INSTRUCTIONS

A. General

1. This booklet is your Question Paper. Do not break the seals of this boolket before being instructed to do so by the invigilators.
2. The question paper CODE is printed on the right hand top corner of this page and on the back page (Page No. 28) of this booklet.
3. Blank spaces and blank pages are provided in this booklet for your rough work. No additional sheets will be provided for rough work.
4. Blank papers, clipboards , log tables, slide rules, calculators, cameras, cellular phones, pagers and electronic gadgets are NOT allowed inside the examination hall.
5. Answer to the questions and personal details are to be filled on a two-part carbon-less paper, which is provided separately. You should not separate these parts. The invigilator will separate them at the end of the examination. The upper sheet is a machine-gradable Objective Response Sheet (ORS) which will be taken back by the invigilator. You will be allowed to take away the bottom sheet at the end of the examination.
6. Using a black ball point pen, darken the bubbles on the upper original sheet. Apply sufficinet pressure so that the impression is created on the bottom sheet.
7. DO NOT TAMPER WITH / MUTILATE THE ORS OR THE BOOKLET.
8. On breaking the seals of the booklet check that it contains 28 pages and all the 60 questions and corresponding answer choices are legible. Read carefully the instructions printed at the beginining of each section.

B. Filling the Right Part of the ORS

9. The ORS has CODES printed on its left and right parts.
10. Check that the same CODE is printed on the ORS and on this boolet. IF IS NOT THEN ASK FOR A CHANGE OF THE BOOKLET. Sign at the place provided on the ORS affirming that you have verified that all the codes are same.
11. Write your name , Registration Number and the name of examination centre and sign with pen in the boxes provided on the right part of the ORS. **Do not write any of this information anywhere else.** Darken the appropriate bubble **UNDER** each digit of your Registration Number in such a way that the impression is created on the bottom sheet. Also darken the paper CODE given on the right side of ORS (R4)

C Question Paper Format

The question paper consists of 3 parts (Physics, Chemistry and Mathematics)> Each part consists of three sections.

12. Section - I contains 8 multiple choice questions. Each question has four choice A,B,C and D out of which ONLY ONE is correct.
13. Section - II contains 3 paragraph each describing theory, experiment, data etc. There are 6 multiple choice questions relating to three paragraphs with 2 questions on each paragraph. Each question of a particular paragraph has four choice A,B,C and D out of which ONLY ONE is correct.
14. Section -III contains 6 multiple choice questions. Each question has four choice A,B,C and D out of which ONE or MORE are correct.

D. Marking Scheme

15. For each question in **Section I** and **Section II** , you will be awarded **3 marks** if you darken the bubble corresponding to the correct answer **ONLY** and **zero (0) marks** if no bubbles are darkened. In all other cases, **minus one (–1) mark** will be awarded in these sections.
16. For each question in **Section III** , you will be awarded **4 marks if** you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY**. In all other cases **zero (0) marks** will be awarded. No **negative marks** wil be awarded for incorrect answer(s) in this section.

PHYSICS

SECTION – I

(SINGLE CORRECT CHOICE TYPE)

This section contains 8 **multiple choicse questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE is correct**

1. A student is performing the experiment of Resosance Column. The diameter of the column tube is 4cm. The frequency of the tuning fork is 512Hz. The air temperature is 38^0C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance Column tube. When the first resonance occurs, the reading of the water level in the column is

a) 14.0 cm b) 15.2 cm c) 16.4 cm d) 17.6 cm

Ans : B

$$\Rightarrow \frac{\lambda}{4} = l + e$$

$$\frac{\lambda}{4} = l + 0.3d$$

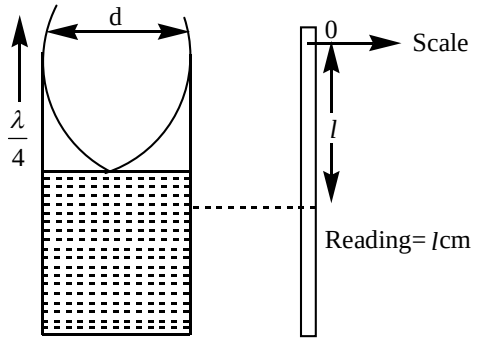
$$l = \frac{\lambda}{4} - 0.3d$$

$$\lambda = \frac{V}{F} = \frac{336}{512} \times 100(cm)$$

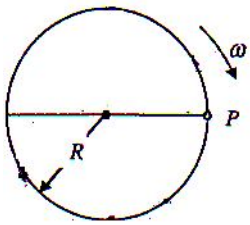
$$\lambda = 16.4\,cm$$

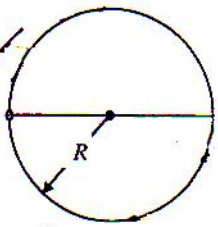
$$l = 16.4 - 0.3(4)$$

$$= 15.2\,cm$$

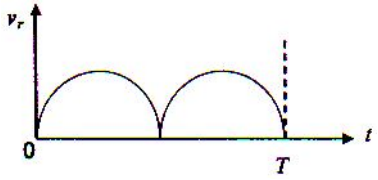


2. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time $t = 0$, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by

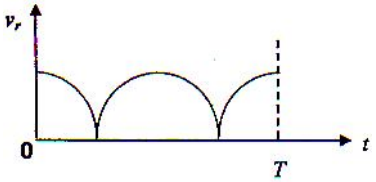




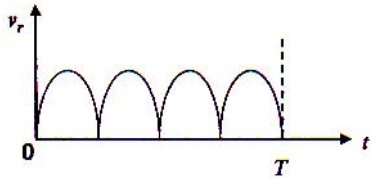
a)



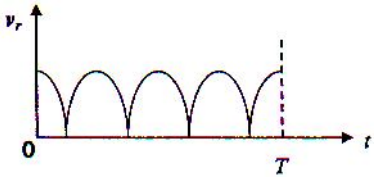
b)



c)



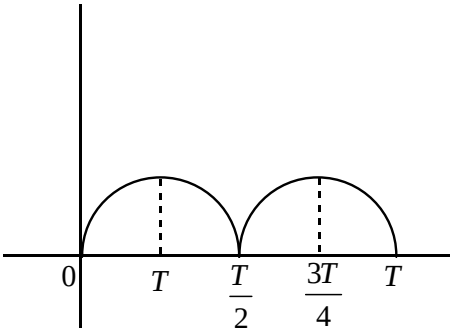
d)

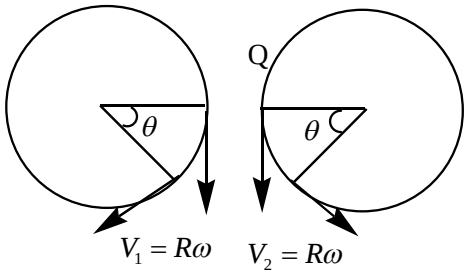


Ans : A

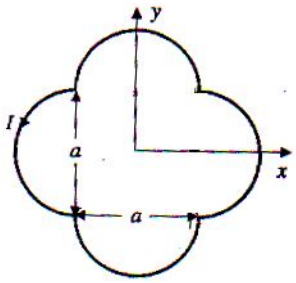
$$\vec{V}_r = V_{1/2} = 2R\omega\sin(\omega t)$$

$$\text{Speed} = \left|\vec{V}_r\right| = 2R\omega\left|\sin(\omega t)\right|$$





3. A loop carrying current I lies in the $x-y$ plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is

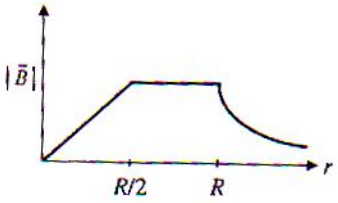


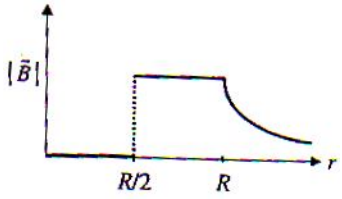
a) $a^2 I\hat{k}$ b) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ c) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ d) $(2\pi+1)a^2 I\hat{k}$

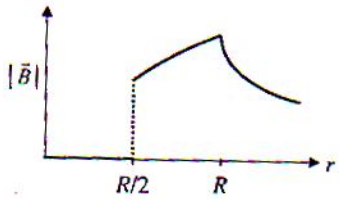
Ans : B

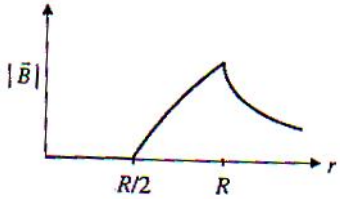
Magnetic moment $\vec{\mu} = I\left[a^2 + \frac{1}{2}\pi\left(\frac{a}{2}\right)^2 \times 4\right]\hat{k} = \left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$

4. An infinitely long hollow conducting cylinder with inner radius $R/2$ and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by

a)


b)


c)


d)


