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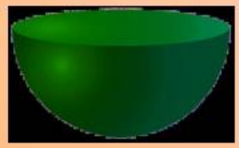
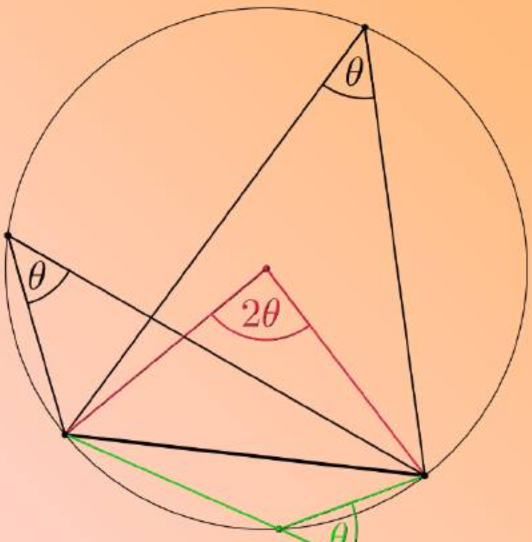
X Maths
Main Concepts





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Chapter – 1
Arithmetic Sequence

Main Concepts

* A sequence got by starting with any number and adding a fixed number repeatedly is called an **arithmetic sequence.**

* The numbers forming a sequence are called its **terms .** The **terms** in a sequence are written in algebra as

$$x_1, x_2, x_3, x_4, \ldots\ldots$$

* The fixed number which is adding repeatedly is called **common difference** and usually denoted using the letter '**d**'

$$d = x_2 - x_1 = x_3 - x_2 = \ldots\ldots.$$

We can find the terms of given A.S when any term and common difference is given

Eg:

$x_8 = x_2 + 6d$	$x_8 = x_{12} - 4d$
$x_9 = x_5 + 4d$	$x_{15} = x_{20} - 5d$
$x_{15} = x_7 + 8d$	$x_6 = x_{12} - 6d$

If a, b, c are 3 consecutive terms of an arithmetic sequence, then $2b = a + c$

In any arithmetic sequence

$$\text{Common difference} = \frac{\text{Term difference}}{\text{Position difference}}$$

So,

Term difference= Position difference × Common difference

∴ **Term difference is a multiple of common difference**

Considering an arithmetic sequence with ***terms and common difference as natural numbers***, the terms of this sequence ***leave same remainder when they are divided by its common difference***

nth term (Algebraic form) of any arithmetic sequence is

$$X_n = f + (n - 1) d$$

or

$$X_n = d n + f - d$$

Where,

f – first term

d – Common difference

n^{th} term (Algebraic form) of an arithmetic sequence can also be written as

$$X_n = an + b$$

where $d = a$, $f = a + b$

No: of terms of an A. S is

$$n = \frac{X_n - X_1}{d} + 1$$

Where,

X_n – n^{th} term / last term

X_1 – first term

d- common difference

Sum of Terms

The **Sum** of any consecutive **odd number of terms** of an arithmetic sequence

Sum of terms = Middle Term x No: of terms

$$\text{Middle Term} = \frac{\text{Sum}}{\text{No: of terms}}$$

Some peculiarities of arithmetic sequence

a) When number of terms odd

Pair sum = 2 × middle term

$$\text{Middle term} = \frac{\text{Pair Sum}}{2}$$

b) When number of terms even

Sum of terms = No: of pairs × Pair sum

$$\text{Pair sum} = \frac{\text{Sum of terms}}{\text{No: of pairs}}$$

Sums

* The sum of any number of **consecutive natural numbers, starting with one** is

$$1 + 2 + 3 + \dots\dots + n = \frac{n(n+1)}{2}$$

* Sum of **first 'n' even natural** numbers

$$2 + 4 + 6 + \dots\dots + 2n = n(n+1)$$

* Sum of **first 'n' odd natural** numbers

$$1 + 3 + 5 + \dots\dots + 2n-1 = n^2$$

The sum of any number of consecutive terms of an arithmetic sequence

(a) (Sum) $S_n = a\frac{n(n+1)}{2} + bn$ Where, n = number of terms

$a = d$

$b = f - d$

(b) $S_n = \frac{n}{2}(X_1 + X_n)$ Where, X_1 = first term

X_n = last term

(c) $S_n = \frac{n}{2}[2f + (n-1)d]$ Where, n = number of terms

f = first term

d = common difference

The algebraic form of the
sum of an arithmetic sequence is

$$p n^2 + q n$$

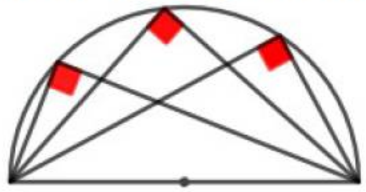
where $p = \frac{d}{2}$ and $p + q = f$

Chapter – 2
Circles

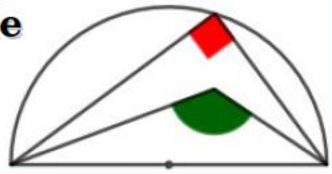
Main Concepts

If we join the ends of a diameter of a circle to a point on the circle , we get a right angle .

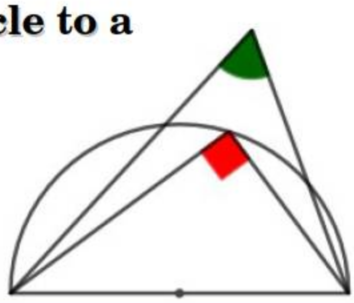
Angle in a semicircle is right.



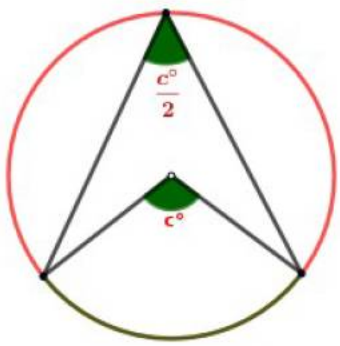
Joining the ends of the diameter of a circle to a point inside the circle gives an angle greater than 90°.



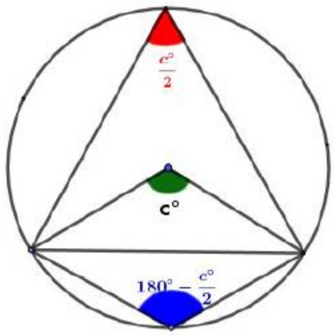
Joining the ends of the diameter of a circle to a point outside the circle gives an angle less than 90°.



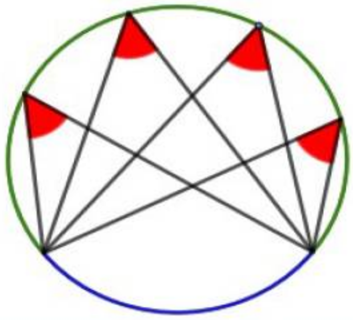
The angle made by an arc of a circle on the alternate arc is half the angle made at the centre.



A pair of an angles on an arc and its alternate arc are supplementary .



