QUESTIONS & SOLUTIONS OF AIEEE 2011

Date : 01-05-2011 **Duration : 3 Hours** **Max. Marks : 360**

IMPORTANT INSTRUCTIONS

1. Immediately fill the particulars on this page of the Test Booklet with Blue / Black Ball Point Pen. Use of pencil is strictly prohibited.

2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.

3. The test is of **3 hours** duration.

4. The Test Booklet consists of **90** questions. The maximum marks are **360.**

5. There are three parts in the question paper A, B, C consisting of **Chemistry, Physics and Mathematics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.

6. Candidates will be awarded marks as stated above in Instructions No. 5 for correct response of each question. **¼ (one fourth)** marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 6 above.

8. Use **Blue/Black Ball Point Pen only** for writing particulars/marking responses on **Side-1** and **Side-2** of the Answer Sheet. **Use of pencil is strictly prohibited.**

9. No candidate is allowed to carry any textual material, printed or written, bits of papers, paper, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.

10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in 3 pages at the end of the booklet.

11. On completion of the test, the candiate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.

12. The CODE for this Booklet is P. Maken sure that the CODE printed on **Side-2** of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the condidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.

13. Do not fold or make any stray marks on the Answer Sheet.

Name of the Candiate (in Capital letters) : _____

Roll Number : in figures : | | | | | | | | | in words : _____

Examination Centre Number : | | | | | | |

Name of Examination Centre (in Capital letters) : _____

Candidate's Signature : _____ Invigilator's Signature : _____

PART-A (CHEMISTRY)

1. The presence or absence of hydroxy group on which carbon atom of sugar differentiates RNA and DNA.
(1) I^{st} (2) 2^{nd} (3) 3^{rd} (4) 4^{th}

Ans. (2)

Sol. RNA and DNA has ribose and deoxyribose sugars, which differs in absence of hydroxy group at 2^{nd} carbon.

2. Among the following the maximum covalent character is shown by the compound :
(1) $FeCl_2$ (2) $SnCl_2$ (3) $AlCl_3$ (4) $MgCl_2$

Ans. (3)

Sol. Covalent character in ionic compounds is governed by Fazan's Rule. $AlCl_3$ will show Maximum covalent character on account of higher polarising power of Al^{3+} because of its having higher positive charge and smaller size.

3. Which of the following statement is wrong ?
(1) The stability of hydrides increase from NH_3 to BiH_3 in group 15 of the periodic table :
(2) Nitrogen cannot form d!-p! bond.
(3) Single N – N bond is weaker than the single P – P bond.
(4) N_2O_4 has two resonance structure

Ans. (1)

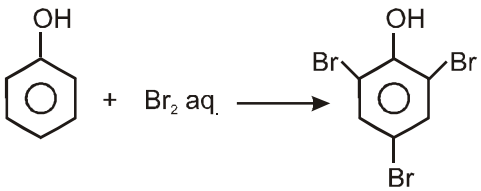
Sol. The stability of hydrides decreases from NH_3 to BiH_3 which can be observed from their bond dissociation enthalpy. The correct order is $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$.

Property	NH_3	PH_3	AsH_3	SbH_3	BiH_3
$\forall_{diss}H^{\#}$(E–H) / kJ mol^{-1}	389	322	297	255	–

4. Phenol is heated with a solution of mixture of KBr and $KBrO_3$. The major product obtained in the above reaction is :
(1) 2-Bromophenol (2) 3-Bromophenol
(3) 4-Bromophenol (4) 2, 4, 6 -Tribromophenol

Ans. **(4)**

Sol. KBr (aq.) + $KBrO_3$ (aq.) ∃ % Br_2 (aq.)



2, 4, 6-tribromophenol

5. A 5.2 molal aqueous solution of methyl alcohol, CH_3OH, is supplied. What is the mole fraction of methyl alcohol in the solution ?

(1) 0.100 (2) 0.190 (3) 0.086 (4) 0.050

Ans. (3)

Sol. $X_{\text{ethyl alcohol}} = \dfrac{5.2}{5.2 + \dfrac{1000}{18}} = 0.086$

6. The hybridisation of orbitals of N atom in NO_3^-, NO_2^+ and NH_4^+ are respectively :

(1) sp, sp^2, sp^3 (2) sp^2, sp, sp^3 (3) sp, sp^3, sp^2 (4) sp^2, sp^3, sp

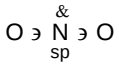
Ans. (2)

Sol. NO_2^+ Number of electron pairs = 2

Number of bond pairs = 2

Number of lone pair = 0

So, the species is linear with sp hybridisation.

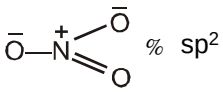


NO_3^- Number of electron pairs = 3

Number of bond pairs = 3

Number of lone pair = 0

So, the species is trigonal planar with sp^2 hybridisation.

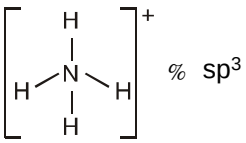


NH_4^+ Number of electron pairs = 4

Number of bond pairs = 4

Number of lone pair = 0

So, the species is tetrahedral with sp^3 hybridisation.



7. Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it form freezing at – 6° C will be : (K_f for water = 1.86 K kg mol^{-1}, and molar mass of ethylene glycol = 62g mol^{-1}) :

(1) 804.32 g (2) 204.30 g (3) 400.00 g (4) 304.60 g

Ans. (1)

Sol. $\Delta T_f = iK_f m$

$\Delta T_f = 6^oC$

$i = 1$

$6 = 1 \times 1.86 \times \dfrac{w}{62 \times 4}$

$w = 804.32$ g.

8. The reduction potential of hydrogen half-cell will be negative if :

(1) $p(H_2)$ = 1 atm and $[H^+]$ = 2.0 M (2) $p(H_2)$ = 1 atm and $[H^+]$ = 1.0 M

(3) $p(H_2)$ = 2 atm and $[H^+]$ = 1.0 M (4) $p(H_2)$ = 2 atm and $[H^+]$ = 2.0 M

Ans. (3)

Sol. $2H^+ (aq) + 2e^- \exists\exists\ \%\ H_2 (g)$

$$E_{red} = E^o_{red} - \frac{0.0591}{n} \log \frac{P_{H_2}}{(H^{\&})^2}$$

$$E_{red} = 0 - \frac{0.0591}{2} \log \frac{2}{(1)^2}$$

$E_{red} = -\frac{0.0591}{2} \log 2$) E_{red} is forund to be negative for (3) option.

9. Which of the following reagents may be used to distinguish between phenol and benzoic acid ?

(1) Aqueous NaOH (2) Tollen's reagent (3) Molisch reagent (4) Neutral $FeCl_3$

Ans. (4)

Sol. Neutral $FeCl_3$ reacts with phenol and give violet coloured complex.

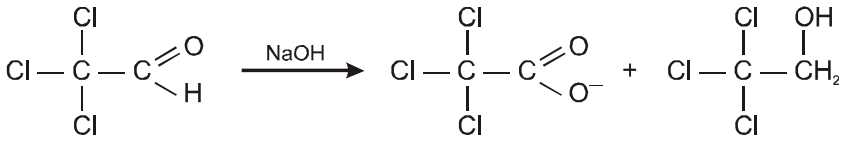
10. Trichloroacetaldehyde was subjected to Cannizzaro's reaction by using NaOH. The mixture of the products contains sodium trichloroacetate ion and another compound. The other compound is :

(1) 2, 2, 2–Trichloroethanol (2) Trichloromethanol

(3) 2, 2, 2–Trichloropropanol (4) Chloroform

Ans. (1)

Sol. The cannizzaro product of given reaction yields 2, 2, 2-trichloroethanol.



11. Which one of the following orders presents the correct sequence of the increasing basic nature of the given oxides ?

(1) $Al_2O_3 < MgO < Na_2O < K_2O$ (2) $MgO < K_2O < Al_2O_3 < Na_2O$

(3) $Na_2O < K_2O < MgO < Al_2O_3$ (4) $K_2O < Na_2O < Al_2O_3 < MgO$

Ans. (1)

Sol. As metallic character of element attached to oxygen atom increases, the difference between the electronegativity values of element and oxygen increases and thus basic character of oxides increases and vice-versa. Hence the increasing correct order of basic nature is $Al_2O_3 < MgO < Na_2O < K_2O$.

12. A gas absorbs a photon of 355 nm and emits at two wavelengths. If one of the emission is at 680 nm, the other is at :

(1) 1035 nm (2) 325 nm (3) 743 nm (4) 518 nm

Ans. (3)

Sol. $E = E_1 + E_2$

$$\frac{hc}{*} = \frac{hc}{*_1} + \frac{hc}{*_2}$$

$$\frac{1}{*} = \frac{1}{*_1} + \frac{1}{*_2}$$

$$\frac{1}{355} = \frac{1}{680} + \frac{1}{*_2}$$

$*_2 = 742.76$ nm.