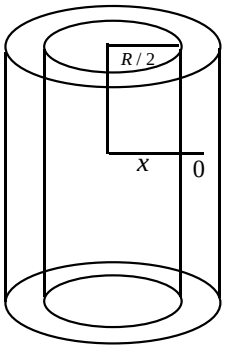
Ans : D

B=0(When $x < R/2$)

$$B(2\pi x) = \mu_0 J\left(\pi x^2 - \frac{\pi R^2}{4}\right)$$

$$B = \mu_0 J\left(x - \frac{R^2}{8x}\right)$$

$$B = \mu_0 J\left(\frac{x}{2} - \frac{R^2}{8x}\right)$$



at $x = \frac{R}{2}$

$$B = \mu_0 J\left(\frac{R}{4} - \frac{R^2}{4R}\right) = 0$$

Where $x > R$

$$B\alpha \frac{1}{x}$$

5. A thin uniform cylinderical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half – submerged state. If ρ_c is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is

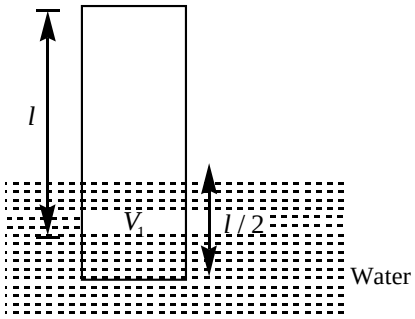
a) more than half–filled if ρ_c is less than 0.5

b) more than half – filled if ρ_c is more than 1.0

c) half – filled if ρ_c is more than 0.5

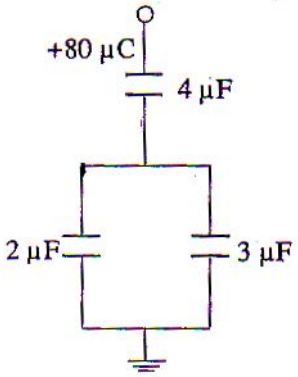
d) less than half – filled if ρ_c is less than 0.5

Ans : D



V_1=Value of water in cylinder. if half volume of cylinder is available in waterl; water can't be filled upto half level because cylinder is not mass less. What ever is the relative density of material as cylinder volume as water in cylinder must be less than that volume as liquid displeased.

6. In the given circuit, a charge of $+80\mu C$ is given to the upper plate of the $4\mu F$ capacitor. Then in the steady state, the charge on the upper plate of the $3\mu F$ capacitor is



a) $+32\mu C$ b) $+40\mu C$ c) $+48\mu C$ d) $+80\mu C$

Ans : C

$q\alpha\, c$

$q_2 = \frac{3}{5}q$

$= \frac{3}{5}\times 80$

$48\mu c$

7. Two moles of ideal helium gas are in a rubber balloon at 30^0C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35^0C. The amount of heat required in raising the temperature is nearly (take R = 8.31 J/mol.K)

a) 62 J b) 104 J c) 124 J d) 208 J

Ans : D

n = 2 moles

$T_i = 30^0C$

$T_f = 35^0C$

Process can be assumed as isobaric

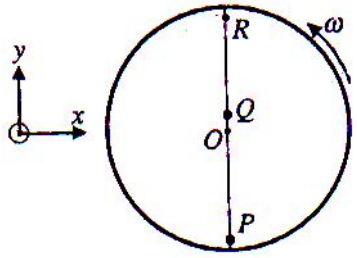
$\Delta Q = 2.\frac{5}{2}R\Delta T$

$5R\Delta T$

$= 5 \times 8.31 \times 5$

$= 207.75$

8. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre *O*. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles *P* and *Q* are simultaneously projected at an angle towards *R*. The velocity of projection is in the $y-z$ plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed $\frac{1}{8}$ rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughtout. Then



a) *P* lands in the shaded region and *Q* in the unshaded region

b) *P* lands in the unshaded region and *Q* in the shaded region

c) Both *P* and *Q* land in the unshaded region

d) Both *P* and *Q* land in the shaded region

Ans : C

t = 0

at $t = \frac{\pi}{4\omega}$

both will fall on disc simultaneously Q will be is unshaded area and P will be in shaded area

SECTION – II

(COMPREHENSION TYPE)

This section contains 3 groups of questions. Each group has 2 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct.

Paragraph for Questions 9 to 10

The β -decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p), and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicated theoretically that the kinetick energy of the electron sould be a constant. But experimentally, it was observed that the electron kinetic energy has continuous spectrum. Considering a three-body decay process, i.e. $n \to p + e^- + \overline{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino $(\overline{\nu}_e)$ to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy consevation principles are applied. From this calculation, the maximum kinetic energy of the electron is $0.8\times10^6 eV$. The kinetic energy carried by the portion is only the recoil energy.

9. If the anti-neutrino had a mass of $3eV/c^2$ (where c is the speed of the light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron?

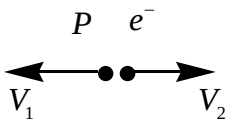
a) $0 \le K \le 0.8\times10^6 eV$

b) $3.0eV \le K \le 0.8\times10^6 eV$

c) $3.0eV \le K < 0.8\times10^6 eV$

d) $0 \le K < 0.8\times10^6 eV$

Ans : D



$P = m_P V_1 = m_e V_0$ ------ (1)

$\frac{1}{2} m_e V_0^2 = 0.8\times10^6 eV$ -- (2)

KE as proton is too small.Q value is Rx^n little more than $0.8\times10^6 eV$.

Q value $\simeq 0.8\times10^6 eV$

Now for least KE of e^- energy released is Rx^n will be shared between anti neutrino and proton and linear momentum will be conserved. KE as e^- will be zero. For max KE as e^- KE as antineutrino is zero all energy released in Rx^n will be shared between e^- and proton like antineutrino is not available

10. What is the maximum energy of the anti-neutrino?

a) Zero

b) Much less than $0.8\times10^{6}eV$

c) Nearly $0.8\times10^{6}eV$

d) Much larger than $0.8\times10^{6}eV$

Ans : ADD

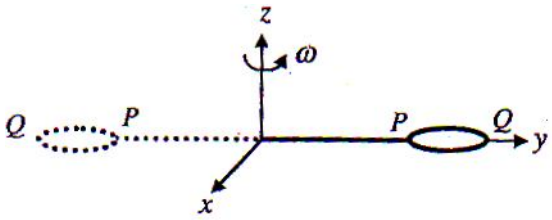
If information in Q. No.9 is not used in Q.No. 10. KE_{max} for neutrino is $0.8\times10^{6}eV$. Which is not there in option.

Or

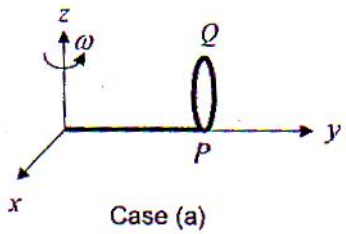
If information of Q.No. 9 is used for Q.No. 10 also max KE as antineutrino will be almost equal ot Q value as Rx^{n} .

Paragraph for Questions 11 and 12

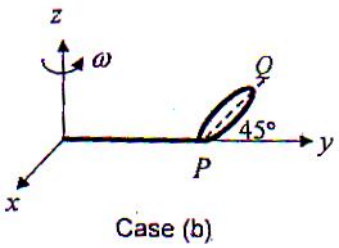
The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω , the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the *z*-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points *P* and *Q*. Both these motions have the same angular speed ω in this case.



Now consider two similar systems as shown in the figure; Case(a) the disc with its face vertical and parallel to *x*-*z* plane; Case(b) the disc with its face making an angle 45^{0} with *x*-*y* plane and its horizontal diameter parallel to *x*-axis. In both the cases, the disc is welded at point *P*, and the systems are rotated with constant angular speed ω about the *z*-axis.



Case (a)



Case (b)

11. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?

a) It is $\sqrt{2\omega}$ for the both the cases

b) It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case(b)

c) It is ω for case(a); and $\sqrt{2}\omega$ for case(b)

d) It is ω for both the cases

Ans : D

Angular speed about the instantaneous axis passing through centre of mass is ω for both the cases.

12. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?

a) It is vertical for the both the cases (a) and (b)

b) It is vertical for case (a); and it is 45^0 to the *x*-*z* plane and lies in the plane of the disc for case (b)

c) It is horizontal for case (a); and is at 45^0 to the *x*-*z* plane and is normal to the plane of the disc for case (b)

d) It is vertical for case (a); and is at 45^0 to the *x*-*z* plane and is normal to the plane of the disc for case (b)

Ans : A

Instantaneous axis passing through c.m. is vertical in both the cases.