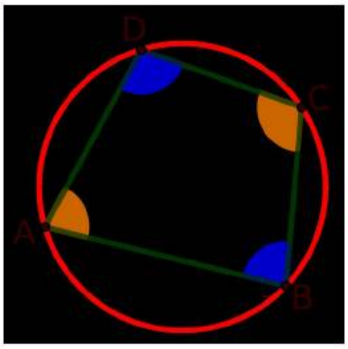
All angles made by an arc on its alternate arc are equal .



If all four vertices of a quadrilateral are on a circle, then its opposite angles are supplementary.

$\angle B + \angle D = 180°$

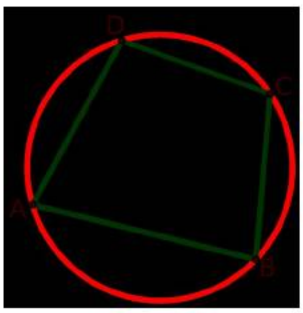
$\angle A + \angle C = 180°$



If the opposite angles of a quadrilateral are supplementary, we can draw a circle passing through all four of its vertices.

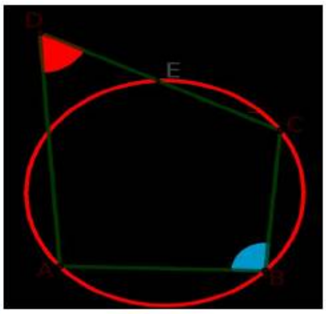
This quadrilateral can be called as a **Cyclic Quadrilateral**

ABCD is a Cyclic Quadrilateral

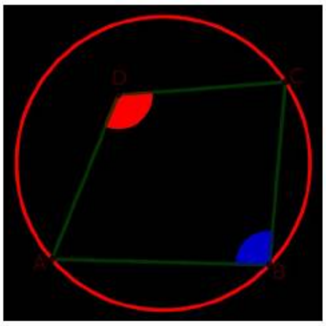


If **vertex D** of quadrilateral ABCD is ,

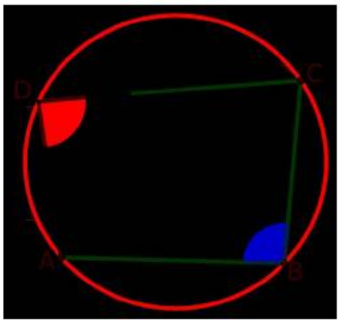
Outside the circle drawn through the other three veritces, then
$\angle B + \angle D < 180^\circ$



Inside the circle drawn through the other three veritces, then
$\angle B + \angle D > 180^\circ$



On the circle drawn through the other three veritces, then
$\angle B + \angle D = 180^\circ$



Cyclic quadrilaterals are those quadrilaterals with opposite angles supplementary.

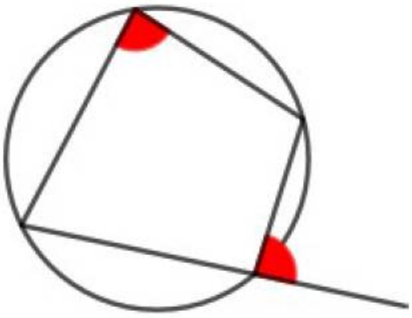
Quadrilaterals which are always cyclic are

(i) Square

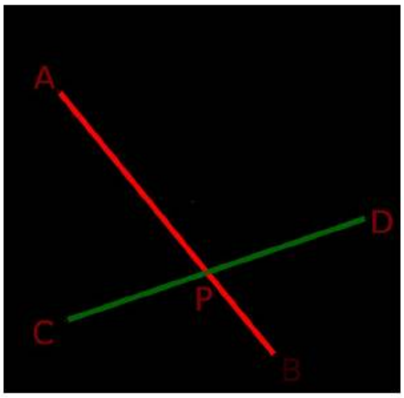
(i) Rectangle

(iii) Isosceles Trapezium

In a cyclic quadrilateral any outer angle is equal to the inner angle at the opposite vertex.

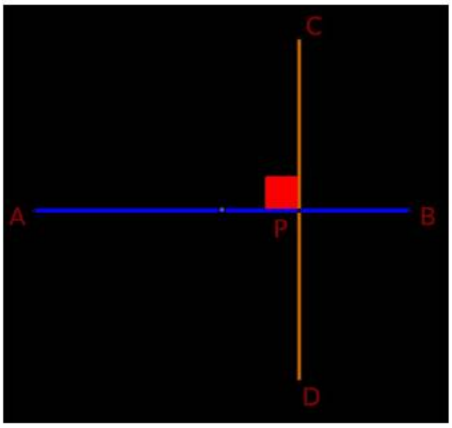


If two chords of a circle intersect within the circle, then the products of the parts of the two chords are equal.



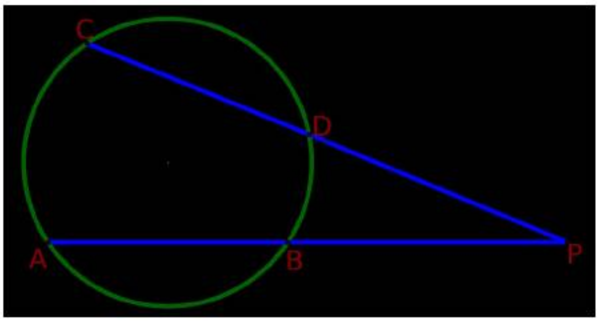
PA x PB = PC x PD

The product of the parts into which a diameter of a circle is cut by a perpendicular chord, is equal to the square of half the chord.



$PA \times PB = PC^2$

If the chords AB and CD of the circle are extended to meet at P.



Then,

$PA \times PB = PC \times PD$

Chapter 3
Mathematics Of Chance

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$\text{Probability of pairs} = \frac{\text{Number of favourable pairs}}{\text{Total number of pairs}}$$

To Remember

* Even numbers 2, 4, 6, 8, 10, 12, ...
* Odd Numbers 1, 3, 5, 7, 9, 11, ...
* Prime numbers 2, 3, 5, 7, 11, 13, 17, 19, ...
* Perfect Squares 1, 4, 9, 16, 25, 36, 49, ...

Total two digit numbers = 90

Total three digit numbers = 900

When a die is thrown , total number of outcomes = 6

When two dice are thrown , total number of outcomes = 6 × 6

= 36

Chapter-4
Second Degree Equations

Main Concepts

Equations of the form $ax^2 + bx + c = 0$, $a \neq 0$ are second degree equations.

The values of x satisfying the equation are called solutions of the equation.

