HSPTA MALAPPURAM

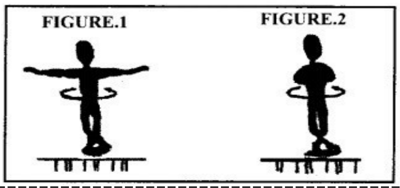
PHYSOL_*The solution for learning Physics*

Question Bank

Systems of Particles and Rotational Motion

Each question scores One

1	The time rate of change of angular momentum is called.... Ans: Torque.
2	Radius of gyration of a disc rotating about an axis passing through the centre and perpendicular to the disc Ans: $\frac{R}{\sqrt{2}}$
3	In pure rotation all particles in a body have the same....... Ans: Angular velocity.
4	Relation between linear velocity and angular velocity is.... Ans: $\omega \times R$
5	Moment of linear momentum is called..... Ans: Angular momentum.
6	Moment of force is......... Ans: Torque.
7	A ring and a disc of same radius are allowed to roll from same height over an inclined plane. Which one will reach the ground first? Ans: Disc.
8	The relation between linear acceleration and angular acceleration is...... Ans: $a=r\alpha$
9	The product of moment of inertia of a body and its angular velocity will give...... Ans: Angular momentum.
10	Unit of angular momentum is..... Ans: kgm^2/s.
11	Unit of torque is.... Ans: Nm or Joule.
12	Unit of Moment of inertia is...... Ans: kgm^2
13	Dimensional Formula of Angular momentum is.... Ans: $[ML^2T^{-1}]$
14	Dimensional Formula of Torque is.... Ans: $[ML^2T^{-2}]$
15	Is Angular momentum a scalar or vector? Ans: Vector. (Axial vector).
16	Angular momentum has the same dimensional formula as that of....... Ans: Plank's constant.

17	Work and......... has the same dimensional formula. Ans: Torque.
18	The quantity MK^2 is called...... Ans: Moment of Inertia
19	If external torque acting on a system is zero, which physical quantity is conserved? (linear momentum /Angular momentum) Ans: Angular momentum
20	What are the rotational equivalents for the physical quantity force? Ans: Torque
21	Write equation connecting torque and force Ans: $\vec{\tau}=\vec{r}\ X\,\vec{F}$
22	Write equation connecting angular momentum linier momentum Ans: $\vec{L}=\vec{r}\ X\,\vec{P}$
23	The rotational analogue of force is----------------- Ans: Torque.
24	In pure rotational motion every particle of the body has the same angular velocity at any instant of time. State whether this statement is True or False. Ans: True
25	The rotational analogue of mass is called............ Ans: Moment of inertia.
26	If M is the mass and R is the radius of the sphere, write an equation for the moment of inertia of the sphere about a diameter. Ans: $I_{sphere}=\frac{2\,MR^2}{5}$
27	In translatory motion, angular momentum......... i) is always zero ii) is always greater than one iii) may be present iv) is always infinite Ans: (iii) may be present.
28	The demonstration of conservation of angular momentum is schematically shown in the figures. Identify the figure which has more angular velocity.  Ans: Figure 2
29	The angular momentum of a particle is the rotational analogue of its ---------------- Ans: linear momentum.
30	The equation connecting angular momentum and linear momentum are .... Ans: $\vec{l}=\vec{r}\,x\,\vec{P}$
31	The moment of inertia of a circular disc about an axis perpendicular to the plane, at the center is given by .... Ans: $I_{disc}=\frac{MR^2}{2}$
32	Classical dancers bring their hands closer to their body to rotate faster. Name the principle employed by them.

	Ans: Law of conservation of angular momentum.
33	Two identical concentric rings each of mass M and radius R are placed perpendicular to each other. What is the moment of inertia about an axis passing through the centre of mass of this system? i) $^3/_2$ MR ^{2} ii) 2 MR ^{2} iii) 3 MR ^{2} iv) 1⁄4 MR ^{2} Ans:i) $^3/_2$ MR ^{2}
34	A solid sphere is rotating about a diameter at an angular velocity ω. If it cools so that the radius reduces to 1/n of its original value, its angular velocity becomes...... (i) ω/n ii) ω/n ^{2} iii) nω iv) n ^{2} ω Ans: n ^{2} ω
35	Moment of inertia of a disc along the diameter is....... Ans: $MR^2/4$
36	The inability to stop rotational motion is called......... Ans: Moment of inertia