Paragraph for Questions 13 and 14

Most materials have the refractive index, $n>1$. So, when a light ray from air entres a naturally occuring material, then by Snell's law, $\frac{\sin\theta_1}{\sin\theta_2}=\frac{n_1}{n_2}$, it is understood that the refracted ray bends towards the normal. But is never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n=\left(\frac{c}{v}\right)\pm\sqrt{\varepsilon_r\mu_r}$, where c is the speed of the electromagnetic waves in vaccum, v its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively. In normal materials, both ε_r and μ_r positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially, prepared and are called meta-materials. They exhibit significantly different optical behaviour, without violating any physical laws. Since n is negative, it results in a change in the direction of propogation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

13. Choose the correct statement.

a) The speed of the light in the meta-material is $v=c|n|$

b) The speed of the light in the meta-material is $v=\frac{c}{|n|}$

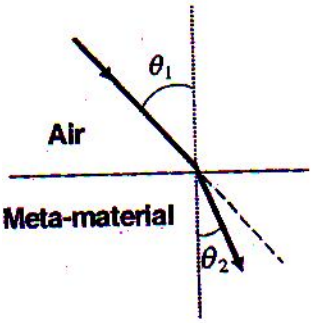
c) The speed of the light in the meta-material is $v=c$

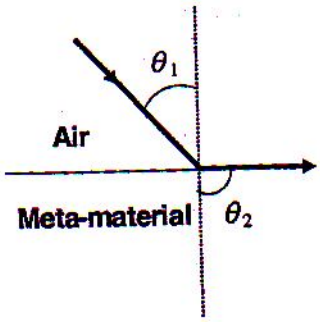
d) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m=\lambda_{air}|n|$, where λ_{air} is the wavelength of the light air

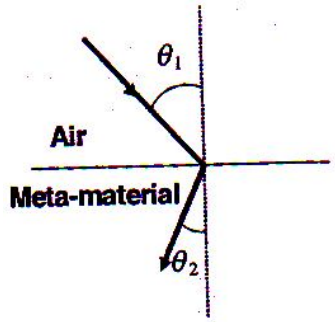
Ans : B

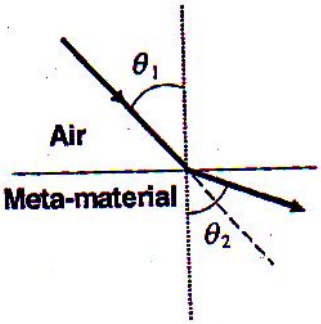
For meta materials n is -ve. Hence speed $V=\frac{c}{|n|}$

14. For light from air on a meta-material, the appropriate ray diagram is

a) 

b) 

c) 

d) 

Ans : C

$-n = \dfrac{\sin\theta_1}{\sin\theta_2}$ where θ_2 is angle made by refracted ray with normal in anticlock wise sense.

$$-\sin\theta_2 = \frac{\sin\theta_1}{n}$$

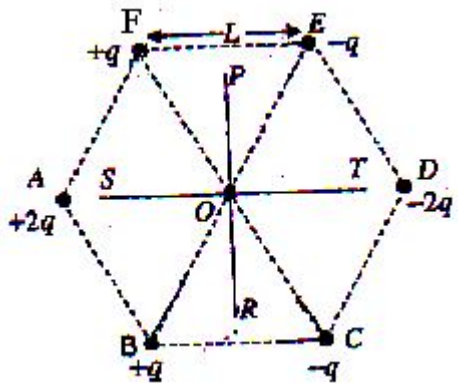
$$\sin(-\theta_2) = \frac{\sin\theta_1}{n}$$

SECTION – III

(MULTIPLE CORRECT CHOICE TYPE)

This section contains **6 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE is/ are correct**

15. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement(s) is (are) correct ?



a) The electric field at O is 6K along OD

b) The potential at O is zero

c) The potential at all points on the line PR is same

d) The potential at all points on the line ST is same

Ans : ABC

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{12} = k$$

Due to F, C $E_p = 2k$ (towards $\vec{E}$)

Due to B, E $E_p = 2k$ (towards $\vec{C}$)

Due to A, D $E_p = 4k$ (towards $\vec{D}$)

$E_P = 4k + k + k = 6k$ along $\left(\vec{D}\right)$

$$E = \frac{-dv}{dx} \Rightarrow dv = -\Delta dx\cos 90^0 = 0$$

So 'PR' is equipotential surface to +q , –q

and $+2q, -2q$

and $+q, -q$

16. Two spherical planets *P* and *Q* have the same uniform density ρ, mass M_P and M_Q, and surface area *A* and 4*A*, respectively. A spherical planet *R* also has uniformly density ρ and its mass is $\left(M_P + M_Q\right)$. The escape velocities from the planets *P*, *Q* and *R*, are V_P,V_Q and V_R respectively. Then

a) $V_Q > V_R > V_P$ b) $V_R > V_Q > V_P$ c) $V_R / V_P = 3$ d) $V_P / V_Q = \frac{1}{2}$

Ans : BD

$$V_e = \sqrt{\frac{2qm}{R}}$$

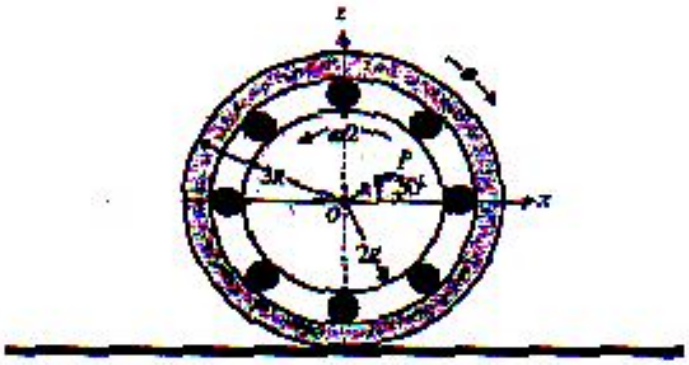
$$V_e = \sqrt{\frac{2q}{R} \times \rho \times \frac{4}{3}\pi R^3}$$

$$V_e \propto \sqrt{A}$$

D) $\frac{V_P}{V_Q} = \frac{\sqrt{A}}{\sqrt{4A}} = \frac{1}{2}$

B) $V_R > V_Q > V_P \Rightarrow V \propto \sqrt{A}$

17. The figure shows a system consisting of (i) a ring of outer radius 3*R* rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2*R* rotating anti–clockwise with angular speed $\frac{\omega}{2}$. The ring and disc are separated by frictionless ball bearings. The system is in the *x*–*z* plane. The point *P* on the inner disc is at a distance *R* from the origin, where *OP* makes an angle of 30^0 with the horizontal. Then with respect to the horizontal surfa



a) the point *O* has a linear velocity $3R\omega\hat{i}$

b) the point *P* has a linear velocity $\frac{11}{4}R\omega\hat{i}+\frac{\sqrt{3}}{4}R\omega\hat{k}$

c) the point *P* has a linear velocity $\frac{13}{4}R\omega\hat{i}-\frac{\sqrt{3}}{4}R\omega\hat{k}$

d) the point *P* has a linear velocity $\left(3-\frac{\sqrt{3}}{4}\right)R\omega\hat{i}+\frac{1}{4}R\omega\hat{k}$

Ans : A,B

$V_0 = 3R\omega\left(\hat{i}\right)$ $\qquad$ $V_{p/0}=\frac{R\omega}{2}$

$\left(3R\omega-\frac{R\omega}{4}\right)\hat{i}+\frac{\sqrt{3}}{4}R\omega\hat{k}$

$\vec{V}_P=\frac{11R\omega}{4}\hat{i}+\frac{\sqrt{3}}{4}R\omega\hat{k}$

18. Two solid cylinders *P* and *Q* of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder *P* has most of its mass concentrated near its surface, while *Q* has most of its mass concentrated near the axis. Which statement(s) is(are) correct ?

a) Both cylinders *P* and *Q* reach the ground at the same time

b) Cylinder *P* has larger linear acceleration than cylinder *Q*.

c) Both cylinders reach the ground with same translational kinetic energy

d) Cylinder *Q* reaches the ground with larger angular speed

Ans : D

$I_P > I_Q$ $\qquad \omega = \dfrac{g\sin\theta}{1+\dfrac{kv}{Rv}}$ $\qquad \omega_P < \omega_Q$

$K_P > K_Q$ $\qquad t = \sqrt{\dfrac{2l}{\omega}}$

$\therefore$ (A) is not event

$\therefore$ (B) is also not event

$\omega = R\alpha$ $\qquad \omega_Q > \omega_P$ $\qquad V_P < V_Q$

$\alpha_Q > \alpha_P$ $\qquad V = r\omega$

$\omega_P < \omega_Q$

$\therefore$ (D) only is correct

19. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is(are)

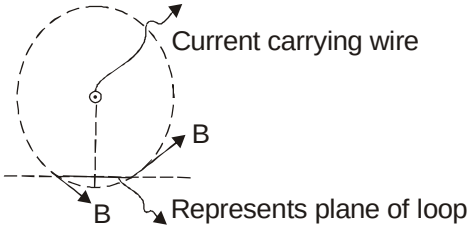
a) The emf induced in the loop is zero if the current is constant

b) The emf induced in the loop is finite if the current is constnat

c) The emf induced in the loop is zero if the current decreases at a steady rate

d) The emf induced in the loop is finite if the current decreases at a steady rate

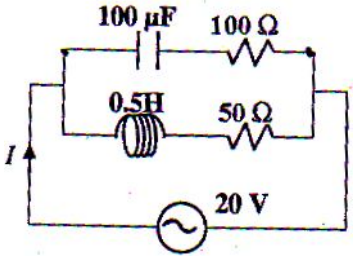
Ans : AC or ABCD



Net flux passing through loop will be zero, in all cases

So, emf induced in loop is zero.