Completing the square method

To convert '$x^2 + 2ax$' to the perfect square$(x + a)^2$

add the square of half the coefficient of 'x'. i.e 'a^2'

$$x^2 + 2ax + a^2 = (x + a)^2$$

$x^2 + 2x$ ⟶ Completing square ⟶ $x^2 + 2x + 1^2 = (x + 1)^2$

$x^2 + 20x$ ⟶ Completing square ⟶ $x^2 + 20x + 10^2 = (x + 10)^2$

$x^2 + 6x$ ⟶ Completing square ⟶ $x^2 + 6x + 3^2 = (x + 3)^2$

$x^2 + 8x$ ⟶ Completing square ⟶ $x^2 + 8x + 4^2 = (x + 4)^2$

Standard form of second degree equation is

$$ax^2 + bx + c = 0 \quad \text{where } a \neq 0$$

To get $ax^2 + bx + c = 0$, we must take

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a = Coefficient of x^2

b = Coefficient of x

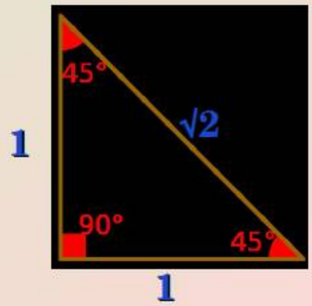
c = Constant

Important points

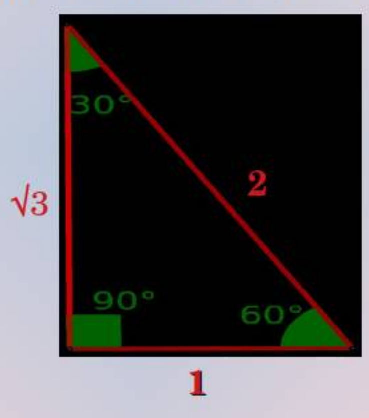
Statement	Algebra
Three more than a number	$x + 3$
Three less than a number	$x - 3$
Two times a number	$2x$
Half of a number	$\frac{x}{2}$
Two consecutive natural numbers	$x , x + 1$
Two consecutive even numbers	$x , x + 2$
Two consecutive odd numbers	$x , x + 2$
A number and its reciprocal	$x , \frac{1}{x}$
A number and is square	x , x^2
Two numbers with sum 10	$x , 10 - x$
Two numbers with difference 10	$x , 10 + x$
Two numbers with product 10	$x , \frac{10}{x}$

Chapter -5
Trigonometry

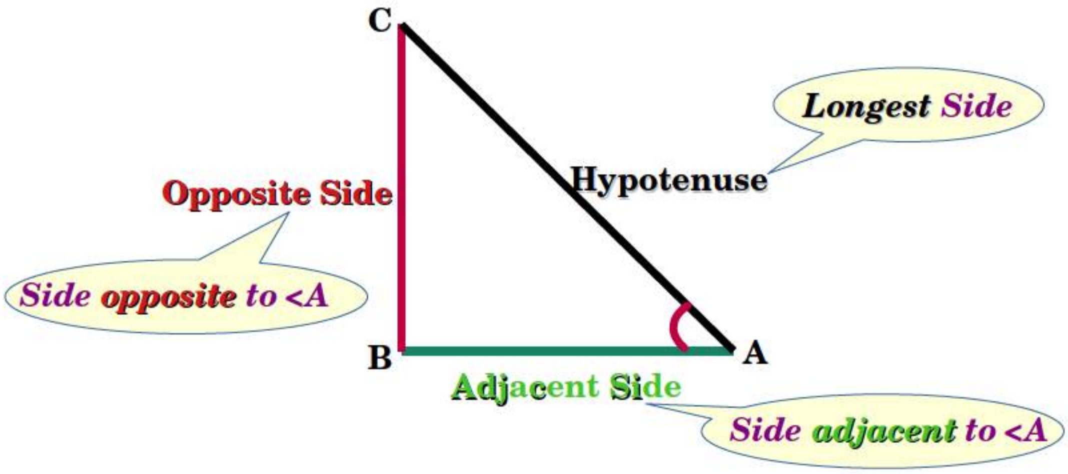
The sides of any triangle of angles 45°, 45°, 90° are in the ratio 1 : 1 : √2



In any triangle of angles 30°, 60°, 90° the sides are in the ratio 1 : √3 : 2



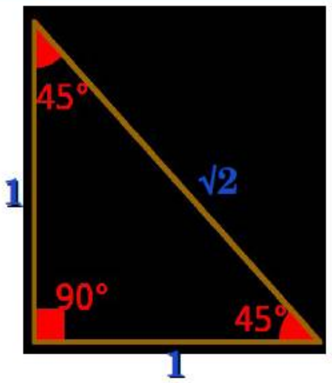
Trigonometric Ratios

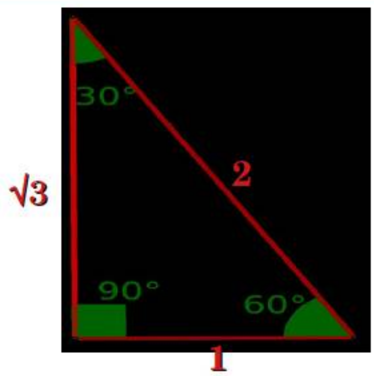


$$\text{Sin } A = \frac{\text{Opposite Side}}{\text{Hypotenuse}}$$

$$\text{Cos } A = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$$

$$\text{Tan } A = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$$





$\text{Sin } 45° = \frac{1}{\sqrt{2}}$

$\text{Cos } 45° = \frac{1}{\sqrt{2}}$

$\text{Tan } 45° = 1$

$\text{Sin } 30° = \frac{1}{2}$

$\text{Cos } 30° = \frac{\sqrt{3}}{2}$

$\text{Tan } 30° = \frac{1}{\sqrt{3}}$

$\text{Sin } 60° = \frac{\sqrt{3}}{2}$

$\text{Cos } 60° = \frac{1}{2}$

$\text{Tan } 60° = \sqrt{3}$

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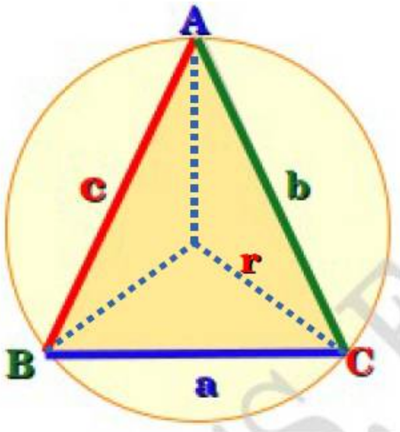
In a circle of radius 'r',

