + \sqrt{-3})}$ or $\boldsymbol{x = (-1 - \sqrt{-3})}$

7	Find the nature of the roots of the equation $4x^2 - 12x + 9 = 0$ $a = 4, \quad b = -12, \quad c = 9$ $b^2 - 4ac = (-12)^2 - 4(4)(9)$ $= 144 - 144$ $b^2 - 4ac = 0$ **∴ Roots are Real and Equal**	8	Find the nature of the roots of the equation $x^2 + 2x - 15 = 0$ $a = 1, \quad b = 2, \quad c = -15$ $b^2 - 4ac = (2)^2 - 4(1)(-15)$ $= 4 + 60$ $= 64$ *Here* $b^2 - 4ac > 0$ **∴ Roots are Real and Distinct**
9	Find the nature of the roots of the equation $x^2 - x + 12 = 0$ $a = 1, \quad b = -1, \quad c = 12$ $b^2 - 4ac = (-1)^2 - 4(1)(12)$ $= 1 - 48$ $= -47$ *Here* $b^2 - 4ac < 0$ **∴ The equation has no real roots.**	10	Find the value of 'k' if the quadratic equation $x^2 - kx + 4 = 0$ has equal roots. $a = 1, \quad b = -k, \quad c = 4$ Given; Roots are Equal $\therefore b^2 - 4ac = 0$ $(-k)^2 - 4(1)(4) = 0$ $k^2 - 16 = 0$ $k^2 = 16$ $k = \pm\sqrt{16}$ $\therefore \boldsymbol{k = \pm 4}$
Three Marks Questions			
1	A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages. Let the present age of sister be 'x' years and girls present age be '$2x$' years Product of their ages 4 years hence = $(x + 4)(2x + 4)$ $\therefore (x + 4)(2x + 4) = 160$ $2x^2 + 12x - 144 = 0$ $x^2 + 6x - 72 = 0$ $x^2 + 12x - 6x - 72 = 0$ $x(x + 12) - 6(x + 12) = 0$ $x = -12 \; or \; x = 6$ Age cannot be negative => $x = 6$ **∴ Girl's present age is 12 years and present age of her sister is 6 years**	2	The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm. find the other two sides. Let the base is 'x' cm and altitude is $(x - 7)$ cm and hypotenuse is 13 cm By Pythagoras theorem. $13^2 = (x - 7)^2 + x^2$ $169 = x^2 + 49 - 14x + x^2$ $2x^2 - 14x - 12 = 0$ $x^2 - 7x - 60 = 0$ $x^2 - 12x + 5x - 60 = 0$ $x(x - 12) + 5(x - 12) = 0$ $x - 12 = 0 \quad or \; x + 5 = 0$ $x = 12 \quad or \; x = -5$  **Base is 12 cm and Altitude is 5 cm**

3	The difference of squares of two positive numbers is 180. The square of small number is 8 times the big number. Find the numbers. Let the bigger number be x and smaller be y Given $x^2 - y^2 = 180$ and $y^2 = 8x$ $\therefore x^2 - 8x = 180$ $x^2 - 8x - 180 = 0$ $x^2 - 18x + 10x - 180 = 0$ $x(x-18) + 10(x-18) = 0$ $(x-18)(x+10) = 0$ => $x = 18$ or $x = -10$ $y^2 = 8(18)$ => $y^2 = 144$ $\therefore y = 12$ ∴ **The numbers are 18 and 12**	4	The sum of the squares of two consecutive positive integers is 13. Find the numbers. Let the numbers be x and $(x+1)$ $x^2 + (x+1)^2 = 13$ $x^2 + x^2 + 1 + 2x = 13$ $2x^2 + 2x - 12 = 0$ $x^2 + x - 6 = 0$ $x^2 + 3x - 2x - 6 = 0$ $x(x+3) - 2(x+3) = 0$ $(x+3) = 0$ or $(x-2) = 0$ $x = -3$ or $x = 2$ The other number = $x + 1 = 3$ ∴**The numbers are 2 and 3**
Four Marks Questions			
5	A person on tour has Rs 4200 for his expenses. If he extends his tour for 3 days, he has to cut down his daily expenses by Rs 70. Find the original duration of the tour. original duration of the tour be ‘x’ days. Given, $\frac{4200}{x} - \frac{4200}{x+3} = 70$ $4200\left(\frac{1}{x} - \frac{1}{x+3}\right) = 70$ $\frac{(x+3)-x}{x(x+3)} = \frac{70}{4200}$ $x(x+3) = 180$ $x^2 + 3x - 180 = 0$ $x^2 + 15x - 12x - 180 = 0$ $(x+15)(x-12) = 0$ $x + 15 = 0$ or $x - 12 = 0$ $x = -15$ or $x = 12$ number of days can’t be negative => $x = 12$ ∴ **Original duration of the tour is 12 days.**	6	A motor boat whose speed in still water is 18km/hr, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream. Let the speed of the stream be x km/hr Speed of the boat in upstream =*(18-x)km/hr* Speed of the boat downstream =*(18+x) km/hr* $Speed = \frac{distance}{time}$ The time taken to go upstream = $\frac{24}{18-x}$ and the time taken to go downstream = $\frac{24}{18+x}$ Given, $\frac{24}{18-x} - \frac{24}{18+x} = 1$ $\frac{24(18+x)-24(18-x)}{(18-x)(18+x)} = 1$ $24(18+x) - 24(18-x) = (18-x)(18+x)$ $x^2 + 48x - 324 = 0$ $x^2 + 54x - 6x - 324 = 0$ $x(x+54) - 6(x+54) = 0$ $(x+54)(x-6) = 0$ $x = -54$ or $x = 6$ speed can’t be negative ∴**The speed of the stream is 6km/hr**

Unit-11 : INTRODUCTION TO TRIGONOMETRY

Multiple Choice Questions

1	If $sin\ \theta\ = \frac{12}{13}$, then the value of $cosec\ \theta$ is			
	A. $\frac{5}{12}$	B. $\frac{5}{13}$	**C. $\frac{13}{12}$**	D. $\frac{12}{13}$
2	The value of $tan\ 45^o$ is			
	A. $\sqrt{3}$	B. 0	**C. 1**	D. $\frac{1}{\sqrt{3}}$
3	If $2cos\ \theta =\ 1$ and θ is an acute angle then the value of θ is			
	A. 0°	B. 30°	C. 45°	**D. 60°**
4	If $cos\theta\ = \frac{1}{2}$, then the value of $tan\theta$ is			
	A. $\frac{1}{\sqrt{3}}$	**B. $\sqrt{3}$**	C. 1	D. 0
5	$\frac{\sin A}{cosA}$ is equal to			
	A. sec A	B. cosec A	**C. tan A**	D. cot A
6	$(1 + cos\theta)\ (1 - cos\theta)\ =$			
	A. $sin^2\theta$	B. $tan^2\theta$	C. $\mathrm{cosec}^2\ A$	D. $\sec^2 A$
7	The value of $(cos\ 48^o -\ sin\ 42^o)\ is$			
	A. 0	B. $\frac{1}{4}$	C. 1	D. $\frac{1}{2}$