20. In the given circuit, the AC source has $\omega = 100\, rad/s$ .Considering the inductor and capacitor to be ideal, the correct choice(s) is(are)



a) The current through the circuit , I is 0.3 A

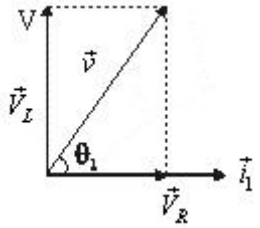
b) The current through the circuit , I is $0.3\sqrt{2}A$

c) The voltage across 100Ω resistor $= 10\sqrt{2}\, A$

d) The voltage across 50Ω resistor = 10 V

Ans : B or AB

For RL



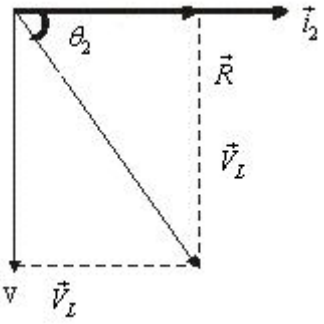
$$TanQ_1 = \frac{\omega L}{R} = \frac{100 \times 0.5}{50} = 1$$

$$Q_1 = 45^0$$

$\vec{v}$ leads current i_1 by 45^0

$$i_1 = \frac{v}{\sqrt{x_L^2 + R^2}} = \frac{20}{50\sqrt{2}} = \frac{\sqrt{2}}{5}$$

For RL



$$TanQ_2 = \frac{\frac{1}{\omega C}}{R} = \frac{100}{100} = 1$$

$$Q_2 = 45^0$$

i_2 leads V_{app} by 45^0

$$i_2 = \frac{V}{\sqrt{x_L^2 + R^2}} = \frac{20}{100\sqrt{2}} = \frac{1}{5\sqrt{2}}$$

$$i = \sqrt{i_2^2 + i_1^2}$$

$$= \sqrt{\frac{2}{25} + \frac{1}{50}} = \sqrt{\frac{5}{50}} = \frac{1}{\sqrt{10}} is\, 0.3A$$

$$V_{100} = (i_2) = 100 = \frac{100}{5\sqrt{2}} = 10\sqrt{2}\,V$$

$$V_{50} = \frac{\sqrt{2}}{5} \times 50 = 10\sqrt{2}\,V$$

CHEMISTRY

SECTION – I

(SINGLE CORRECT CHOICE TYPE)

This section contains **8 multiple choicse questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE is correct**

21. Using the data provided, calculate the multiple bond energy $\left(kJ\ mol^{-1}\right)$ of a $C \equiv C$ bond in C_2H_2. That energy is (take the bond energy of a C – H bond as $350 kJ\ mol^{-1}$.)

$2C(s) + H_2(g) \rightarrow C_2H_2(g)$ $\quad \Delta H = 225\, kJ\ mol^{-1}$

$2C(s) \rightarrow 2C(g)$ $\quad \Delta H = 1410\, kJ\ mol^{-1}$

$H_2(g) \rightarrow 2H(g)$ $\quad \Delta H = 330 kJ\ mol^{-1}$

A) 1165 B) 837 C) 865 D) 815

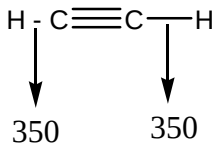
Ans : D

$2C(s) + H_2(g) \rightarrow C_2H_2(g)$ $\quad \Delta H = 225\ kJ\ mol^{-1}$

$2C(g) \rightarrow 2C(s)$ $\quad \Delta H = -1410\ kJ\ mol^{-1}$

$2H(g) \rightarrow H_2(g)$ $\quad \Delta H = -330\ kJ\ mol^{-1}$

$2C(S) + 2H(g) \rightarrow C_2H_2(g)$ $\quad \Delta H = -1515\ kJ\ mol^{-1}$



$C \equiv C$ bond energy is 815 kJ/mole.

22. The major product H of the given reaction sequence is

$$CH_3-CH_2-CO-CH_3 \xrightarrow{\ominus_{CN}} G \xrightarrow[Heat]{95\%H_2SO_4} H$$

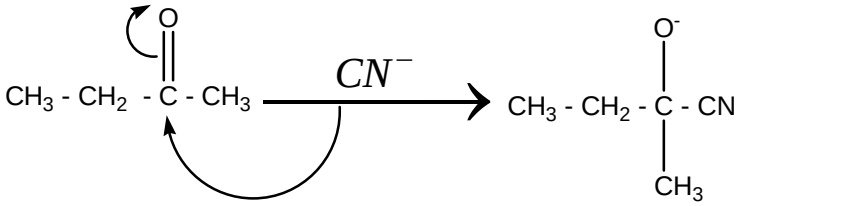
A) $CH_3-CH=\underset{\displaystyle CH_3}{\overset{\displaystyle |}{C}}-COOH$

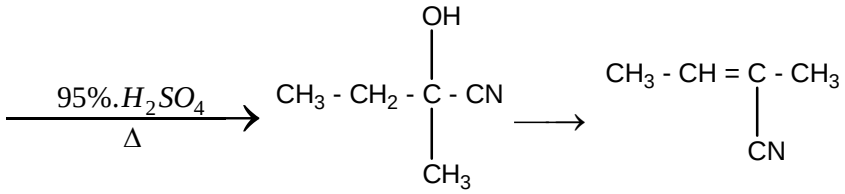
B) $CH-CH=\underset{\displaystyle CH_3}{\overset{\displaystyle |}{C}}-CN$

C) $CH_3-CH_2-\underset{\displaystyle CH_3}{\overset{\displaystyle OH\ |}{C}}-COOH$

D) $CH_3-CH=\underset{\displaystyle CH_3}{\overset{\displaystyle |}{C}}-CO-NH_2$

Ans : B





23. $NiCl_2\left\{P\left(C_2H_5\right)_2\left(C_6H_5\right)\right\}_2$ exhibits temperature dependent magnetic behaviour (paramagnetic / diamagnetic) . The coordination geometries of Ni^{2+} in the paramagnetic and diamagnetic states are respectively

A) tetrahedral and tetrahedral

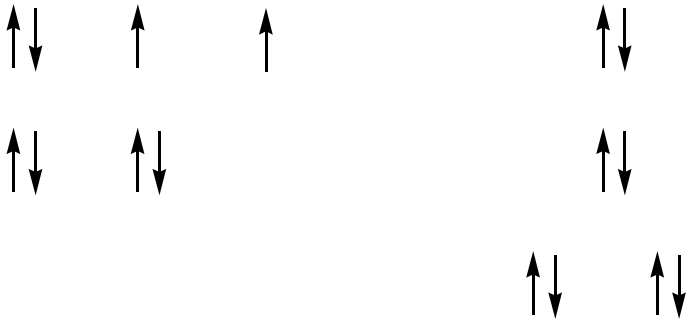
B) square planar and square planar

C) tetrahedral and square planar

D) square planar and tetrahedral

Ans : C

Tetrahedral and square planar



24. In the cyanide extraction process of silver from argentite ore, the oxidizing and reducing agent used are

A) O_2 and CO respectively

B) O_2 and Zn dust respectively

C) HNO_3 and Zn dust respectively

D) HNO_3 and CO respectively

Ans : B

$O_2 \rightarrow$ oxidising agent

Zn $\rightarrow$ reducing agent

25. The reaction of white phosphorus with aqueous NaOH gives phosphine along with another phosphorus containing compound. The reaction type; the oxidation states of phosphorus in phosphine and the other product are respectively

A) redox reaction ; –3 and –5

B) redox reaction; +3 and +5

C) disproportionation reaction; –3 and +5

D) disproportionation reaction; –3 and +3

Ans : D

$$P_4 + NaOH \rightarrow H_2PO_2^- + PH_3$$

$$\downarrow \Delta$$

$$Na_2HPO_3 + PH_3$$

26. The shape of XeO_2F_2 molecule is

A) trigonal bipyramidal

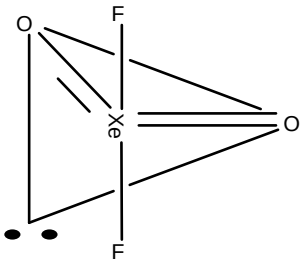
B) Square planar

C) tetrahedral

D) see - saw

Ans : D

see-saw



27. For a dilute solution containing 2.5 g of a non – volatile non – electrolyte solute in 100 g of water, the elevation in boiling point at 1 atm pressure is $2°C$. Assuming concentration of solute is much lower than the concentration of solvent, the vapour pressure (mm of Hg) of the solution is (take $K_b = 0.76\,K\,kg\,mol^{-1}$)

A) 724 B) 740 C) 736 D) 718

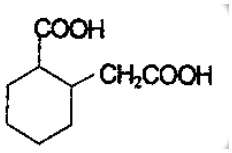
Ans : A

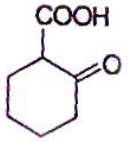
$$X_{solute} = \frac{m}{m+55.5} = \frac{p^0 - p}{p^0}$$

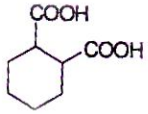
$$\frac{m}{55.5} = \frac{760 - p}{760}$$

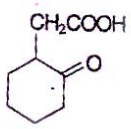
P = 724 mm of Hg

28. The compound that undergoes decarboxylation most readily under mild condition is

A) 

B) 

C) 

D) 

Ans : B

β -keto acids readily undergo decarboxylation

SECTION – II

(COMPREHENSION TYPE)

This section contains 3 groups of questions. Each group has 2 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct.