13. Which of the following statements regarding sulphur is **incorrect** ?

(1) S_2 molecule is paramagnetic.

(2) The vapour at 200ºC consists mostly of S_8 rings.

(3) At 600ºC the gas mainly consists of S_2 molecules.

(4) The oxidation state of sulphur is never less than +4 in its compounds.

Ans. (4)

Sol. Sulphur exhibit + 2, + 4, + 6 oxidation states but + 4 and + 6 are more common.

14. The entropy change involved in the isothermal reversible expansion of 2 moles of an ideal gas from a volume of 10 dm^3 to a volume of 100 dm^3 at 27°C is :

(1) 38.3 J mol^{-1} K^{-1} (2) 35.8 J mol^{-1} K^{-1} (3) 32.3 J mol^{-1} K^{-1} (4) 42.3 J mol^{-1} K^{-1}

Ans. (1)

Sol. $\forall S = nR \ln \frac{V_2}{V_1}$

$= 2.303\ nR \log \frac{V_2}{V_1}$

$= 2.303 \times 2 \times 8.314 \times \log \frac{100}{10}$

$= 38.3$ J mol^{-1} K^{-1}

15. Which of the following facts about the complex $[Cr(NH_3)_6]Cl_3$ is **wrong** ?

(1) The complex involves d^2sp^3 hybridisation and is octahedral in shape.

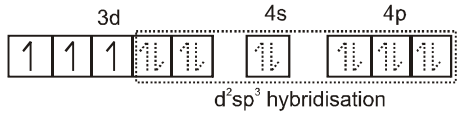
(2) The complex is paramagnetic.

(3) The complex is an outer orbital complex.

(4) The complex gives white precipitate with silver nitrate solution.

Ans. (3)

Sol. In case of d^3 configuration, the number of unpaired electrons remains 3 whether the ligand is strong field or weak field. The hybridisation scheme can be shown as follow :



Hence the complex is inner orbital complex as it involves (n – 1) d orbitals for hybridisation,

$3.93 = \sqrt{n(n+2)}$; so n = 3 (here n is number of unpaired electron(s)).

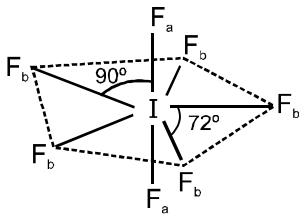
16. The structure of IF_7 is :

(1) square pyramid (2) trigonal bipyramid

(3) octahedral (4) pentagonal bipyramid

Ans. (4)

Sol. The structure is pentagonal bipyramid having sp^3d^3 hybridisation as given below :



$F_b - I - F_b = 72^o$ (5 number) ; $F_b - I - F_a = 90^o$ (10 number)

$F_b - I$ bond length = 1.858 ± 0.004 Å ; $F_a - I$ bond length = 1.786 ± 0.007 Å.

17. The rate of a chemical reaction doubles for every 10ºC rise of temperature. If the temperature is raised by 50ºC, the rate of the reaction increases by about :

(1) 10 times (2) 24 times (3) 32 times (4) 64 times

Ans. (3)

Sol. $\frac{\text{Rate at } 50^oC}{\text{Rate at } T_1{}^oC} = (2)^{\frac{\forall T}{T_1}} = (2)^{\frac{50}{10}} = 2^5$

= 32 times

18. The strongest acid amongst the following compounds is :

(1) CH_3COOH (2) HCOOH

(3) $CH_3CH_2CH(Cl)CO_2H$ (4) $ClCH_2CH_2CH_2COOH$

Ans. (3)

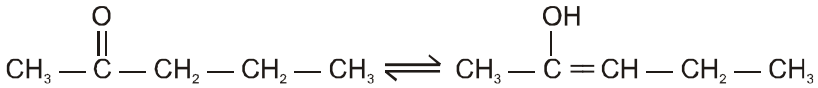
Sol. +-chlorobutyric acid is more stronger acid than others due to $-I$ effect of Cl.

19. Identify the compound that exhibits tautomerism.

(1) 2-Butene (2) Lactic acid (3) 2-Pentanone (4) Phenol

Ans. (3)

Sol.



20. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of the CO_2 is converted into CO on the addition of graphite. If the total pressure at equilibrium is 0.8 atm, the value of K is :

(1) 1.8 atm (2) 3 atm (3) 0.3 atm (4) 0.18

Ans. (1)

Sol. $CO_2(g) + C(s) \rightleftharpoons 2CO(g)$

0.5 atm

0.5–p 2p

Total pressure $= 0.5 - P + 2P = 0.8$

$P = 0.3$

$$K_p = \frac{P_{CO}^2}{P_{CO_2}} = \frac{(2P)^2}{(0.5\ ,\ P)} = \frac{(0.6)^2}{(0.5\ ,\ 0.3)}$$

$K_p = 1.8$

21. In context of the lanthanoids, which of the following statement is not correct ?

(1) There is a gradual decrease in the radii of the members with increasing atomic number in the series.

(2) All the member exhibit +3 oxidation state.

(3) Because of similar properties the separation of lanthanoids is not easy.

(4) Availability of 4f electrons results in the formation of compounds in +4 state for all the members of the series.

Ans. (4)

Sol. Availability of 4f electrons donot results in the formation of compounds in +4 state for all the members of the seires.

22. 'a' and 'b' are van der Waals' constants for gases. Chlorine is more easily liquefied than ethane because :

(1) a and b for Cl_2 > a and b for C_2H_6

(2) a and b for Cl_2 < a and b for C_2H_6

(3) a and Cl_2 < a for C_2H_6 but b for Cl_2 > b for C_2H_6

(4) a for Cl_2 > a for C_2H_6 but b for Cl_2 < b for C_2H_6

Ans. (4)

Sol.

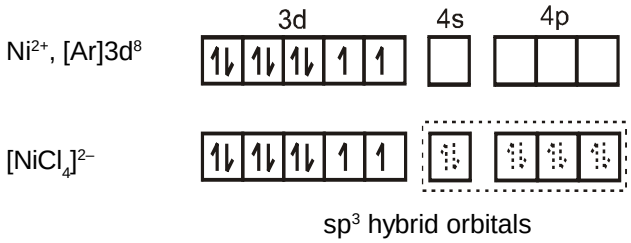
	a	b
Cl_2	6.579 L^2 bar mol^{-2}	0.05622 L mol^{-1}
C_2H_5	5.562 L^2 bar mol^{-2}	0.06380 L mol^{-1}

23. The magnetic moment (spin only) of $[NiCl_4]^{2-}$ is :

(1) 1.82 BM (2) 5.46 BM (3) 2.82 BM (4) 1.41 BM

Ans. (3)

Sol. In the paramagnetic and tetrahedral complex $[NiCl_4]^{2-}$, the nickel is in +2 oxidation state and the ion has the electronic configuration $3d^8$. The hybridisation scheme is as shown in figure.



sp^3 hybrid orbitals

$\mu_{B.M.} = \sqrt{n(n+2)} = \sqrt{2(2+2)} = \sqrt{8} = 2.82$ BM

24. In a face centred cubic lattice, atom A occupies the corner positions and atom B occupies the face centre positions. If one atom of B is missing from one of the face centred points, the formula of the compound is :

(1) A_2B (2) AB_2 (3) A_2B_3 (4) A_2B_5

Ans. (4)

Sol. A B

$8\times\frac{1}{8}$ $5\times\frac{1}{2}$

Formula of compound A_2B_5.

25. The outer electron configuration of Gd (Atomic No : 64) is :

(1) $4f^3\ 5d^5\ 6s^2$ (2) $4f^8\ 5d^0\ 6s^2$ (3) $4f^4\ 5d^4\ 6s^2$ (4) $4f^7\ 5d^1\ 6s^2$

Ans. (4)

Sol. Gadolinium ($_{64}Gd$) = $[Xe]^{54}\ 4f^7 5d^1 6s^2$

26. Boron cannot form which one of the following anions ?

(1) BF_6^{3-} (2) BH_4^- (3) $B(OH)_4^-$ (4) BO_2^-

Ans. (1)

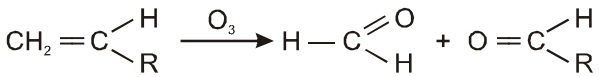
Sol. Due to non-availability of d-orbitals, boron is unable to expand its octet. Therefore, the maximum covalence of boron cannot exceed 4.

27. Ozonolysis of an organic compound gives formaldehyde as one of the products. This confirms the presence of :

(1) two ethylenic double bonds (2) a vinyl group

(3) an isopropyl group (4) an acetylenic triple bond

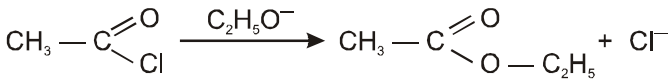
Ans. **(2)**

Sol. 

