MATHEMATICS

SECTION – I

(SINGLE CORRECT CHOICE TYPE)

This section contains 10 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct

41. Let z be a complex number such that the imaginary part of z is nonzero and and $a = z^2 + z + 1$ is real. Then a connot take the value

a) -1 b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{3}{4}$

Ans. D

Let $Z = x + iy \quad (y \neq 0)$

$a = z^2 + z + 1$ becomes

$a = (x+iy)^2 + (x+iy) + 1$

$(x^2 - y^2 + x + 1) + i(2xy + y) = a + i0$

Comparing real & imaginary parts

$x^2 - y^2 + x + 1 = a$ ——— (1)

$2xy + y = 0$ ——— (2)

From (2) $x = \frac{-1}{2}$ as $y \neq 0$

Put value of x in (1)

$\left(-\frac{1}{2}\right)^2 - y^2 - \frac{1}{2} + 1 = a$

$\frac{1}{4} + \frac{1}{2} - y^2 = a$

$y^2 = \frac{3}{4} - a > 0$

$a < \frac{3}{4}$

42. If $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then

a) a = 1, b = 4 b) a = 1, b = –4 c) a = 2, b = –3 d) a = 2, b = 3

Ans. B

$$\lim_{x\to\infty}\frac{x^2+x+1}{x+1}-ax-b=4$$

$$\lim_{x\to\infty}\frac{x^2(1-a)+x(1-a-b)+1-b}{x+1}=4$$

For existense of limit coefficient of $x^2=0$

1 – a = 0

a = 1

$$\lim_{x\to\infty}\frac{x(1-a-b)+1-b}{1+x}=4$$

$1-a-b=4$

b = – 4

43. Let $P=[a_{ij}]$ be a 3×3 matrix and let $Q=[b_{ij}]$, where $b_{ij}=2^{i+j}a_{ij}$ for $1\le i, j\le 3$. If the determinant of P is 2, then the determinant of the matrix Q is

a) 2^{10} b) 2^{11} c) 2^{12} d) 2^{13}

Ans. D

as $b_{ij}=2^{i+j}a_{ij}$

$b_{11}=4a_{11},\ b_{12}=8a_{12}\ ,b_{13}=16a_{13}$

$b_{21}=8a_{2},\ b_{22}=16a_{22},\ b_{23}=32a_{23}$

$b_{31}=16a_{31},\ b_{32}=32a_{32},\ b_{33}=64a_{33}$

$$A=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}=2$$

$$B=\begin{bmatrix}4a_{11} & 8a_{12} & 16a_{13}\\ 8a_{21} & 16a_{22} & 32a_{23}\\ 16a_{31} & 32a_{32} & 64a_{33}\end{bmatrix}$$

$$=4\times8\times16\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ 2a_{21} & 2a_{22} & 2a_{23}\\ 4a_{31} & 4a_{32} & 4a_{33}\end{bmatrix}$$

$$=4\times8\times16\times2\times4\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$$

$$=4\times8\times16\times2\times4\times2$$

$$B=2^{13}$$

44. The ellipse $E_1:\frac{x^2}{9}+\frac{y^2}{4}=1$ is inscribed in a rectangle R whose sides are parallel to the coordinates axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is

a) $\frac{\sqrt{2}}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{1}{2}$ d) $\frac{3}{4}$

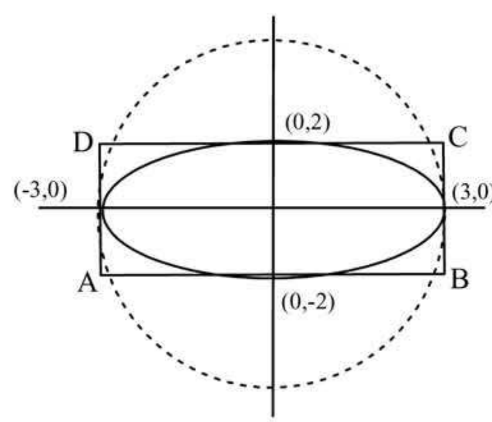
Ans. C

Given Ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$

As ellipse is bounded by $x=\pm a$ & $y=\pm b$

Sides of rectangle are

$x=\pm3$ & $y=\pm2$



$C(3,2),D(-3,2)$

$A(-3,-2),B(3,-2)$

As new ellipse

Circumscribe the rectangle & passes through (0 ,4).