Each question scores Two

1	The possibility of falling backward with the ladder is more when you are high up on the ladder than when you just begin to climb. Explain why? {NB:-Ladder is placed vertically near a wall} Ans: torque increases. As we climb up, torque with respect to lower edge of the ladder increases which may turn the ladder backwards.
2	Write an expression for the moment of inertia of the sphere about its axis passing through the centre. What is its radius of gyration? Ans: Moment of inertia of sphere $I = \frac{2}{5} M R^2$ $M K^2 = \frac{2}{5} M R^2 \Rightarrow K = \sqrt{\frac{2}{5}} R$
3	Moment of inertia can be regarded as a measure of rotational inertia. Why? Write any two factors on which the moment of inertia of a rigid body depends. Ans: Moment of inertia resists any change in the rotational motion of the body. So it is called rotational inertia. (Note: Inertia means “resistance to change”)
4	The moments of inertia of two rotating bodies A and B are IA and IB (IA > IB) and their angular Momentum are equal. Which one has a greater kinetic energy? Explain. Ans: We have $K.E = \frac{L^2}{2I}$ $K.E_A = \frac{L^2}{2I_A}$ and $K.E_B = \frac{L^2}{2I_B}$. Since $I_A > I_B$ $K.E_A < K.E_B$
5	Remya stands at the centre of a turntable with her two arms outstretched. The table with an angular speed of 40 revolutions / minute. a) What will happen to the moment of inertia if she folds her hands back? b) If the angular speed is increased to 100 revolutions / minute, what will be the new moment of inertia? Ans: a) Moment of inertia decreases. b) We have according to the law of conservation of angular momentum, $I_1 \omega_1 = I_2 \omega_2$ $I_1 \times 40 = I_2 \times 100$ or $I_2 = I_1 \times 0.4$. Moment of inertia will be 0.4 times initial value.
6	Explain Parallel axes theorem

Ans: Parallel axes theorem statement

$$I' = I_c + MR^2$$

7 Distinguish linear motion and Rotational motion

Ans:

linear motion: All particles in the system have same velocity.

Rotational motion : All particles in the system have same angular velocity

8 Derive the relation between torque and angular momentum

Ans:

Angular momentum $\vec{L} = \vec{r} \times \vec{P}$ P=m×v

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{P}) = \vec{r} \times \frac{d\vec{P}}{dt} + \frac{d\vec{r}}{dt} \times \vec{P}$$

Where $\frac{d\vec{r}}{dt} \times \vec{P} = \vec{v}\, x\, m\, x\, \vec{v} = 0$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F}$$ =τ (Torque)

9 In the case of a ring, show that both the translational and rotational kinetic energy have the same value.

Ans: Translational Kinetic energy, $KE_t = \frac{1}{2} mv^2$

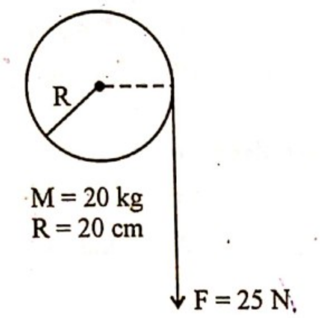
Rotational kinetic energy, $KE_r = \frac{1}{2} I\,\omega^2$

For a ring $I = MR^2, \omega = \frac{v}{R}$

Therefore $KE_r = \frac{1}{2} MR^2 \left(\frac{v}{R}\right)^2 = \frac{1}{2} Mv^2 = KE_t$

That is both the translational and rotational kinetic energy have the same value.

10 A cord of negligible mass is wound round the rim of a fly wheel mounted on a horizontal axis as shown in figure. Calculate the angular acceleration of the wheel if steady pull of 25 N is applied on the cord.



Moment of inertia of fly wheel about its axis $I = \frac{MR^2}{2}$

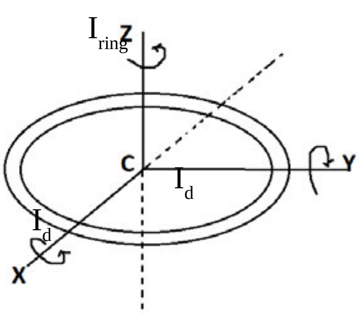
Ans: We have torque $\tau = I\,\alpha = FR$

Therefore $\frac{MR^2}{2}\alpha = FR$

$$\frac{MR}{2}\alpha = F$$

$$\alpha = \frac{2F}{MR} = \frac{2\, x\, 25}{20\, x\, 20\, x\, 10^{-2}} = 12.5\, rad\, s^{-2}$$

11 Find the moment of inertia of the ring about its diameter.



Ans: We have $I_{ring} = MR^2$

By Perpendicular axes theorem

$I_d + I_d = MR^2$

$2I_d = MR^2$

$$I_d = \frac{MR^2}{2}$$

This is the Moment of inertia of a thin circular ring of radius 'R' and mass 'M' about an axis passing through diameter.

