## PAPER-1: PHYSICS, MATHEMATICS \& CHEMISTRY <br> Code: <br>  <br> SOLUTIONS TO AIEEE - 2011

Time: 3 hours
Maximum Marks: $\mathbf{3 6 0}$
IMPORTANT INSTRUCTIONS:

1. Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of $\mathbf{3}$ hours duration.
4. The Test Booklet consists of $\mathbf{9 0}$ questions. The maximum marks are $\mathbf{3 6 0}$.
5. There are three parts in the questions paper $A, B, C$ consisting of Physics, Mathematics and Chemistry having 30 questions in each part of equal weightage. Each question is allotted 4(four) marks for each correct response.
6. Candidates will be awarded marks as stated above in instruction No. 5 for correct response of each question. $1 / 4$ (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. Use Blue/Black Ball Point Pen only for writing particulars / marking response on side-1 and Side - $\mathbf{2}$ of the Answer Sheet. Use of pencil is strictly prohibited.
9. No candidate is allowed to carry any textual material printed or written, bits of papers, pager, mobile phone, any electronic device etc; except the Admit Card inside the examination hall/room.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in 3 pages at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.
12. The CODE for this Booklet is Q. Make sure that the CODE printed on side -2 of the Answer Sheet is the same as that on this booklet. In case of discrepancy the candidate should immediately report the matter to the invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. Do not fold make any stray marks on the Answer Sheet.

Name of the Candidate (in Capital letters):

Roll Number: in figures $\quad$|  |  |  |  |  |  |  |  |
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: in words
Examination Centre Number:


Name of Examination Centre (in Capital letters):
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## SOLUTIONS TO AIEEE 2011 PHYSICS: (CODE: Q) PART - A

Note: Questions with (*) mark are from syllabus of class XI.

1. The transverse displacement $y(x, t)$ of a wave on a string is given by $y(x, t)=e^{-\left(a x^{2}+b t^{2}+2 \sqrt{a b} x t\right)}$. This represents a :
(1) wave moving in $-x$ direction with speed $\sqrt{\frac{b}{a}}$
(2) standing wave of frequency $\sqrt{b}$
(3) standing wave of frequency $\frac{1}{\sqrt{b}}$
(4) wave moving in $+x$ direction with speed $\sqrt{\frac{a}{b}}$

Sol.: $\quad y(x, t)=e^{-(\sqrt{a} x+\sqrt{b} t)^{2}}$, it is a function of type $y=f(x+v t)$
$\Rightarrow$ Speed of wave $=\sqrt{\frac{b}{a}}$
Correct choice: (1)
2. A screw gauge gives the following reading when used to measure the diameter of a wire.

Main scale reading $: 0 \mathrm{~mm}$
Circular scale reading : 52 divisions
Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.
The diameter of wire from the above data is :
(1) 0.052 cm
(2) 0.026 cm
(3) 0.005 cm
(4) 0.52 cm

Sol.: $\quad$ Diameter of wire $=M S R+C S R \times$ leastcount $=0+\frac{1}{100} \times 52=0.052 \mathrm{~cm}$
Correct choice: (1)
3. A mass $m$ hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass $m$ and radius $R$. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass $m$, if the string does not slip on the pulley, is :
(1) $g$
(2) $\frac{2}{3} g$
(3) $\frac{g}{3}$
(4) $\frac{3}{2} g$

Sol.: $\quad a=\frac{m g}{\left(m+\frac{I}{R^{2}}\right)}=\frac{m g}{m+\frac{m R^{2}}{2 R^{2}}}=\frac{2 m g}{3 m}=\frac{2 g}{3}$
Correct choice: (2)
4. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution $=0.03 \mathrm{Nm}^{-1}$ )
(1) $0.2 \pi \mathrm{~mJ}$
(2) $2 \pi \mathrm{~mJ}$
(3) $0.4 \pi \mathrm{~mJ}$
(4) $4 \pi \mathrm{~mJ}$

Sol.: $\quad W=2 T 4 \pi\left[\left(5^{2}\right)-(3)^{2}\right] 10^{-4}=2 \times 0.03 \times 4 \pi[25-9] \times 10^{-4} \mathrm{~J}=0.4 \pi \mathrm{~mJ}$
Correct choice: (3)
5. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc :
(1) continuously decreases
(2) continuously increases
(3) first increases and then decreases
(4) remains unchanged

Sol.: Since M.I. is first decreasing then increasing so from principle of conservation of angular momentum, angular speed, first increase then decreases
Correct choice: (3)
6. Two particles are executing simple harmonic motion of the same amplitude $A$ and frequency $\omega$ along the $x$-axis. Their mean position is separated by distance $X_{0}\left(X_{0}>A\right)$. If the maximum separation between them is $\left(X_{0}+A\right)$, the phase difference between their motion is :
(1) $\frac{\pi}{3}$
(2) $\frac{\pi}{4}$
(3) $\frac{\pi}{6}$
(4) $\frac{\pi}{2}$

Sol.: Let, $x_{1}=A \sin \omega t$ and $x_{2}=A \sin (\omega t+\phi)$
$x_{2}-x_{1}=2 A \cos \left(\omega t+\frac{\phi}{2}\right) \sin \frac{\phi}{2}$
But $2 A \sin \frac{\phi}{2}=A \Rightarrow \phi=\frac{\pi}{3}$
Correct choice: (1)
7. Two bodies of masses $m$ and $4 m$ are placed at a distance $r$. The gravitational potential at a point on the line joining them where the gravitational field is zero is :
(1) $-\frac{4 G m}{r}$
(2) $-\frac{6 G m}{r}$
(3) $-\frac{9 G m}{r}$
(4) zero

Sol.: $\quad \frac{G m}{x^{2}}=\frac{4 G m}{(r-x)^{2}} \Rightarrow \quad \frac{1}{x}=\frac{2}{r-x} \quad \therefore \quad r-x=2 x$

$$
x=\frac{r}{3}
$$

Gravitational potential $V=-\frac{G m}{\frac{r}{3}}-\frac{4 G m}{\frac{2 r}{3}}=-\frac{9 G m}{r}$

## Correct choice: (3)

8. Two identical charged spheres suspended from a common point by two massless strings of length $l$ are initially a distance $d(d \ll l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result charges approach each other with a velocity $v$. Then as a function of distance $x$ between them,
(1) $v \propto x^{-1}$
(2) $v \propto x^{1 / 2}$
(3) $v \propto x$
(4) $v \propto x^{-1 / 2}$

Sol.: At any instant $\frac{k q^{2}}{x^{2}} \cos \theta=m g \sin \theta$
$\Rightarrow q^{2} \propto x^{2} \tan \theta$

$\Rightarrow q^{2} \propto x^{3}$
$\Rightarrow q \frac{d q}{d t} \propto x^{2} \frac{d x}{d t}$
$\Rightarrow v \propto x^{-1 / 2} \quad\left[\because q^{2} \propto x^{3}\right]$
Correct choice: (4)

9. A boat is moving due east in a region where the earth's magnetic field is $5.0 \times 10^{-5} \mathrm{NA}^{-1} \mathrm{~m}^{-1}$ due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is $1.50 \mathrm{~ms}^{-1}$, the magnitude of the induced emf in the wire of aerial is :
(1) 0.75 mV
(2) 0.50 mV
(3) 0.15 mV
(4) 1 mV

Sol.: $\quad$ emf $=v B_{H} l=1.5 \times 5 \times 10^{-5} \times 2=15 \times 10^{-5}=0.15 \mathrm{mV}$

## Correct choice: (3)

10. An object, moving with a speed of $6.25 \mathrm{~m} / \mathrm{s}$, is decelerated at a rate given by :
$\frac{d v}{d t}=-2.5 \sqrt{v}$ where $v$ is the instantaneous speed. The time taken by the object, to come to rest, would be :
(1) 2 s
(2) 4 s
(3) 8 s
(4) 1 s

Sol.: $\quad \frac{d v}{d t}=-2.5 \sqrt{v}$
$\int_{6.25}^{0} v^{-1 / 2} d v=-2.5 \int_{0}^{t} d t$
$t=2 \mathrm{sec}$

## Correct choice: (1)

11. A fully charged capacitor $C$ with initial charge $q_{0}$ is connected to a coil of self inductance $L$ at $t=0$. The time at which the energy is stored equally between the electric and the magnetic fields is :
(1) $\frac{\pi}{4} \sqrt{L C}$
(2) $2 \pi \sqrt{L C}$
(3) $\sqrt{L C}$
(4) $\pi \sqrt{L C}$