Presence of one vinyl group gives formaldehyde as one of the product in ozonolysis.

28. Sodium ethoxide has reacted with ethanoyl chloride. The compound that is produced in the above reaction is :
(1) Diethyl ether (2) 2-Butanone (3) Ethyl chloride (4) Ethyl ethanoate

Ans. (4)

Sol. 

Ethylethanoate

29. The degree of dissociation (+) of a weak electrolyte, A_xB_y is related to van't Hoff factor (i) by the expression:

(1) $+ = \frac{i-1}{(x \,\&\, y-1)}$ (2) $+ = \frac{i-1}{x \,\&\, y \,\&\, 1}$ (3) $+ = \frac{x \,\&\, y-1}{i-1}$ (4) $+ = \frac{x \,\&\, y \,\&\, 1}{i-1}$

Ans. (1)

Sol. A_xB_y ∃∃ % xA^{y+} + yB^{x-}

1–+ .....x+ y+

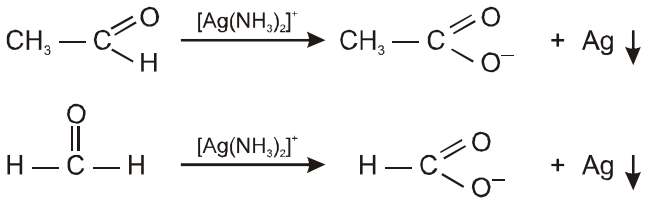
i = 1–+.+ x+.+ y+

i = 1 + +.(x + y – 1)

$+ = \frac{i-1}{(x \,\&\, y-1)}$

30. Silver Mirror test is given by which one of the following compounds ?
(1) Acetaldehyde (2) Acetone (3) Formaldehyde (4) Benzophenone

Ans. (1, 3)

Sol. 

PART-B (PHYSICS)

31. 100g of water is heated from 30ºC to 50ºC ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/Kg/K) :

(1) 4.2 kJ (2) 8.4 kJ (3) 84 kJ (4) 2.1 kJ

Ans. (2)

Sol. $\forall Q = M,S,\forall T$

$= 100 \times 10^{-3} \times 4.184 \times 20 = 8.4 \times 10^{3}$

$\forall Q = 84$ kJ, $\forall W = 0$

$\forall Q = \forall V + \forall W$

) $\forall V = 8.4$ kJ. **Ans.**

32. The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is :

(1) 7 min (2) 14 min (3) 20 min (4) 28 min

Ans. (3)

Sol. $\frac{2}{3}N_0 = N_0 e^{-*t_1}$

$\frac{1}{3}N_0 = N_0 e^{-*t_2}$

$2 э e^{*(t_2 - t_1)}$

$*(t_2 - t_1) = !n\ 2$

$(t_2 - t_1) = \frac{!n2}{*} = 20$ min. **Ans.**

33. A mass M, attached to a horizontal spring, executes SHM with a amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\frac{A_1}{A_2}$ is :

(1) $\frac{M}{M\&m}$ (2) $\frac{M\&m}{M}$ (3) $\left(\frac{M}{M\&m}\right)^{1/2}$ (4) $\left(\frac{M\&m}{M}\right)^{1/2}$

Ans. (4)

Sol. C.O.L.M. $MV_{max} = (m + M)V_{new}$, $V_{max} = A_1 5_1$

$V_{new} = \frac{MV_{max}}{(m\&M)}$

Now, $V_{new} = A_2.5_2$

$\frac{M.A_1}{(m\&M)}\sqrt{\frac{K}{M}} = A_2\sqrt{\frac{K}{(m\&M)}}$

$A_2 = A_1\sqrt{\frac{M}{(m\&M)}}$ $\frac{A_1}{A_2} э \left(\frac{m\&M}{M}\right)^{1/2}$ **Ans.**

34. Energy required for the electron excitation in Li^{++} from the first to the thrid Bohr orbit is :
(1) 12.1 eV (2) 36.3 eV (3) 108.8 eV (4) 122.4 eV

Ans. (3)

Sol. $E_1 = -\frac{13.6(3)^2}{(1)^2}$

$E_3 = -\frac{13.6(3)^2}{(3)^2}$

$\Delta E = E_3 - E_1$

$= 13.6(3)^2\left[\frac{1}{9}-\frac{1}{9}\right]$

$= \frac{13.6\times 9\times 8}{9}$

$\Delta E = 108.8$ eV. **Ans.**

35. The transverse displacement y(x,t) of a wave on a string is given by

$y(x,t) = e^{-(ax^2+bt^2+2\sqrt{ab}xt)}$

This represents a :

(1) wave moving in +x–direction with speed $\sqrt{\frac{a}{b}}$

(2) wave moving in –x-direction with speed $\sqrt{\frac{b}{a}}$

(3) standing wave of frequency $\sqrt{b}$

(4) standing wave of frequency $\frac{1}{\sqrt{b}}$

Ans. (2)

Sol. $y(x,t) = e^{-[\sqrt{a}x+\sqrt{b}t]^2}$

It is transverse type $y(x,t) = e^{-(ax+bt)^2}$

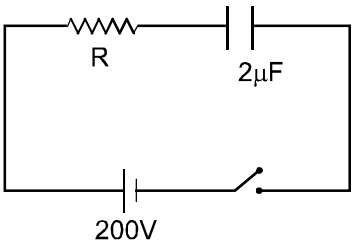
Speed $v = \frac{\sqrt{b}}{\sqrt{a}}$

and wave is moving along –x direction.

36. A resistor 'R' and 2–F capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5s after the switch has been closed. ($\log_{10}2.5 = 0.4$)
(1) $1.3\times10^4\Omega$ (2) $1.7\times10^5\Omega$ (3) $2.7\times10^6\Omega$ (4) $3.3\times10^7\Omega$

Ans. (3)

Sol.



$v = 200(1 - e^{-t/t})$

$120 = 200(1 - e^{-t/?})$

$e^{-t/?} = \frac{200 - 120}{200} = \frac{80}{200}$

$t/? = \log(2.5) = 0.4$

$5 = (0.4) \times R \times 2 \times 10^{-6}$

$\cong \quad R = \frac{5}{(0.4)(2(10^{-6}}$

$= \quad R = 2.7 \times 10^{6}$ **Ans.**

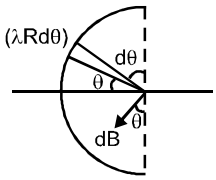
37. A current A flows in an infinitely long wire with cross–section in the form of a semicircular ring of radius R. The magnitude of the magnetic induction along its axis is :

(1) $\frac{-_0 A}{!^2 R}$ (2) $\frac{-_0 A}{2!^2 R}$ (3) $\frac{-_0 A}{2! R}$ (4) $\frac{-_0 A}{4! R}$

Ans. (1)

Sol. $v = \frac{A}{! R}$

$dB = \frac{4-_0}{2} / \frac{12A}{34! \ 0 R}$ $\quad A = * R \, dB$



) $\quad B = \int_{-!/2}^{!/2} dB \cos B$

$= \frac{-_0 *}{2!} \int_{-!/2}^{!/2} \cos B \, dB$

$= \frac{-_0 *}{!} = \frac{-_0 A}{!^2 R}$ **Ans.**

38. A Carnot engine operating between temperatures T_1 and T_2 has effeiciency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively :

(1) 372 K and 310 K (2) 372 K and 330 K (3) 330 K and 268 K (4) 310 K and 248 K

Ans. (1)

Sol. $\Delta = 1 - \frac{T_2}{T_1} = \frac{1}{6} \quad \cong \quad \frac{T_2}{T_1} = 1 - \frac{1}{6} = \frac{5}{6}$

$\frac{1}{3} = 1, \frac{(T_2\ ,\ 62)}{T_1} \quad \cong \quad \frac{T_2\ ,\ 62}{T_1} = \frac{2}{3}$

$\frac{5(T_2\ ,\ 62)}{T_2} = \frac{2}{3}$

$5T_2 - 310 = 4T_2$

$T_2 = 310$ and $T_1 = \frac{6(\ 310}{5}$

$T_1 = 372$ K **Ans.**

39. An object moving with a speed of 6.25 m/s, is decelerated at a rate given by :

$$\frac{dE}{dt} э , 2.5\sqrt{E}$$

where E is the instantaneous speed. The time taken by the object, to come to rest, would be :
(1) 1 s (2) 2 s (3) 4 s (4) 8 s

Ans. (2)

Sol. $\int_{6.25}^{0} \frac{dv}{\sqrt{v}} = , 2.5 \int_{0}^{t} dt$

$\left|2\sqrt{E}\right|_{6.25}^{0} = -2.5\ t$

$2.\sqrt{6.25} = 2.5\ t$

$t = 2$ sec. **Ans.**

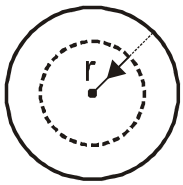
40. The electrostatic potential inside a charged spherical ball is given by $\Phi = ar^2 + b$ where r is the distance from the centre; a,b are constants. Then the charge density inside the ball is :
(1) $-24!\ a\Gamma_0 r$ (2) $-6!\ a\Gamma_0 r$ (3) $-24!\ a\Gamma_0$ (4) $-6\ a\Gamma_0$