Let new ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Passes through $(0,4) \Rightarrow \frac{16}{b^2}=1 \Rightarrow b^2=16$

Passes through $(3,2)$ $\frac{9}{a^2}+\frac{4}{16}=1$

$$a^2=12$$

So, Vertical ellipse $\frac{x^2}{12}+\frac{y^2}{16}=1$

$a^2=b^2\left(1-e^2\right)$

$12=16\left(1-e^2\right)$

$e^2=1-\frac{12}{16}=\frac{1}{4}$

$e=\frac{1}{2}$

45. The function $f:[0,3] \rightarrow [1,29]$, defined by $f(x)=2x^3-15x^2+36x+1$ is

a) one-one and onto

b) onto but not one-one

c) one-one but not onto

d) neither one-one nor onto

Ans. B

$f:[0,3] \rightarrow [1,29]$

$f(x)=2x^3-15x^2+36x+1$

$f'(x)=6x^2-30x+36$

$=6(x-2)(x-3)$

$f''(x)=6(2x-5)$

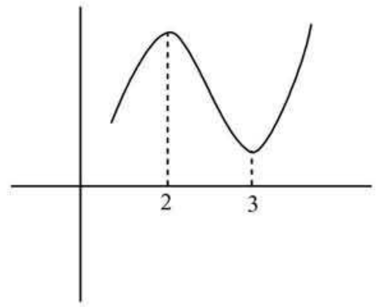
$\therefore x=2$ is pt of maxima.

$x = 3$ is pt of minima.

$\& f(2) = 29 > 0$

$f(3) > 0$

so curve is



$\therefore f^n$ is not one one

Min value $= f(0) = 1$

Max value $= f(2) = 29$

$\therefore$ so f^n is Onto.

46. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is

a) $20(x^2 + y^2) - 36x + 45y = 0$

b) $20(x^2 + y^2) + 36x - 45y = 0$

c) $36(x^2 + y^2) - 20x + 45y = 0$

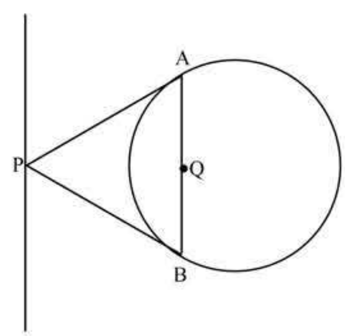
d) $36(x^2 + y^2) + 20x - 45y = 0$

Ans. A

Let $P(h,k) \& Q(x_1, y_1)$

As P lies of *4x-5y=20*

$\therefore P\left(h, \frac{4h-20}{5}\right)$



Equation of chord of contact

$$AB \Rightarrow S_1 = 0$$

$xh + y\left(\frac{4h-20}{5}\right) = 9$

$5xh + y(4h - 20) = 45$ ------------------(1)

Equation of chord AB whose mid point is $Q(x_1, y_1)$

$xx_1 + yy_1 = x_1^2 + y_1^2$ ----------------(2)

Compare (1) & (2)

$$\frac{5h}{x_1} = \frac{4h-20}{y_1} = \frac{45}{x_1^2 + y_1^2}$$

$$h = \frac{9x_1}{x_1^2 + y_1^2} \;\; \& \; 4h - 20 = \frac{45y_1}{x_1^2 + y_1^2}$$

On eliminating h

$$\frac{49x_1}{x_1^2 + y_1^2} - 20 = \frac{45y_1}{x_1^2 + y_1^2}$$

$$20\left(x_1^2 + y_1^2\right) = 36x_1 - 45y_1$$

$\therefore$ Locus is

$$20\left(x^2 + y^2\right) - 36x + 45y = 0$$

47. The point P is the intersection of the straight line joining the points Q (2, 3, 5) and R (1, –1, 4) with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point T (2, 1, 4) to QR, then the length of the line segement PS is

a) $\frac{1}{\sqrt{2}}$ b) $\sqrt{2}$ c) 2 d) $2\sqrt{2}$

Ans. A

Equation of $QR \rightarrow \frac{x-1}{1} = \frac{y+1}{4} = \frac{z-4}{1} = \lambda$