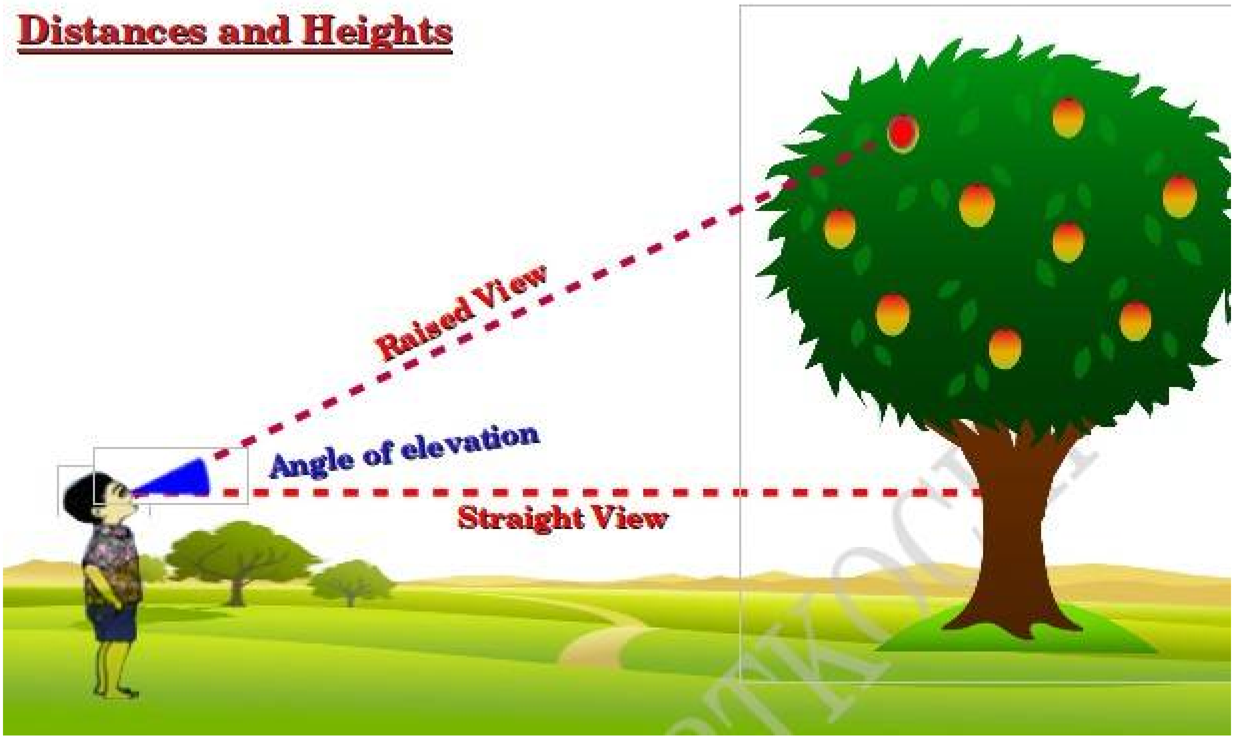
Length of a chord of central angle x° $= 2r \, \text{Sin}\left(\frac{x^\circ}{2}\right)$

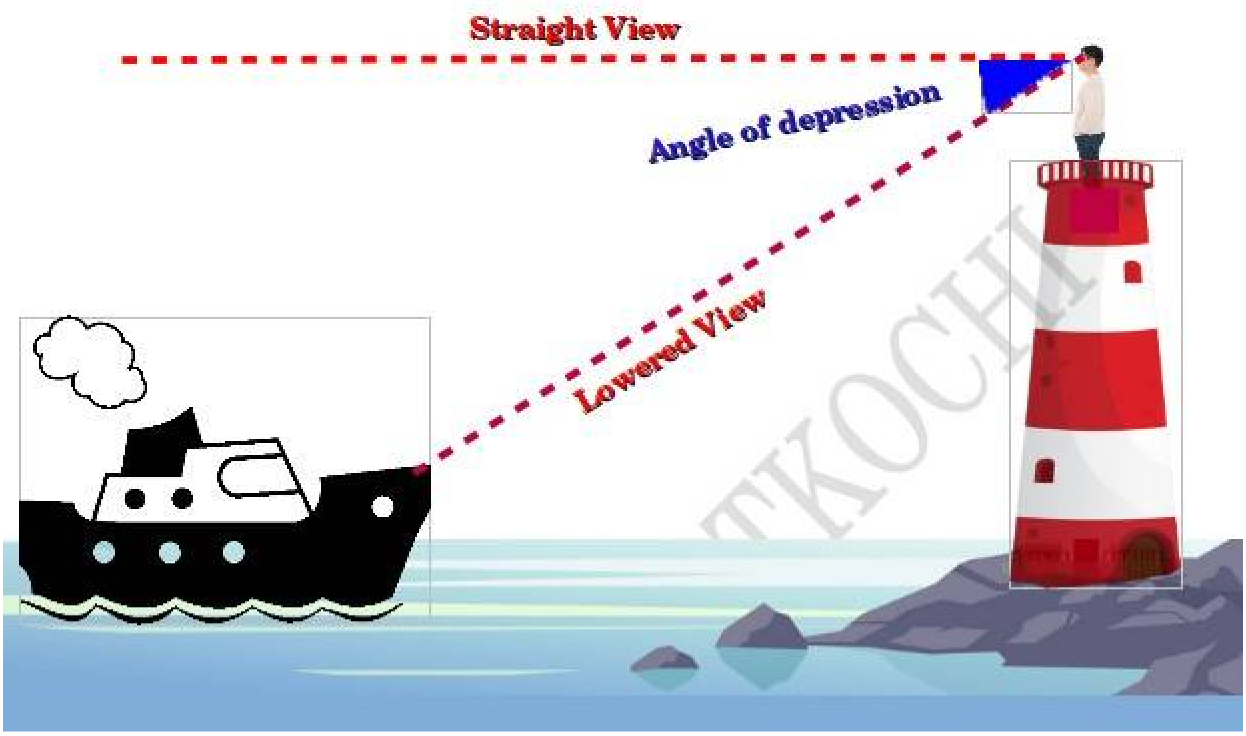
Consider ∆ ABC , let the sides be a, b, c
If ' r ' is the circumradius,



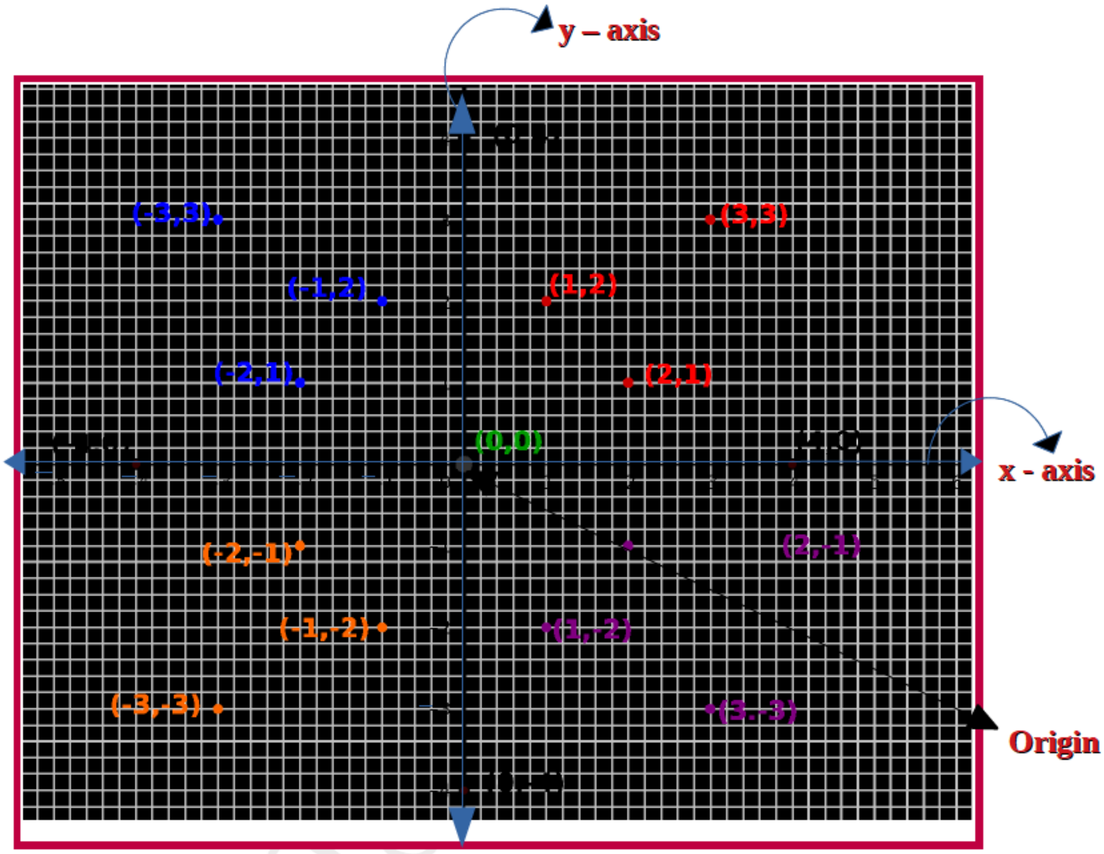
$$\frac{a}{\text{SinA}} = \frac{b}{\text{SinB}} = \frac{c}{\text{SinC}} = 2r$$