One Mark Questions

1. Find the value of $sin^2 25^0 + sin^2 65^0$.

$$sin^2 25^0 + sin^2 65^0 = sin^2 25^0 + sin^2(90^0 - 25^0)$$
$$= sin^2 25^0 + cos^2 25^0.$$
$$= 1$$

2. If $sinA\ = \frac{1}{2}$ where A is an acute angle then find the value of A.

$$sinA\ = \mathbf{\frac{1}{2}}$$
$$sinA\ = \sin 60^o$$
$$=> \boldsymbol{A = 60^o}$$

3. Find the value of $(1 + tan^2\theta).\ cos^2\theta$.

$$(1 + tan^2\theta).\ cos^2\theta = \sec^2\theta\ \text{x}\ \frac{1}{sec^2\theta}$$
$$= 1$$

4. If $cos\ A = sin\ B$, then find the value $A + B$.

$$sin(90^o - A) = sinB$$
$$90^0 - A = B$$
$$=> \boldsymbol{A + B = 90^o}$$

Two Marks Questions

1 Evaluate: $sin18^o - cos72^o - cos18^o + sin72^o$.

$$sin18^o - cos72^o - cos18^o + sin72^o = sin(90^o - 72^0) - cos72^o - cos(90^o - 72^0) + sin72^o$$
$$= cos72^o - cos72^o - sin72^o + sin72^o$$
$$= 0$$

2 If $\tan 2A = cot\,(A - 18^o)$, where 2A is an acute angle. Find the value of A.

$cot(90^0 - 2A) = cot(A - 18^0)$

$90^o - 2A = A - 18^o$

$90^o + 18^o = A + 2A$

$3A = 108^o$

$\boldsymbol{A = 36^o}$

3 If A=60^0, B=30^0 then show that $cos(A+B) = cosA.cosB - sinA.sinB$

$cos(A+B) = cosA.cosB - sinA.sinB$

$cos(60^o + 30^o) = cos60^o.cos30^o - sin\,60^o.sin\,30^o$

$$cos\,90° = \frac{1}{2} \text{ x } \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \text{ x } \frac{1}{2}$$

$$0 = 0$$

4 Show that $(tanA\,.sinA) + cosA = secA$

$$LHS = \left(\frac{sinA}{cosA} \text{ x } sinA\right) + cos\,A$$

$$= \frac{sin^2\text{A}}{\text{cosA}} + cosA$$

$$= \frac{sin^2\text{A} + cos^2\text{A}}{\text{cosA}} = \frac{1}{\text{cosA}}$$

$$= \text{sec A}$$

5 If $A, B, and\ C$ are interior angles of a triangle ABC, then show that, $sin\left(\frac{A+B}{2}\right) = cos\frac{A}{2}$

We know ,

Sum of the interior angles of a Triangle =180°

=> $A + B + C = 180°$

$B + C = 180° - A$

$$\frac{B+C}{2} = \frac{180° - A}{2}$$

Taking sin on both sides

$$sin\left(\frac{B+C}{2}\right) = sin\left(90° - \frac{A}{2}\right)$$

$$\therefore\ \boldsymbol{sin\left(\frac{B+C}{2}\right) = cos\left(\frac{A}{2}\right)}$$

6 Prove that $tan10°.tan15°.tan75°.tan80° = 1$

$LHS = tan10°.tan15°.tan75°.tan80°$

$= tan(90° - 80°) \text{ x } tan(90° - 75°) \text{ x } tan75° x\ tan80°$

$$= cot80° \text{ x } \cot 75° \text{ x } \frac{1}{cot75°} \text{ x } \frac{1}{cot80°}$$

$= 1$

Three Marks Questions

1

Show that, $\frac{\sin\theta}{1-\cos\theta} = cosec\,\theta + \cot\theta$

L.H.S = $\frac{\sin\theta}{1-\cos\theta}$

$= \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$

$= \frac{\sin\theta\ (1+\cos\theta)}{1-\cos^2\theta}$

$= \frac{\sin\theta\ (1+\cos\theta)}{\sin^2\theta}$

$= \frac{(1+\cos\theta)}{\sin\theta}$

$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$

$= cosec\theta + \cot\theta$

= RHS

2

Show that, $\frac{1+\cot^2 A}{1+\tan^2 A} = \cot^2 A.$

LHS $= \frac{1+\cot^2 A}{1+\tan^2 A}$

$= \frac{cosec^2 A}{\sec^2 A}$

$= \frac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A}}$

$= \frac{1}{\sin^2 A} \times \frac{\cos^2 A}{1}$

$= \frac{\cos^2 A}{\sin^2 A}$

$= \cot^2 A$

= RHS

3

Prove that $\frac{\cos\theta - 2\cos^3\theta}{2\sin^3\theta + \sin\theta} = \cot\theta$

L.H.S = $\frac{\cos\theta - 2\cos^3\theta}{2\sin^3\theta - \sin\theta}$

$= \frac{\cos\theta\,(1-2\cos^2\theta)}{\sin\theta(2\sin^2\theta - 1)}$

$= \frac{\cos\theta\,(1-\cos^2\theta-\cos^2\theta)}{\sin\theta(\sin^2\theta+\sin^2\theta-1)}$

$= \frac{\cos\theta\,(\sin^2\theta-\cos^2\theta)}{\sin\theta(\sin^2\theta-\cos^2\theta)}$

$= \frac{\sin\theta}{\cos\theta}$

$= \cot\theta$

$= R.H.S$

4

If $\sin\theta = \frac{1}{2}$, then show that $3\cos\theta - 4\cos3\theta = 0.$

Given, $\sin\theta = \frac{1}{2}$

$\sin\theta = \sin 30^0$

$\Rightarrow \theta = 30^0$

$LHS = 3\cos\theta - 4\cos^3\theta$

$= 3\cos30^0 - 4\cos^3 30^0$

$= 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3$

$= 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{3\sqrt{3}}{8}\right)$

$= 3\left(\frac{\sqrt{3}}{2}\right) - 3\left(\frac{\sqrt{3}}{2}\right)$

$= 0$

$= RHS$

5	Prove that $\sqrt{\frac{1+sinA}{1-sinA}} = secA + tanA$. LHS $= \sqrt{\frac{1+sinA}{1-sinA}}$ $= \sqrt{\frac{1+sinA}{1-sinA}}$ x $\sqrt{\frac{1-sinA}{1-sinA}}$ $= \sqrt{\frac{(1+sinA)(1+sinA)}{(1-sinA)(1-sinA)}}$ $= \sqrt{\frac{(1+sinA)^2}{1-sin^2A}}$ $= \sqrt{\frac{(1+sinA)^2}{cos^2A}}$ $= \frac{1+sin\mathrm{A}}{\cos\mathrm{A}}$ $= \frac{1}{\cos\mathrm{A}} + \frac{sin\mathrm{A}}{\cos\mathrm{A}}$ $= secA + tanA$ $= RHS$	6	Prove that $\frac{sin\,\theta}{1+\cos\theta} + \frac{1+cos\theta}{sin\theta} = 2\,cosec\,\theta$. LHS $= \frac{sin\,\theta}{1+cos\theta} + \frac{1+cos\theta}{sin\theta}$ $= \frac{sin^2\theta+(1+cos\theta)^2}{(1+cos\theta)sin\theta}$ $= \frac{sin^2\theta+1^2+cos^2\theta+2cos\theta}{(1+cos\theta)sin\theta}$ $= \frac{1+1+2cos\theta}{(1+cos\theta)sin\theta}$ $= \frac{2+2cos\theta}{(1+cos\theta)sin\theta}$ $= \frac{2(1+cos\theta)}{(1+cos\theta)sin\theta}$ $= \frac{2}{sin\theta}$ $= 2\,cosec\,\theta$ $= RHS$

