SECOND YEAR HIGHER SECONDARY MODEL EXAMINATION , MARCH 2022

Part I

A . Answer any five questions (5 x 1 = 5)

1. x

2. $\cos\left(sec^{-1}x+cosec^{-1}x\right)=\cos\frac{\pi}{2}=0$

3. c) $k^3|A|$

4. Area of circle = πr^2

$$\frac{dA}{dr}=2\pi r$$

$$= 12\pi$$

5. f(x) = sin x

f(-x) = sin(-x) = - sin x

f(x) is an odd function.

Therefore $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin x\, dx=0$

6. Degree = 1

7. $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & 3\\ 8 & 4 & 12\end{vmatrix}=4\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & 3\\ 2 & 1 & 3\end{vmatrix}=0$

8. $\vec{r}=\lambda\left(2\hat{i}+4\hat{j}+2\hat{k}\right)$

9. $P\left(B/A\right)=\frac{P\left(A\cap B\right)}{P\left(A\right)}=\frac{P\left(A\right)}{P\left(A\right)}=1$

B. Answer all questions (4 x 1 = 4)

10. $\sin x=\frac{1}{2}$

Principal values is $\frac{\pi}{6}$

11. 0

12. $y=e^{\log x}$

that is, y = x

$$\frac{dy}{dx}=1$$

13. 1

Part II

A. Answer any two questions (2 x 2 = 4)

14. $A=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}$

$a_{11}=1, a_{12}=0, a_{21}=3, a_{22}=2$

$A=\begin{bmatrix}1 & 0\\ 3 & 2\end{bmatrix}$

15. $f(x)=x^2-4x+6$

$f'(x)=2x-4$

$f'(x)=0\Rightarrow 2x-4=0$

x = 2

2 divides R into $(-\infty,2)$ and $(2,\infty)$

In $(2,\infty)$, $f'(x)>0$,

f(x) is strictly increasing in $(2,\infty)$

16. $\frac{dy}{dx}=3x^2$

slope , at (1,1) = 3

slope of normal = $-\frac{1}{3}$

Equation of normal,

$y-1=-\frac{1}{3}(x-1)$

ie, x + 3y – 4 = 0

17. Let y=mx

$\frac{dy}{dx}=m$

So, $y=\frac{dy}{dx}x$

B. Answer any two questions (2 x 2 = 4)

18. $x-y=\pi$

differentiating w.r.t x

$1-\frac{dy}{dx}=0$

$\frac{dy}{dx}=1$

19. $\frac{dy}{dx} = 1 + \frac{y}{x}$

$\frac{dy}{dx} - \frac{y}{x} = 1$

$P = -\frac{1}{x}\ ,\ Q = 1$

$I\ .\ F = e^{\int -\frac{1}{x}\,dx} = e^{-\log x} = \frac{1}{x}$

General solution is,

$y\ .\ IF = \int Q\ .\ IF\ \ dx + C$

$y\ .\ \frac{1}{x} = \int \frac{1}{x}\ dx + C$

$\frac{y}{x} = \log\ |x| + C$

20. Vectors are coplanar if $[\vec{a}\,\vec{b}\,\vec{c}]$=0

$$\begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}$$

$= 1\ (15 - 12) + 2\ (-10 + 4) + 3\ (6 - 3)$

$= 3 - 12 + 9 = 0$

Therefore the vectors are coplanar.

Part III

A. Answer any three questions (3 x 3 = 9)

21. (1,1), (2,2), (3,3) $\in$ R

therefore R is reflexive.

(1,2) $\in$ R but (2,1) $\notin$ R

therefore R is not reflexive.

(1,2), (2,3) $\in$ R but (1,3) $\notin$ R

Therefore R is not transitive.

22. $A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$= \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$kA - 2\ I = k \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k \end{bmatrix}$

$A^2 = kA - 2\ I$

$\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k \end{bmatrix}$

ie, 3 k – 2 = 3

k= 1

23. Area of parallelogram = $|\vec{a} \times \vec{b}|$

$\vec{a} \times \vec{b} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{bmatrix}$

$= \hat{i}\ (1 + 4) - \hat{j}\ (3 - 4) + \hat{k}\ (-3 - 1)$

$= 5\ \hat{i} + \hat{j} - 4\ \hat{k}$

Area = $\sqrt{25 + 1 + 16} = \sqrt{42}$

24. Probability of solving A , P(A) = $\frac{1}{2}$

Probability of solving B, P(B) = $\frac{1}{3}$

A and B are independent events.