12 What do you mean by the radius of gyration of a rolling body?

Ans:

Radius of gyration is defined as the distance from the axis of rotation to a point where the total mass of the body is supposed to be concentrated, so that the moment of inertia about the axis may remain the same about the same axis of rotation

That is $I=MK^2$

Therefore , Radius of gyration $K=\sqrt{\frac{I}{M}}$

13 Fill in the blanks :

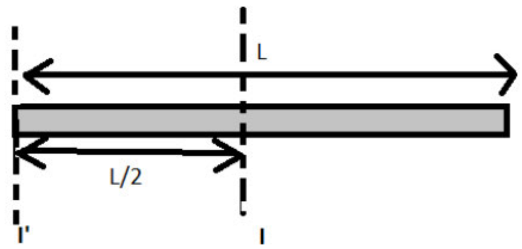
Body	**Axis**	**Moment of inertia**
Circular disc radius R	Perpendicular to disc at centre	-------------------------------------
Thin Circular ring radius R	-----------------------------------	$\frac{MR^2}{2}$
Thin rod length L	Perpendicular to rod, at mid-point	-------------------------------------
-------------------------------------- -	Perpendicular to plane, at centre	MR^2

Ans:

Body	**Axis**	**Moment of inertia**
Circular disc radius R	Perpendicular to disc at centre	$\frac{MR^2}{2}$
Thin Circular ring radius R	About diameter	$\frac{MR^2}{2}$
Thin rod length L	Perpendicular to rod, at mid-point	$\frac{ML^2}{12}$
Circular ring of radius R	Perpendicular to plane, at centre	MR^2

14 The moment of inertia of a thin rod of mass M and length l about an axis perpendicular to the rod at its mid point is $\frac{ML^2}{12}$. Find the moment of inertia of the rod about an axis perpendicular to it and passing through one end of the rod.

Ans:



Moment of inertia of a thin rod (scale) of length 'L' about an axis passing through mid point and perpendicular is given by $I_{rod}=\frac{ML^2}{12}$

By Parallel axis theorm,

$$I_{end}=I_{rod}+M\left(\frac{L}{2}\right)^2$$

$$I_{end}=\frac{ML^2}{12}+\frac{ML^2}{4}$$

$$I_{end}=\frac{ML^2}{3}$$

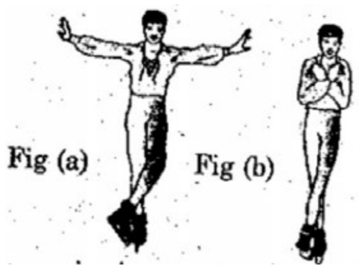
This is the moment of inertia of the rod about an axis perpendicular to it and passing through one end of the rod.

15 A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad. s^{-1} . The radius of the cylinder is 0.25 m. What is the magnitude of angular momentum of the cylinder about its axis?

Ans: Angular momentum $L=I\,\omega=\frac{MR^2}{2}\,\omega$

Therefore $L=\frac{20\,x\,0.25^2}{2}100=62.5\,Js$

16 The figure shows two different spinning poses of a ballet dancer.
In which spinning pose does the ballet dancer have less angular velocity? Justify your answer.



Ans:

Figure (a).
Angular momentum L =Iω is a constant .
Thus when she stretches his hands
moment of inertia 'I' increases and hence angular velocity 'ω' decreases.