Ans. (4)

Sol. $\Phi = ar^2 + b$

$E = -\frac{d\Phi}{dt} = -2ar$

$\oint \vec{E}.\vec{dS} э \frac{q}{\Gamma_0}$



$-2ar \cdot 4!\ r^2 = \frac{q}{\Gamma_0}$

$q = -8\ \Gamma_0 a!\ r^3$

$H = \frac{q}{\frac{4}{3}!\ r^3}$

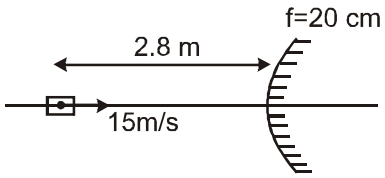
$H = -6a\Gamma_0$ **Ans.**

41. A car is fitted with a convex side–view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is :

(1) $\frac{1}{10}$ m/s (2) $\frac{1}{15}$ m/s (3) 10 m/s (4) 15 m/s

Ans. (2)

Sol. Mirror formula :



$\frac{1}{v} - \frac{1}{-280} = \frac{1}{20}$

$\frac{1}{v} = \frac{1}{20} - \frac{1}{280}$

$\frac{1}{v} = \frac{14-1}{280}$

$v = \frac{280}{15}$

$v_A = -\left(\frac{v}{u}\right)^2 . v_{om}$

$v_A = -\left(\frac{280/15}{-280}\right)^2 .15$

$v_A = \frac{15}{15^2}$

$v_A = \frac{1}{15}$ m/s **Ans.**

42. If a wire is stretched to make it 0.1% longer, its resistance will :
(1) increase by 0.05% (2) increase by 0.2% (3) decrease by 0.2% (4) decrease by 0.05%

Ans. (2)

Sol. $R = \frac{\rho l}{A}$ (# V = Al const.)

V = Al

By differentiation $0 = ldA + Adl$ ....(1)

By differentiation $dR = \frac{\rho(Adl - ldA)}{A^2}$ ....(2)

$dR = \rho\frac{2Adl}{A^2}$

$dR = \frac{2\rho dl}{A}$ or $\frac{dR}{R} = 2.\frac{dl}{l}$

So, $\frac{dR}{R}\% = 2.\frac{dl}{l}\%$ = 2 × 0.1%

$\frac{dR}{R}\% = 0.2\%$ **Ans.**

43. Three perfect gases at absolute temperature T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is :

(1) $\frac{(T_1 \& T_2 \& T_3)}{3}$ (2) $\frac{n_1T_1 \& n_2T_2 \& n_3T_3}{n_1 \& n_2 \& n_3}$

(3) $\frac{n_1T_1^2 \& n_2T_2^2 \& n_3T_3^2}{n_1T_1 \& n_2T_2 \& n_3T_3}$ (4) $\frac{n_1^2T_1^2 \& n_2^2T_2^2 \& n_3^2T_3^2}{n_1T_1 \& n_2T_2 \& n_3T_3}$

Ans. (2)

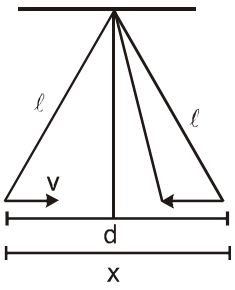
Sol. $T = \frac{n_1T_1 \& n_2T_2 \& n_3T_3}{n_1 \& n_2 \& n_3}$ **Ans.**

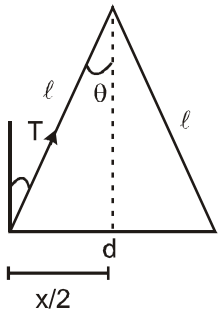
44. Two identical charged spheres suspended from a common point by two massless strings of length ! are initially a distance d(d < < !) apart becuase of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity E. Then as a function of distance x between them :

(1) E.I $x^{-1/2}$ (2) E.I x^{-1} (3) E.I $x^{1/2}$ (4) E.I x

Ans. (1)

Sol.





$\sin B = \frac{kq^2}{d^2}$

$\cos B = mg$

$\tan B = \frac{k}{mg} \cdot \frac{q^2}{x^2}$

$\frac{x}{2!} \cdot ə \frac{k}{mg} \cdot \frac{q^2}{x^2}$

$x^3 = \frac{2k!}{mg} q^2$

$q^2 \text{ I } x^3$

$q \text{ I } x^{3/2}$

$\frac{dq}{dt} \text{ I } \frac{3}{2} x^{1/2} \frac{dx}{dt}$ (dq/dt is constant)

$c \text{ I } x^{1/2} v$

$v \text{ I } x^{-1/2}$.

45. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly. (Surface tension of soap solution = 0.03 Nm^{-1})

(1) 4! mJ (2) 0.2! mJ (3) 2! mJ (4) 0.4! mJ

Ans. (4)

Sol. $W = T\forall A$

$= 0.03\ (2 \times 4! \times (5^2 - 3^2)\ 10^{-4}$

$= 24!\ (16) \times 10^{-6}$

$= 0.384\ ! \times 10^{-3}$ Joule

$\Im$ 0.4 ! mJ **Ans.**

46. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at t = 0. The time at which the energy is stored equally between the electric and the magnetic fields is :

(1) $!\sqrt{LC}$ (2) $\frac{!}{4}\sqrt{LC}$ (3) $2!\sqrt{LC}$ (4) $\sqrt{LC}$

Ans. (2)

Sol. In LC oscillation energy is transfered C toL

or L to C maximum energy in L is = $\frac{1}{2}LA_{max}^2$

Maximum energy in C is = $\frac{q_{max}^2}{2C}$

Equal energy will be when

$$\frac{1}{2}LA^2 = \frac{1}{2}\frac{1}{2}LA_{max}^2$$

$$A = \frac{1}{\sqrt{2}}A_{max}$$

$$A = A_{max}\sin 5t = \frac{1}{\sqrt{2}}A_{max}$$

$$5t = \frac{!}{4}$$

or $\frac{2!}{T}t = \frac{!}{4}$ or $t = \frac{T}{8}$

$t = \frac{1}{8}2!\sqrt{LC}$ = $\frac{!}{4}\sqrt{LC}$ **Ans.**

47. Two bodies of masses m and 4 m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is :

(1) zero (2) $-\frac{4Gm}{r}$ (3) $-\frac{6Gm}{r}$ (4) $-\frac{9Gm}{r}$

Ans. (4)

Sol. $\frac{Gm}{x^2} = \frac{G(4m)}{(r\ ,\ x)^2}$

$\frac{1}{x} = \frac{2}{r\ ,\ x}$

$r - x = 2x$

$3x = \frac{r}{3}$

$x = \frac{r}{3}$

m ——— r/3 ——— 2r/3 ——— 4m

$, \frac{Gm}{r/3}, \frac{G(4m)}{2r/3}$

$, \frac{3Gm}{r}, \frac{6Gm}{r} = \frac{9Gm}{r}$ **Ans.**

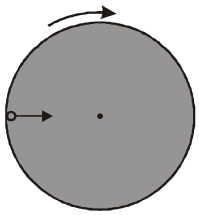
48. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc :

(1) remains unchanged (2) continuously decreases

(3) continuously increases (4) first increases and then decreases

Ans. (4)

Sol.



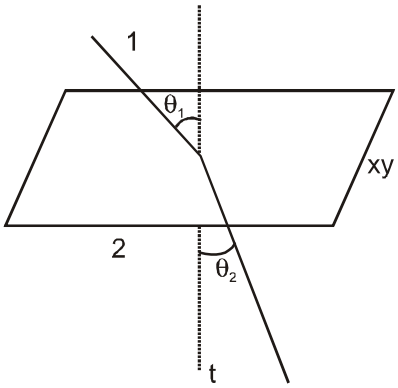
From angular momentum conservation about vertical axis passing through centre. When insect is coming from circumference to center. Moment of inertia first decrease then increase. So angular velocity inecrease then decrease.