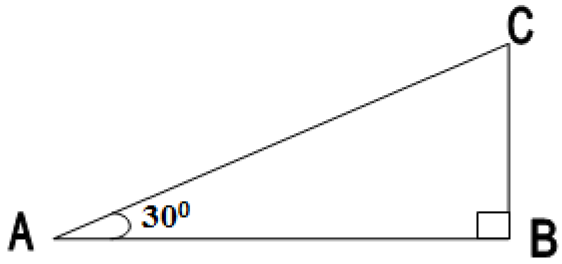
Unit-12 : SOME APPLICATIONS OF TRIGONOMETRY

Two Marks Questions

1 The top of a building is observed from a point on the ground $100\sqrt{3}ft$. away from its base. If the angle of elevation is 30^0, then find the height of the building.

Let A be the point of observation and C be the top of the building.

Then AB=$100\sqrt{3}ft$ and $\angle A=30^0$



$tanA = \frac{BC}{AB}$

$tan30^0 = \frac{BC}{100\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \frac{BC}{100\sqrt{3}}$

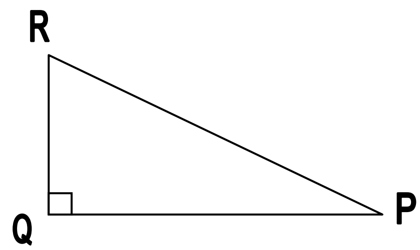
= > BC= 100m

∴ **height of the building is 100ft**.

2 A kite flying at a height of $50\sqrt{3}m$ above the ground is tied to a point on the ground by a thread of 100m length without any slack. Find the angle formed by the thread with the ground.

Let P be the point on the ground where thread is tied and
R be the position of kite.

Then QR=$50\sqrt{3}m$ and PR= 100m



$sinP = \frac{QR}{PR}$

$sinP = \frac{50\sqrt{3}}{100}$

$sinP = \frac{\sqrt{3}}{2}$

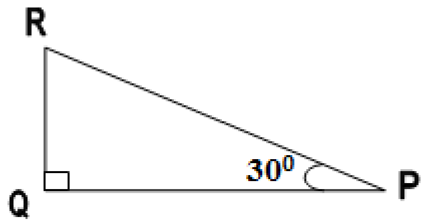
= > $\angle P= 60^0$

∴**Thread makes an angle of 60^0 with the ground.**

3 In an amusement park, there is a slide of height $6m$, which is inclined at an angle of 30^0 to the ground. Then, find the length of the slide.

Let QR be the height of the slide and $\angle P$ is the angle of inclination
PR is the length of the slide

Then QR=6m and $\angle P=30^0$



$sinP = \frac{QR}{PR}$

$sin30^0 = \frac{6}{PR}$

$\frac{1}{2} = \frac{6}{PR}$

= > PR= 12m

∴**The length of the slide is 12m**.

Three Marks Questions

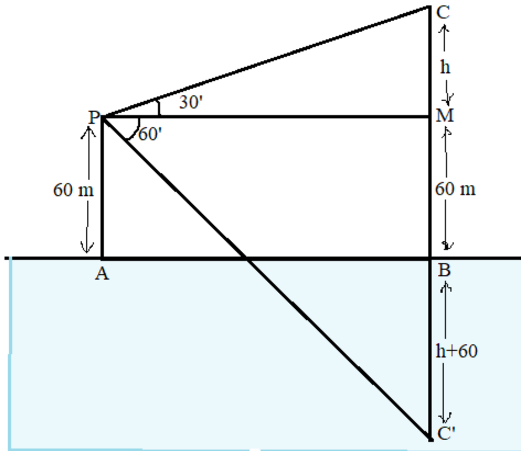
1 The angle of elevation of a cloud is 30° from a point 60 m above a lake and from the same point, the angle of depression of the reflection of cloud in the lake is 60^0. Find the height of the cloud.

Let AB be the surface of lake.

P be the point of observation. AP=60 m

Let C be the position of cloud. C' be its reflection in the lake.

CB=C'B



Let $CM = h$, then C'B= $(h + 60)$

In Δ CMP, $\tan 30^0 = \frac{h}{PM}$

$$PM = \sqrt{3}h \; ------ (1)$$

In Δ PMC' $\tan 60^0 = \frac{C'M}{PM}$

$$\sqrt{3} = \frac{h+60+60}{PM}$$

$$PM = \frac{h+120}{\sqrt{3}} \; -------(2)$$

From (1) and (2) $\sqrt{3}h = \frac{h+120}{\sqrt{3}}$ => $h = 60\ m$

CB=CM+MB = 60+60 = 120 m

Height of the cloud from the surface of the lake is 120 m.

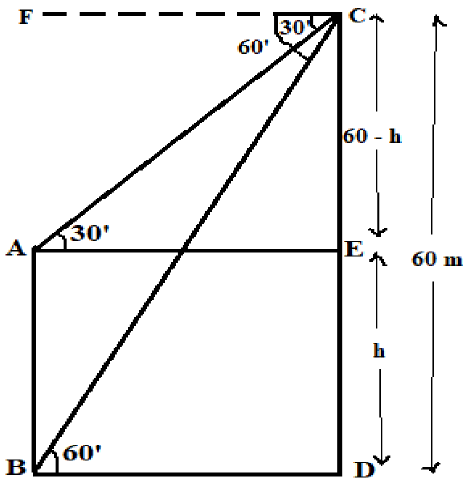
2 The top of a tower is observed from two points on the same straight line on the ground. The distances of these points from the base of the tower is a *and* b meters. If the angles of elevation are complementary prove that the height of the tower is $\sqrt{ab}$ meter.