Distances and Heights





Chapter – 6
Coordinates



The x coordinate of any point on the y axis is 0

The y coordinate of any point on the x axis is 0

The x coordinate of any point on a line parallel to y axis are equal

The y coordinate of any point on a line parallel to x axis are equal

For any two points A(x_1 , y_1) , B(x_2, y_2) on a plane,

Distance AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Distance of any point (x, y) from the origin is

$\sqrt{(x)^2+(y)^2}$

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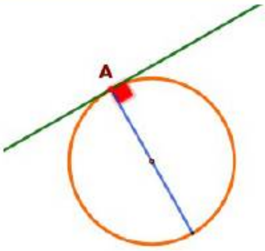
Chapter – 7
Tangents

Main Concepts

The tangent line to a circle at a given point is the straight line that "just touches" the circle at that point.

The tangent at a point on a circle is perpendicular to the diameter through that point.

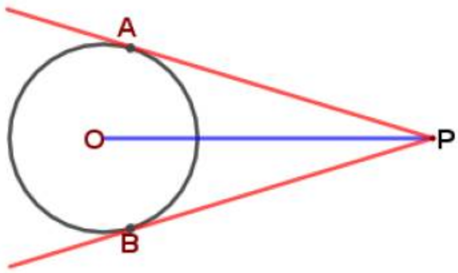


From a point outside a circle, two tangents can be drawn.

&

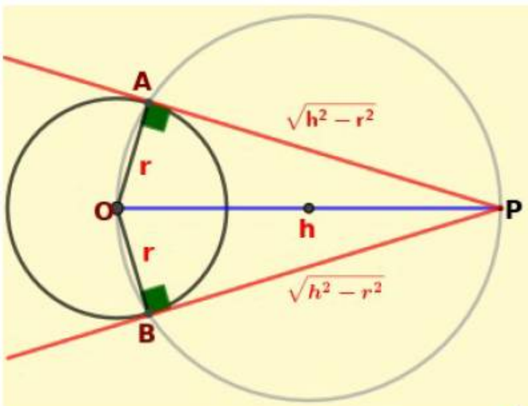
The tangents to a circle from a point are of the same length

PA = PB

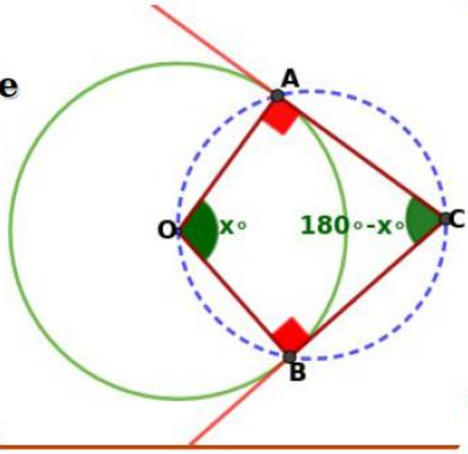


Lengths of the tangents

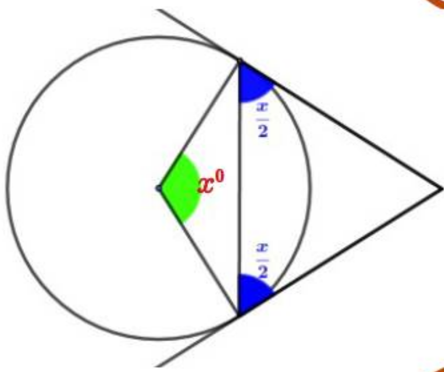
$$PA = PB = \sqrt{h^2 - r^2}$$



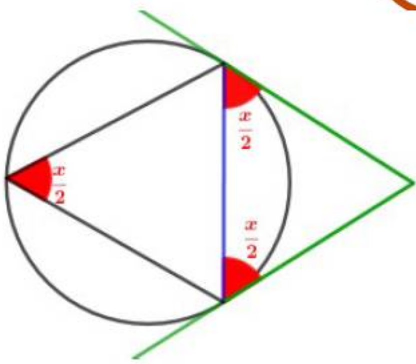
In a circle, the angles between the radii through two points and the angle between the tangents at these points are supplementary.



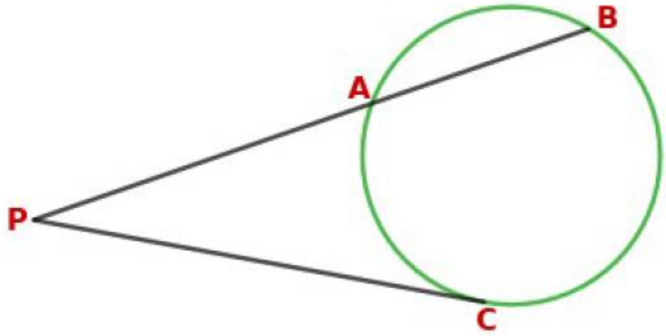
In a circle, the angle between a chord and tangent at either end is half the central angle of the chord.



In a circle, the angle which a chord makes with the tangent at one end on any side is equal to the angle which it makes on the part of the circle on the other side.

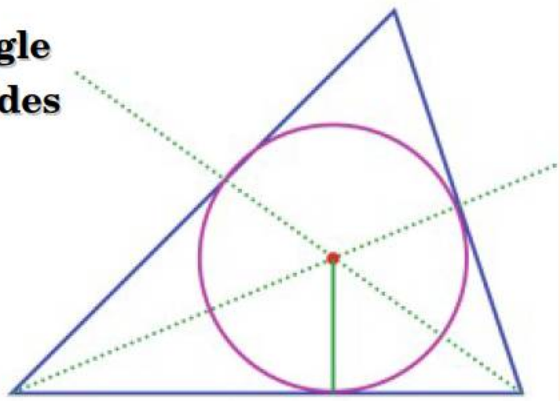


The product of an intersecting line and the part of it outside the circle is equal to the square of the tangent.



$$PA \times PB = PC^2$$

A circle is drawn inside a triangle such that it touches all three sides of the triangle is called the incircle of a triangle.