49. Let the x - z plane be the boundary between two transparent media. Medium 1 in z ϑ 0 has refractive index of $\sqrt{2}$ and medium 2 with z < 0 has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $A э 6\sqrt{3}\,\hat{i} \&8\sqrt{3}\,\hat{j} - 10\,\hat{k}$ in incident on the plane of separation. The angle of refraction in medium 2 is :

(1) 30º (2) 45º (3) 60º (4) 75º

Ans. (2)

Sol. X–Y Plane



$-_1 \sin B_1 = -_2 \sin B_2$

$\cos B_1 = \frac{10}{\sqrt{(6\sqrt{3})^2 \,\&(8\sqrt{3})^2 \,\&100}} \ni \frac{10}{\sqrt{400}} \ni \frac{10}{20}$

$\cos B_1 = \frac{1}{2}$

$B_1 = 60°$

$\sqrt{2}\ \sin 60° = \sqrt{3}\ \sin B_2$

$\sqrt{2}\,(\ \frac{\sqrt{3}}{2}\ = \sqrt{3}\ \sin B_2$

$\sin B_2 = \frac{1}{\sqrt{2}}$

$B_2 = 45°$

50. Two particles are executing simple harmonic motion of the same amplitude A and frequency 5 along the x - axis. Their mean position is separated by distance X_0 ($X_0 > A$). If the maximum separation between them is ($X_0 + A$), the phase difference between their motion is :

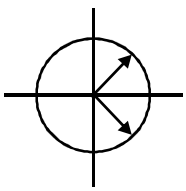
(1) $\frac{!}{2}$ (2) $\frac{!}{3}$ (3) $\frac{!}{4}$ (4) $\frac{!}{6}$

Ans. (2)

Sol. $x_1 = A \sin(5t + \Phi_1)$

$x_2 = A \sin(5t + \Phi_2)$

$x_1 - x_2 = A\,2\sin\left(5t \,\&\, \frac{\Phi_1 \,\&\, \Phi_2}{2}\right)\sin\left(\frac{\Phi_1 \,,\, \Phi_2}{2}\right)$



$A = 2A \sin\left(\frac{\Phi_1 \,,\, \Phi_2}{2}\right)$

$\frac{\Phi_1 \,,\, \Phi_2}{2} = \frac{!}{6}$

$\Phi_1 = \frac{!}{3}$ **Ans.**

51. **Direction :**

The question has a paragraph followed by two statements, Statement –1 and Statement –2. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement –1 :

When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of !

Statement –2 :

The centre of the interference pattern is dark.

(1) Statement –1 is true, statement –2 is false.

(2) Statement –1 is true, Statement –2 is true, Statement –2 is the correct explanation of Statement –1

(3) Statement –1 is true, Statement –2 is true, Statement –2 is not the correct explanation of Statement –1

(4) Statement–1 is false, Statement –2 is true

Ans. (1)

Sol. **S_1 :** When light reflects from deuser med. (Glass) a phase dift of ! is generated.

S_2 : Centre maxima or minima depends on thickness of the lens.

52. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats K It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by :

(1) $\frac{(K, 1)}{2(K\&1)R}Mv^2K$ (2) $\frac{(K, 1)}{2KR}Mv^2K$ (3) $\frac{KMv^2}{2R}K$ (4) $\frac{(K, 1)}{2R}Mv^2K$

Ans. (4)

Sol. $\frac{1}{2}M\Lambda^2 \ni C_V.\forall T$

$\frac{1}{2}M\Lambda^2 \ni \frac{R}{K, 1}.\forall T$

$\forall T = \frac{M.\Lambda^2(K, 1)}{2R} = \frac{(K, 1)M\Lambda^2}{2R}$

53. A screw gauge gives the following reading when used to measure the diameter of a wire .

Main scale reading : 0 mm

Circular scale reading : 52 division

Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.

The diameter of wire from the above data is :

(1) 0.52 cm (2) 0.052 cm (3) 0.026 cm (4) 0.005 cm

Ans. (2)

Sol. Least count of screw gauge = $\frac{1}{100}$mm = 0.01 mn

Diameter - Divisions on cirular scale × least count + main scale reading

= 52 × $\frac{1}{100}$ &0

= 0.52 mm

diameter = 0.052 cm

54. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} $NA^{-1}m^{-1}$ due north and horizontal. The boat carries a vertical aerial 2m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is :

(1) 1 mV (2) 0.75 mV (3) 0.50 mV (4) 0.15 mV

Ans. (4)

Sol. $E_{ind} = B \times v \times !$

$= 5.0 \times 10^{-5} \times 1.50 \times 2$

$= 10.0 \times 10^{-5} \times 1.5$

$= 15 \times 10^{-5}$ vot.

= 0.15 mv

55. This question has Statement –1 and Statement –2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement –1

Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

Statement –2 :

The state of ionosphere varies from hour to hour, day to day and season to season.

(1) Statement –1 is true, statement –2 is false.

(2) Statement –1 is true, Statement –2 is true, Statement –2 is the correct explanation of Statement –1

(3) Statement –1 is true, Statement –2 is true, Statement –2 is not the correct explanation of Statement–1

(4) Statement–1 is false, Statement –2 is true

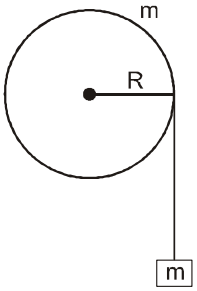
Ans. (4)

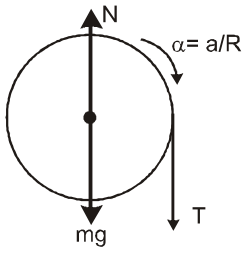
56. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is :

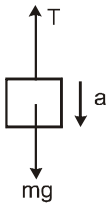
(1) $\frac{3}{2}g$ (2) g (3) $\frac{2}{3}g$ (4) $\frac{g}{3}$

Ans. (3)

Sol. $mg - T = ma$







$TR = \frac{mR^2+}{2}$

$T = \frac{mR+}{2} = \frac{ma}{2}$

$mg - \frac{ma}{2} = ma$

$\frac{3ma}{2} = mg$

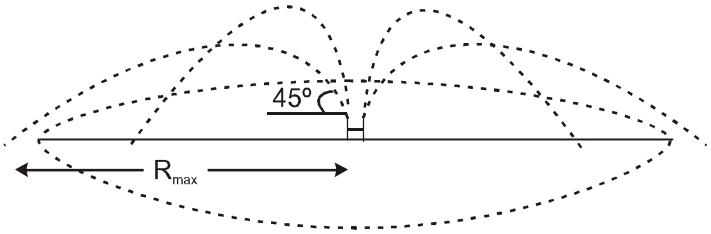
$a = \frac{2g}{3}$ **Ans.**

57. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is :

(1) $\pi \frac{v^2}{g}$ (2) $\pi \frac{v^4}{g^2}$ (3) $\frac{\pi}{2}\frac{v^4}{g^2}$ (4) $\pi \frac{v^2}{g^2}$

Ans. (2)

Sol.



$R_{max} = \frac{v^2}{g} \sin 2\theta = \frac{v^2}{g}$

area = πR^2

$= \pi \frac{v^4}{g^2}$ **Ans.**

58. This questions has Statement –1 and statement –2. Of the four choices given after the statements, choose the one that best describes the two statements :

Statement –1 :

A metallic surface is irradiated by a monochromatic light of frequency $\lambda > \lambda_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement –2 :

The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

(1) Statement –1 is true, statement –2 is false.
(2) Statement –1 is true, Statement –2 is true, Statement –2 is the correct explanation of Statement –1
(3) Statement –1 is true, Statement –2 is true, Statement –2 is not the correct explanation of Statement–1
(4) Statement–1 is false, Statement –2 is true

Ans. (4)

Sol. $h\nu = h\nu_0 + k_{max}$
$k_{max} = h\nu - h\nu_0$

59. A pulley of radius 2m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion if reversed, is :

(1) less than 3 (2) more than 3 but less than 6
(3) more than 6 but less than 9 (4) more than 9

Ans. (2)

Sol. To reverse the direction $\int dB = 0$ (work done is zero)

$? = (20\,t - 5t^2)\ 2 = 40t - 10t^2$

$+ = \frac{?}{A} = \frac{40t,\ 10t^2}{10} = 4t,\ t^2$

$5 = \int_0^t dt = 2t^2 - \frac{t^3}{3}$

5 is zero at

$2t^2 - \frac{t^3}{3} = 0$

$t^3 = 6t^2$

$t = 6$ sec.

$B = \int_0^6 dt$

$= \int_0^6 (2t^2 , \frac{t^3}{3})dt$

$\left[\frac{2t^3}{3} , \frac{t^4}{12}\right]_0^6 = 216 \left[\frac{2}{3} , \frac{18}{27}\right] = 36$ rad.

No of revolution $\frac{36}{2!}$ Less than 6

60. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to :

(1) 5.0×10^{-3} m (2) 7.5×10^{-3} m (3) 9.6×10^{-3} m (4) 3.6×10^{-3} m

Ans. (4)

Sol. Diameter = 8×10^{-3} m

v = 0.4 m/s

$v = \sqrt{u^2 + 2gh}$

$= \sqrt{(0.4)^2 + 2 \times 10 \times 0.2}$

= 2 m/s

$A_1v_1 = A_2v_2$

$\pi \left(\frac{8 \times 10^{-3}}{4}\right)^2 \times 2 = \pi \times \frac{d^2}{4} \times 0.4$

$d = 3.6 \times 10^{-3}$ m.