17 Match the following:

A		B	
a)	Torque $(\vec{\tau})$ ടോർക്ക് $(\vec{\tau})$	i)	Perpendicular to $\vec{r}$ and $\vec{P}$ $\vec{r}$ നും $\vec{P}$ യ്ക്കും ലംബമായിരിക്കും.
b)	Angular Momentum $(\vec{L})$ കോണീയ ആക്കം $(\vec{L})$	ii)	$\sum\vec{F}=0$
c)	Rotational equilibrium പരിക്രമണ സംതുലിതാവസ്ഥ	iii)	$\vec{\omega}\times\vec{r}$
d)	Linear velocity $(\vec{v})$ രേഖീയ പ്രവേഗം $(\vec{v})$	iv)	$\vec{r}\times\vec{F}$
		v)	$\sum\vec{\tau}=0$

Ans:

A	B
Torque τ	$\vec{r} \times \vec{F}$
Angular momentum (L)	Perpendicular to r and P
Rotational equilibrium	$\Sigma \tau = 0$
Linear velocity	$\vec{\omega} \times \vec{r}$

18 State the theorem of parallel axes on a moment of inertia.

Ans: Theorem of parallel axes states that "The moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes".

Each question scores Three

1 a) The rotational analogue of force is ....

b) The rotational analogue of mass is ....

Ans: a) Torque b) Moment of inertia

c) The turning effect of force is maximum when the angle between $\vec{r}$ and $\vec{f}$ is……………….

Ans: 90^0

2 A wheel of mass 1000 kg and radius 1 m is rotating at the rate of 420 r.p.m. What is the constant torque required to stop the wheel in 14 rotations, assuming the mass to be concentrated at the rim of the wheel?

Ans: A wheel is a ring. For ring $I = MR^2 = 1000 \times 1^2 = 1000$ kg m^2

Given initial frequency υ_i = 420 revolution per minute = $\frac{420}{60}$ = 7 revolution per second.

Initial angular velocity $\omega_i = 2\pi\upsilon_i = 2 \times \frac{22}{7} \times 7 = 44$ rad/s

Final angular velocity $\omega_i = 0$

Total angular displacement before stopping $\theta = 14 \times 2\pi = 28\pi$ rad

According to work energy theorem, the work done = the total change of kinetic energy

That is, $W = \tau\theta = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$ $\tau = \frac{\frac{1}{2} \times 1000 \times (44)^2 - 0}{28 \times \frac{22}{7}}$ = 11000 Nm

3 A wheel starting from rest acquires an angular velocity of 10 rad/s in two seconds. The moment of inertia of the wheel is 0.4 kg m^2. Calculate the torque acting on it.

Ans: Given, $\omega_i = 0$ $\omega_f = 10$ rad/s $t = 2$ s $I = 0.4$ kg m^2

We have $\tau = I\alpha = I \times \frac{\omega_f - \omega_i}{t} = 0.4 \times \frac{10-0}{2} = 2$ Nm

4 A solid cylinder of mass 20kg rotates about its axis with an angular speed of 100 rad s^{-1}. The radius of the cylinder is 0.25m. What is the magnitude of angular momentum of the cylinder about its axis?

Given, M = 20 kg, ω = 100 rad/s, R = 0.25 m

We have $L = I\omega = \frac{MR^2}{2}\omega = \frac{20 \times (0.25)^2}{2} \times 100$m = 62.5 kg m^2/s

5 Remya stands at the centre of a turntable with her two arms outstretched. The table with an angular speed of 40 revolutions / minute.

a) What will happen to the moment of inertia if she folds her hands back?

	b) If the angular speed is increased to 100 revolutions / minute, what will be the new moment of inertia? Ans: a) Moment of inertia increases. b) We have according to the law of conservation of angular momentum, $I_1 \omega_1 = I_2 \omega_2$ $I_1 \times 40 = I_2 \times 100$ or $I_2 = I_1 \times 0.4$. Moment of inertia will be 0.4 times initial value.
6	Moment of inertia of a uniform disc about an axis passing through the centre and perpendicular to the plane is $MR^2/2$ a) State Perpendicular axes theorem (1 score) b) Derive the expression for moment of inertia of a uniform disc about an axis passing through the diameter. (2 score) Ans: a) Statement $I_z = I_x + I_y$ b) $I_x = I_y = I_d$ $I_z = 2 I_d$ $I_d = \frac{I_z}{2}$ $I_d = \frac{\frac{MR^2}{2}}{2}$ $I_d = \frac{MR^2}{4}$
7	A girl rotates on a swivel chair as shown below. a.)What happens to her angular speed when she stretches her arms? b.) Name and state the conservation law applied for your justification. Ans: (a) Angular speed decreases. (b) Conservation of Angular momentum. If the total external torque on a system of particles is zero, then the total angular momentum of the system is conserved. 