$P\ (A') = 1 - \frac{1}{2} = \frac{1}{2}$

$P\ (B') = 1 - \frac{1}{3} = \frac{2}{3}$

Probability of exactly one of them solves the problem = $P\ (A\ \cap B') + P\ (A' \cap B)$

$= P\ (A)\ .\ P\ (B') + P\ (A')\ P\ (B)$

$= \frac{1}{2}\ .\ \frac{2}{3} + \frac{1}{2}\ .\ \frac{1}{3} = \frac{3}{6} = \frac{1}{2}$

B. Answer any two questions (2 x 3 = 6)

25.

$\wedge$	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

26. A = I A

$$\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} A$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix}1 & 2\\0 & -5\end{bmatrix} = \begin{bmatrix}1 & 0\\-2 & 1\end{bmatrix} A$$

$R_2 \to \frac{R_2}{-5}$

$$\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\\frac{2}{5} & -\frac{1}{5}\end{bmatrix} A$$

$R_1 \to R_1 - 2R_2$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}\frac{1}{5} & \frac{2}{5}\\\frac{2}{5} & -\frac{1}{5}\end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix}\frac{1}{5} & \frac{2}{5}\\\frac{2}{5} & -\frac{1}{5}\end{bmatrix}$$

27. By definition

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} h[f(a) + f(a+h)..+f(a+(n-1)h$$

where $h = \frac{b-a}{n} \to 0$ as $n \to \infty$

Here $h = \frac{2-0}{n} = \frac{2}{n}$

$f(a) = f(0) = 0$

$f(a+h) = (h)^2$

$f(a+2h) = (2h)^2$

$f(a+3h) = (3h)^2$

....

$f(a+(n-1)h) = ((n-1)h)^2$

So,

$$\int_0^2 x^2\,dx = \lim_{n\to\infty} \frac{2}{n}\left[h^2 + 2^{2\cdot} h^2 + 3^{2\cdot} h^2 + ... + (n-1)^{2\cdot} h^2\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[h^2\left(1^2 + 2^2 + 3^2 + ... + (n-1)^2\right)\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\left(\frac{2}{n}\right)^2 \frac{(n-1)n(2n-1)}{6}$$

$$= \lim_{n\to\infty} \frac{4}{3}\left(1 - \frac{1}{n}\right)\left(2 - \frac{1}{n}\right)$$

$$= \frac{4}{3} x\, 1\, x\, 2 = \frac{8}{3}$$

Part IV

A. Answer any three questions (3 x 4 = 12)

28. (i) $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1}\left(\frac{\frac{1}{2} + \frac{2}{11}}{1 - \frac{1}{2} x \frac{2}{11}}\right)$

$$= \tan^{-1}\left(\frac{\frac{15}{22}}{\frac{20}{22}}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right)$$

(ii) Put x = cos θ ,then $\theta = \cos^{-1} x$

$$\cos^{-1}(4x^3 - 3x) = \cos^{-1}(4\cos^3\theta - 3\cos\theta)$$

$$= \cos^{-1}(\cos 3\theta)$$

$$= 3\theta$$

$$= 3\cos^{-1} x$$

29. f(x) is continuous on [2,4]

f(x) is differentiable on (2,4)

$f(a) = f(2) = 2^2 = 4$

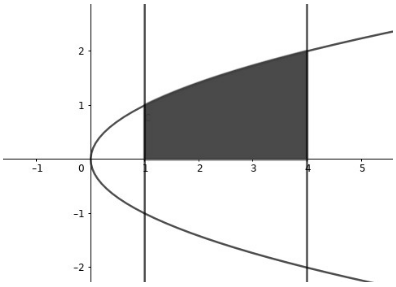
$f(b) = f(4) = 4^2 = 16$

$f'(c) = \frac{f(b) - f(a)}{b-a} = \frac{16-4}{4-2} = \frac{12}{2} = 6$

But $f'(c) = 2c$

$2c = 6 \Rightarrow c = 3 \in (2,4)$. Hence verified.