Let CD be the building of height 60 m and

AB be the tower

$\underline{|FCA}$ = $\underline{|CAE}$ = 30^0

$\underline{|FCB}$ = $\underline{|CBD}$ = 60^0



In ΔACE, $\tan 30° = \frac{CE}{AE}$

$$\frac{1}{\sqrt{3}} = \frac{60-h}{AE}$$

$$AE = (60-h)\sqrt{3}$$

$$AE = BD = (60-h)\sqrt{3}$$

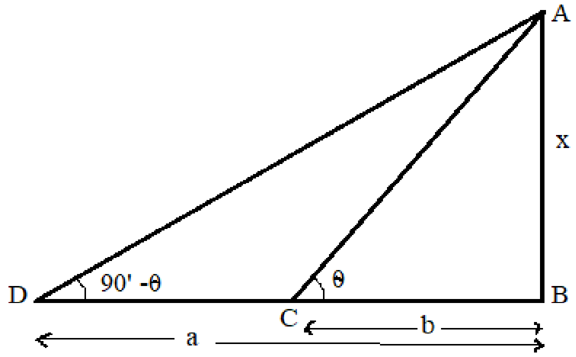
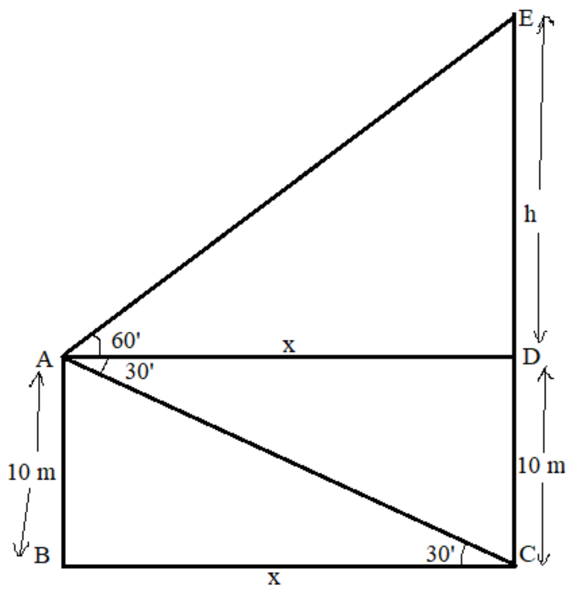
In ΔBCD, $\tan 60° = \frac{60}{BD}$

$\sqrt{3} = \frac{60}{(60-h)\sqrt{3}}$ => $(60-h)3 = 60$

$60 - h = 20$

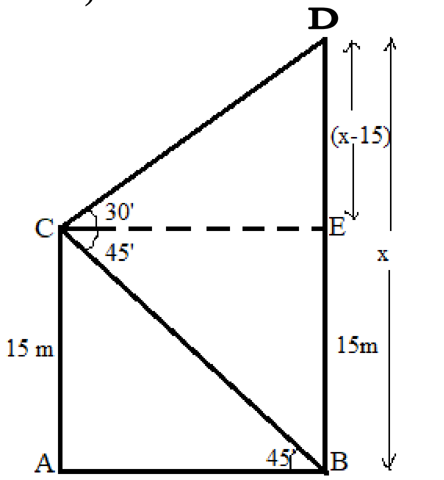
$h = 60 - 20$

$\therefore$ ***height of the tower = 40 m***

3	The angle of elevation of the top of a tower from two points on the ground at distances 'a' and 'b' meters from the base of a tower and in the same straight line with it are complementary. Prove that height of the tower is $\sqrt{ab}$ meter.
	Height of the tower be 'x' m $tan\theta = \frac{x}{b} ----(i)$ $tan(90° - \theta) = \frac{x}{a}$ $cot\theta = \frac{x}{a} -------(ii)$ Multiplying (i) and (ii) $tan\theta x\ cot\theta = \frac{x}{b} \text{ x } \frac{x}{a}$ $1 = \frac{x^2}{ab}$ $x^2 = ab$ => $x = \sqrt{ab}$ **∴Height of the tower is $\sqrt{ab}$ meter.** 
4	The deck of a ship is 10m high from the level of water. A man standing on it observes the top of a hill with an angle of elevation 60° and from the same point, he observes the base of the same hill at an angle of depression 30°. Then, find the distance of the ship from the hill and also the height of the hill.
	In ΔADE, $tan60° = \frac{h}{AD}$ $\sqrt{3} = \frac{h}{x}$ $h = x\sqrt{3}$ -------(i) In ΔABC, $tan30° = \frac{AB}{BC}$ $\frac{1}{\sqrt{3}} = \frac{10}{x}$ => $x = 10\sqrt{3}$ ----(ii) **Distance of the ship from the hill =10$\sqrt{3}$ m** Substituting (ii) in (i) gives $h = 10\sqrt{3}\ x\ \sqrt{3}$ $h = 30\ m$ => **Height of the hill = 30+10 = 40 m.** 

Four Marks Questions

1 From a window 15 m high above the ground in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 30^0 and 45^0 respectively. Show that the height of the opposite house is 23.66m. (take $\sqrt{3}$=1.73)



AB – ground, C- position of window

BD – house in the opposite side of the street

<u>|DCE</u> = 30^0, <u>|ECB</u> = <u>|CBA</u> =45^0

AC = BE = 15 m, Let BD = $x\ m, \therefore DE = (x - 15)\ m$

In ΔCDE, $tan30° = \frac{x-15}{CE}$

$$\frac{1}{\sqrt{3}} = \frac{x-15}{CE} \quad => CE = \sqrt{3}(x - 15)$$

In ΔACB, $tan45° = \frac{AC}{AB}$

$$1 = \frac{15}{\sqrt{3}(x-15)} \qquad \text{(since AB =CE)}$$

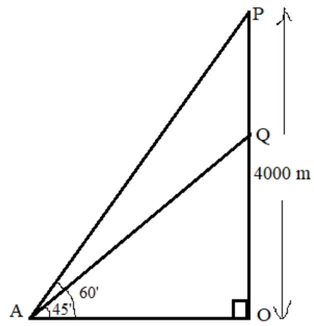
$$\sqrt{3}(x - 15) = 15$$

$$(x - 15) = \frac{15}{\sqrt{3}}$$

$$x - 15 = 8.66$$

$$x = 23.66\ m$$

2 An aero plane when flying at a height of 4000 m from the ground passes vertically above another aero plane at an instant when the angles of the elevation of the two planes from the same point on the ground are 60^0 and 45^0 respectively. Find the vertical distance between the aero planes at that instance.



Let P and Q be the positions of two aero planes,

when Q is vertically below P and OP=4000 m

A be the point of observation on the ground

In ΔAOP and in Δ AOQ

$tan60° = \frac{OP}{OA}$ $\qquad$ $tan45° = \frac{OQ}{OA}$

$\sqrt{3} = \frac{4000}{OA}$ $\qquad$ $1 = \frac{OQ}{OA}$

OA = $\frac{4000}{\sqrt{3}}$ $\qquad$ OQ = OA

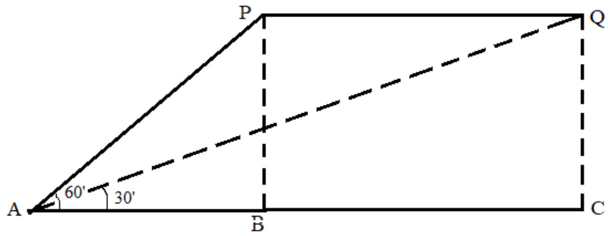
Vertical distance PQ = OP – OQ

$$PQ = 4000 - \frac{4000}{\sqrt{3}} \quad => \frac{4000\sqrt{3}-4000}{\sqrt{3}}$$

$$=> \frac{4000(\sqrt{3}-1)}{\sqrt{3}}$$

$$\therefore \textbf{\textit{The vertical distance}} = \mathbf{1690.53\ m}$$

3 The angle of elevation of a jet plane from a point A on the ground is 60^0. After a flight of 30 seconds, the angle of elevation changes to 30^0. if the jet plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed of the plane.