The radius of the incircle of a triangle is its area divided by half the perimeter.

$$r = \frac{A}{s}$$

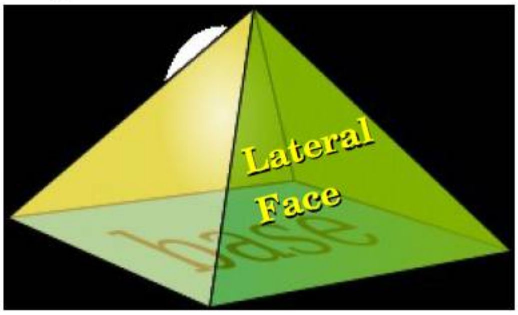
r = Radius of incircle
A = Area of triangle
S = Half the perimeter of the triangle

Chapter 8
Solids

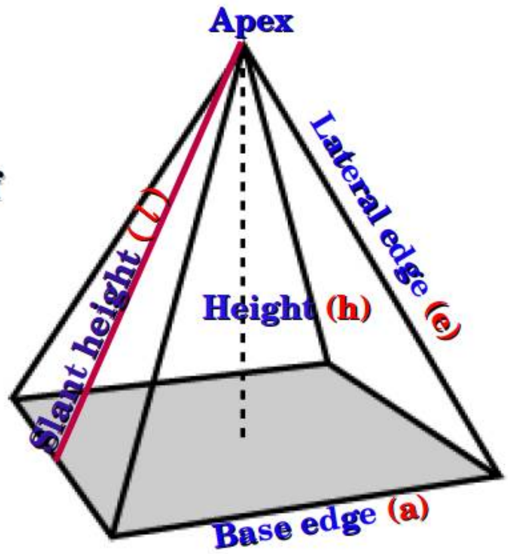
Main Concepts

Square Pyramid

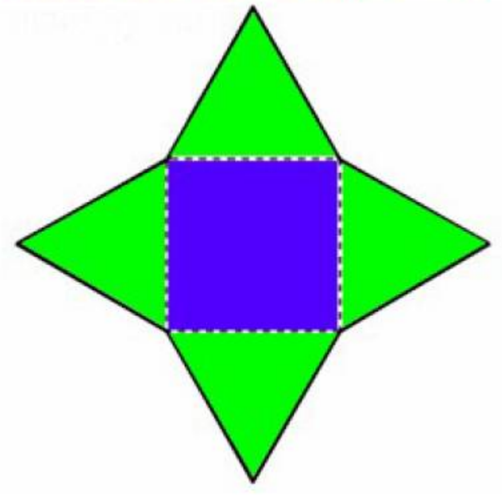
A square pyramid is a pyramid having a ***square base***.
In a square pyramid there are ***four lateral faces*** which are equal isosceles triangles.



The sides of the polygon forming the base of a pyramid are called **base edges (a)** and the other sides of the triangles are called **lateral edges (e)**.
The topmost point of a pyramid is called its **apex**.
The **height (h)** of a pyramid is the perpendicular distance from the apex to the base.
The height of the triangle is called the **slant height (l)** of the pyramid.

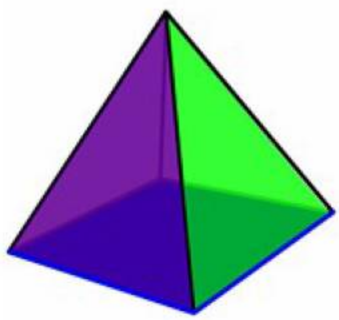


Net of a square pyramid



A square pyramid has a square in the middle and four triangles around it; all four of them are isosceles triangles and they are equal.

Folding the four triangles gives the square pyramid.



Click here to view pyramid and its net

Base area = $(\text{Base edge})^2 = a^2$

Area of one lateral face = $\frac{1}{2} \times$ base edge × slant height

$= \frac{1}{2} \times a \times l$

Lateral surface area = $\overset{2}{\cancel{4}} \times \frac{1}{\cancel{2}} \times a \times l$

$= 2 \times a \times l = 2al$

Total surface area = Base area + Lateral surface area

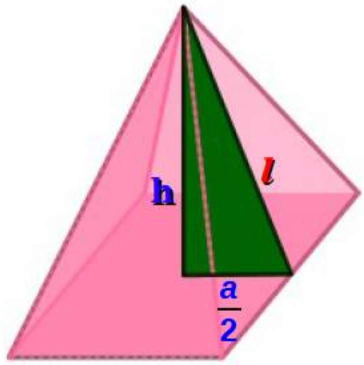
$= a^2 + 2al$

3 right triangles in a square pyramid and relation between different measures using pythagores theorem

i)

Right triangle consisting of slant height (l) half of base edge $\frac{a}{2}$ & height(h),

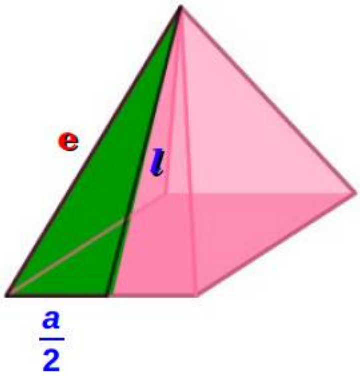
$$l^2 = \left(\frac{a}{2}\right)^2 + h^2$$



ii)

Right triangle consisting of lateral edge(e), half of base edge $\frac{a}{2}$ & slant height (l)

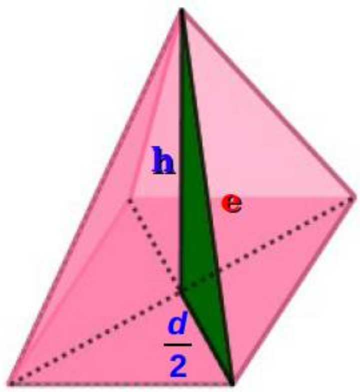
$$e^2 = \left(\frac{a}{2}\right)^2 + l^2$$



iii)

Right triangle consisting of lateral edge (e), half of diagonal $\left(\frac{d}{2}\right)$ & height(h)

$$e^2 = \left(\frac{d}{2}\right)^2 + h^2$$

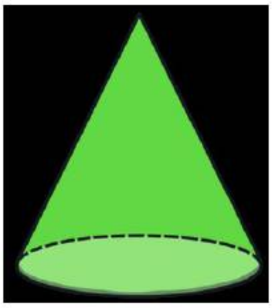


$$\text{Volume of square pyramid} = \frac{1}{3} \times \text{base area} \times \text{height}$$
$$= \frac{1}{3} \times a^2 \times h$$

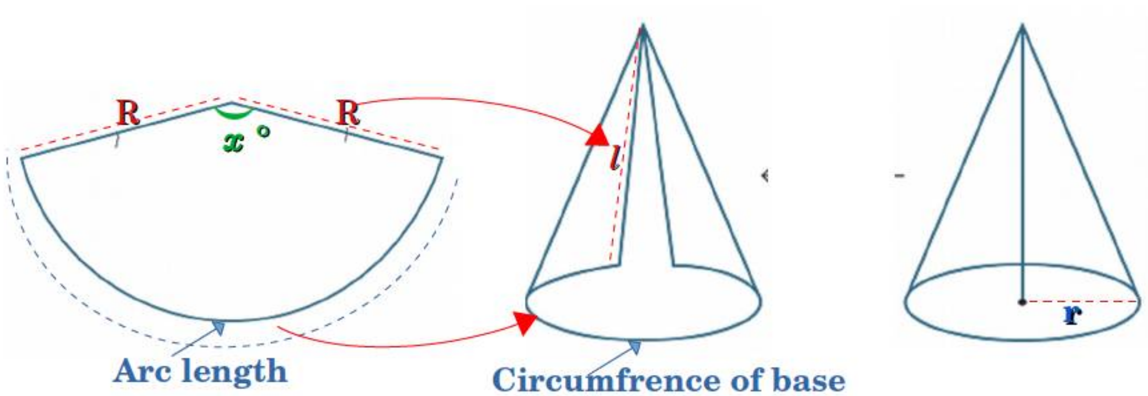
Click here to view animation on volume

Cone

Pyramid like solids with circular bases are called cones.