Sol.: $\quad \frac{1}{2} L i^{2}=\frac{1}{2} \frac{q^{2}}{C}$ also $q=q_{0} \cos \omega t$ and $\omega=\frac{1}{\sqrt{L C}}$
On solving $t=\frac{\pi}{4} \sqrt{L C}$

## Correct choice: (1)

12. Let the $x-z$ plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z<0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A}=6 \sqrt{3} \hat{i}+8 \sqrt{3} \hat{j}-10 \hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is :
(1) $45^{\circ}$
(2) $60^{\circ}$
(3) $75^{\circ}$
(4) $30^{\circ}$

Sol.: Angle of incidence is given by
$\cos (\pi-i)=\frac{(6 \sqrt{3} \hat{i}+8 \sqrt{3} \hat{j}-10 \hat{k}) \cdot \hat{k}}{20}$
$-\cos i=-\frac{1}{2}$
$\angle i=60^{\circ}$
From Snell's law, $\sqrt{2} \sin i=\sqrt{3} \sin r$
$\angle r=45^{\circ}$
Correct choice: (1)

13. A current $I$ flows in an infinitely long wire with cross section in the form of a semi-circular ring of radius $R$. The magnitude of the magnetic induction along its axis is :
(1) $\frac{\mu_{0} I}{2 \pi^{2} R}$
(2) $\frac{\mu_{0} I}{2 \pi R}$
(3) $\frac{\mu_{0} I}{4 \pi R}$
(4) $\frac{\mu_{0} I}{\pi^{2} R}$

Sol.: $\quad d I=\frac{d \theta}{\pi} I, d B=\frac{\mu_{0}}{4 \pi} \frac{2 d I}{R}$
$B_{\text {net }}=\int d B \sin \theta=\frac{\mu_{0} I}{2 \pi^{2} R} \int_{0}^{\pi} \sin \theta d \theta=\frac{\mu_{0} I}{\pi^{2} R}$
Correct choice: (4)

14. A thermally insulated vessel contains an ideal gas of molecular mass $M$ and ratio of specific heats $\gamma$. It is moving with speed $v$ and its suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by :
(1) $\frac{(\gamma-1)}{2 \gamma R} M v^{2} \mathrm{~K}$
(2) $\frac{\gamma M v^{2}}{2 R} \mathrm{~K}$
(3) $\frac{(\gamma-1)}{2 R} M v^{2} \mathrm{~K}$
(4) $\frac{(\gamma-1)}{2(\gamma+1) R} M v^{2} \mathrm{~K}$

Sol.: Work done is zero
So loss in kinetic energy = change in internal energy of gas
$\frac{1}{2} m v^{2}=n C_{v} \Delta T=n \frac{R}{\gamma-1} \Delta T$
$\frac{1}{2} m \nu^{2}=\frac{m}{M} \frac{R}{\gamma-1} \Delta T$
$\therefore \quad \Delta T=\frac{M \nu^{2}(\gamma-1)}{2 R} \mathrm{~K}$
Correct choice: (3)
15. A mass $M$, attached to a horizontal spring, executes S.H.M. with amplitude $A_{1}$. When the mass $M$ passes through its mean position then a smaller mass $m$ is placed over it and both of them move together with amplitude $A_{2}$. The ratio of $\left(\frac{A_{1}}{A_{2}}\right)$ is :
(1) $\frac{M+m}{M}$
(2) $\left(\frac{M}{M+m}\right)^{1 / 2}$
(3) $\left(\frac{M+m}{M}\right)^{1 / 2}$
(4) $\frac{M}{M+m}$

Sol.: At mean point $F_{\text {net }}=0$, so linear momentum must be conserved
$M v_{1}=(M+m) v_{2}$
$M A_{1} \sqrt{\frac{K}{M}}=(M+m) A_{2} \sqrt{\frac{K}{m+M}}$
$A_{1} \sqrt{M}=A_{2} \sqrt{M+m}$
$\therefore \frac{A_{1}}{A_{2}}=\sqrt{\frac{m+M}{M}}$
Correct choice: (3)
16. Water is flowing continuously from a tap having an internal diameter $8 \times 10^{-3} \mathrm{~m}$. The water velocity as it leaves the tap is $0.4 \mathrm{~ms}^{-1}$. The diameter of the water stream at a distance $2 \times 10^{-1} \mathrm{~m}$ below the tap is close to :
(1) $7.5 \times 10^{-3} \mathrm{~m}$
(2) $9.6 \times 10^{-3} \mathrm{~m}$
(3) $3.6 \times 10^{-3} \mathrm{~m}$
(4) $5.0 \times 10^{-3} \mathrm{~m}$

Sol.: From Bernoulli's theorem
$P_{0}+\frac{1}{2} \rho v_{1}^{2}+\rho g h=P_{0}+\frac{1}{2} \rho v_{2}^{2}+0$
$v_{2}=\sqrt{v_{1}^{2}+2 g h}=\sqrt{0.16+2 \times 10 \times 0.2}=2.03 \mathrm{~m} / \mathrm{s}$
From continuity equation $A_{2} v_{2}=A_{1} v_{1}$
$\pi \frac{D_{2}^{2}}{4} \times v_{2}=\pi \frac{D_{1}^{2}}{4} v_{1}$
$\Rightarrow D_{2}=D_{1} \sqrt{\frac{v_{1}}{v_{2}}}=3.55 \times 10^{-3} \mathrm{~m}$

## Correct choice: (3)

17. This question has Statement -1 and Statement -2 . Of the four choices given after the statements, choose the one that best describes the two statements.

## Statement-1 :

Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.
Statement - 2 :
The state of ionosphere varies from hour to hour, day to day and season to season.
(1) Statement -1 is true, Statement -2 is true, Statement -2 is the correct explanation of Statement -1 .
(2) Statement -1 is true, Statement -2 is true, Statement -2 is not the correct explanation of Statement -1 .
(3) Statement -1 is false, Statement -2 is true.
(4) Statement -1 is true, Statement -2 is false.

Sol.: Correct choice: (2)
18. Three perfect gases at absolute temperatures $T_{1}, T_{2}$ and $T_{3}$ are mixed. The masses of molecules are $m_{1}, m_{2}$ and $m_{3}$ and the number of molecules are $n_{1}, n_{2}$ and $n_{3}$ respectively. Assuming no loss of energy, the final temperature of the mixture is :
(1) $\frac{n_{1} T_{1}+n_{2} T_{2}+n_{3} T_{3}}{n_{1}+n_{2}+n_{3}}$
(2) $\frac{n_{1} T_{1}^{2}+n_{2} T_{2}^{2}+n_{3} T_{3}^{2}}{n_{1} T_{1}+n_{2} T_{2}+n_{3} T_{3}}$
(3) $\frac{n_{1}^{2} T_{1}^{2}+n_{2}^{2} T_{2}^{2}+n_{3}^{2} T_{3}^{2}}{n_{1} T_{1}+n_{2} T_{2}+n_{3} T_{3}}$
(4) $\frac{\left(T_{1}+T_{2}+T_{3}\right)}{3}$

Sol.: If atomicity of gas is same
So $\frac{f}{2}\left(n_{1}+n_{2}+n_{3}\right) k T=\frac{f}{2} n_{1} k T_{1}+\frac{f}{2} n_{2} k T_{2}+\frac{f}{2} n_{3} k T_{3}$
$\therefore T=\frac{n_{1} T_{1}+n_{2} T_{2}+n_{3} T_{3}}{n_{1}+n_{2}+n_{3}}$

## Correct choice: (1)

19. A pulley of radius 2 m is rotated about its axis by a force $F=\left(20 t-5 t^{2}\right)$ Newton (where $t$ is measured in seconds) applied tangentially. It the moment of inertia of the pulley about its axis of rotation is $10 \mathrm{~kg}-\mathrm{m}^{2}$ the number of rotations made by the pulley before its direction of motion if reserved, is:
(1) more than 3 but less than 6
(2) more than 6 but less than 9
(3) more than 9
(4) less than 3