Let P and Q be the two positions of plane

A be the point of observation

In Δ ABP, $tan60° = \frac{PB}{AB}$ => $\sqrt{3} = \frac{3600\sqrt{3}}{AB}$

$$AB = 3600\,m$$

In Δ ACQ, $tan30° = \frac{CQ}{AC}$ => $\frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{AC}$

AC = 10800 m

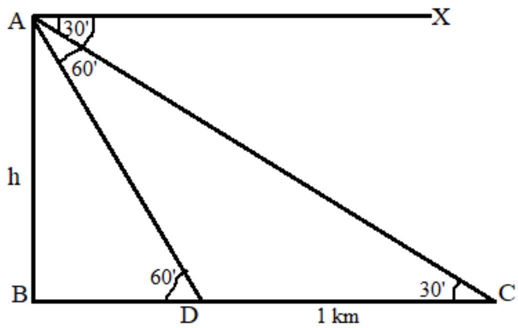
BC = 10800 – 3600

BC = 7200 m

but BC=PQ => Distance travelled is 7200 m

Speed of the plane = $\frac{\mathbf{7200}}{\mathbf{30}}$ = $\boldsymbol{240\ m/s}$

4 A person at the top of a hill observes that the angles of depression of two consecutive kilo metre stones on a road leading to the foot of the hill and on the same vertical plane containing the position of the observer are 30^0 and 60^0. Find the height of the hill.



AB – hill

C and D are kilometer stones

AX is the horizontal through A

A is the position of observation

|XAC = |ACD = 30^0, |XAD = |ADB = 60^0

In ΔABC, $tan30° = \frac{AB}{BC}$ => $\frac{1}{\sqrt{3}} = \frac{h}{BC}$

$BC = h\sqrt{3}$ -----(i)

In ΔABD, $tan60° = \frac{h}{BD}$ => $\sqrt{3} = \frac{h}{BD}$

$BD = \frac{h}{\sqrt{3}}$ -------(ii)

$BC = BD + DC$ => $h\sqrt{3} = \frac{h}{\sqrt{3}} + 1$

$$h\sqrt{3} - \frac{h}{\sqrt{3}} = 1$$

$$\frac{3h-h}{\sqrt{3}} = 1$$

$\therefore$ ***H*eight of the hill.** $= \frac{\sqrt{3}}{2}$ ***km.***

Unit 13: STATISTICS

Multiple Choice Questions

1 The mean value of 10, 15, 5, 20 and 50 is

(A) 10 (B) 5 (C) 15 **(D) 20**

2 The median of 7, 3, 6, 14, 13, 11, 19 is

(A) 7 (B) 13 **(C) 11** (D) 19

3 The mode of 6, 7, 2, 4, 2, 8, 5, 2, 2, 7 is

(A) 7 (B) 6 (C) 4 **(D) 2**

4 The measure of central tendency that gives the middle most value of the data is

A. midpoint B. mean **C. median** D. mode

5 Mode of the given set of scores is

A) Middle most value B) Least frequent value
C) Most frequent value D) None of these

One Mark Questions

1. Write the empirical relationship between the three measures of central tendency.

3Median = Mode + 2Mean

2.Find the median of 24, 31, 17, 29, 36, 39

17, 24, 29, 31, 36, 39

$Median = \frac{29+31}{2}$

$\therefore \mathbf{Median = 30}$

3. Find the class mark of the class interval 40-50

$Class\ mark = \frac{lower\ limit + upper\ limit}{2}$

$Class\ mark = \frac{40+50}{2}$

$\therefore \mathbf{Class\ mark = 45}$

Three Marks Questions

1) Find mean for the following frequency distribution.

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	3	5	9	5	3

Class Interval	**Frequency**	x	fx
0-10	3	5	15
10-20	5	15	75
20-30	9	25	225
30-40	5	35	175
40-50	3	45	135
	$\Sigma f = 25$		$\Sigma fx = 625$

$$Mean = \frac{\Sigma fx}{\Sigma f} = \frac{625}{25}$$

$\therefore$ **Mean = 25**

2) Find the Median of the following frequency distribution.

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	4	7	13	9	3

Class Interval	Frequency	Cumulative Frequency
0-10	4	4
10-20	7	4+7=**11**
20-30	**13**	11+13=24
30-40	9	24+9=33
40-50	3	33+3=36

$nn = 36, \quad \frac{n}{2} = 18, \quad f = 13, \quad cf = 11,$

$h = 10, \quad l = 20$

$$Median = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

$$Median = 20 + \left[\frac{18 - 11}{13}\right] \times 10$$

$$Median = 20 + 5.38$$

$$\therefore \boldsymbol{Median = 25.38}$$

3) Find the mode of the following frequency distribution.

Class interval	*Frequency*
30-40	4
40-50	7
50-60	9
60-70	11
70-80	6
80-90	2

$f_1 = 11, \quad f_0 = 9, \quad f_2 = 6, \quad l = 60, \quad h = 10$

$$Mode = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$Mode = 60 + \left[\frac{11-9}{2(11)-9-6}\right] \times 10$$

$$Mode = 60 + 2.86$$

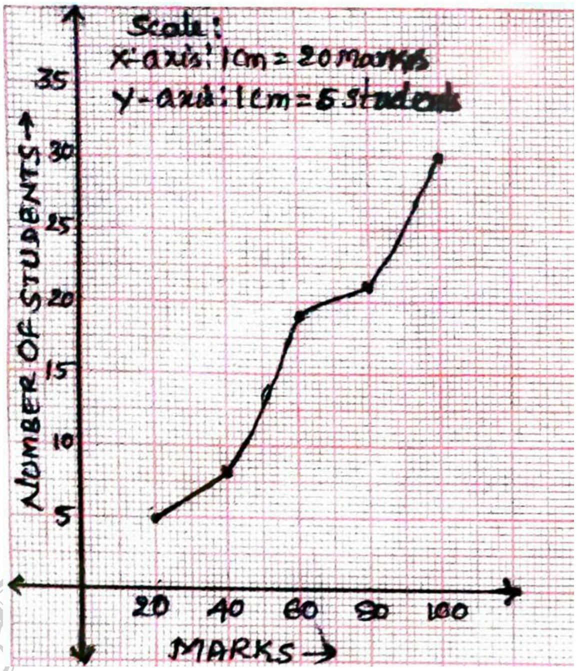
$$\therefore \boldsymbol{Mode = 62.86}$$

Three Marks Questions

4) The marks scored by 30 Students of class X, in the Mathematics are given below. Draw a less than type ogive.