We can make a cone by rolling up a sector of a circle.



Relation between the dimensions of the sector we start with and the cone we end up with.

Radius (R) of the sector = Slant height (l) of the cone

ie, $R = l$

Arc length of the sector = Circumference of base of the cone

$$\frac{x^\circ}{360^\circ} \times 2\pi R = 2\pi r$$

$$\therefore \quad \frac{x^\circ}{360^\circ} = \frac{r}{R}$$

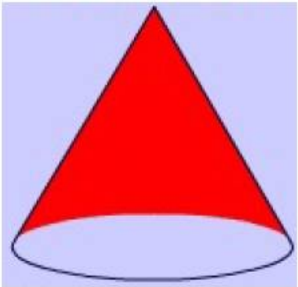
or

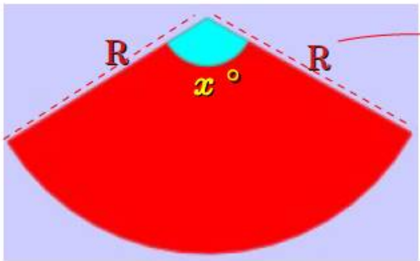
$$\frac{x^\circ}{360^\circ} = \frac{r}{l}$$

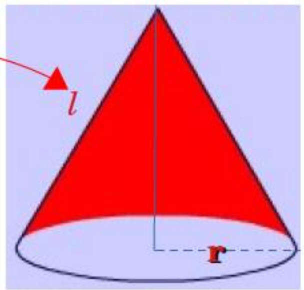
- x° = Central angle of the sector
- R = Radius of the sector
- l = Slant height of the cone
- r = radius of base circle of cone

Curved surface area of a cone

It is the area of the curved part of the cone (Excluding the circular base)







Area of the sector used to make the cone = Curved surface area of the cone

∴ **Curved surface area of the cone** $= \frac{x°}{360°} \times \pi R^2$

Since $R = l$

Curved surface area of the cone $= \frac{x°}{360°} \times \pi l^2$

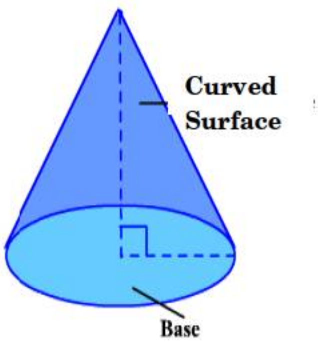
We have $\frac{x°}{360°} = \frac{r}{l}$

∴ **Curved surface area of the cone** $= \frac{r}{l} \times \pi l^2$

$= \pi r l$

$x°$ = Central angle of the sector
R = Radius of the sector
l = Slant height of the cone
r = radius of base circle of cone

Surface area of the cone

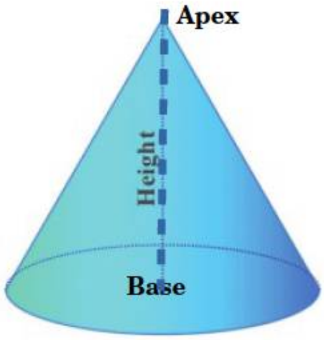


Surface area of a Cone = Curved Surface area + Base Area

$= \pi r l + \pi r^2$

Height of a cone

The height of a cone is the perpendicular distance from the apex to the base

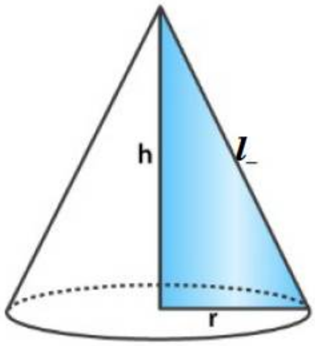


Relation between height (h), slant height (l) & base-radius(r) of a cone

$$(\text{Slant height})^2 = (\text{height})^2 + (\text{base-radius})^2$$

$$l^2 = h^2 + r^2$$

$$l = \sqrt{h^2 + r^2}$$



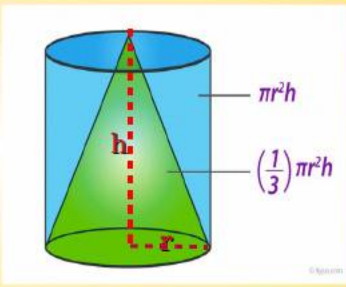
Volume of a cone

$$\text{Volume of cone} = \frac{1}{3} \times \text{base area} \times \text{height}$$

$$= \frac{1}{3} \times \pi r^2 \times h$$

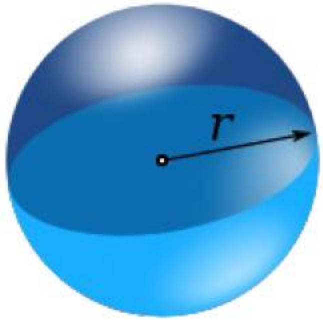
Where, r = base-radius of cone

h = height of cone



Sphere

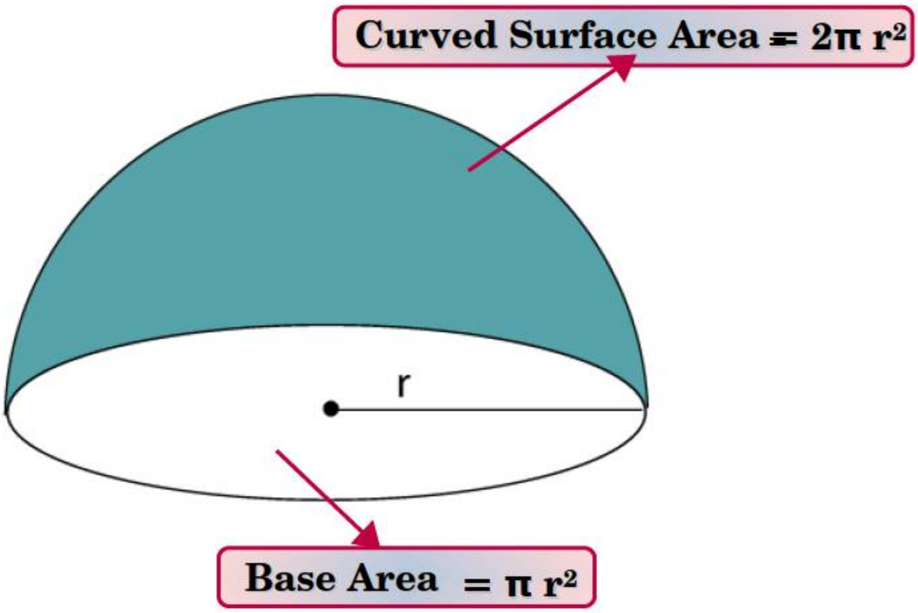
A sphere has only one face.