30.



Area =

$$\int_1^4 y\,dx = \int_1^4 \sqrt{(x)}\,dx = \left[\frac{x^{\left(\frac{3}{2}\right)}}{\frac{3}{2}}\right] = \frac{2}{3}\left(4^{\frac{3}{2}}-1\right) = \frac{14}{3}$$

31. $\vec{a}_1 = \hat{i} + \hat{j}$

$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}$

$\vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$

$\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\ \hat{i} - \hat{j} - 7\ \hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{9+1+49} = \sqrt{59}$$

$$(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1) = (3\hat{i} - \hat{j} - 7\hat{k}).(\hat{i} - \hat{k}) = 10$$

$$d = \left|\frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right| = \frac{10}{\sqrt{59}}$$

B. Answer any one question (1 x 4 = 4)

32. a) 0.1 + k + 2k + 2k + k = 1

0.1 + 6k = 1

k = 0.15

b) $P(x<3) = P(x=0) + P(x=1) + P(x=2)$

= 0.1 + k + 2k

= 0.1 + 3 x 0.15

= 0.55

33. Cartesian form is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$$\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$$

$$\frac{6x + 4y + 3z}{12} = 1$$

ie $6x + 4y + 3z = 12$

Vector form is $\vec{r}.\hat{n} = d$

$$\vec{r}.(6\hat{i} + 4\hat{j} + 3\hat{k}) = 12$$

Part V

Answer any two questions (2 x 6 = 12)

34. A X = B

$$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{vmatrix} = -17$$

Co-factor matrix = $\begin{bmatrix} -1 & -8 & -10 \\ -5 & -6 & 1 \\ -1 & 9 & 7 \end{bmatrix}$

adj A = $\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$A^{-1} = \frac{1}{-17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$= \frac{1}{-17}\begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

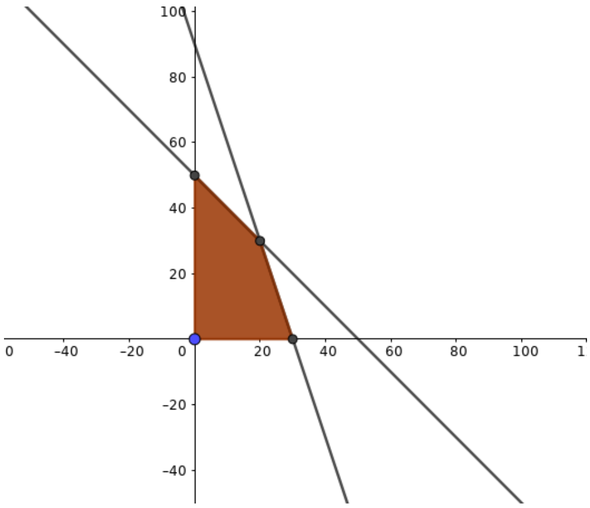
x = 1 , y = 2 , z = 3

35. x + y = 50

x	y
0	50
50	0

3x + y = 90

x	y
0	50
50	0



Point	**Z = 4x + y**
(0,0)	0
(30,0)	120
(20, 30)	110
(0,50)	50

Maximum of Z is 120 at (30,0).

36. i) put $t = \tan^{-1} x$, *then* $dt = \frac{1}{1+x^2} dx$

$$\int \frac{\sin(\tan^{-1} x)}{1+x^2} dx = \int \sin t \, dt$$

$$= -\cos t$$

$$= -\cos(\tan^{-1} x)$$

ii) Let $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

Then

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} x + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

Adding them, we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 \, dx = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

iii) $\int \frac{dx}{x^2 - 16} = \int \frac{dx}{x^2 - 4^2}$

$$= \frac{1}{2 X 4} \log\left|\frac{x-4}{x+4}\right| + c$$

$$= \frac{1}{8} \log\left|\frac{x-4}{x+4}\right| + c$$

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