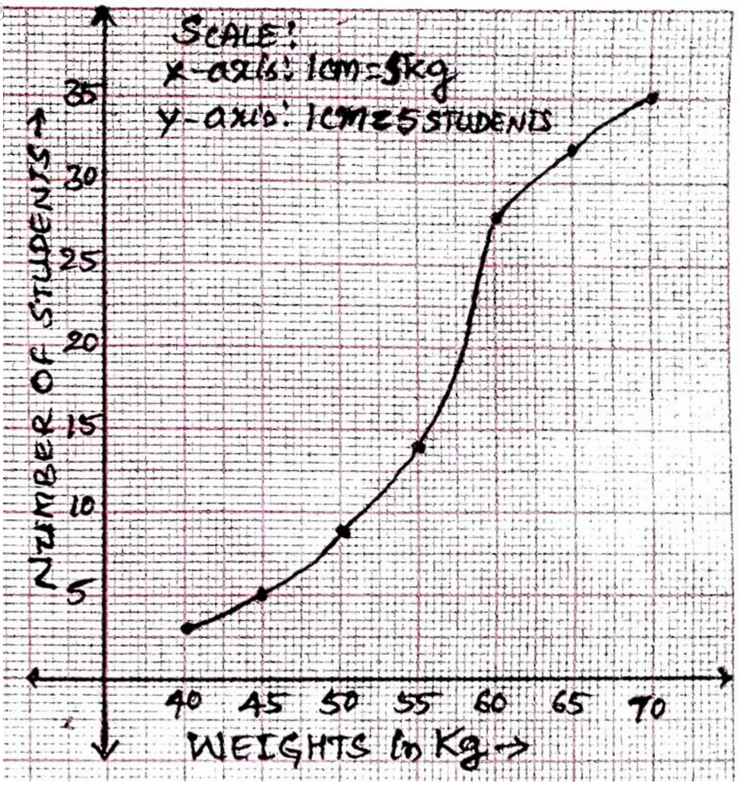
Marks	0-20	20-40	40-60	60-80	80-100
Number of students	5	3	11	2	9

Marks	Number of students
Less than 20	5
Less than 40	5+3=8
Less than 60	8+11=19
Less than 80	19+2=21
Less than 100	21+9=30



5) During the medical check-up of 35 students of a class, their weights were recorded as follows. Draw a less than type ogive for the given data.

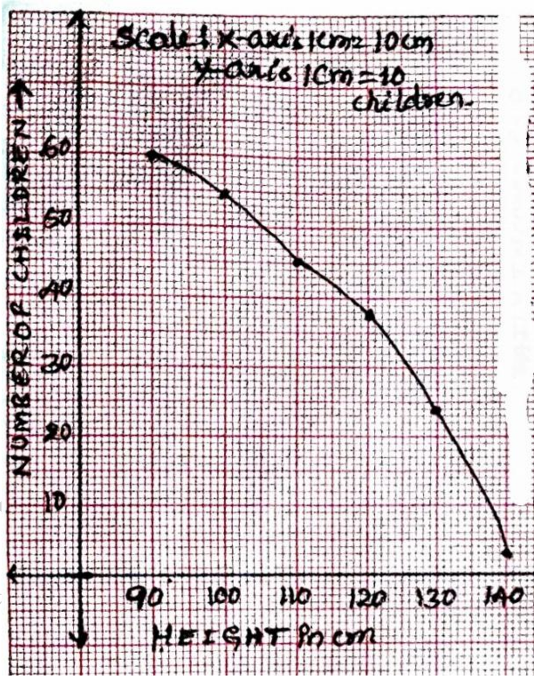
Weights (in kg)	Number of students
Less than 40	3
Less than 45	5
Less than 50	9
Less than 55	14
Less than 60	28
Less than 65	32
Less than 70	35



6)Heights of 60 children are given below. Draw a more than type ogive.

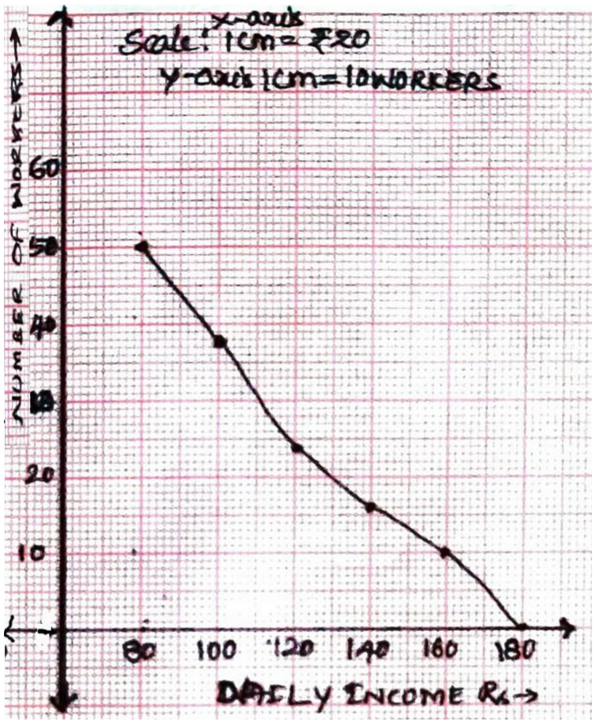
Height(in cm)	90-100	100-110	110-120	120-130	130-140	140-150
Number of children	5	10	7	24	11	3

Height(in cm)	Number of children
More than or equal to 90	60
More than or equal to 100	60 – 5 = 55
More than or equal to 110	55 – 10 = 45
More than or equal to 120	45 – 7 = 38
More than or equal to 130	38 – 24 = 14
More than or equal to 140	14 – 11 = 3



7) Details of daily income of 50 workers in a food industry are given below. Draw a more than type ogive for the following data.

Daily Income (in Rs.)	Number of workers
More than or equal to 80	50
More than or equal to 100	38
More than or equal to 120	24
More than or equal to 140	16
More than or equal to 160	10
More than or equal to 180	0



Unit 15: SURFACE AREA AND VOLUME

Multiple Choice Questions

1. The volume of a hemisphere of radius 'r' is

(A) πr^2 (B) $\frac{4}{3}\pi r^3$ (C) $4\pi r^3$ **(D) $\frac{2}{3}\pi r^3$**

2. If two solid hemispheres with same radii of their bases are joined together along their bases, then, the curved surface area of the new solid formed is

(A) $3\pi r^2$ **(B) $4\pi r^2$** (C) $5\pi r^2$ (D) $6\pi r^2$

3. A cylinder and a cone are of same heights and same radii of their bases. If the volume of the cylinder is 924cm^3 then, the volume of the cone is

(A) 924 cm^3 **(B) 308 cm^3** (C) 462 cm^3 (D) 38 cm^3

4. While conversion of a solid from one shape to another, the volume of the new shape will

(A) increases (B) decreases **(C) remain unaltered** (D) doubled

5. The surface area of a sphere of radius $7cm$ is

(A) 308 cm^2 (B) 154 cm^2 **(C) 616 cm^2** (D) 462 cm^2

6. If the slant height of a frustum of a cone is 4cm and radii of its two circular ends are 5cm and 2cm, then its curved surface area is

(A) 88 cm^2 (B) 22 cm^2 (C) 48 cm^2 (D) 26 cm^2

7. Three cubes of edge 4 cm are joined end to end, then the volume of the cuboid so formed is

(A) 162 cm^3 (B) 172 cm^3 (C) 182 cm^3 **(D) 192 cm^3**

8. The radius of the base of a cone is 9cm and slant height is 15cm, then its height is

(A) 6cm (B) 3cm (C) 5cm **(D) 12cm**

One Mark Questions

1. A frustum of a cone is of radii of circular ends r_1 and r_2 and height 'h'. Then write the formula to find its volume. Ans: $\mathbf{V = \frac{1}{3}\pi h(r_1{}^2 + r_2{}^2 + r_1 r_2)}$

2. Find the ratio of the total surface areas of a sphere and a solid hemisphere having equal radii.

$\frac{\text{Area of sphere}}{\text{Area of solid hemisphere}} = \frac{4\pi r^2}{3\pi r^2}$ $\frac{A_1}{A_2} = \frac{4}{3}$ $\therefore \boldsymbol{A_1 : A_2 = 4 : 3}$

3. If the area of base of a right circular cylinder is $38.5cm^2$ and its height is 6cm, then find its volume.

Given: $area = \pi r^2 = 38.5cm^2, h = 6cm, V = ?$

$Volume\ of\ a\ cylinder = \pi r^2 h = 38.5 \times 6$

$\therefore \boldsymbol{Volume\ of\ a\ cylinder = 231\ cm^3}$

Two Marks Questions

1. Two cubes of edge 8cm each are kept together joining their faces to form a cuboid. Find the total surface area of the cuboid.