Sol.: $\quad F=20 t-5 t^{2}$
$\therefore \alpha=\frac{F R}{I}=4 t-t^{2} \quad \Rightarrow \frac{d \omega}{d t}=4 t-t^{2} \quad \Rightarrow \int_{0}^{\omega} d \omega=\int_{0}^{t}\left(4 t-t^{2}\right) d t$
$\Rightarrow \omega=2 t^{2}-\frac{t^{3}}{3}$
(as $\omega=0$ at $t=0,6 \mathrm{~s}$ )
$\int_{0}^{\theta} d \theta=\int_{0}^{6}\left(2 t^{2}-\frac{t^{3}}{3}\right) d t \quad \Rightarrow \theta=36 \mathrm{rad} \quad \Rightarrow n=\frac{36}{2 \pi}<6$
Correct choice: (1)
20. A resistor ' $R$ ' and $2 \mu F$ capacitor in series in connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V . Calculate the value of $R$ to make the bulb light up 5 s after the switch has been closed. $\left(\log _{10} 2.5=0.4\right)$
(1) $1.7 \times 10^{5} \Omega$
(2) $2.7 \times 10^{6} \Omega$
(3) $3.3 \times 10^{7} \Omega$
(4) $1.3 \times 10^{4} \Omega$

Sol.: $\quad V_{c}=V_{\max }\left(1-e^{-t / R C}\right) ; \quad 120=200\left(1-e^{-t / R C}\right) \Rightarrow t=R C \ln (2.5) \Rightarrow R=2.71 \times 10^{6} \Omega$
Correct choice: (2)
21. A Carnot engine operating between temperatures $T_{1}$ and $T_{2}$ has efficiency $\frac{1}{6}$. When $T_{2}$ is lowered by 62 K its efficiency increases to $\frac{1}{3}$. Then $T_{1}$ and $T_{2}$ are, respectively:
(1) 372 K and 330 K
(2) 330 K and 268 K
(3) 310 K and 248 K
(4) 372 K and 310 K

Sol.: Efficiency of engine $\eta=1-\frac{T_{2}}{T_{1}}$
$\Rightarrow \frac{T_{2}}{T_{1}}=\frac{5}{6}$
$\eta_{2}=1-\frac{T_{2}-62}{T_{1}}=\frac{1}{3}$
Solving we get,

$$
T_{1}=372 \mathrm{~K} \text { and } T_{2}=\frac{5}{6} \times 372=310 \mathrm{~K}
$$

Correct choice: (4)
22. If a wire is stretched to make it $0.1 \%$ longer, its resistance will :
(1) increase by $0.2 \%$
(2) decrease by $0.2 \%$
(3) decrease by $0.05 \%$
(4) increase by $0.05 \%$

Sol.: Resistance of wire $R=\frac{\rho l}{A}=\frac{\rho l^{2}}{C}$ (volume constant or $A l=C$ )
$\therefore$ Fractional change in resistance $\frac{\Delta R}{R}=2 \frac{\Delta l}{l}$
Correct choice: (1)
23. This question has a paragraph followed by two statements, Statement -1 and Statement -2 . Of the given four alternatives after the statements, choose the one that describes the statements.
A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement - $\mathbf{1}$ : When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of $\pi$.
Statement-2: The centre of the interference pattern is dark.
(1) Statement -1 is true, Statement -2 is true, Statement -2 is the correct explanation of Statement -1 .
(2) Statement -1 is true, Statement -2 is true, Statement -2 is not the correct explanation of Statement -1 .
(3) Statement - 1 is false, Statement -2 is true.
(4) Statement -1 is true, Statement -2 is false.

Sol.: Correct choice: (2)
24. A car is fitted with a convex side-view mirror of focal length 20 cm . A second car 2.8 m behind the first car is overtaking the first car at a relative speed of $15 \mathrm{~m} / \mathrm{s}$. The speed of the image of the second car as seen in the mirror of the first one is :
(1) $\frac{1}{15} \mathrm{~m} / \mathrm{s}$
(2) $10 \mathrm{~m} / \mathrm{s}$
(3) $15 \mathrm{~m} / \mathrm{s}$
(4) $\frac{1}{10} \mathrm{~m} / \mathrm{s}$

Sol.: From mirror formulae
$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ so, $\frac{d v}{d t}=-\frac{v^{2}}{u^{2}}\left(\frac{d u}{d t}\right) \Rightarrow \frac{d v}{d t}=-\left(\frac{f}{u-f}\right)^{2} \frac{d u}{d t} \Rightarrow \frac{d v}{d t}=\frac{1}{15} \mathrm{~m} / \mathrm{s}$
Correct choice: (1)
25. Energy required for the electron excitation in $\mathrm{Li}^{++}$from the first to the third Bohr orbit is :
(1) 36.3 eV
(2) 108.8 eV
(3) 122.4 eV
(4) 12.1 eV

Sol.: $\quad \Delta E=13.6(3)^{2}\left(\frac{1}{1^{2}}-\frac{1}{3^{2}}\right)=108.8 \mathrm{eV}$
Correct choice: (2)
26. The electrostatic potential inside a charged spherical ball is given by $\phi=a r^{2}+b$ where $r$ is the distance from the centre $a, b$ are constants. Then the charge density inside the ball is :
(1) $-6 a \varepsilon_{0} r$
(2) $-24 \pi a \varepsilon_{0}$
(3) $-6 a \varepsilon_{0}$
(4) $-24 \pi a \varepsilon_{0} r$

Sol.: Electric field $E=-\frac{d \phi}{d r}=-2 a r$
By Gauss's theorem
$E=\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{r^{2}}$
From (i) and (ii) $q=-8 \pi \varepsilon_{0} a r^{3}, d q=-24 \pi \varepsilon_{0} a r^{2} d r$
Charge density $\rho=\frac{d q}{4 \pi r^{2} d r}=-6 \varepsilon_{0} a$
Correct choice: (3)
27. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is $v$, the total area around the fountain that gets wet is :
(1) $\pi \frac{v^{4}}{g^{2}}$
(2) $\frac{\pi}{2} \frac{v^{4}}{g^{2}}$
(3) $\pi \frac{v^{2}}{g^{2}}$
(4) $\pi \frac{v^{2}}{g}$

Sol.: Total area around fountain $A=\pi R_{\max }^{2}$
Where $R_{\max }=\frac{v^{2} \sin 90^{0}}{g}=\frac{v^{2}}{g} \quad \therefore \quad A=\pi \frac{v^{4}}{g^{2}}$
Correct choice: (1)
28. 100 g of water is heated from $30^{\circ} \mathrm{C}$ to $50^{\circ} \mathrm{C}$. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is $4184 \mathrm{~J} / \mathrm{kg} / \mathrm{K}$ ):
(1) 8.4 kJ
(2) 84 kJ
(3) 2.1 kJ
(4) 4.2 kJ

Sol.: $\quad \Delta U=\Delta Q=m c \Delta T=100 \times 10^{-3} \times 4184(50-30) \approx 8.4 \mathrm{~kJ}$
Correct choice: (1)
29. The half life of a radioactive substance is 20 minutes. The approximate time interval $\left(t_{2}-t_{1}\right)$ between the time $t_{2}$ when $\frac{2}{3}$ of it has decayed and time $t_{1}$ when $\frac{1}{3}$ of it had decayed is :
(1) 14 min
(2) 20 min
(3) 28 min
(4) 7 min

Sol.: No of undecayed atom after time $t_{2}$; $\quad \frac{N_{0}}{3}=N_{0} e^{-\lambda t_{2}} \quad \ldots$ (i)
Number of undecayed atom after time $t_{1} ; \quad \frac{2 N_{0}}{3}=N_{0} e^{-\lambda t_{1}}$
Solving (i) and (ii) $\quad t_{2}-t_{1}=20 \mathrm{~min}$
Correct choice: (2)
30. This question has Statement -1 and Statement - 2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement - 1 :
A metallic surface is irradiated by a monochromatic light of frequency $v>v_{0}$ (the threshold frequency). The maximum kinetic energy and the stopping potential are $K_{\max }$ and $V_{0}$ respectively. If the frequency incident on the surface is doubled, both the $K_{\max }$ and $V_{0}$ are also doubled.
Statement-2 :
The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.
(1) Statement -1 is true, Statement -2 is true, Statement -2 is the correct explanation of Statement -1 .
(2) Statement -1 is true, Statement -2 is true, Statement -2 is not the correct explanation of Statement -1 .
(3) Statement -1 is false, Statement -2 is true.
(4) Statement -1 is true, Statement -2 is false.