PART-C (MATHEMATICS)

61. Let $+, N$ be real and z be a complex number. If $z^2 + +z + N = 0$ has two distinct roots on the line Re z = 1, then it is necessary that :

(1) $NO (0, 1)$ (2) $NO (-1, 0)$ (3) $|N| = 1$ (4) $NO (1, \Pi)$

Ans. (4)

Sol. Let roots be p + iq and p – iq $p, q O R$

root lie on line Re(z) = 1

$\cong$ p = 1

product of roots = $p^2 + q^2 = N = 1 + q^2$

$\cong$ $NO.(1, \Pi$ ($q P.0$, #$ roots are distinct)

62. The value of $\int_0^1 \frac{8\log(1\&x)}{1\&x^2}\,dx$ is :

(1) ! log 2 (2) $\frac{!}{8}$ log 2 (3) $\frac{!}{2}$ log 2 (4) log 2

Ans. (1)

Sol. x = tanB

dx = $\sec^2 B.dB$

$$I = \int_0^{!/4} \frac{8!n(1\&\tan B)}{\sec^2 B}\sec^2 dB = 8\int_0^{!/4} !n(1\&\tan B)dB$$

$\cong$ $$I = 8\int_0^{!/4} !n(1\&\tan(!/4 - B))dB$$

$$= 8\int_0^{!/4} !n\left(\frac{4}{3}\frac{2}{1\&\tan B}\right)dB$$

$\cong$ $$2I = 8\int_0^{!/4} !n(2)dB$$

$$= 8\,\frac{!}{4}\,!n2 = 2!\,!n2$$

$\cong$ $I = !\,!n2$ **Ans.**

63. $\frac{d^2x}{dy^2}$ equals :

(1) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (2) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{3}$

(3) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (4) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

Ans. (4)

Sol. $\frac{dy}{dx} \cdot \frac{1}{\frac{dx}{dy}}$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{dx/dy}\right) = \frac{d}{dy}\left(\frac{1}{dx/dy}\right)\cdot\frac{dy}{dx} = -\frac{1}{\left(\frac{dx}{dy}\right)^2}\cdot\frac{\frac{d^2x}{dy^2}}{\frac{dx}{dy}} = \frac{-\frac{d^2x}{dy^2}}{\left(\frac{dx}{dy}\right)^3}$$

64. Let A be the purchase value of an equipment and V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T-t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is :

(1) $T^2 - \frac{1}{k}$ (2) $A - \frac{kT^2}{2}$ (3) $A - \frac{k(T-t)^2}{2}$ (4) e^{-kT}

Ans. (2)

Sol. $\frac{dv(t)}{dt} = k(T-t)$

$\int dv(t) = \int (-kT)dt + \int kt\,dt$

$V(t) = -kTt + k\frac{t^2}{2} + c$

at $t = 0$ $C = I$

$V(T) = -kTt + \frac{kt^2}{2} + I$

Now at $t = T$

$V(T) = -kT^2 + k\frac{T^2}{2} + I$

$V(T) = I - \frac{1}{2}kT^2$

65. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is :

(1) 144 (2) – 132 (3) – 144 (4) 132

Ans. (3)

Sol. $(1 - x - x^2 + x^3)^6$

$(1 - x)^6 (1 - x^2)^6$

$({}^6C_0 - {}^6C_1 x^1 + {}^6C_2 x^2 - {}^6C_3 x^3 + {}^6C_4 x^4 - {}^6C_5 x^5 + {}^6C_6 x^6)\ ({}^6C_0 - {}^6C_1 x^2 + {}^6C_2 x^4 - {}^6C_3 x^6 + {}^6C_4 x^8 + \ldots\ldots\ldots\ldots + {}^6C_6 x^{12})$

Now coefficient of $x^7 = {}^6C_1\,{}^6C_3 - {}^6C_3\,{}^6C_2 + {}^6C_5\,{}^6C_1$

$= 6 \times 20 - 20 \times 15 + 36$

$= 120 - 300 + 36$

$= 156 - 300$

$= -144$ **Ans.**

66. For $x \in \left[0, \frac{5\pi}{2}\right]$, define $f(x) = \int_0^x \sqrt{t}\ \sin t\ dt$. Then f has :

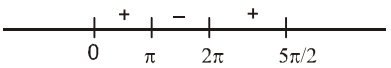
(1) local maximum at π and 2π. (2) local minimum at π and 2π

(3) local minimum at π and local maximum at 2π. (4) local maximum at π and local minimum at 2π.

Ans. (4)

Sol. $f(x) = \int_0^x \sqrt{t} \sin t\, dt$

$f'(x) = \sqrt{x}\ \sin x$



local maximum at π

and local minimum at 2π **Ans.**

67. The area of the region enclosed by the curves $y = x$, $x = e$, $y = \frac{1}{x}$ and the positive x-axis is

(1) $\frac{1}{2}$ square units (2) 1 square units (3) $\frac{3}{2}$ square units (4) $\frac{5}{2}$ square units

Ans. (3)

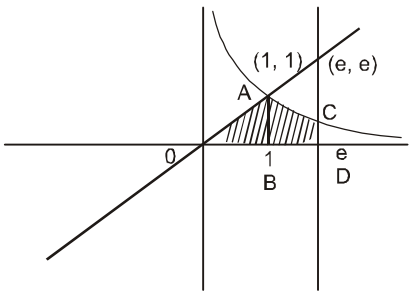
Sol. Required area

= OAB + ACDB

$= \frac{1}{2}(1)(1) + \int_1^e \frac{1}{x}\,dx$

$= \frac{1}{2} + (\ln x)_1^e$

$= \frac{3}{2}$ square unit **Ans.**



68. The line $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement-1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

Statement-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(1) Statement-1 is true, Statement-2 is true ; Statement-2 is correct explanation for Statement-1

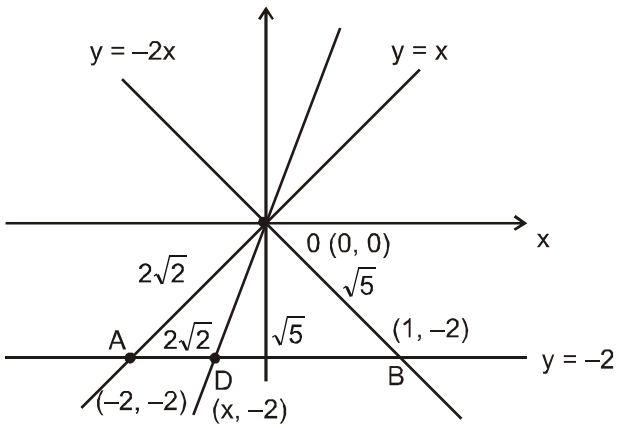
(2) Statement-1 is true, Statement-2 is true ; Statement-2 is **not** a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

Ans. (3)

Sol.



) AD : DB = $2\sqrt{2} : \sqrt{5}$

OD is angle bisector of angle AOB

) St : 1 true

St. 2 false (obvious) **Ans.**

69. The value of p and q for which the function $f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$ is continuous for all x in R, are:

(1) $p = \frac{1}{2}, q = -\frac{3}{2}$

(2) $p = \frac{5}{2}, q = \frac{1}{2}$

(3) $p = -\frac{3}{2}, q = \frac{1}{2}$

(4) $p = \frac{1}{2}, q = \frac{3}{2}$

Ans. (3)

Sol. $f(0) = q$

$$f(0^+) = \lim_{x \to 0^+} \frac{(1+x)^{1/2} - 1}{x} = \lim_{x \to 0^+} \frac{1 + \frac{1}{2}x + \ldots - 1}{x} = \frac{1}{2}$$

$$f(0^-) = \lim_{x \to 0^-} \frac{\sin(p+1)x + \sin x}{x}$$

$$f(0^-) = \lim_{x \to 0^-} \frac{(\cos(p+1)x)(p+1) + (\cos x)}{1}$$

$= (p + 1) + 1 = p + 2$

$\therefore \quad p + 2 = q = \frac{1}{2} \quad \Rightarrow \quad p = -\frac{3}{2}, q = \frac{1}{2}$ **Ans.**

70. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\frac{2}{3}\sqrt{\frac{5}{14}}\right)$, then λ equals:

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\frac{2}{5}$ (4) $\frac{5}{3}$

Ans. (1)

Sol. $\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-3}{\lambda}$ ..... (1)

$x + 2y + 3z = 4$ ..... (2)

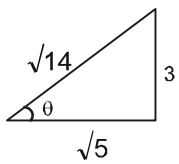
Angle between the line and plane is

$$\cos(90 - \beta) = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\Rightarrow \quad \sin\beta = \frac{1 + 4 + 3\lambda}{\sqrt{14}\sqrt{5 + \lambda^2}} = \frac{5 + 3\lambda}{\sqrt{14}\sqrt{5 + \lambda^2}} \quad \text{..... (3)}$$