Given: $l = 8 + 8 = 16cm, b = 8cm,\ h = 8cm,$ $T.S.A\ Of\ cuboid = ?$

$$T.S.A.\ of\ a\ cuboid = 2[lb + bh + hl]$$

$$= 2[(16)(8) + (8)(8) + (8)(16)]$$

$$\therefore \boldsymbol{T.S.A.\ of\ a\ cuboid = 640cm^2}$$

2. If the total surface area of a cube is $150cm^2$, find its volume.

$$T.S.A\ Of\ a\ cube = 6a^2$$

$$150 = 6a^2$$

$$a = 5cm$$

$$Volume\ of\ a\ cube = a^3 = 5^3$$

$$\therefore \textbf{Volume of a cube} = \mathbf{125cm^3}$$

3. A metal container is in the shape of a frustum of a cone of height 21 cm and radii of its circular ends are 8 cm and 20 cm. Find its capacity.

$r_1 = 20cm, r_2 = 8cm,\ \ h = 21cm$

$$Capacity = \mathrm{V} = \frac{1}{3}\pi\mathrm{h}(\mathrm{r_1}^2 + \mathrm{r_2}^2 + \mathrm{r_1 r_2})$$

$$= \frac{1}{3} \times \frac{22}{7} \times 21(20^2 + 8^2 + 20 \times 8)$$

$\therefore \boldsymbol{Volume = 13728cm^3}$

4. If the total surface area of a hemispherical bowl is $462cm^2$, then find its radius.

$$TSA\ of\ hemisphere = 2\pi r^2 = 462$$

$$2 \times \frac{22}{7} \times r^2 = 462$$

$$r^2 = \frac{462 \times 7}{2 \times 22}$$

$\therefore \boldsymbol{Radius\ of\ the\ bowl = 7cm}$

Three Marks Questions

1. The diameter of a solid metallic sphere is 6cm. It is melted and drawn into a wire having diameter of the uniform cross-section is 0.2cm. Find the length of the wire.

$radius\ of\ the\ sphere\ R = 3cm,$
$radius\ of\ the\ wire\ (cylinder) r = 0.1cm$
$length\ of\ the\ wire\ (cylinder) h = ?$

$$Volume\ of\ cylinder = Volume\ of\ sphere$$

$$\pi r^2 h = \frac{4}{3}\pi R^3$$

$$\pi(0.1)^2 h = \frac{4}{3}\pi(3)^3$$

$$0.01\pi h = 36\pi$$

$$\therefore \boldsymbol{h = 3600cm = 36m}$$

2. A big solid metal sphere of diameter 48cm is melted and casted into small solid spheres of radius 3cm. Find the number of small solid spheres so formed.

$radius\ of\ big\ solid\ sphere\ R = 24\ cm$
$radius\ of\ small\ solid\ sphere\ r = 3cm$
$Number\ of\ small\ solid\ spheres = ?$

$$Number\ of\ small\ spheres = \frac{V(\ big\ sphere)}{V(a\ small\ sphere}$$

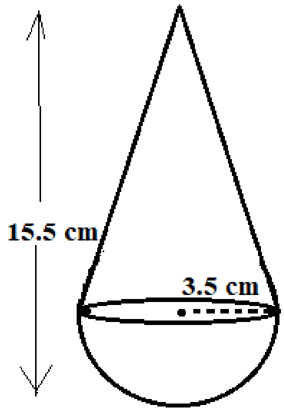
$$= \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{R^3}{r^3}$$

$$= \frac{24^3}{3^3}$$

$\therefore$ **The number of small solid sphere = 512**

Four Marks Questions

1. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.



$Cone: h = 15.5 - 3.5 = 12cm, r = 3.5cm$

$Hemisphere: R = 3.5cm$

$Slant\ height: l = \sqrt{h^2 + r^2}$

$= \sqrt{(12)^2 + (3.5)^2}$

$\therefore l = 12.5cm$

TSA of a toy = CSA of cone + CSA of hemisphere

$= \pi rl + 2\pi R^2$

$= \frac{22}{7} \times 3.5 \times 12.5 + 2 \times \frac{22}{7} \times 3.5 \times 3.5$

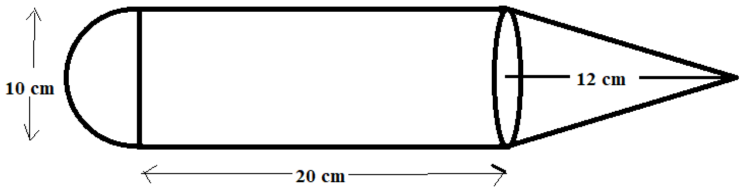
$\therefore$ **TSA of the toy = 214.5 cm²**

2. A Toy is made in the shape of a cylinder with one hemisphere stuck to one end and a cone to the other end. The length of the cylindrical part of the toy is 20cm and its diameter is 10 cm. If the height of the cone is 12 cm. Find the surface area of the toy.

$Hemisphere: r_{hs} = 5cm$

$Cylinder: r_{cylinder} = 5cm, h_{cylinder} = 20cm$

$Cone: r_{cone} = 5cm, h_{cone} = 12cm$



$Slant\ height: l_{cone} = \sqrt{{r_{cone}}^2 + {h_{cone}}^2}$

$= \sqrt{5^2 + 12^2}$

$\therefore l_{cone} = 13cm$

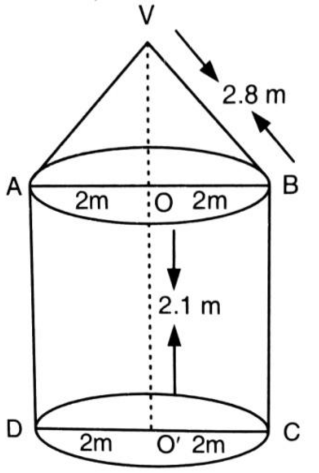
$TSA\ of\ the\ toy = CSA\ of\ hemisphere + CSA\ of\ cylinder + CSA\ of\ cone$

$= 2\pi {r_{hs}}^2 + 2\pi {r_{cylinder}}^2 + \pi r_{cone} l_{cone}$

$= 2 \times \frac{22}{7} \times 5^2 + 2 \times \frac{22}{7} \times 5^2 + \frac{22}{7} \times 5 \times 13$

$\therefore \boldsymbol{TSA\ of\ the\ toy = 518.57cm^2}$

3. A tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical parts are 2.1 m and 4 m respectively and the slant height of conical part is 2.8 m. Find the area of the canvas used for making the tent. Also find the cost of canvas of the tent at the rate of Rs. 500 per m^2.