Sol.: $\quad K_{\max }=\mathrm{e} V_{0}=h v-h v_{0}$
When $v$ is doubled, $K_{\max }$ and $V_{0}$ become more that double.
Correct choice: (3)

## SOLUTIONS TO AIEEE 2011 MATHEMATICS (CODE: Q) PART - B

Note: Questions with (*) mark are from syllabus of class XI.
*31. The lines $L_{1}: y-x=0$ and $L_{2}: 2 x+y=0$ intersect the line $L_{3}: y+2=0$ at $P$ and $Q$ respectively. The bisector of the acute angle between $L_{1}$ and $L_{2}$ intersects $L_{3}$ at $R$.
Statement-1: The ratio PR: RQ equals $2 \sqrt{2}: \sqrt{5}$
Statement-2: In any triangle, bisector of an angle divides the triangle into two similar triangles.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol.: $\quad P=(-2,-2)$
$Q=(1,-2)$
$\frac{P R}{R Q}=\frac{A P}{A Q}=\frac{2 \sqrt{2}}{\sqrt{5}}$
But statement 2 is not always true
Correct choice: (2)
*32. If $A=\sin ^{2} x+\cos ^{4} x$, then for all real $x$ :
(1) $\frac{13}{16} \leq A \leq 1$
(2) $1 \leq A \leq 2$
(3) $\frac{3}{4} \leq A \leq \frac{13}{16}$
(4) $\frac{3}{4} \leq A \leq 1$

Sol.: $\quad A=\left(1-\sin ^{2} x\right)^{2}+\sin ^{2} x$
$=\sin ^{4} x-\sin ^{2} x+1$
$=\left(\sin ^{2} x-\frac{1}{2}\right)^{2}+\frac{3}{4}$
$0 \leq \sin ^{2} x \leq 1$
$0 \leq\left(\sin ^{2} x-\frac{1}{2}\right)^{2} \leq \frac{1}{4}$
$\frac{3}{4} \leq A \leq 1$.
Correct choice: (4)
*33. The coefficient of $x^{7}$ in the expansion of $\left(1-x-x^{2}+x^{3}\right)^{6}$ is
(1) -132
(2) -144
(3) 132
(4) 144

Sol.: $\quad\left(1-x-x^{2}+x^{3}\right)^{6}=(1-x)^{6}\left(1-x^{2}\right)^{6}$
$=\left(\sum_{r=0}^{6}{ }^{6} C_{r}(-x)^{r}\right)\left(1-6 x^{2}+15 x^{4}-20 x^{6}+\ldots\right)$
$=-6 \times{ }^{6} C_{5} \times(-1)^{5}+15 \times{ }^{6} C_{3} \times(-1)^{3}-20 \times{ }^{6} C_{1} \times(-1)^{1}$
$=36-300+120=-144$
Correct choice: (2)
34. $\lim _{x \rightarrow 2}\left(\frac{\sqrt{1-\cos \{2(x-2)\}}}{x-2}\right)$
(1) equals $\sqrt{2}$
(2) equals $-\sqrt{2}$
(3) equals $\frac{1}{\sqrt{2}}$
(4) does not exist

Sol.: $\quad \lim _{x \rightarrow 2} \sqrt{\frac{1-\cos 2(x-2)}{x-2}}=\lim _{x \rightarrow 2} \frac{\sqrt{2}|\sin (x-2)|}{x-2}$
LHL $=-\sqrt{2}$
RHL $=\sqrt{2}$
Correct choice: (4)
*35. Statement-1: The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${ }^{9} C_{3}$.
Statement-2: The number of ways of choosing any 3 places from 9 different places is ${ }^{9} C_{3}$.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement- 1 is false, Statement- 2 is true.
(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol.: $\quad x_{1}+x_{2}+x_{3}+x_{4}=10, x_{i} \geq 1, i=1,2,3,4$
Number of ways ${ }^{10-1} C_{4-1}={ }^{9} C_{3}$
This can also be explained as : Between 10 balls, there are 9 places out of which 3 are to be selected to make 4 partitions.
Correct choice: (4)
36. $\frac{d^{2} x}{d y^{2}}$ equals :
(1) $-\left(\frac{d^{2} y}{d x^{2}}\right)^{-1}\left(\frac{d y}{d x}\right)^{-3}$
(2) $\left(\frac{d^{2} y}{d x^{2}}\right)\left(\frac{d y}{d x}\right)^{-2}$
(3) $-\left(\frac{d^{2} y}{d x^{2}}\right)\left(\frac{d y}{d x}\right)^{-3}$
(4) $\left(\frac{d^{2} y}{d x^{2}}\right)^{-1}$

Sol.: $\quad \frac{d^{2} x}{d y^{2}}=\frac{d}{d y}\left(\frac{d x}{d y}\right)=\frac{d}{d x}\left(\frac{d x}{d y}\right) \frac{d x}{d y}$

$$
\begin{aligned}
& =\frac{d}{d x}\left(\frac{1}{d y / d x}\right) \frac{d x}{d y} \\
& =-\frac{1}{\left(\frac{d y}{d x}\right)^{3}} \frac{d^{2} y}{d x^{2}}
\end{aligned}
$$

## Correct choice: (3)

37. If $\frac{d y}{d x}=y+3>0$ and $y(0)=2$, then $y(\ln 2)$ is equal to :
(1) 5
(2) 13
(3) -2
(4) 7

Sol.: $\frac{d y}{d x}=y+3$
$\int \frac{d y}{y+3}=\int d x \Rightarrow \ln (y+3)=x+c$
$\ln 5=0+c$
$\therefore \ln (y+3)=x+\ln 5$
$\ln (y+3)=\ln 2+\ln 5 \Rightarrow y+3=10 \Rightarrow y=7$
Correct choice: (4)
*38. Let R be the set of real numbers.
Statement-1: $A=\{(x, y) \in R \times R: y-x$ is an integer $\}$ is an equivalence relation on $R$.
Statement-2: $B=\{(x, y) \in R \times R: x=\alpha y$ for some rational number $\alpha\}$ is an equivalence relation on $R$.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol.: Let for statement $1 x R y=x-y \in I$. As $x R x$ is an integer and $y R x$ as well as $x R z$ (for $x R y$ and $y R z$ ) is also an integer Hence equivalence.
Similarly as $x=\alpha y$ hence $\alpha=1$ for reflexive and product of rationals also being rational $\Rightarrow$ transitive
But not symmetric because of $\alpha=0$ case. e.g. for $(0,1), \alpha=0$ but no $\alpha$ for $(1,0)$.
Correct choice: (2)
39. The value of $\int_{0}^{1} \frac{8 \log (1+x)}{1+x^{2}} d x$ is
(1) $\frac{\pi}{8} \log 2$
(2) $\frac{\pi}{2} \log 2$
(3) $\log 2$
(4) $\pi \log 2$

Sol.: $\quad I=8 \int_{0}^{1} \frac{\log (1+x)}{1+x^{2}} d x$
Put $x=\tan \theta$

$$
\begin{aligned}
I & =8 \int_{0}^{\pi / 4} \log (1+\tan \theta) d \theta \\
& =8 \times \frac{\pi}{8} \log 2=\pi \log 2
\end{aligned}
$$

Correct choice: (4)
*40. Let $\alpha, \beta$ be real and $z$ be a complex number. If $z^{2}+\alpha z+\beta=0$ has two distinct roots on the line $\operatorname{Re} z=1$, then it is necessary that :
(1) $\beta \in(-1,0)$
(2) $|\beta|=1$
(3) $\beta \in(1, \infty)$
(4) $\beta \in(0,1)$

Sol.: $\quad z^{2}+\alpha z+\beta=0$
Since the coefficients are real, complex roots must occur in conjugate pairs.
Let the roots be $1+i x, 1-i x, x \neq 0 . \beta=(1+i x)(1-i x)=1+x^{2}>1$
Correct choice: (3)
41. Consider 5 independent Bernoulli's trials each with probability of success $p$. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then $p$ lies in the interval
(1) $\left(\frac{3}{4}, \frac{11}{12}\right]$
(2) $\left[0, \frac{1}{2}\right]$
(3) $\left(\frac{11}{12}, 1\right]$
(4) $\left(\frac{1}{2}, \frac{3}{4}\right]$

Sol.: $\quad P(X=r)={ }^{5} C_{r} p^{r} q^{5-r}$
$1-P(X=5) \geq \frac{31}{32} \Rightarrow p^{5} \leq \frac{1}{32}$
$p \leq \frac{1}{2}$
Correct choice: (2)
*42. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after
(1) 19 months
(2) 20 months
(3) 21 months
(4) 18 months

Sol.: $200+200+200+(240+280+\ldots n$ terms $)=11040$
$n=18$
$\therefore 21$ months
Correct choice: (3)
43. The domain of the function $f(x)=\frac{1}{\sqrt{|x|-x}}$ is
(1) $(0, \infty)$
(2) $(-\infty, 0)$
(3) $(-\infty, \infty)-\{0\}$
(4) $(-\infty, \infty)$