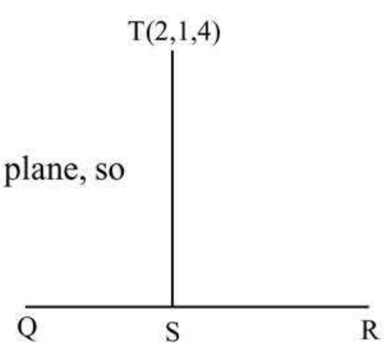
Dr's of $QR(1,4,1)$

Co-ordinates of $P(1+\lambda, -1+4\lambda, 4+\lambda)$

As P is point of Intersection of QR & given plane, so

$5(1+\lambda) - 4(-1+4\lambda) - (4+\lambda) = 1$

$\lambda = \frac{1}{3}$



$\therefore P\left(\frac{4}{3},\frac{1}{3},\frac{13}{3}\right)$

Let $S(1+\mu,-1+4\mu,4+\mu)$

Dr's of $TS(\mu-1,4\mu-2,\mu)$

$TS\perp QR$ So,

$1(\mu-1)+4(4\mu-2)+\mu=0$

$\mu=\frac{1}{2}$

$\therefore S\left(\frac{3}{2},1,\frac{9}{2}\right)$

$\therefore PS=\sqrt{\left(\frac{4}{3}-\frac{3}{2}\right)^2+\left(1-\frac{1}{3}\right)^2+\left(\frac{13}{3}-\frac{9}{2}\right)^2}$

$=\frac{1}{\sqrt{2}}$

48. Let $f(x)=\begin{cases}x^2\left|\cos\frac{\pi}{x}\right|, & x\neq 0\\ 0, & x=0\end{cases}$, $x\in IR$, then f is

a) differentiable both at x = 0 and at x = 2

b) differentiable at x = 0 but not differentiable at x = 2

c) not differentiable at x = 0 but differentiable at x = 2

d) differentiable neither at x = 0 nor at x = 2

Ans. B

$f(x)=\begin{cases}x^2\left|\cos\frac{\pi}{x}\right| ; x\neq 0\\ 0 ; x=0\end{cases}$

<u>At $x=0$</u>

$$RHD \Rightarrow \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{h^2\cos\frac{\pi}{h}-0}{h}$$

$$=\lim_{h\to 0}\left(h\cos\frac{\pi}{h}\right)=0$$

$$LHD \Rightarrow \lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}=\lim_{h\to 0}\frac{h^2\left|\cos\left(-\frac{\pi}{h}\right)\right|-0}{-h}$$

$$=\lim_{h\to 0}\left(-h\cos\left(\frac{\pi}{h}\right)\right)=0$$

$\therefore$ Differentiable at $x=0$

<u>At $x=2$</u>

$$RHD \Rightarrow \lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{(2+h)^2\left|\cos\frac{\pi}{2+h}\right|-4\left|\cos\frac{\pi}{2}\right|}{h}=\infty$$

$$LHD \Rightarrow \lim_{h\to 0}\frac{f(2-h)-f(2)}{-h}=\lim_{h\to 0}\frac{(2-h)^2\left|\cos\frac{\pi}{2-h}\right|-4\left|\cos\frac{\pi}{2}\right|}{-h}=-\infty$$

$\therefore$ Not Differentiable at $x=2$.

49. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

a) 75 b) 150 c) 210 d) 243

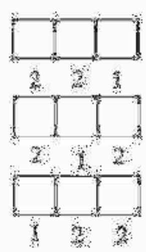
Ans. B

5 Balls to 3 persons, so that all persons get atleast one ball.

<u>Case-I</u>

Total ways

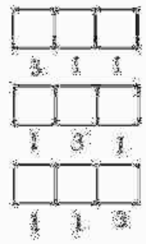
$$=\left({}^5C_2\times{}^3C_2\times{}^1C_1\right)\times 3$$



$=10\times 3\times 1\times 3=90$

<u>Case-II</u>

Total ways



$=\left({}^5C_3\times {}^2C_1\times {}^1C_1\right)\times 3$

$=10\times 2\times 1\times 3=60$

$\therefore$ Total ways = 90 + 60 = 150.