Paragraph for Questions 29 and 30

The electrochemical cell shown below is a concentration cell.

$M \mid M^{2+}$ (saturated solution of a sparingly soluble salt, MX_2) $\| M^{2+}\left(0.001\,mol\,dm^{-3}\right) \mid M$

The emf of the cell depends on the difference in concentrations of M^{2+} ions at the two electrodes. The emf of the cell at 298 K is 0.059 V.

29. The value of $\Delta G\left(kJ\ mol^{-1}\right)$ for the given cell is (take $1F = 96500\,C\,mol^{-1}$)

A) –5.7 B) 5.7 C) 11.4 D) –11.4

Ans : D

$\Delta G = -nFE$

$= -2 \times 96500 \times 0.059$

$= -11.4\ kJ\ /\ mol$

30. The solubility product $\left(K_{sp}; mol^3 dm^{-9}\right)$ of MX_2 at 298 K based on the information available for the given concentration cell is (take $2.303 \times R \times 298 / F = 0.059V$)

A) 1×10^{-15} B) 4×10^{-15} C) 1×10^{-12} D) 4×10^{-12}

Ans : B

$$0.059 = E = 0 - \frac{0.059}{2}\ \log\frac{0.001}{s}$$

$S = 10^{-5}$

$K_{SP} = 4s^3 = 4\times10^{-15}$

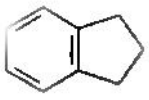
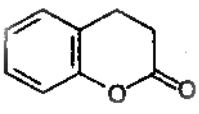
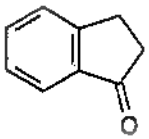
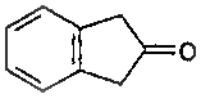
Paragraph for Questions 31 and 32

In the following reaction sequence, the compound J is an intermediate.

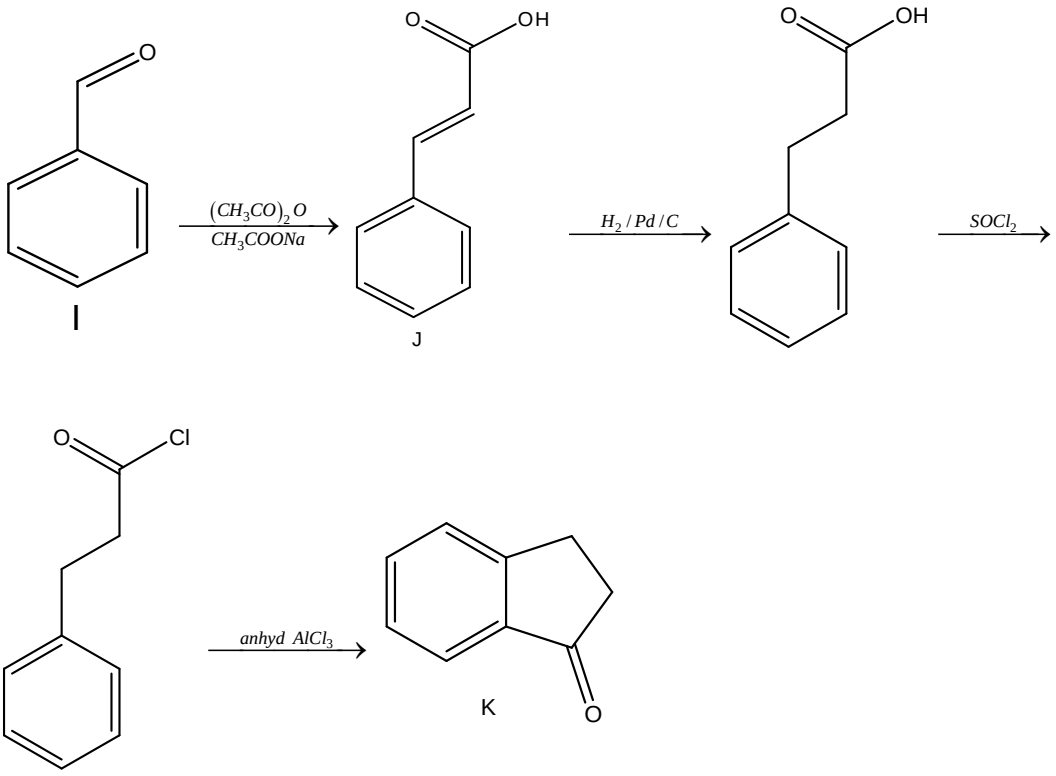
$$I \xrightarrow[CH_3COONa]{(CH_3CO)_2O} J \xrightarrow[\substack{(ii)SOCl_2\\(iii)anhyd.AlCl_3}]{(i)H_2, Pd/C} K$$

$J\left(C_9H_8O_2\right)$ gives effervescence on treatment with $NaHCO_3$ and a positive Baeyer's test.

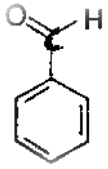
31. The compound K is

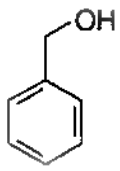
A)  B)  C)  D) 

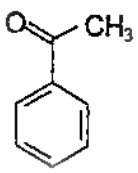
Ans : C

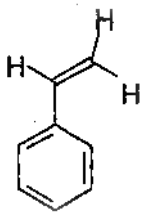


32. The compound I is

A) 

B) 

C) 

D) 

Ans : A

Paragraph for Questions 33 and 34

Bleaching powder and bleach solution are produced on a large scale and used in several house hold products. The effectiveness of bleach solution is often measured by iodometry.

33. 25 mL of household bleach solution was mixed with 30 mL of 0.50 M KI and 10 mL of 4 N acetic acid. In the titration of the liberated iodine, 48 mL of 0.25 N $Na_2S_2O_3$ was used to reach the end point. The molarity of the household bleach solution is

A) 0.48 M B) 0.96 M C) 0.24 M D) 0.024 M

Ans : C

$CaOCl_2 + I^- \rightarrow I_2 + \xrightarrow{hypo} NaI + Na_2S_4O_6$

12 meq. 30 ml 48 ml 0.25 N = 12 m eq

0.5 N

15 m eq

limiting reagent is beaching powder.

$25 \times N = 12$

$N = \frac{12}{25} = 0.48\,N$

$M = 0.24\,M$

34. Bleaching powder contains a salt of an oxoacid as one of its components. The anhydride of that oxoacid is

A) Cl_2O B) Cl_2O_7 C) ClO_2 D) Cl_2O_6

Ans : A

oxoacid is HOCl

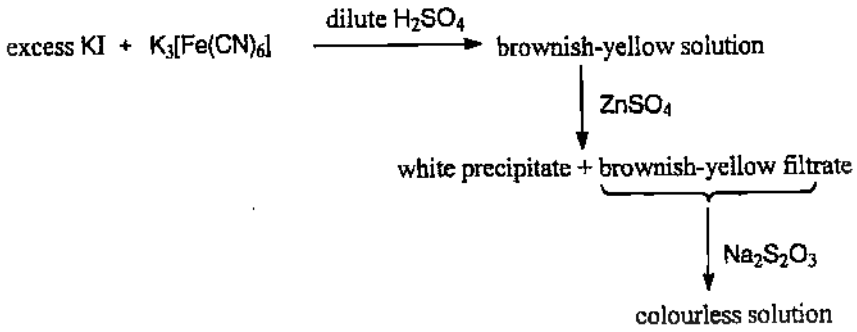
$2HOCl \rightarrow Cl_2O + H_2O$

SECTION – III

(MULTIPLE CORRECT CHOICE TYPE)

This section contains **6 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE is/ are correct**

35. For the given aqueous reactions, which of the statement(s) is (are) true?



A) The first reaction is a redox reaction

B) White precipitate is $Zn_3\left[Fe(CN)_6\right]_2$.

C) Addition of filtrate to starch solution gives blue colour

D) White precipitate is soluble in NaOH solution.