Sol.: $\quad|x|-x>0$
$|x|>x$
$\Rightarrow x<0$

## Correct choice: (2)

44. If the angle between the line $x=\frac{y-1}{2}=\frac{z-3}{\lambda}$ and the plane $x+2 y+3 z=4$ is $\cos ^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then $\lambda$ equals
(1) $\frac{3}{2}$
(2) $\frac{2}{5}$
(3) $\frac{5}{3}$
(4) $\frac{2}{3}$

Sol.: $\quad \sin \theta=\frac{1 \times 1+2 \times 2+\lambda \times 3}{\sqrt{5+\lambda^{2}} \sqrt{14}}$
$\sin \cos ^{-1} \sqrt{\frac{5}{14}}=\frac{5+3 \lambda}{\sqrt{14} \sqrt{5+\lambda^{2}}}$
$\lambda=\frac{2}{3}$
Correct choice: (4)
45. If $\vec{a}=\frac{1}{\sqrt{10}}(3 \hat{i}+\hat{k})$ and $\vec{b}=\frac{1}{7}(2 \hat{i}+3 \hat{j}-6 \hat{k})$, then the value of $(2 \vec{a}-\vec{b}) \cdot[(\vec{a} \times \vec{b}) \times(\vec{a}+2 \vec{b})]$ is
(1) -3
(2) 5
(3) 3
(4) -5

Sol.: $\quad(2 \vec{a}-\vec{b}) \cdot[(\vec{a} \times \vec{b}) \times(\vec{a}+2 \vec{b})]$
$=(\vec{b}-2 \vec{a}) \cdot[(\vec{a}+2 \vec{b}) \times(\vec{a} \times \vec{b})]$
$=(\vec{b}-2 \vec{a})[\vec{a} \times(\vec{a} \times \vec{b})+2 \vec{b} \times(\vec{a} \times \vec{b})]$
$=(\vec{b}-2 \vec{a}) \cdot[(\vec{a} \cdot \vec{b}) \vec{a}-(\vec{a} \cdot \vec{a}) \vec{b}+2(\vec{b} \cdot \vec{b}) \vec{a}-2(\vec{b} \cdot \vec{a}) \vec{b}]$
$=(\vec{b}-2 \vec{a}) \cdot(-\vec{b}+2 \vec{a})=-(2 \vec{a}-\vec{b}) \cdot(2 \vec{a}-\vec{b})$
$=-(4+1-4 \vec{a} \cdot \vec{b})=-5$

## Correct choice: (4)

*46. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point $(-3,1)$ and has eccentricity $\sqrt{\frac{2}{5}}$ is
(1) $5 x^{2}+3 y^{2}-48=0$
(2) $3 x^{2}+5 y^{2}-15=0$
(3) $5 x^{2}+3 y^{2}-32=0$
(4) $3 x^{2}+5 y^{2}-32=0$

Sol.: $\quad \frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$
As $b^{2}=a^{2}\left(1-\frac{2}{5}\right)$
$\frac{9}{a^{2}}+\frac{1}{b^{2}}=1$
$\frac{9}{a^{2}}+\frac{5}{a^{2}(3)}=1$
$\frac{1}{a^{2}}\left(9+\frac{5}{3}\right)=1 \Rightarrow a^{2}=\frac{32}{3}$
$b^{2}=\frac{32}{3} \times \frac{3}{5}=\frac{32}{5}$
$\Rightarrow 3 x^{2}+5 y^{2}-32=0$.
There is one more case of vertical ellipse which gives $5 x^{2}+3 y^{2}-48=0$. So two options are correct.
Correct choice: (1) and (4)
47. Let $I$ be the purchase value of an equipment and $V(t)$ be the value after it has been used for $t$ years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{d V(t)}{d t}=-k(T-t)$, where $k>0$ is a constant and $T$ is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is
(1) $I-\frac{k T^{2}}{2}$
(2) $I-\frac{k(T-t)^{2}}{2}$
(3) $e^{-k T}$
(4) $T^{2}-\frac{I}{k}$

Sol.: $\quad \frac{d V(t)}{d t}+k T-k t=0$
$\frac{d V(t)}{d t}-k t=-k T \Rightarrow L D E \quad($ at $t=0, V=I)$
$\Rightarrow V(T)=I-\frac{k T^{2}}{2} \quad($ scrap value obtained by putting $t=T)$

## Correct choice: (1)

48. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying $\vec{b} \times \vec{c}=\vec{b} \times \vec{d}$ and $\vec{a} \cdot \vec{d}=0$. Then the vector $\vec{d}$ is equal to
(1) $\vec{c}+\left(\frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right) \vec{b}$
(2) $\vec{b}+\left(\frac{\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right) \vec{c}$
(3) $\vec{c}-\left(\frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right) \vec{b}$
(4) $\vec{b}-\left(\frac{\vec{b} \cdot \vec{c}}{\vec{a} . \vec{b}}\right) \vec{c}$

Sol.: $\quad \vec{a} \cdot \vec{b} \neq 0$
$\vec{b} \times \vec{c}=\vec{b} \times \vec{d}$ and $\vec{a} \cdot \vec{d}=0$
$\Rightarrow \vec{a} \times(\vec{b} \times \vec{c})=\vec{a} \times(\vec{b} \times \vec{d})$
$\Rightarrow(\vec{a} \cdot \vec{c}) \vec{b}-(\vec{a} \cdot \vec{b}) \vec{c}=(\vec{a} \cdot \vec{d}) \vec{b}-(\vec{a} \cdot \vec{b}) \vec{d}$
$\Rightarrow(\vec{a} \cdot \vec{b}) \vec{d}=-(\vec{a} \cdot \vec{c}) \vec{b}+(\vec{a} \cdot \vec{b}) \vec{c}$
$\vec{d}=\vec{c}-\left(\frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right) \vec{b}$
Correct choice: (3)
*49. The two circles $x^{2}+y^{2}=a x$ and $x^{2}+y^{2}=c^{2}(c>0)$ touch each other if
(1) $|a|=c$
(2) $a=2 c$
(3) $|a|=2 c$
(4) $2|a|=c$

Sol.:

$\Rightarrow|a|=C$
Correct choice: (1)
50. If $C$ and $D$ are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is
(1) $P(C \mid D) \geq P(C)$
(2) $P(C \mid D)<P(C)$
(3) $P(C \mid D)=\frac{P(D)}{P(C)}$
(4) $P(C \mid D)=P(C)$

Sol.: In this case
$P\left(\frac{C}{D}\right)=\frac{P(C \cap D)}{P(D)}=\frac{P(C)}{P(D)}$
where $P(D) \leq 1$ hence $P\left(\frac{C}{D}\right) \geq P(C)$

## Correct choice: (1)

51. The number of values of $k$ for which the linear equations $4 x+k y+2 z=0, k x+4 y+z=0$ and $2 x+2 y+z=0$ possess a non-zero solution is
(1) 2
(2) 1
(3) zero
(4) 3

Sol.: $\Delta=0$
$\Rightarrow\left|\begin{array}{lll}4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1\end{array}\right|=0 \Rightarrow 4(4-2)-k(k-2)+2(2 k-8)=0$
$\Rightarrow 8-k^{2}+2 k+4 k-16=0$
$k^{2}-6 k+8=0 \Rightarrow(k-4)(k-2)=0, k=4,2$
Correct choice: (1)
*52. Consider the following statements
$\mathbf{P}$ : Suman is brilliant
Q : Suman is rich
$\mathbf{R}$ : Suman is honest
The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as
(1) $\sim\left(Q \leftrightarrow\left(P^{\wedge} \sim R\right)\right)$
(2) $\sim Q \leftrightarrow \sim P \wedge R$
(3) $\sim\left(P^{\wedge} \sim R\right) \leftrightarrow Q$
(4) $\sim P^{\wedge}(Q \leftrightarrow \sim R)$

Sol.: Correct choice: (1)
*53. The shortest distance between line $y-x=1$ and curve $x=y^{2}$ is
(1) $\frac{3 \sqrt{2}}{8}$
(2) $\frac{8}{3 \sqrt{2}}$
(3) $\frac{4}{\sqrt{3}}$
(4) $\frac{\sqrt{3}}{4}$

Sol.: $\quad y=m x-2 a m-a m^{3}$ as $m=-1$
$y=-x-2 \frac{1}{4}(-1)-\frac{1}{4}(-1)^{3}$
$y=-x+\frac{1}{2}+\frac{1}{4}=\frac{3}{4}-x$
Common normal
$y=\frac{3}{4}-x \Rightarrow A B \Rightarrow A=\left(-\frac{1}{8}, \frac{7}{8}\right) ; B=\left(\frac{1}{4}, \frac{1}{2}\right)$
$=\frac{3}{8} \sqrt{2}$
Correct choice: (1)
*54. If the mean deviation about the median of the numbers $a, 2 a, \ldots \ldots, 50 a$ is 50 , then $|a|$ equals
(1) 3
(2) 4
(3) 5
(4) 2

Sol.: Median (M) $=\frac{25 a+26 a}{2}=\frac{51 a}{2}$ and Now M.D. $=\sum \frac{\left|x_{i}-M\right|}{n}$
Correct choice: (2)
55. Statement-1: The point $A(1,0,7))$ is the mirror image of the point $B(1,6,3)$ in the line $: \frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.