50. The integral $\int \frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$ equals (for some arbitrary constant K)

a) $-\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

b) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

c) $-\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

d) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

Ans. *C*

$\int \frac{\sec x}{(\sec x+\tan x)^{9/2}}dx$

Let $\sec x+\tan x=t$ ----------------(1)

So, $\left(\sec x\tan x+\sec^2 x\right)dx=dt$

$\sec x\,dx=\frac{dt}{t}$ ----------------(2)

$\sec x - \tan x = \frac{1}{t}$ ----------(3)

From (1) & (3)

$\sec x = \frac{1}{2}\left(t + \frac{1}{t}\right)$

So, Integral becomes

$$\int \frac{\sec x . \sec x}{(\sec x + \tan x)^{9/2}} dx$$

$$\int \frac{\frac{1}{2}\left(t + \frac{1}{t}\right) dt/t}{t^{9/2}}$$

$$\frac{1}{2}\int \frac{t^2 + 1}{t^{13/2}} dt$$

$$= \frac{1}{2}\left[\int t^{-9/2} dt + \int t^{-13/2} dt\right]$$

$$= \frac{1}{2}\left[\frac{t^{-7/2}}{-7/2} + \frac{t^{-11/2}}{-11/2}\right] + K$$

$$= \frac{t^{-7/2}}{-7} + \frac{t^{-11/2}}{-11} + K$$

$$= -\frac{1}{7(\sec x + \tan x)^{7/2}} - \frac{1}{11(\sec x + \tan x)^{11/2}}$$

$$= -\frac{1}{(\sec x + \tan x)^{11/2}}\left(\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right) + K$$

SECTION – II

(MULTIPLE CORRECT CHOICE TYPE)

This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONE OR MORE is/ are correct

51. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true ?

a) $P\left[x_1^c \mid X\right] = \frac{3}{16}$ b) P [Exactly two engines of the ship are functioning | X] $= \frac{7}{8}$

c) $P\left[X \mid X_2\right] = \frac{5}{16}$ d) $P\left[X \mid X_1\right] = \frac{7}{16}$

Ans. B,D

A) $$\frac{P\left(X_1^c \cap X\right)}{P(X)} = \frac{\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}}{\frac{1}{4}} = \frac{1}{8}$$

$$P(X) = \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right) \times 2 + \left(\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}\right) \times 2 = \frac{1}{4}$$

B) $$P\left[Exactly\ two\ engines\ of\ the\ ship\ are\ functioning / X\right] = \frac{\left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right) \times 2 + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}}{\frac{1}{4}} = \frac{7}{8}$$

C) $$P\left(X / X_2\right) = \frac{P\left[X_2 \cap X\right]}{P\left(X_2\right)} = \frac{\left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}\right) \times 2}{\frac{1}{4}} = \frac{5}{8}$$

D) $$P\left(X / X_1\right) = \frac{P\left[X \cap X_1\right]}{P\left(X_1\right)} = \frac{\left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right) \times 2 + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}}{\frac{1}{2}} = \frac{7}{16}$$

52. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, parallel to the straight line $2x-y=1$. The points of contact of the tangents on the hyperbola are

a) $\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ b) $\left(-\frac{9}{2\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$ c) $\left(3\sqrt{3},-2\sqrt{2}\right)$ d) $\left(-3\sqrt{3},2\sqrt{2}\right)$

Ans. A, B

$$\frac{x^2}{9}-\frac{y^2}{4}=1$$

Tangent to the hyperbola is of the form

$$y=2x+\lambda$$

$$\lambda^2=9(4)-4=36-4=32$$

$$\lambda=\pm 4\sqrt{2}$$

Tangent is : $2x-y\pm 4\sqrt{2}=0$

Point of contact is $\left(-\frac{a^2l}{n},\frac{b^2m}{n}\right)=\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right),\left(-\frac{9}{2\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$

53. Let S be the area of the region enclosed by $y=e^{-x^2}$, y = 0, x = 0, and x = 1. Then

a) $S\geq\frac{1}{e}$ b) $S\geq 1-\frac{1}{e}$ c) $S\leq\frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$ d) $S\leq\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$