But given that angle between line and plane is

$\beta = \cos^{-1}\left(\frac{2}{3}\sqrt{\frac{5}{14}}\right) = \sin^{-1}\left(\frac{2}{3}\frac{3}{\sqrt{14}}\right)$



$$\Rightarrow \quad \sin\beta = \frac{3}{\sqrt{14}}$$

$\therefore$ from (3)

$$\frac{3}{\sqrt{14}} = \frac{5 + 3\lambda}{\sqrt{14}\sqrt{5 + \lambda^2}}$$

$\Rightarrow \quad 9(5 + \lambda^2) = 25 + 9\lambda^2 + 30\lambda$

$\Rightarrow \quad 30\lambda = 20$

$\lambda = \frac{2}{3}$ **Ans.**

71. The domain of the function $f(x) = \frac{1}{\sqrt{|x|, x}}$ is :

(1) $(-\Pi, \Pi)$ (2) $(0, \Pi)$ (3) $(-\Pi, 0)$ (4) $(-\Pi, \Pi) - \{0\}$

Ans. (3)

Sol. $f(x) = \frac{1}{\sqrt{|x|, x}}$

$|x| - x > 0$

$|x| > x$

$\cong x < 0$

$) x O (-\Pi, 0)$ **Ans.**

72. The shortest distance between line $y - x = 1$ and curve $x = y^2$ is :

(1) $\frac{\sqrt{3}}{4}$ (2) $\frac{3\sqrt{2}}{8}$ (3) $\frac{8}{3\sqrt{2}}$ (4) $\frac{4}{\sqrt{3}}$

Ans. (2)

Sol. $y - x = 1$

$y^2 = x$

$2y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y} = 1$

$y = \frac{1}{2}$

$x = \frac{1}{4}$

tangent at $\left(\frac{1}{4}, \frac{1}{2}\right)$

$\frac{1}{2}y = \frac{1}{2}(2x + \frac{1}{4})$

$y = x + \frac{1}{4}$

$y - x = \frac{1}{4}$

distance $= \left|\frac{1, \frac{1}{4}}{\sqrt{2}}\right| = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$ **Ans.**

73. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after :

(1) 18 months (2) 19 months (3) 20 months (4) 21 months

Ans. (4)

Sol. a = Rs. 200

d = Rs. 40

savings in first two months = Rs. 400

remained savings = 200 + 240 + 280 + ... upto n terms

$= \frac{n}{2}[400 \&(n, 1)40. = 11040 - 400$

$200n + 20n^2 - 20n = 10640$

$20n^2 + 180n - 10640 = 0$

$n^2 + 9n - 532 = 0$

$(n + 28)(n - 19) = 0$

$n = 19$

) no. of months = 19 + 2 = **21** .

74. Consider the following statements

P : Suman is brilliant

Q : Suman is rich

R : Suman is honest.

The negation of the statement " Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as :

(1) ~ P ^ (Q] ~ R) (2) ~ (Q] (P ^ ~R))

(3) ~ Q] ~ P ^ R (4) ~ (P ^ ~ R)] Q

Ans. (2)

Sol. Negation of ⇔P⊥~ R=] Q is ~⇔(P⊥~ R)] Q=

It may also be written as ~⇔Q] (P⊥~ R)=

75. If $5(P1)$ is a cube root of unity, and $(1 + 5)^7 = A + B5$. Then (A, B) equals

(1) (0, 1) (2) (1, 1) (3) (1, 0) (4) (–1, 1)

Ans. (2)

Sol. $(1 + 5)^7 = A + B5$

$(-5^2)^7 = A + B5$

$-5^{14} = A + B5$

$-5^2 = A + B5$

$1 + 5 = A + B5$

) (A, B) = (1, 1)

76. If $\vec{a} = \frac{1}{\sqrt{10}}(3\hat{i} - \hat{k})$ and $\vec{b} = \frac{1}{7}(2\hat{i} - 3\hat{j} - 6\hat{k})$, then the value of $(2\vec{a} \times \vec{b}) \cdot [(\vec{a} \times \vec{b}) \times (\vec{a} - 2\vec{b})]$ is:

(1) – 5 (2) –3 (3) 5 (4) 3

Ans. (1)

Sol. $(2\vec{a} - \vec{b}) \cdot [(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})]$

$= -(2\vec{a} - \vec{b}) \cdot [(\vec{a} + 2\vec{b}) \times (\vec{a} \times \vec{b})]$

$= -(2\vec{a} - \vec{b}) \cdot [(\vec{a} + 2\vec{b}) \cdot \vec{b})\vec{a} - ((\vec{a} + 2\vec{b}) \cdot \vec{a})\vec{b}]$

$= -(2\vec{a} - \vec{b}) \cdot [(\vec{a} \cdot \vec{b}) + 2\vec{b} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a} + 2\vec{b} \cdot \vec{a})\vec{b})]$

$= -(2\vec{a} - \vec{b}) \cdot [0 + 2\vec{a} - (0 + \vec{b})]$

$= -(2\vec{a} - \vec{b}) \cdot (2\vec{a} - \vec{b})$

$= -(2\vec{a} - \vec{b})^2 = -4\vec{a}^2 + 4\vec{a} \cdot \vec{b} - \vec{b}^2$

$= -4 + 0 - 1 = -5$

77. If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(\ln 2)$ is equal to :

(1) 7 (2) 5 (3) 13 (4) –2

Ans. (1)

Sol. $\frac{dy}{dx} = y + 3$

$\frac{dy}{y - 3} = dx$

$\ln(y+3) = x + c$

given at $x = 0$, $y = 2$

$\ln 5 = c$

$\therefore$ $\ln(y+3) = x + \ln 5$

$\ln\left(\frac{y - 3}{5}\right) = x$

$y + 3 = 5e^x$

$y = 5e^x - 3$

$\therefore$ $y(\ln 2) = 5e^{\ln 2} - 3 = 7$ **Ans.**

78. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is :

(1) $3x^2 + 5y^2 - 32 = 0$ (2) $5x^2 + 3y^2 - 48 = 0$

(3) $3x^2 + 5y^2 - 15 = 0$ (4) $5x^2 + 3y^2 - 32 = 0$

Ans. (1, 2)

Sol. $\frac{x^2}{a^2} \& \frac{y^2}{b^2} = 1$

$\frac{9}{a^2} \& \frac{1}{b^2} = 1$ ..... (1)

case - 1 when a > b

$b^2 = a^2 (1 - e^2)$

$b^2 = a^2 (1 - 2/5)$

$5b^2 = 3a^2$ ..... (2)

from (1) & (2)

$\frac{9(3}{5b^2} + \frac{1}{b^2} = 1 \quad \cong \quad b^2 = \frac{32}{5}$

) $a^2 = \frac{32}{3}$

) $\frac{3x^2}{32} + \frac{5y^2}{32} = 1 \quad \cong \quad 3x^2 + 5y^2 - 32 = 0$ **Ans.**

case - 2 when b > a

$a^2 = b^2 (1 - e^2)$

$= \frac{3}{5} b^2$ ..... (3)

from (1) & (3)

$a^2 = \frac{48}{5}$, $b^2 = 16$

) $\frac{5x^2}{48} \& \frac{y^2}{16} = 1$

$\cong \quad 5x^2 + 3y^2 - 48 = 0$ **Ans.**

79. If the mean deviation about the median of the numbers a, 2a, ..., 50a is 50, then |a| equals :
(1) 2 (2) 3 (3) 4 (4) 5

Ans. (3)

Sol. Median = 25.5 a

Mean deviation about median = 50

$\cong \quad \frac{-\left|x_i , 25.5a\right|}{50} = 50$

$\cong$ 24.5 a + 23.5a + ... + 0.5a + 0.5a + ... + 24.5a = 2500

$\cong$ a + 3a + 5a + ... + 49a = 2500

$\cong .. \frac{25}{2} (50a) = 2500 \quad \cong \quad a = 4$

Ans. 4

80. $\lim\limits_{x \to 2} \left(\frac{\sqrt{1 - \cos\{2(x-2)\}}}{x-2} \right)$

(1) does not exist (2) equals $\sqrt{2}$ (3) equals $-\sqrt{2}$ (4) equals $\frac{1}{\sqrt{2}}$

Ans. (1)

Sol. $\lim\limits_{x \to 2} \sqrt{2} \ \frac{|\sin(x-2)|}{(x-2)}$

) does not exist

81. **Statement-1 :** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^{9}C_{3}$.