Statement-2: The line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ bisects the line segment joining $A(1,0,7)$ and $B(1,6,3)$.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement- 1 is false, Statement- 2 is true.
(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol.: As the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ is the right bisector of line segment joining $(1,0,7)$ with $(1,6,3)$.
But right bisector is not mentioned in statement 2 .
Correct choice: (1)
56. Let $A$ and $B$ be two symmetric matrices of order 3 .

Statement-1: $A(B A)$ and $(A B) A$ are symmetric matrices.
Statement-2: $A B$ is symmetric matrix if matrix multiplication of $A$ with $B$ is commutative.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement- 1 is false, Statement- 2 is true.
(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol.: $\quad(A(B A))^{T}=(B A)^{T} A^{T}=\left(A^{T} B^{T}\right) A^{T}=(A B) A=A(B A)$
Correct choice: (1)
*57. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^{7}=A+B \omega$. Then $(A, B)$ equals
(1) $(1,1)$
(2) $(1,0)$
(3) $(-1,1)$
(4) $(0,1)$

Sol.: $\quad \Rightarrow(1+\omega)^{7}=A+B \omega$
$\left(-\omega^{2}\right)^{7}=1+\omega$

$$
A=1, B=1
$$

## Correct choice: (1)

58. The values of $p$ and $q$ for which the function $f(x)= \begin{cases}\frac{\sin (p+1) x+\sin x}{x} & , x<0 \\ q & x=0 \text { is continuous for all } x \text { in } R, \text { are } \\ \frac{\sqrt{x+x^{2}}-\sqrt{x}}{x^{3 / 2}} & , x>0\end{cases}$
(1) $p=\frac{5}{2}, q=\frac{1}{2}$
(2) $p=-\frac{3}{2}, q=\frac{1}{2}$
(3) $p=\frac{1}{2}, q=\frac{3}{2}$
(4) $p=\frac{1}{2}, q=-\frac{3}{2}$

Sol.: LHL:-
$\lim _{x \rightarrow 0^{-}} f(x)=\lim _{h \rightarrow 0} \frac{\sin \{(p+1)(-h)\}-\sinh }{-h}=p+1+1=p+2$
RHL:-
$\lim _{x \rightarrow 0^{+}} f(x)=\lim _{h \rightarrow 0}=\frac{1}{1+1}=\frac{1}{2} \quad$ (by rationalization)
$f(0)=q \Rightarrow p=-\frac{3}{2}, q=\frac{1}{2}$
Correct choice: (2)
59. The area of the region enclosed by the curves $y=x, x=e, y=\frac{1}{x}$ and the positive x -axis is
(1) 1 square unit
(2) $\frac{3}{2}$ square units
(3) $\frac{5}{2}$ square units
(4) $\frac{1}{2}$ square unit

Sol.: Required region $A O B C$ :-
$\int_{0}^{1} x d x+\int_{1}^{e} \frac{1}{x} d x$
$=\frac{1}{2}+1=\frac{3}{2}$
Correct choice: (2)

60. For $x \in\left(0, \frac{5 \pi}{2}\right)$, define $f(x)=\int_{0}^{x} \sqrt{t} \sin t d t$. Then $f$ has
(1) local minimum at $\pi$ and $2 \pi$
(2) local minimum at $\pi$ and local maximum at $2 \pi$
(3) local maximum at $\pi$ and local minimum at $2 \pi$
(4) local maximum at $\pi$ and $2 \pi$

Sol.: $\quad f^{\prime}(x)=\sqrt{x} \sin x$
$f^{\prime}(x)=0$
$\Rightarrow x=0$ or $\sin x=0$
$\Rightarrow \quad x=2 \pi, \pi$
$f^{\prime \prime}(x)=\sqrt{x} \cos x+\frac{1}{2 \sqrt{x}} \sin x$
$\Rightarrow \frac{1}{2 \sqrt{x}}(2 x \cos x+\sin x)$
at $x=\pi \Rightarrow f^{\prime \prime}(x)<0 \Rightarrow$ maxima
at $x=2 \pi \Rightarrow f^{\prime \prime}(x)>0 \Rightarrow$ minima
Correct choice: (3)

## SOLUTIONS TO AIEEE 2011 CHEMISTRY: (CODE: Q) <br> PART - C

Note: Questions with (*) mark are from syllabus of class XI.
*61. Among the following the maximum covalent character is shown by the compound:
(1) $\mathrm{SnCl}_{2}$
(2) $\mathrm{AlCl}_{3}$
(3) $\mathrm{MgCl}_{2}$
(4) $\mathrm{FeCl}_{2}$

Sol.: Higher charge density of cation favours covalency.
Correct choice: (2)
62. The presence or absence of hydroxy group on which carbon atom of sugar differentiates RNA and DNA?
(1) $2^{\text {nd }}$
(2) $3^{\text {rd }}$
(3) $4^{\text {th }}$
(4) $1^{\text {st }}$

Sol.: RNA has hydroxy group attached at the $2^{\text {nd }} \mathrm{C}$-atom while DNA does not.
Correct choice: (1)
63. Trichloroacetaldehyde was subjected to Cannizzaro's reaction by using NaOH . The mixture of the products contains sodium trichloroacetate and another compound. The other compound is:
(1) Trichloromethanol
(2) 2,2,2-Trichloropropanol
(3) Chloroform
(4) 2,2,2- Trichloroethanol

Sol.:


Correct choice: (4)
64. Sodium ethoxide has reacted with ethanoyl chloride. The compound that is produced in the above reaction is:
(1) 2-Butanone
(2) Ethyl chloride
(3) Ethyl ethanoate
(4) Diethyl ether

Sol.:


Correct choice: (3)
65. The reduction potential of hydrogen half-cell will be negative if:
(1) $\mathrm{p}\left(\mathrm{H}_{2}\right)=1 \mathrm{~atm}$ and $\left[\mathrm{H}^{+}\right]=1.0 \mathrm{M}$
(2) $\mathrm{p}\left(\mathrm{H}_{2}\right)=2 \mathrm{~atm}$ and $\left[\mathrm{H}^{+}\right]=1.0 \mathrm{M}$
(3) $\mathrm{p}\left(\mathrm{H}_{2}\right)=2 \mathrm{~atm}$ and $\left[\mathrm{H}^{+}\right]=2.0 \mathrm{M}$
(4) $\mathrm{p}\left(\mathrm{H}_{2}\right)=1 \mathrm{~atm}$ and $\left[\mathrm{H}^{+}\right]=2.0 \mathrm{M}$

Sol.: $\quad \mathrm{Pt}\left|\mathrm{H}_{2}(\mathrm{~g})\right| \mathrm{H}^{+}(\mathrm{aq})$
$\mathrm{H}^{+}+\mathrm{e}^{-} \longrightarrow 1 / 2 \mathrm{H}_{2}(\mathrm{~g})$
$\mathrm{E}=\mathrm{E}^{\circ}-\frac{0.059}{1} \log \frac{\left(\mathrm{P}_{\mathrm{H}_{2}}\right)^{1 / 2}}{\left[\mathrm{H}^{+}\right]} \quad ; \mathrm{E}=0-\frac{0.059}{1} \log \frac{2^{1 / 2}}{1}=-\mathrm{ve}$.

## Correct choice: (2)

*66. The strongest acid amongst the following compounds is:
(1) HCOOH
(2) $\mathrm{CH}_{3} \mathrm{CH}_{2} \mathrm{CH}(\mathrm{Cl}) \mathrm{CO}_{2} \mathrm{H}$
(3) $\mathrm{ClCH}_{2} \mathrm{CH}_{2} \mathrm{CH}_{2} \mathrm{COOH}$
(4) $\mathrm{CH}_{3} \mathrm{COOH}$

Sol.: $\quad \mathrm{CH}_{3} \mathrm{CH}_{2} \mathrm{CH}(\mathrm{Cl}) \mathrm{CO}_{2} \mathrm{H}$ is the strongest acid due to -I effect of Cl .