Ans. A, B, D

$$I=\int_0^1 e^{-x^2}dx\geq\int_0^1 e^{-x}dx=1-\frac{1}{e}$$

$$I\geq 1-\frac{1}{e}$$

$$e^{-1}\leq e^{-x^2}\leq 1$$

$$\int_0^1 e^{-1}dx\leq\int_0^1 e^{-x^2}dx\leq\int_0^1 dx$$

$$\therefore\int_0^1 e^{-x^2}dx\geq\frac{1}{e}$$

54. If y (x) satisfies the differential equation $y' - y\tan x = 2x\sec x$ and y (0) = 0, then

a) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$ b) $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$ c) $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$ d) $y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$

Ans. A,D

$$\frac{dy}{dx} - y\tan x = 2x\sec x$$

$$I.F = e^{-\int \tan dx} = \cos x$$

General solution is $y(\cos x) = \int (2x\sec x)\cos x \, dx$

$$= x^2 + c$$

$$y = x^2 \sec x + c\sec x$$

$$\longrightarrow (0,0)$$
$$0 = 0 + c \Rightarrow c = 0$$

$$y = x^2 \sec x$$

$$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16}\left(\sqrt{2}\right)$$

$$= \frac{\pi^2}{8\sqrt{2}}$$

$$y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9} \times 2$$

$$\frac{dy}{dx} = x^2 \sec x \tan x + 2x\sec x.$$

$$y'\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}(2)\sqrt{3} + 2\left(\frac{\pi}{3}\right)2$$

$$= \frac{2\pi^2}{3\sqrt{3}} + \frac{4\pi}{3}$$

55. Let $\theta,\ \phi \in [0, 2\pi]$ be such that

$$2\cos\theta(1-\sin\phi) = \sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\phi - 1,$$

$\tan(2\pi-\theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$.

Then ϕ cannot satisfy

a) $0 < \phi < \frac{\pi}{2}$ b) $\frac{\pi}{2} < \phi < \frac{4\pi}{3}$ c) $\frac{4\pi}{3} < \phi < \frac{3\pi}{2}$ d) $\frac{3\pi}{2} < \phi < 2\pi$

Ans. A, C, D

$$\tan(2\pi-\theta) > 0, -1 < \sin\theta < -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right) \Rightarrow 0 < \cos\theta < \frac{1}{2} \quad \text{\_\_\_\_\_\_ (1)}$$

$$2\cos\theta(1-\sin\phi) = \sin^2\theta(\tan\theta/2+\cot\theta/2)\cos\phi - 1$$

$$\Rightarrow \sin(\theta+\phi) = \cos\theta + \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} < \sin(\theta+\phi) < 1 \quad \text{(from (1))}$$

$$\frac{13\pi}{6} < \theta+\phi < \frac{17\pi}{6}$$

$$\Rightarrow \frac{2\pi}{3} < \phi < \frac{7\pi}{6} \quad \text{(from (1))}$$

cannot satisfied by A, C, D

SECTION –III

(INTEGER ANSWER TYPE)

This section contains 5 questions . The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened.

56. Let p (x) be a real polynomial of least degree which has a local maximum at x = 1 and a local minimum at x =3. If p (1) = 6 and p (3) = 2, then $p'(0)$ is

Ans. 9

$$P(x) = ax^3 + bx^2 + cx + d$$

$$P(1) = 6 \Rightarrow a + b + c + d = 6$$

$$P(3) = 2 \Rightarrow 27a + 9b + 3c + d = 2$$

$$P^1(1) = 0 \Rightarrow 3a + 2b + c = 0$$

$$P^1(3) = 0 \Rightarrow 27a + 6b + c = 0$$

$$a = 1, b = -6, c = 9$$

$$P^1(x) = 3ax^2 + 2bx + c$$

$$P^1(0) = c = 9$$

57. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\left|\vec{a} - \vec{b}\right|^2 + \left|\vec{b} - \vec{c}\right|^2 + \left|\vec{c} - \vec{a}\right|^2 = 9$,then $\left|2\vec{a} + 5\vec{b} + 5\vec{c}\right|$ is\