Statement-2 : The number of ways of choosing any 3 places from 9 different places is ${}^{9}C_{3}$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

Ans. (2)

Sol. **Statement - 1 :**
$B_1 + B_2 + B_3 + B_4 = 10$
= coefficient of x^{10} in $(x^1 + x^2 + ...+ x^7)^4$
= coefficient of x^6 in $(1 - x^7)^4 (1 - x)^{-4}$
$= {}^{4+6-1}C_6 = {}^{9}C_3$
Statement - 2 :
Obviously ${}^{9}C_3$

82. Let R be the set of real numbers.
Statement-1 : A = {(x, y) O R × R : y – x is an integer} is an equivalence relation on R.
Statement-2 : B = {(x, y) O R × R : x = +y for some rational number +} is an equivalence relation on R.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

Ans. (3)

Sol. **Statement - 1 :**
(i) x – x is an integer ¯x O R so A is reflexive relation.
(ii) y – x O.A..≅ x – y O.A so A is symmetric relation.
(iii) y – x O.A.and .z – y O.A .≅ y – x + z – y O.A
≅ z – x O.A.so A is transitive relation.
Therefore A is equivalence relation.
Statement - 2 :
(i) x = +x when + = 1 .≅ B is reflexive relation
(ii) for x = 0 and y = 2, we have 0 = +(2) for + = 0
But 2 = +(0) for no +
so B is not symmetric so not equivalence.

83. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval :

(1) $\left[\frac{1}{2}, \frac{3}{4}\right]$ (2) $\left[\frac{3}{4}, \frac{11}{12}\right]$ (3) $\left[0, \frac{1}{2}\right]$ (4) $\left[\frac{11}{12}, 1\right]$

Ans. (3)

Sol. $1 - P^5 \geq \frac{31}{32}$

$P^5 \leq \frac{1}{32}$

$P \leq \frac{1}{2}$

$P \in \left[0, \frac{1}{2}\right]$

84. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if :
(1) $2|a| = c$ (2) $|a| = c$ (3) $a = 2c$ (4) $|a| = 2c$

Ans. (2)

Sol. $x^2 + y^2 = ax$ .....(1)

$\Rightarrow$ centre $c_1 \left(\frac{a}{2}, 0\right)$ and radius $r_1 = \left|\frac{a}{2}\right|$

$x^2 + y^2 = c^2$ .....(2)

$\Rightarrow$ centre c_2 (0, 0) and radius $r_2 = c$

both touch each other iff

$|c_1c_2| = r_1 \pm r_2$

$\frac{a^2}{4} = \left(\frac{a}{2} \pm c\right)^2 \quad \Rightarrow \quad \frac{a^2}{4} = \frac{a^2}{4} \pm |a|\, c + c^2 \quad \Rightarrow \quad |a| = c$

85. Let A and B be two symmetric matrices of order 3.
Statement-1 : A(BA) and (AB)A are symmetric matrices.
Statement-2 : AB is symmetric matrix if matrix multiplication of A with B is commutative.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

Ans. (2)

Sol. $A^T = A$, $B^T = A$

$P = A(BA)$

$P^T = (A(BA))^T$

$= (BA)^T A^T$

$= (A^T B^T) A^T$

$= (AB) A$

$= A(BA)$

$\therefore$ A(BA) is symmetric

similarly (AB) A is symmetric

Statement(2) is correct but not correct explanation of statement (1).

86. If C and D are two events such that $C \neq D$ and $P(D) \neq 0$, then the correct statement among the following is :

(1) $P(C|D) = P(C)$ (2) $P(C|D) \geq P(C)$

(3) $P(C|D) < P(C)$ (4) $P(C|D) = \dfrac{P(D)}{P(C)}$

Ans. (2)

Sol. $P\left(\dfrac{C}{D}\right) = \dfrac{P(C \cap D)}{P(D)} = \dfrac{P(C)}{P(D)}$

$\dfrac{1}{P(D)} \geq 1$

$\dfrac{P(C)}{P(D)} \geq P(C)$

$P(C) \propto P\left(\dfrac{C}{D}\right)$

87. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a} \cdot \vec{d} = 0$. Then the vector $\vec{d}$ is equal to :

(1) $\vec{b} + \left(\dfrac{\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{d}}\right)\vec{c}$ (2) $\vec{c} + \left(\dfrac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right)\vec{b}$ (3) $\vec{b} + \left(\dfrac{\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right)\vec{c}$ (4) $\vec{c} + \left(\dfrac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right)\vec{b}$

Ans. (4)

Sol. $\vec{a} \cdot \vec{b} \neq 0$, $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$, $\vec{a} \cdot \vec{d} = 0$

$(\vec{b} \times \vec{c}) \times \vec{a} = (\vec{b} \times \vec{d}) \times \vec{a}$

$(\vec{b} \cdot \vec{a})\vec{c} - (\vec{c} \cdot \vec{a})\vec{b} = (\vec{b} \cdot \vec{a})\vec{d} - (\vec{d} \cdot \vec{a})\vec{b}$

$\vec{d} = \vec{c} + \left(\dfrac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right)\vec{b}$

88. **Statement-1 :** The point A(1, 0, 7) is the mirror image of the point B(1, 6, 3) in the line :

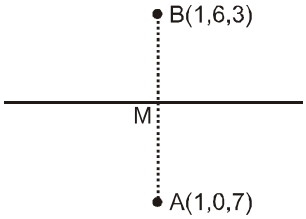
$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

Statement-2 : The line : $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining A(1, 0, 7) and B(1, 6, 3).

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

Ans. (2)

Sol. Mid- point of AB ε M(1,3,5)
M lies on line
Direction ratios of AB is < 0, 6, – 4 >
Direction ratios of given line is < 1, 2, 3 >
As AB is perpendicular to line
) 0.1 + 6.2 – 4.3 = 0



89. If A = $\sin^2 x + \cos^4 x$, then for all real x :

(1) $\frac{3}{4} \le A \le 1$ (2) $\frac{13}{16} \le A \le 1$ (3) $1 \le A \le 2$ (4) $\frac{3}{4} \le A \le \frac{13}{16}$

Ans. (1)

Sol. $A = \sin^2 x + \cos^4 x$
$= \sin^2 x + (1 - \sin^2 x)^2$
$= \sin^4 x - \sin^2 x + 1$
$= \frac{4}{3}\sin^2 x - \left(\frac{1}{2}\right)^2 + \frac{3}{4}$
$= \frac{3}{4} \le A \le 1$

90. The number of values of k for which the linear equations

$4x + ky + 2z = 0$
$kx + 4y + z = 0$
$2x + 2y + z = 0$

posses a non-zero solution is :

(1) 3 (2) 2 (3) 1 (4) zero

Ans. (2)

Sol. $\Delta = \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow 8 - k(k-2) - 2(2k-8) = 0$
$\Rightarrow 8 - k^2 + 2k - 4k + 16 = 0$
$\Rightarrow -k^2 - 2k + 24 = 0$
$\Rightarrow k^2 + 2k - 24 = 0$
$\Rightarrow (k+6)(k-4) = 0$
$\Rightarrow k = -6, 4$
Number of values of k is 2

Read the following instructions carefully:

1. The candidates should fill in the required particulars on the Test Booklet and Answer Sheet ***(Side—I)*** with ***Blue/Black Ball Point Pen.***

2. For writing/marking particulars on ***Side-2*** of the Answer Sheet, use ***Blue/Black Ball Point*** *Pen* only.

3. The candidates should not write their Roll Numbers anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.

4. Out of the four options given for each question, only one option is the correct answer.

5. For each *incorrect response, one-fourth* (¼) of the total marks allotted to the question would be deducted froth the total score. ***No deduction*** from the total score, however, will be made ***if no response*** is indicated for an item in the Answer Sheet.

6. Handle the Test Booklet and Answer Sheet with care, *as under no circumstance (except for discrepancy in Test Booklet Code and Answer Sheet Code), will another set be provided.*

7. The candidates are not allowed to do any rough work or writing work on the Answer Sheet. All calculations/ writing work are to be done in the space provided for this purpose in the Test Booklet itself, marked 'Space for Rough Work'. This space is given at the bottom of each page and in 3 pages at the end of the booklet.

8. On completion of the test, the candidates must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. **However, the candidates are allowed to take away this Test Booklet with them.**

9. Each candidate must show on demand his/her Admit Card to the Invigilator.

10. No candidate, without special permission of the Superintendent or Invigilator, should leave his/her seat.

11. The candidates should not leave the Examination Hall without handing over their Answer Sheet to the Invigilator on duty and sign the Attendance Sheet again. Cases where a candidate has not signed **the Attendance Sheet a** second time will be deemed not to have handed over the Answer Sheet and dealt with as an unfair means case. The candidates are **also required to put their left** hand **THUMB impression in the space provided in the Attendance Sheet.**

12. Use of Electronic/Manual Calculator and any Electronic Item like mobile phone, pager etc. is prohibited.

13. The candidates are governed by all Rules and Regulations of the Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the Board.

14. No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.

15. **Candidates** are **not allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, electronic device or** any **other material except the Admit Card inside the examination hall/room.**