Ans : ACD

$$KI + K_3\left[Fe(CN)_6\right] \xrightarrow{di/H_2SO_4} K_4\left[Fe(CN)_6\right] + I_2$$

$\downarrow ZnSO_4$

$$I_3^- + K_2Zn_3\left[Fe(CN)_6\right]_2$$

Brownish yellow ppt white ppt

$\downarrow Na_2S_2O_3$

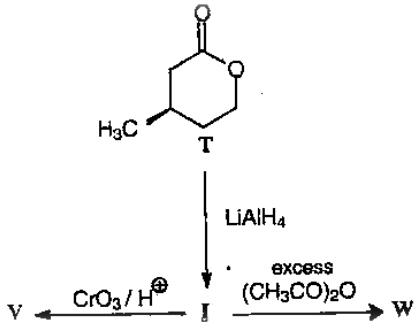
(colourless solution)

$NaI + Na_2S_4O_6$

The white ppt is soluble in sodium hydroxide

$$K_2Zn_3\left[Fe(CN)_6\right]_2 + 12OH^- \rightarrow 2\left[Fe(CN)_6\right]^{4-} + 3\left[Zn(OH)_4\right]^{2-} + 2K^+$$

36. With reference to the scheme given, which of the given statement(s) about T, U, V and W is(are) correct?

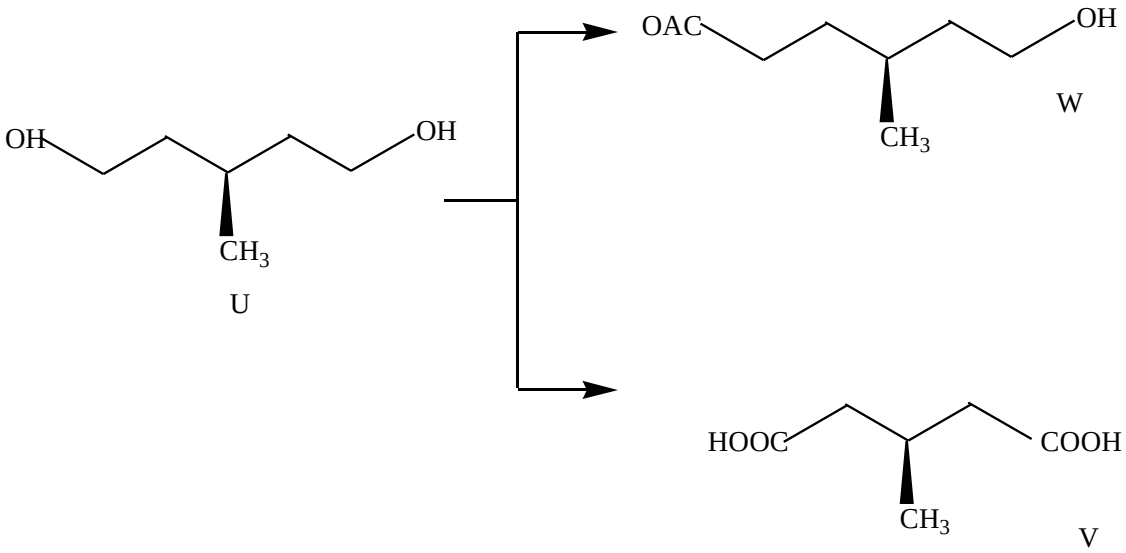


A) T is soluble in hot aqueous NaOH B) U is optically active

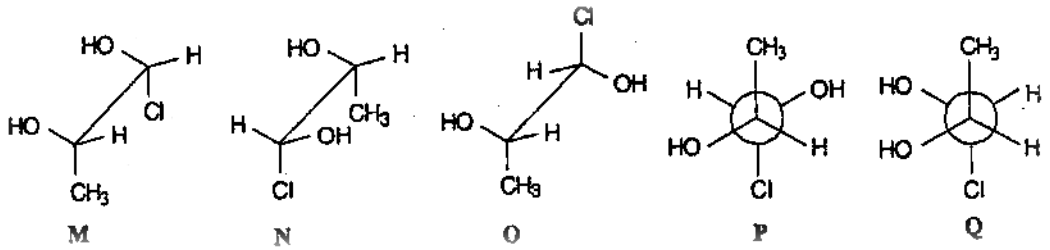
C) Molecular formula of W is $C_{10}H_{18}O_4$

D) V gives effervescence on treatment with aqueous $NaHCO_3$

Ans : ACD



37. Which of the given statement (s) about N, O, P and Q with respect to M is (are) correct?



A) M and N are non–mirror image stereoisomers

B) M and O are identical

C) M and P are enantiomers

D) M and Q are identical

Ans : ABC

38. With respect to graphite and diamond, which of the statement(s) given below is(are) correct?

A) Graphite is harder than diamond.

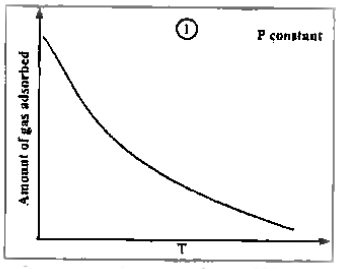
B) Graphite has higher electrical conductivity than diamond.

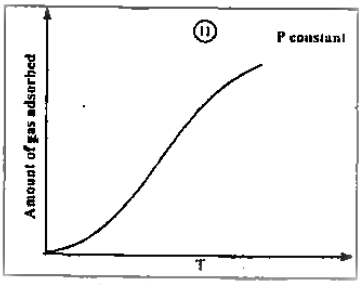
C) Graphite has higher thermal conductivity than diamond.

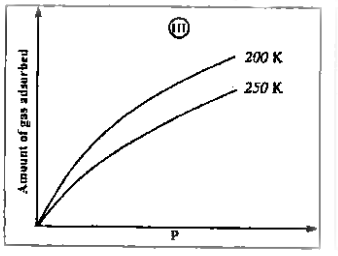
D) Graphite has higher C – C bond order than diamond.

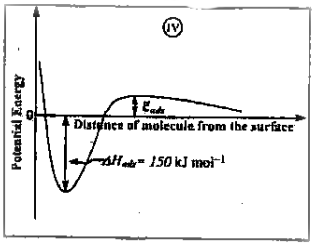
Ans : BD

39. The given graphs / data I, II, III and IV represent general trends observed for different physisorption and chemisorption processes under mild conditions of temperature and pressure. Which of the following choice(s) about I, II, III and IV is(are) correct?









A) I is physisorption and II is chemisorption

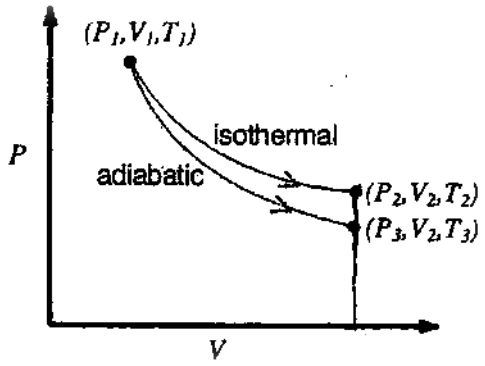
B) I is physisorption and III is chemisorption

C) IV is chemisorption and II is chemisorption

D) IV is chemisorption and III is chemisorption

Ans : AC

40. The reversible expansion of an ideal gas under adiabatic and isothermal conditions is shown in the figure. Which of the following statement(s) is (are) correct?



A) $T_1 = T_2$

B) $T_3 > T_1$

C) $w_{isothermal} > w_{adiabatic}$

D) $\Delta U_{isothermal} > \Delta U_{adiabatic}$

Ans : AD

MATHEMATICS

SECTION – I

(SINGLE CORRECT CHOICE TYPE)

This section contains 8 **multiple choicse questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE is correct**

41. The value of the integral

$$\int_{-\pi/2}^{\pi/2}\left(x^2+\ln\frac{\pi+x}{\pi-x}\right)\cos x\ dx \text{ is}$$

A) 0 B) $\frac{\pi^2}{2}-4$ C) $\frac{\pi^2}{2}+4$ D) $\frac{\pi^2}{2}$

Ans. B

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x^2\cos x\,dx \quad \left(\because \log\left(\frac{\pi+x}{\pi-x}\right) \textit{is an odd function}\right)$$

$$=2\int_0^{\frac{\pi}{2}} x^2\cos x dx$$

$$=2\left[x^2(\sin x)-(2x)(-\cos x)+(2)(-\sin x)\right]_0^{\frac{\pi}{2}}$$

$$=2\left[\frac{\pi^2}{4}-2\right]$$

$$=\frac{\pi^2}{2}-4$$

42. Let $a_1, a_2, a_3, \ldots$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is

A) 22 B) 23 C) 24 D) 25

Ans. D

$$\frac{1}{a} = 5, \quad \frac{1}{a+19d} = 25$$

$$a = \frac{1}{5} \qquad a + 19d = \frac{1}{25}$$

$$19d = \frac{1}{25} - \frac{1}{5}$$

$$= \frac{-4}{25}$$

$$d = \frac{-4}{25(19)}$$

$$a_n = \frac{1}{a + (n-1)d} < 0$$

$$\Rightarrow a + (n-1)d < 0$$

$$\frac{1}{5} + (n-1)\left(\frac{-4}{5(19)}\right) < 0$$

$95 - 4n + 4 < 0$

$99 < 4n$

$$n > \frac{99}{4}$$

$$n > 24\frac{3}{4}$$

n = 25, 26, 27, ...

least n = 25

43. The equation of a plane passing through the line of intersection of the planes $x+2y+3z=2$ and $x-y+z=3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3,1,-1)$ is