Ans. 3

sol. $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$$\left|\bar{a} - \bar{b}\right|^2 + \left|\bar{b} - \bar{c}\right|^2 + \left|\bar{c} - \bar{a}\right|^2 = 9$$

$$\Rightarrow \ \bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} . \bar{a} = \frac{-3}{2}$$

$$3 + 2\left(\bar{a} \cdot \bar{b} + \bar{b} . \bar{c} + \bar{c} . \bar{a}\right) = 0$$

$$\Rightarrow \ |\bar{a}|^2 + \left|\bar{b}\right|^2 + |\bar{c}|^2 + 2\left|\bar{a} . \bar{b} + \bar{b} . \bar{c} + \bar{c} . \bar{a}\right| = 0$$

$$\Rightarrow \ \left|\bar{a} + \bar{b} + \bar{c}\right|^2 = 0$$

$\Rightarrow \ \bar{a}+\bar{b}+\bar{c}=\bar{0}$ ——— **(1)**

Taking dot product with (1) by $\bar{a}, \bar{b} \ \& \ \bar{c}$

then $\bar{a}.\bar{b}=\frac{-1}{2}, \ \bar{b}.\bar{c}=\frac{-1}{2}, \ \bar{c}.\bar{a}=\frac{-1}{2}$

$\therefore \ \left|2\bar{a}+5\bar{b}+5\bar{c}\right|=3$

58. The value of $6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\cdots}}}\right)$ is

Ans. 4

sol. $x=\frac{1}{3\sqrt{2}}\sqrt{4-x}$

$3\sqrt{2}\ x=\sqrt{4-x}$

squaring on both sides

$18x^2=4-x$

$18x^2+x-4=0$

$18x^2+9x-8x-4=0$

$9x(2x+1)-4(2x+1)=0.$

$(2x+1)(9x-4)=0$

$x=-\frac{1}{2},\frac{4}{9}$

$\Rightarrow x=\frac{4}{9} \quad (\because x>0)$

$\therefore$ Required answer is $6+\log_{3/2}\left(\frac{3}{2}\right)^{-2}$

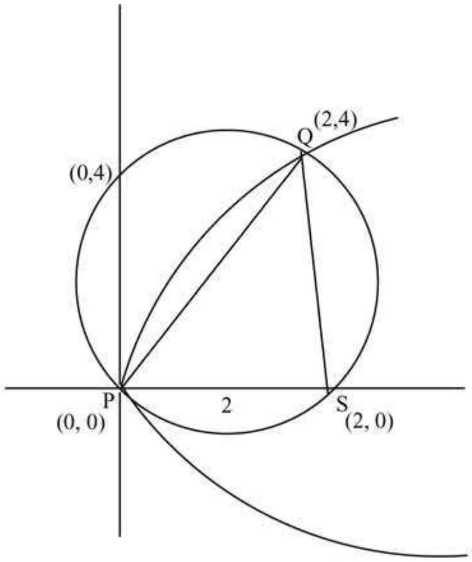
$=6-2$

$=4$

59. Let S be the focus of the parabola $y^2=8x$ and let PQ be the common chord of the circle $x^2+y^2-2x-4y=0$ and the given parabola. The area of the triangle PQS is

Ans. 4

sol.



$$x^2+8x-2x-4\sqrt{2x}=0$$
$$\Rightarrow x=0,2 \Rightarrow P=(0,0),\ Q=(2,4)$$

area of ΔSPQ

$= \frac{1}{2}\times 2\times 4$

$= 4$

60. Let $f: IR \to IR$ be defined as $f(x)=|x|+|x^2-1|$. The total number of points at which f attains either a local maximum or a local minimum is

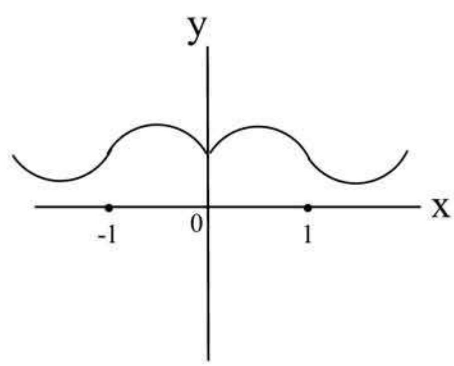
Ans. 5

sol. $f(x)=-x+x^2-1,\ x\le -1$

$=-x+1-x^2,\ -1<x<0$

$=x+1-x^2,\ 0\le x<1$

$=x+x^2-1,\ x\ge 1$



$f'(x)=2x-1,\quad x<-1$

$=-2x-1, -1<x<0$

$=1-2x, 0<x<1$

$=1+2x,\ x>1$

differentiable at $x=-\frac{1}{2},\frac{1}{2},$

not differentiable at -1, 0, 1

The number of points = 5