Surface area of a sphere of radius 'r' = $4\pi r^2$

Volume of a sphere of radius 'r' = $\frac{4}{3}\pi r^3$

Hemisphere



Total Surface Area sphere of radius 'r' = $3\pi r^2$

Volume of a sphere of radius 'r' = $\frac{2}{3}\pi r^3$

Chapter– 9
Geometry and Algebra

Main Concepts

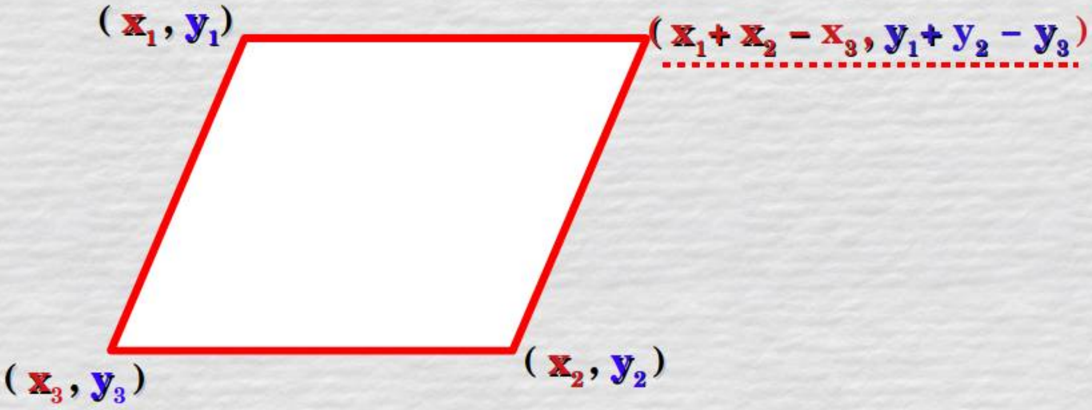
Mid Point of line joining the points (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Slope of line joining the points (x_1, y_1) and (x_2, y_2)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

If (x_1, y_1), (x_2, y_2), (x_3, y_3) are thre vertices of a parallelogram, then coordinates of its **fourth vertex is** $(x_1 + x_2 - x_3, y_1 + y_2 - y_3)$



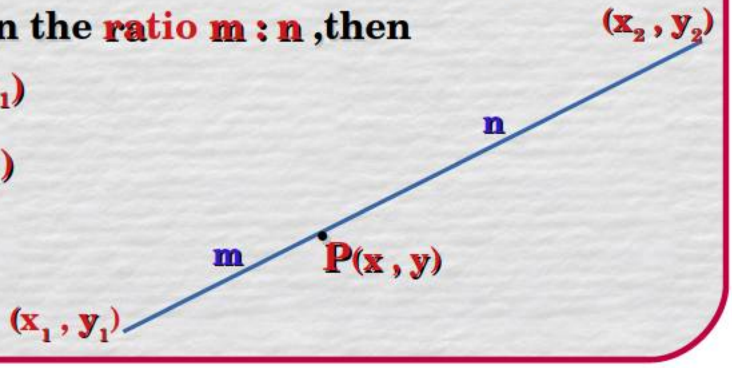
Equation of a line

$y - y_1 = Slope(x - x_1)$ where (x_1, y_1) is a point on the line

If the point $P(x, y)$ divides the line joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$, then

$$x = x_1 + \frac{m}{m+n}(x_2 - x_1)$$

$$y = y_1 + \frac{m}{m+n}(y_2 - y_1)$$

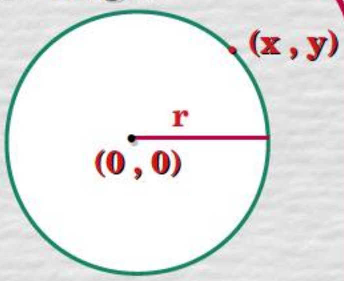


If (x_1, y_1), (x_1, y_1), (x_1, y_1) are three vertices of a triangle

Coordinates of Centroid = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

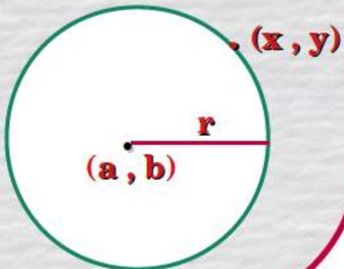
The equation of a circle with centre at the origin and radius 'r' is

$$x^2 + y^2 = r^2$$



The equation of a circle with centre (a , b) and radius 'r' is

$$(x - a)^2 + (y - b)^2 = r^2$$



Chapter – 10
Polynomials

Main Concepts

Polynomials and Factors

* For the second degree polynomial $P(x)$

If $P(x) = q(x)\,r(x)$, then $q(x)$, $r(x)$ are factors of $P(x)$.

$$x^2 + ax = x(x + a)$$
$$x^2 + (a + b)x + ab = (x + a)(x + b)$$
$$x^2 - a^2 = (x + a)(x - a)$$
$$x^2 + 2ax + b^2 = (x + a)^2$$
$$x^2 - 2ax + b^2 = (x - a)^2$$

Remainders and Factors

* For the second degree polynomial $P(x)$

The remainder when $P(x)$ is divided by $x - a$ is $P(a)$

The remainder when $P(x)$ is divided by $x + a$ is $P(-a)$

If $P(a) = 0$ then $x - a$ will be a factor of $P(x)$

If $P(-a) = 0$ then $x + a$ will be a factor of $P(x)$

On the other hand,

If $x - a$ is a factor of $P(x)$, then $P(a) = 0$

If $x + a$ is a factor of $P(x)$, then $P(-a) = 0$

If P(x) is a polynomial and a is a number , then

x – a will be a factor of P(x) – P(a)

x + a will be a factor of P(x) – P(–a)

If x = a and x = – b are solutions of P(x) = 0
then x – a and x + b are the 2 factors of P(x)

Chapter – 11
Statistics

Main Concepts

Mean

$$\text{Arithmetic mean (Mean)} = \frac{\text{Sum of items}}{\text{Number of items}}$$

Median

Median is the middle most number, when the numbers are arranged in ascending or descending order.

* If the number of terms is odd

Median = Middle Number

* If the number of terms is even

Here two numbers comes in the middle
Half of their sum is taken as the median

Median in frequency table

Prepare cumulative frequency table.

If the total number of terms(n) is odd

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ term}$$

If the total number of terms(n) is even

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{ term} + \left(\frac{n}{2}\right)+1^{th} \text{ term}}{2}$$