## Correct choice: (2)

67. The degree of dissociation ( $\alpha$ ) of a weak electrolyte, $A_{x} B_{y}$ is related to van't Hoff factor ( $i$ ) by the expression:
(1) $\alpha=\frac{i-1}{x+y+1}$
(2) $\alpha=\frac{x+y-1}{i-1}$
(3) $\alpha=\frac{x+y+1}{i-1}$
(4) $\alpha=\frac{i-1}{(x+y-1)}$

Sol.: $\quad \mathrm{A}_{x} \mathrm{~B}_{\mathrm{y}} \rightleftharpoons \mathrm{xA}^{\mathrm{y}+}+\mathrm{yB}^{x-}$
Moles at
equilibrium $\quad 1-\alpha \quad x \alpha \quad y \alpha$
$i=1-\alpha+x \alpha+y \alpha$
$(i-1)=\alpha(x+y-1)$
$\alpha=\frac{i-1}{x+y-1}$
Correct choice: (4)
*68. 'a' and ' $b$ ' are van der Waal's constants for gases. Chlorine is more easily liquefied than ethane because
(1) $a$ and $b$ for $\mathrm{Cl}_{2}<a$ and $b$ for $\mathrm{C}_{2} \mathrm{H}_{6}$.
(2) a for $\mathrm{Cl}_{2}<\mathrm{a}$ for $\mathrm{C}_{2} \mathrm{H}_{6}$ but b for $\mathrm{Cl}_{2}>\mathrm{b}$ for $\mathrm{C}_{2} \mathrm{H}_{6}$
(3) a for $\mathrm{Cl}_{2}>\mathrm{a}$ for $\mathrm{C}_{2} \mathrm{H}_{6}$ but b for $\mathrm{Cl}_{2}<$ b for $\mathrm{C}_{2} \mathrm{H}_{6}$
(4) $a$ and $b$ for $\mathrm{Cl}_{2}>a$ and $b$ for $\mathrm{C}_{2} \mathrm{H}_{6}$

Sol.: van der Waal's constant ' $a$ ' is a measure of intermolecular force of attraction while ' $b$ ' is a measure of size of the gaseous molecule.
Correct choice: (3)
*69. A vessel at 1000 K contains $\mathrm{CO}_{2}$ with a pressure of 0.5 atm . Some of the $\mathrm{CO}_{2}$ is converted into CO on the addition of graphite. If the total pressure at equilibrium is 0.8 atm , the value of K is:
(1) 3 atm
(2) 0.3 atm
(3) 0.18 atm
(4) 1.8 atm

Sol.: $\quad \mathrm{CO}_{2}(\mathrm{~g})+\mathrm{C}(\mathrm{s}) \rightleftharpoons 2 \mathrm{CO}(\mathrm{g})$
$0.5-x \quad 2 x$
$0.5-x+2 x=0.8$
$x=0.3$
$\mathrm{K}=\frac{0.6 \times 0.6}{0.2}=1.8 \mathrm{~atm}$.
Correct choice: (4)
*70. Boron cannot form which one of the following anions?
(1) $\mathrm{BH}_{4}^{-}$
(2) $\mathrm{B}(\mathrm{OH})_{4}^{-}$
(3) $\mathrm{BO}_{2}^{-}$
(4) $\mathrm{BF}_{6}^{3-}$

Sol.: Boron cannot form $\mathrm{BF}_{6}^{3-}$ due to non availability of d-orbital.
Correct choice: (4)
71. Which of the following facts about the complex $\left[\mathrm{Cr}\left(\mathrm{NH}_{3}\right)_{6}\right] \mathrm{Cl}_{3}$ is $\boldsymbol{w r o n g}$ ?
(1) The complex is paramagnetic.
(2) The complex is an outer orbital complex.
(3) The complex gives white precipitate with silver nitrate solution.
(4) The complex involves $\mathrm{d}^{2} \mathrm{sp}^{3}$ hybridisation and is octahedral in shape.

Sol.: The complex involves $\mathrm{d}^{2} \mathrm{sp}^{3}$ hybridisation, and hence it is an inner orbital complex.
Correct choice: (2)
72. Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it from freezing at $-6^{\circ} \mathrm{C}$ will be: ( $\mathrm{K}_{\mathrm{f}}$ for water $=1.86 \mathrm{~K} \mathrm{~kg} \mathrm{~mol}^{-1}$, and molar mass of ethylene glycol $=62 \mathrm{~g} \mathrm{~mol}^{-1}$ )
(1) 204.30 g
(2) 400.00 g
(3) 304.60 g
(4) 804.32 g

Sol.: $\quad \Delta T_{f}=K_{f} \times m$
$6=1.86 \times \frac{\mathrm{w}}{62 \times 4}$
$\mathrm{w}=\frac{6 \times 62 \times 4}{1.86}=800 \mathrm{~g}$.

## Correct choice: (4)

*73. Which one of the following orders presents the correct sequence of the increasing basic nature of the given oxides?
(1) $\mathrm{MgO}<\mathrm{K}_{2} \mathrm{O}<\mathrm{Al}_{2} \mathrm{O}_{3}<\mathrm{Na}_{2} \mathrm{O}$
(2) $\mathrm{Na}_{2} \mathrm{O}<\mathrm{K}_{2} \mathrm{O}<\mathrm{MgO}<\mathrm{Al}_{2} \mathrm{O}_{3}$
(3) $\mathrm{K}_{2} \mathrm{O}<\mathrm{Na}_{2} \mathrm{O}<\mathrm{Al}_{2} \mathrm{O}_{3}<\mathrm{MgO}$
(4) $\mathrm{Al}_{2} \mathrm{O}_{3}<\mathrm{MgO}<\mathrm{Na}_{2} \mathrm{O}<\mathrm{K}_{2} \mathrm{O}$

Sol.: The basic character of metal oxide increases with increase in electropositivity of metal.

## Correct choice: (4)

74. The rate of a chemical reaction doubles for every $10^{\circ} \mathrm{C}$ rise of temperature. If the temperature is raised by $50^{\circ} \mathrm{C}$, the rate of the reaction increases by about:
(1) 24 times
(2) 32 times
(3) 64 times
(4) 10 times

Sol.: $\quad$ Increase in reaction rate $=2^{5}=32$ times.

## Correct choice: (2)

75. The magnetic moment (spin only) of $\left[\mathrm{NiCl}_{4}\right]^{2-}$ is:
(1) 5.46 BM
(2) 2.82 BM
(3) 1.41 BM
(4) 1.82 BM

Sol.: $\quad \mathrm{Ni}^{2+} 3 \mathrm{~d}^{8} 4 \mathrm{~s}^{0}$
The hybridisation of central metal ion is $\mathrm{sp}^{3}$ in $\mathrm{NiCl}_{4}^{-2}$.
Therefore number of unpaired electrons $=2$.
$\mu=\sqrt{\mathrm{n}(\mathrm{n}+2)}=\sqrt{2 \times 4}=2 \sqrt{2}=2 \times 1.41=2.82 \mathrm{BM}$.
Correct choice: (2)
*76. The hybridisation of orbitals of N atom in $\mathrm{NO}_{3}^{-}, \mathrm{NO}_{2}^{+}$and $\mathrm{NH}_{4}^{+}$are respectively:
(1) $\mathrm{sp}^{2}, \mathrm{sp}, \mathrm{sp}^{3}$
(2) $\mathrm{sp}, \mathrm{sp}^{3}, \mathrm{sp}^{2}$
(3) $\mathrm{sp}^{2}, \mathrm{sp}^{3}, \mathrm{sp}$
(4) $\mathrm{sp}, \mathrm{sp}^{2}, \mathrm{sp}^{3}$

Sol.:



Hybridization of N atom in $\mathrm{NO}_{3}^{-}, \mathrm{NO}_{2}^{+}$and $\mathrm{NH}_{4}^{+}$are respectively $\mathrm{sp}^{2}$, sp , $\mathrm{sp}^{3}$.
Correct choice: (1)
77. In context of the lanthanoids, which of the following statements is not correct?
(1) All the members exhibit +3 oxidation state.
(2) Because of similar properties the separation of lanthanoids is not easy.
(3) Availability of 4 f electrons results in the formation of compounds in +4 state for all the members of the series.
(4) There is a gradual decrease in the radii of the members with increasing atomic number in the series.

Sol.: The common oxidation state of lanthanoids is +3 . Some of the lanthanoids also show +2 and +4 oxidation state but they easily revert to the more stable +3 state.