A) $5x-11y+z=17$ B) $\sqrt{2}x+y=3\sqrt{2}-1$

C) $x+y+z=\sqrt{3}$ D) $x-\sqrt{2}y=1-\sqrt{2}$

Ans. A

Required plane is

$(x+2y+3z-2)+\lambda(x-y+z-3)=0 \longrightarrow (1)$

$\Rightarrow x(1+\lambda)+y(2-\lambda)+z(3+\lambda)+(-2-3\lambda)=0$

Given

$$\frac{|3(1+\lambda)+(2-\lambda)(1)(3+\lambda)(-1)-2-3\lambda|}{\sqrt{(1+\lambda)^2+(2-\lambda)^2+(3+\lambda)^2}}=\frac{2}{\sqrt{3}}$$

$\lambda=\frac{-7}{2}$

$(1)\Rightarrow 5x-11y+z-17=0$

44. Let PQR be a triangle of area Δ with $a=2, b=\frac{7}{2}$ and $c=\frac{5}{2}$, where a,b and c are the lengths of the sides of the triangle opposite to the angles at P,Q and R respectively. Then $\frac{2\sin P-\sin 2P}{2\sin P+\sin 2P}$ equals

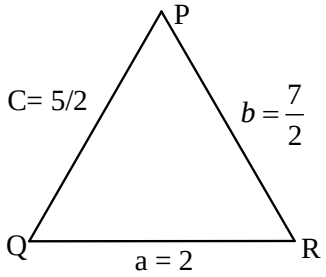
A) $\frac{3}{4\Delta}$ B) $\frac{45}{4\Delta}$ C) $\left(\frac{3}{4\Delta}\right)^2$ D) $\left(\frac{45}{4\Delta}\right)^2$

Ans. C

Hence $\Delta=\sqrt{6}$

$$\frac{2\sin p-\sin 2p}{2\sin p+\sin 2p}=\frac{1-\cos p}{1+\cos p}=\frac{1-\frac{29}{35}}{1+\frac{29}{35}}=\frac{3}{32}$$

Clearly From (C) : $\left(\frac{3}{40}\right)^2=\frac{3}{32}$



45. If $\vec{a}$ and $\vec{b}$ are vectors such that $\left|\vec{a}+\vec{b}\right|=\sqrt{29}$ and $\vec{a}\times\left(2\hat{i}+3\hat{j}+4\hat{k}\right)=\left(2\hat{i}+3\hat{j}+4\hat{k}\right)\times\vec{b}$, then a possible value of $\left(\vec{a}+\vec{b}\right)\cdot\left(-7\hat{i}+2\hat{j}+3\hat{k}\right)$ is

A) 0 B) 3 C) 4 D) 8

Ans. C

$\bar{a}\times(\bar{r})=\bar{r}\times\bar{b}$, where $\bar{r}=2i+3j+4k$

$\bar{a}\times\bar{r}-\bar{r}\times\bar{b}=\bar{0}$

$\bar{r}\times\left(\bar{a}+\bar{b}\right)=0$

$\Rightarrow \bar{a}\times\bar{b}=\lambda\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$

$\left|\bar{a}\times\bar{b}\right|=\sqrt{29}$

$\lambda\sqrt{4+9+16}=\sqrt{29}\Rightarrow\lambda=1$

$\Rightarrow\left(\bar{a}+\bar{b}\right)\cdot\left(-7i+2j+3k\right)=\left(2i+3j+4k\right)\cdot\left(-7i+2j+3k\right)$

$\Rightarrow$ $-14+6+12=4$

46. If P is a 3×3 matrix such that $P^T=2P+I$, where P^T is the transpose of P and I is the 3×3 identity matrix, then there exists a column matrix $X=\begin{bmatrix}x\\y\\z\end{bmatrix}\neq\begin{bmatrix}0\\0\\0\end{bmatrix}$ such that

A) $PX=\begin{bmatrix}0\\0\\0\end{bmatrix}$ B) $PX=X$ C) $PX=2X$ D) $PX=-X$

Ans. D

$P^T=2P+I$

$P=2P^T+I$

$=4P+3I$

$\Rightarrow P=-I$

$\therefore PX=-X$

47. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right)=0$ where $a>-1$. Then $\lim_{a\to 0^+}\alpha(a)$ and $\lim_{a\to 0^+}\beta(a)$ are

A) $-\frac{5}{2}$ and 1 B) $-\frac{1}{2}$ and -1 C) $-\frac{7}{2}$ and 2 D) $-\frac{9}{2}$ and 3

Ans. B

$$\lim_{a\to 0}\frac{(1+a)^{\frac{1}{3}}-1}{(1+a)-1}x^2+\frac{(1+a)^{1/2}-1}{1+a-1}x+\frac{(1+a)^{\frac{1}{6}}-1}{1+a-1}=0$$

$$\frac{1}{3}x^2+\frac{1}{2}x+1=0$$

$$\Rightarrow\frac{2x^2+3x+1}{6}=0$$

$$\Rightarrow 2x^2+2x+x+1=0$$

$$\Rightarrow 2x(x+1)+1(x+1)=0$$

$$x=-\frac{1}{2}\ \text{(or)}\ -1.$$

48. Four fair dice D_1, D_2, D_3 and D_4, each having six faces numbered 1, 2, 3, 4, 5 and 6, are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is

A) $\frac{91}{216}$ B) $\frac{108}{216}$ C) $\frac{125}{216}$ D) $\frac{127}{216}$

Ans. A

Required probability

= 1 – probability of D_4 does not show a number appearing on any of D_1, D_2, D_3.

$$=1-\frac{6\times 5\times 5\times 5}{6\times 6\times\times 6\times 6}$$

$$=1-\frac{125}{216}$$

$$=\frac{91}{216}$$

SECTION – II

(COMPREHENSION TYPE)

This section contains 3 groups of questions. Each group has 2 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct.

Paragraph for Questions 49 and 50

Let $f(x)=(1-x)^2\sin^2 x+x^2$ for all $x\in IR$, and let $g(x)=\int_1^x\left(\frac{2(t-1)}{t+1}-\ln t\right)f(t)\,dt$ for all $x\in(1,\infty)$

49. Consider the statements:

P : There exists some $x\in IR$ such that $f(x)+2x=2(1+x^2)$

Q : There exists some $x\in IR$ such that $2f(x)+1=2x(1+x)$

Then

A) Both **P** and **Q** are true

B) **P** is true and **Q** is false

C) **P** is false and **Q** is true

D) both **P** and **Q** are false

Ans. C

$f(x)=(x-1)^2\sin^2 x+x^2 \quad \forall x\in R$

$f(x)+2x=2(1+x^2)$

$(x-1)^2.\sin^2 x+x^2+2x=2+2x^2$

$(x-1)^2\sin^2 x+2x=x^2+1+1$

$(x-1)^2\sin^2 x=x^2-2x+1+1$

$(x-1)^2\sin^2 x=(x-1)^2+1$

$(x-1)^2\left[\sin^2 x-1\right]=1$

$\sin^2 x-1=\frac{1}{(x-1)^2}$

But $\sin^2 x - 1 \le 0 \quad \forall x \in R$

and $\frac{1}{(x-1)^2} > 0 \quad \forall x \in R$

No such x exist satisfying P.

Q: $2f(x) + 1 = 2x + 2x^2$

$2\left[(x-1)^2 \sin^2 x + x^2\right] + 1 = 2x^2 + 2x$

$2(x-1)^2 \sin^2 x + 2x^2 + 1 = 2x^2 + 2x$

$2(x-1)^2 \sin^2 x + 1 = 2x$

$2(x-1)^2 \sin^2 x = 2x - 1$

$\sin^2 x = \frac{2x-1}{2(x-1)^2} \Rightarrow \frac{2x-1}{2(x-1)^2} \ge 0$

$x \ge \frac{1}{2}$

Q is true.

50. Which of the following is true?

A) g is increasing on $(1, \infty)$

B) g is decreasing on $(1, \infty)$

C) g is increasing on $(1, 2)$ and decreasing on $(2, \infty)$

D) g is decreasing on $(1, 2)$ and increasing on $(2, \infty)$

Ans. B

$$g'(x)=\left[\frac{2(x-1)}{x+1}-\ln x\right]f(x)$$

$f(x)>0$ x > 1

$$h(x)=\frac{2(x-1)}{x+1}-\ln x$$

$$h'(x)=\frac{4}{(x+1)^2}-\frac{1}{x}<0 \quad \forall x>1$$

h (x) is decreasing.