$Cylinder: H = 2.1m, D = 4m, R = 2m$

$Cone: l = 2.8m, r = 2m$

$TSA\ of\ the\ canvas = CSA\ of\ cylinder + CSA\ of\ cone$

$= 2\pi RH + \pi rl$

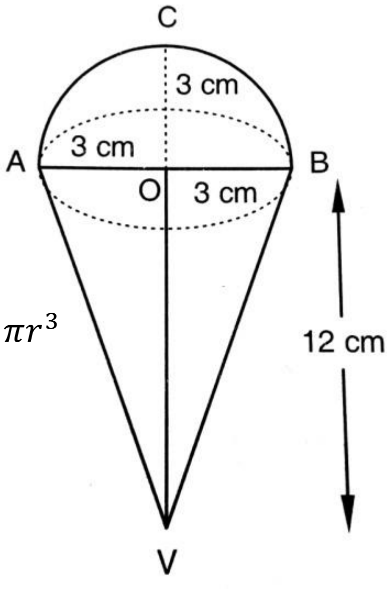
$= 2 \times \frac{22}{7} \times 2 \times 2.1 + \frac{22}{7} \times 2 \times 2.8$

$\therefore \boldsymbol{TSA\ of\ the\ canvas = 44m^2}$

$Total\ cost\ of\ the\ canvas\ at\ the\ rate\ of\ Rs. 500\ per\ m^2 = Rs. (500 \times 44)$

$\therefore \boldsymbol{Total\ cost\ of\ the\ canvas\ = Rs. 22000}$

4. A container is shaped like a right circular cylinder having radius of the base 6 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled into cones of height 12 cm and radius 3 cm, having a hemispherical shape of same radius on the top as in the figure. Find the number of such cones which can be filled with ice-cream.



$Cylindrical\ container: R = 6cm, H = 15cm$
$Cone: r = 3cm, h = 12cm$

$V_1 = Volume\ of\ container = \pi R^2 H$
$V_1 = \pi \times 6^2 \times 15 = 540\pi\ cm^3$

$$V_2 = Volume\ of\ cone + Volume\ of\ hemisphere = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 (h + 2r)$$

$$= \frac{1}{3} \times \pi \times 3^2 (12 + 2 \times 3)$$

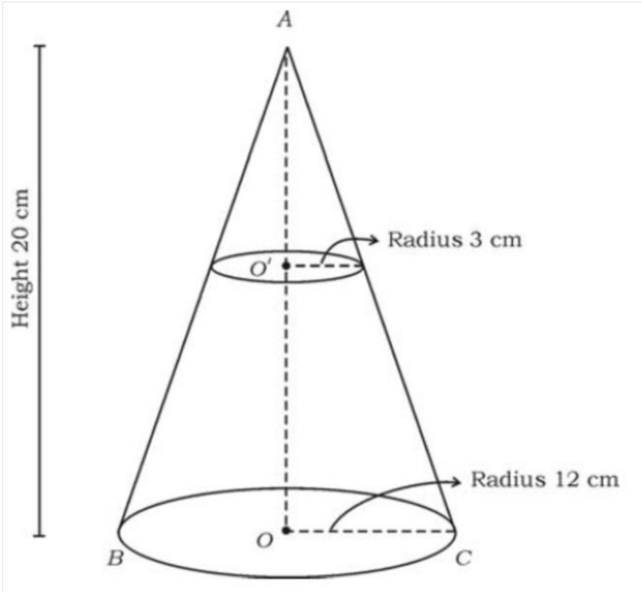
$$V_2 = 54\pi cm^3$$

$$Number\ of\ ice\ cream\ cones = \frac{V_1}{V_2} = \frac{540\pi}{54\pi}$$

$\therefore \boldsymbol{Number\ of\ ice\ cream\ cones\ = 10}$

Five marks questions

1. A cone is of the radius of its base 12 cm and height 20 cm. If the top of this cone is cut to form a small cone of radius of base 3 cm, then the remaining part of the solid cone becomes a frustum. Calculate the volume of the frustum.



$Original\ cone: r_1 = 12cm, h_1 = 20cm$

$Removed\ cone: r_2 = 3cm, h_2 = ?$

$\frac{h_2}{h_1} = \frac{r_2}{r_1}$ $\qquad \frac{h_2}{20} = \frac{3}{12}$

$h_1 = 5cm$

$h = 20 - 5 = 15cm$

$V = \frac{1}{3}\pi h(r_1{}^2 + r_2{}^2 + r_1 r_2)$

$= \frac{1}{3} \times \frac{22}{7} \times 15(12^2 + 3^2 + 12 \times 3)$

$\therefore$ **Volume of the frustum** $= \mathbf{2970cm^3}$

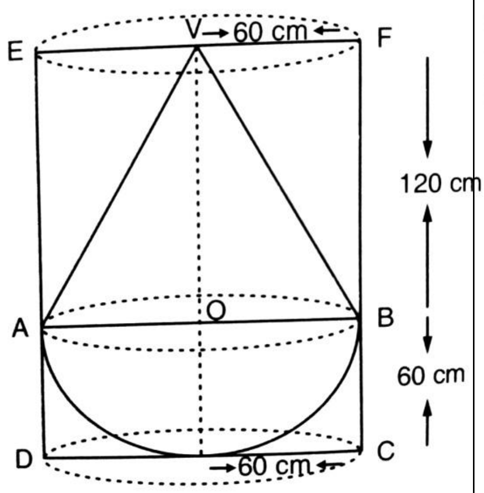
2. A solid consisting of a right cone standing on a hemisphere is placed upright in a right circular cylinder full of water and touches the bottom as shown in the figure. Find the volume of water left in the cylinder, if the radius of the cylinder is 60cm and its height is 180cm, the radius of the hemisphere is 60cm and height of the cone is 120cm, assuming that the hemisphere and the cone have common base.

$Cylinder: r_{cy} = 60cm, h_{cy} = 180cm \qquad Cone: r_{co} = 60cm, h_{co} = 120cm$

$Hemisphere: r_{hs} = 60cm$

$The\ volume\ of\ the\ water\ left\ out\ in\ the\ cylinder = V$

$V_{water} = V_{cylinder} - V_{cone} - V_{hemisphere}$



$= \pi r_{cy}{}^2 h_{cy} - \frac{1}{3}\pi r_{co}{}^2 h_{co} - \frac{2}{3}\pi r_{hs}{}^3$

$= \pi \times 60^2 \times 180 - \frac{1}{3} \times \pi \times 60^2 \times 120 - \frac{2}{3} \times \pi \times 60^3$

$= \pi \times 60^2[180 - 40 - 40]$

$= \frac{22}{7} \times 60 \times 60 \times 100 = \frac{22 \times 360000}{7} cm^3$

$$V = \frac{22 \times 360000}{7 \times (100)^3} m^3$$

$\therefore \boldsymbol{The\ volume\ of\ the\ water\ leftout\ in\ the\ cylinder = 1.1314m^3}$