## Correct choice: (3)

*78. A 5.2 molal aqueous solution of methyl alcohol, $\mathrm{CH}_{3} \mathrm{OH}$, is supplied. What is the mole fraction of methyl alcohol in the solution?
(1) 0.190
(2) 0.086
(3) 0.050
(4) 0.100

Sol.: $\quad \mathrm{n}_{\mathrm{CH}_{3} \mathrm{OH}}=5.2$ moles
$\mathrm{n}_{\mathrm{H}_{2} \mathrm{O}}=\frac{1000}{18}=55.55$ moles
$\mathrm{X}_{\mathrm{CH}_{3} \mathrm{OH}}=\frac{5.2}{5.2+55.55}=0.086$
Correct choice: (2)
79. Which of the following statement is wrong?
(1) Nitrogen cannot form $\mathrm{d} \pi-\mathrm{p} \pi$ bond.
(2) Single N-N bond is weaker than the single $\mathrm{P}-\mathrm{P}$ bond.
(3) $\mathrm{N}_{2} \mathrm{O}_{4}$ has two resonance structures.
(4) The stability of hydrides increases from $\mathrm{NH}_{3}$ to $\mathrm{BiH}_{3}$ in group 15 of the periodic table.

Sol.: The stability of hydrides decreases from $\mathrm{NH}_{3}$ to $\mathrm{BiH}_{3}$ due to increase in size of the central atom.
Correct choice: (4)
80. The outer electron configuration of Gd (Atomic number: 64) is:
(1) $4 \mathrm{f}^{8} 5 \mathrm{~d}^{0} 6 \mathrm{~s}^{2}$
(2) $4 \mathrm{f}^{4} 5 \mathrm{~d}^{4} 6 \mathrm{~s}^{2}$
(3) $4 \mathrm{f}^{7} 5 \mathrm{~d}^{1} 6 \mathrm{~s}^{2}$
(4) $4 f^{3} 5 d^{5} 6 s^{2}$

Sol.: Outer electronic configuration of Gd is $4 f^{7} 5 d^{1} 6 s^{2}$
Correct choice: (3)
81. Which of the following statements regarding sulphur is incorrect?
(1) The vapour at $200^{\circ} \mathrm{C}$ consists mostly of $\mathrm{S}_{8}$ rings.
(2) At $600^{\circ} \mathrm{C}$ the gas mainly consists of $\mathrm{S}_{2}$ molecules.
(3) The oxidation state of sulphur is never less than +4 in its compounds.
(4) $S_{2}$ molecule is paramagnetic.

Sol.: The oxidation state of sulphur in its various compounds varies from -2 to +6 .
Correct choice: (3)
*82. The structure of $\mathrm{IF}_{7}$ is:
(1) trigonal bipyramid
(2) octahedral
(3) pentagonal bipyramid
(4) square pyramid

Sol.: The hybridisation of central atom is $\mathrm{sp}^{3} \mathrm{~d}^{3}$ and its structure is pentagonal bipyramid.
Correct choice: (3)
*83. Ozonolysis of an organic compound gives formaldehyde as one of the products. This confirms the presence of:
(1) a vinyl group.
(2) an isopropyl group.
(3) an acetylenic triple bond.
(4) two ethylenic double bonds.

Sol.: Vinyl group on ozonolysis gives formaldehyde as one of the products.
Correct choice: (1)
*84. A gas absorbs a photon of 355 nm and emits at two wavelengths. If one of the emissions is at 680 nm , the other is at:
(1) 325 nm
(2) 743 nm
(3) 518 nm
(4) 1035 nm

Sol.: $\frac{1}{\lambda}=\frac{1}{\lambda_{1}}+\frac{1}{\lambda_{2}} ; \quad \frac{1}{355}=\frac{1}{680}+\frac{1}{\lambda_{2}} \quad ; \quad \lambda_{2}=743 \mathrm{~nm}$.

## Correct choice: (2)

85. Silver mirror test is given by which one of the following compounds?
(1) Acetone
(2) Formaldehyde
(3) Benzophenone
(4) Acetaldehyde

Sol.: Correct choice: (4)
86. Which of the following reagents may be used to distinguish between phenol and benzoic acid?
(1) Tollen's reagent
(2) Molisch reagent
(3) Neutral $\mathrm{FeCl}_{3}$
(4) Aqueous NaOH

Sol.: Phenol gives a violet colour with neutral ferric chloride solution.
Correct choice: (3)
87. Phenol is heated with a solution of mixture of KBr and $\mathrm{KBrO}_{3}$. The major product obtained in the above reaction is:
(1) 3-Bromophenol
(2) 4-Bromophenol
(3) 2,4,6-Tribromophenol
(4) 2-Bromophenol

Sol.: $5 \mathrm{Br}^{-}+\mathrm{BrO}_{3}^{-}+3 \mathrm{H}_{2} \mathrm{O} \rightleftharpoons 3 \mathrm{Br}_{2}+6 \mathrm{OH}^{-}$


Correct choice: (3)
88. In a face centred cubic lattice, atom $A$ occupies the corner positions and atom $B$ occupies the face centre positions. If one atom of $B$ is missing from one of the face centred points, the formula of the compound is
(1) $\mathrm{AB}_{2}$
(2) $A_{2} B_{3}$
(3) $\mathrm{A}_{2} \mathrm{~B}_{5}$
(4) $\mathrm{A}_{2} \mathrm{~B}$

Sol.: Contribution by $A$ atoms at corners $=8 \times \frac{1}{8}=1$.
Contribution by B atoms at face centre $=5 \times \frac{1}{2}=\frac{5}{2}$.
So, formula is $\mathrm{AB}_{5 / 2}$ or $\mathrm{A}_{2} \mathrm{~B}_{5}$.
Correct choice: (3)
*89. The entropy change involved in the isothermal reversible expansion of 2 moles of an ideal gas from a volume of $10 \mathrm{dm}^{3}$ to a volume of $100 \mathrm{dm}^{3}$ at $27^{\circ} \mathrm{C}$ is:
(1) $35.8 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$
(2) $32.3 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$
(3) $42.3 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$
(4) $38.3 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$

Sol.: $\quad \Delta \mathrm{S}$ for isothermal reversible expansion is given by
$\Delta \mathrm{S}=\mathrm{nR} \ln \frac{\mathrm{V}_{2}}{\mathrm{~V}_{1}}=2 \times 8.314 \ln \frac{100}{10}=2 \times 8.314 \times 2.303 \times 1=38.3 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$.
Correct choice: (4)
*90. Identify the compound that exhibits tautomerism.
(1) Lactic acid
(2) 2-Pentanone
(3) Phenol
(4) 2-Butene

Sol.:


Correct choice: (2)
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## LEVEL OF DIFFICULTY



## CLASSWISE




## Read the following instructions carefully

1. The candidates should fill in the required particulars on the Test Booklet and Answer Sheet (side - 1) with Blue/Black Ball Point Pen.
2. For writing / marking particulars on side - 2 of the Answer Sheet, use Blue/Black Ball Point Pen only.
3. The candidate should not write their Roll Numbers anywhere else (except in the specific space) on the Test Booklet/Answer Sheet.
4. Out of the four options given for each question only one option is the correct answer.
5. For each incorrect response, one-fourth (1/4) of the total marks allotted to the question would be deducted from the total score. No deduction from the total score, however, will be made if no response is indicated for an item in the Answer Sheet.
6. Handle the Test Booklet and Answer Sheet with care, as under no circumstance (except for discrepancy in Test Booklet Code and Answer Sheet Code) will another set be provided.
7. The candidates are not allowed to do any rough work or writing work on the Answer Sheet. All calculations/ writing work are to be done in the space provided for this purpose in the Test Booklet itself, marked 'Space for Rough Work'. This space is given at the bottom of each page and in 3 pages (Pages 21-23) at the end of the booklet.
8. On completion of the test, the candidates must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.
9. Each candidate must show on demand his/her Admit Card to the Invigilator.
10. No candidate, without special permission of the Superintendent or Invigilator, should leave his/ her seat.
11. The candidates should not leave the Examination Hall without handing over their Answer Sheet to the Invigilator on duty and sign the Attendance Sheet again. Cases where a candidate has not signed the Attendance Sheet a second time will be deemed not to have handed over the Answer Sheet and dealt with as an unfair means case. The candidates are also required to put their left hand THUMB impression in the space provided in the Attendance Sheet.
12. Use of electronic/ Manual Calculator and any electronic Item like mobile phone, pager etc. is prohibited.
13. The candidates are governed by all Rules and Regulations of the Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the Board.
14. No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.

Candidates are not allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, electronic device or any other material except the Admit Card inside the examination hall/ room.