$x>1\Rightarrow h(x)<h(1)$

$$\frac{2(x-1)}{x+1}-\ln x<0$$

$\therefore g'(x)<0 \quad \forall x>1$

g is decreasing. on $(1,\infty)$

Paragraph for Questions 51 and 52

A tangent PT is drawn to the circle $x^2+y^2=4$ at the point $P(\sqrt{3},1)$. A straight line L, perpendicular to PT is a tangent to the circle $(x-3)^2+y^2=1$

51. A common tangent of the two circles is

A) $x=4$ B) $y=2$ C) $x+\sqrt{3}y=4$ D) $x+2\sqrt{2}y=6$

Ans. D

Common tangent B Transversal

$$\therefore I_s=\frac{2(3,0)-(0,0)}{2-1}=(6,0)$$

Common tangent y–0 = m (x-6)

mx-y-6m=0

$$\frac{|-3m|}{\sqrt{1+m^2}}=1$$

$$m=\pm\frac{1}{2\sqrt{2}}$$

$$\therefore\ y=\pm\frac{1}{2\sqrt{2}}(x-6)$$

$$\therefore\ y2\sqrt{2}=-x+6$$

$$x+2\sqrt{2}y=6$$

52. A possible equation of L is

A) $x-\sqrt{3}y=1$ B) $x+\sqrt{3}y=1$ C) $x-\sqrt{3}y=-1$ D) $x+\sqrt{3}y=5$

Ans. A

$$L\equiv x-\sqrt{3}y+k=0$$

is tangent to $(x-3)^2+y^2=1$

$$\frac{3+k}{2}=\pm1$$

$$\therefore\ k=-5, k=-1$$

$$\therefore\ x-\sqrt{3}y=1$$

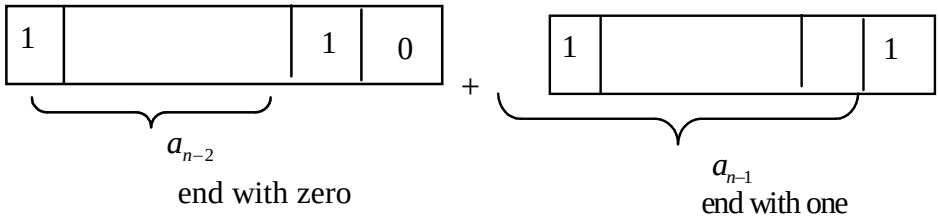
Paragraph for Questions 53 and 54

Let a_n denote the number of all n – digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let $b_n=$ the number of such n – digit integers ending with digit 1 and c_n = the number of such n – digit integers ending with digit 0.

53. Which of the following is correct?

A) $a_{17}=a_{16}+a_{15}$ B) $c_{17}\neq c_{16}+c_{15}$ C) $b_{17}\neq b_{16}+c_{16}$ D) $a_{17}=c_{17}+b_{16}$

Ans. A

a_n = 

$\therefore\ a_n = a_{n-2} + a_{n-1}$

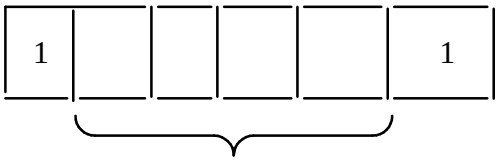
$a_{17} = a_{15} + a_{16}$

54. The value of b_6 is

A) 7 B) 8 C) 9 D) 11

Ans. B

6 digits the integer ending with one = b_6

The no of cases 

cases with 4 ones : 1 1 1 1 one

3 ones :

$$\left.\begin{matrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{matrix}\right\} \rightarrow 4$$

2 ones :

$$\left.\begin{matrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{matrix}\right\} \rightarrow 3$$

$b_6 = 8$

SECTION – III

(MULTIPLE CORRECT CHOICE TYPE)

This section contains **6 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE is/ are correct**

55. If the straight lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is(are)

A) $y+2z=-1$ B) $y+z=-1$ C) $y-z=-1$ D) $y-2z=-1$

Ans. B,C

The lines are coplanar

$$\Leftrightarrow \begin{vmatrix} -2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0$$

$$\Leftrightarrow \quad k=\pm 2$$

Case (i) If $k=2$ *then equation of plane is* $\begin{vmatrix} x-1 & y+1 & z \\ 1 & 1 & 1 \\ 5 & 2 & 2 \end{vmatrix} = 0$

$$\&(x-5)(0)-(y+1)(-3)+z(-3)=0$$

$$y+1=z$$

$$y-z=-1$$

Case (ii) If $k=-2$ *then equation of plane is* $\begin{vmatrix} x-1 & y+1 & z \\ 1 & -1 & 1 \\ 5 & 2 & -2 \end{vmatrix} = 0$

$$(x-1)(0)-(y+1)(-7)+z(7)=0$$

$$7y+7+7z=0$$

$$y+z=-1$$

56. If the adjoint of a 3×3 matrix P is $\begin{bmatrix}1 & 4 & 4\\ 2 & 1 & 7\\ 1 & 1 & 3\end{bmatrix}$, then the possible value(s) of the determinant of P is (are)

A) -2 B) -1 C) 1 D) 2

Ans. A,D

$$|adjp| = \begin{vmatrix}1 & 4 & 4\\ 2 & 1 & 7\\ 1 & 1 & 3\end{vmatrix}^2 = 4 = |P|^2$$

$$|P| = \pm 2$$

57. Let $f:(-1,1)\to IR$ be such that $f(\cos 4\theta) = \dfrac{2}{2-\sec^2\theta}$ for $\theta \in \left(0,\dfrac{\pi}{4}\right)\cup\left(\dfrac{\pi}{4},\dfrac{\pi}{2}\right)$. Then the value(s) of $f\left(\dfrac{1}{3}\right)$ is (are).

A) $1-\sqrt{\dfrac{3}{2}}$ B) $1+\sqrt{\dfrac{3}{2}}$ C) $1-\sqrt{\dfrac{2}{3}}$ D) $1+\sqrt{\dfrac{2}{3}}$

Ans. A,B

$$f(\cos 4\theta) = \frac{1+\cos 2\theta}{\cos 2\theta} = 1+\sec 2\theta$$

$$\therefore \cos 4\theta = \frac{1}{3} \Rightarrow 2\cos^2 2\theta = \frac{4}{3} \Rightarrow \cos^2 2\theta = \frac{2}{3}$$

$$\therefore \sec 2\theta = \pm\sqrt{\frac{3}{2}}$$

$$\therefore f\left(\frac{1}{3}\right) = 1\pm\sqrt{\frac{3}{2}}$$

$\therefore A, B$

58. Let X and Y be two events such that $P(X/Y)=\frac{1}{2}, P(Y/X)=\frac{1}{3}$ and $P(X\cap Y)=\frac{1}{6}$. Which of the following is (are) correct?

A) $P(X\cup Y)=\frac{2}{3}$

B) X and Y are independent

C) X and Y are not independent

D) $P(X^c\cap Y)=\frac{1}{3}$

Ans. A,B

$$p\left(\frac{X}{Y}\right)=\frac{p(X\cap Y)}{p(Y)}\Rightarrow p(Y)=\frac{1}{3}$$
$$p\left(\frac{Y}{X}\right)=\frac{p(X\cap Y)}{p(X)}\Rightarrow p(Y)=\frac{1}{2}$$

$\therefore p(X\cap Y)=p(X).p(Y)\therefore X,Y$ *independent*

$$\therefore p(X\cup Y)=\frac{1}{3}+\frac{1}{2}-\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$$
$$p(\bar{X}\cap Y)=p(X)-p(X\cap Y)=\frac{1}{2}-\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$$
$\therefore A,B$

59. If $f(x)=\int_0^x e^{t^2}(t-2)(t-3)\,dt$ for all $x\in(0,\infty)$, then

A) f has a local maximum at $x=2$

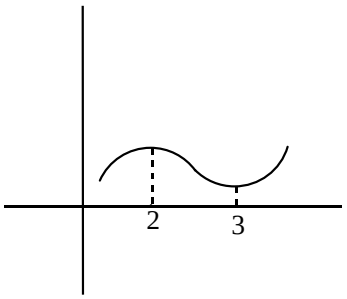
B) f is decreasing on $(2,3)$

C) there exists some $c\in(0,\infty)$ such that $f''(c)=0$

D) f has a local minimum at $x=3$

Ans. ABCD

$f'(x) = e^{x^2}(x-2)(x-3)$



max . at x = 2

min. at x = 3

decreasing on (2, 3)

$f''(x) = e^{x^2}\left(x^2 - 5x + 6 + 2x - 5\right) = 0$

$\Rightarrow x^2 - 3x + 1 = 0$

$\Rightarrow x = \dfrac{3 \pm \sqrt{5}}{2} \in (0, \infty)$

60. For every integer n, let a_n and b_n be real numbers. Let function $f : IR \to IR$ be given by

$f(x) = \begin{cases} a_n + \sin \pi x, & \text{for } x \in [2n, 2n+1] \\ b_n + \cos \pi x, & \text{for } x \in (2n-1, 2n) \end{cases}$, for all integers n.

If f is continuous, then which of the following hold(s) for all n?

A) $a_{n-1} - b_{n-1} = 0$ B) $a_n - b_n = 1$ C) $a_n - b_{n+1} = 1$ D) $a_{n-1} - b_n = -1$

Ans. B, D

continuous at x = 2n

$\Rightarrow b_n + 1 = a_n$

$\Rightarrow a_n - b_n = 1 \quad \Rightarrow$ (B)

continuous at x = 2 (n –1)

$\Rightarrow a_{n-1} - b_{n-1} = 1$

continuous at x = 2n – 1

$\Rightarrow a_{n-1} = b_n - 1$ —— (D)

continuous at x = 2n + 1

$\Rightarrow a_n = b_{n+1} - 1$