Each question scores Four

1	We know that angular momentum is a quantity associated with rotation of a body. a) Write down the dimensional formula of Angular momentum. (1score). b) When polar ice melts what will happen to the duration of a day? (3 score) Ans: a) ML^2T^{-1}. b)According to law of conservation of Angular momentum Iω = a constant. $I = MR^2$ When polar ice is in solid form r is small so I is small, then w will be large. When ice melts r increases so I will increase and hence w will decrease. So when polar ice melts rotation of earth become slow, so duration of a day will increase.
2	a)Write down the equation for Moment of inertia of a disc passing through its centre of mass and perpendicular to the disc? (1 score) b) Find the Moment of inertia of the disc tangential to the surface and parallel to the disc? (3 score) Ans: a) $MR^2/2$. b) Moment of inertia of the disc through its diameter is = $MR^2/4$. According to parallel axes theorem. $I' = I_{cm} + Ma^2$ $I' = MR^2/4 + MR^2$ $I' = 5/4\ MR^2$

3 a)In the absence of external torque........ Of an isolated system remains constant (1 score).

b) Why planets move faster at near region of sun and slower when they are far away?(3 score)

Ans: a) Angular momentum.

b) we know that when external torque is zero, Angular momentum remains constant.

Iw= a constant.

When planets are at near region of sun their r will be small. So I will be small. ($I=MR^2$). So their w will be large. When planets are at far regions, r is large, so I is large, then w will be small. So planets are slow at far regions.

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4 a) Can centre of mass of a body be a point outside the body?

b) Find moment of inertia of a disc of mass 9.5Kg having radius0.4m about an axis passing through centre of mass and perpendicular to the surface? (2 score).

c) Find its moment of inertia about its diameter? (1 score)

Ans:

a) Yes, in the case of a ring.

b) $I= MR^2/2$

$I= 9.5×0.4×0.4/2$

$I= 0.76 Kgm^2$

c) $I'= 0.76/2$

$I'= 0.38 Kgm^2$

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5 State theorem of perpendicular axes on moment of inertia. Derive an expression to find the moment of inertia of a disc about one of its diameters with the help of a neat diagram.

Ans: Theorem of Perpendicular axes states that "The moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body."

Here $I_Z= I_X +I_Y$

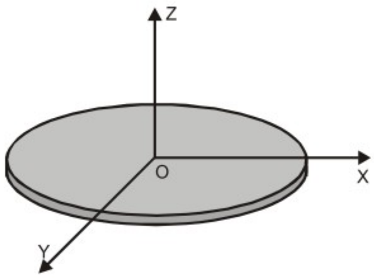
Where I_Z --> Moment of Inertia about Z-axis.

I_X --> Moment of Inertia about X-axis.

I_Y --> Moment of Inertia about Y-axis.

Moment of inertia of a thin circular disc of radius 'R' and mass 'M' about an axis passing through diameter:

We have $I_{disc}=\frac{MR^2}{2}$



By Perpendicular axes theorem

$I_d+I_d=\frac{MR^2}{2}$

$2I_d=\frac{MR^2}{2}$

$I_d=\frac{MR^2}{4}$

This is the Moment of inertia of a thin circular disc of radius 'R' and mass 'M' about an axis passing through diameter.

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6 (a)Show that $\tau=\frac{dl}{dt}$ for rotational motion.

(b) State the law of conservation of angular momentum.

(c) Write an example for the motion in which angular momentum is conserved.

Ans: (a) We have $\vec{l}=\vec{r} \, x \, \vec{P}$

Therefore $\frac{d\vec{l}}{dt}=\frac{d(\vec{r} \, x \, \vec{P})}{dt}$

$$\frac{d\vec{l}}{dt} = \vec{r}\, x \frac{d\vec{P}}{dt} + \frac{d\vec{r}}{dt}\, x \vec{P}$$

$$\frac{d\vec{l}}{dt} = \vec{r}\, x \vec{F} + \vec{v}\, x (m\vec{v})$$

Therefore $\frac{d\vec{l}}{dt} = \vec{\tau}$ (Because $\vec{r}\, x \vec{F} = \tau$ and $\vec{v}\, x \vec{v} = 0$)

Thus Torque is equal to the rate of change of angular momentum.

(b) **Law of conservation of Angular momentum:**

If the total external torque on a system of particles is zero, then the total angular momentum of the system is conserved.

(c)Planetary motion.

Each question scores Five

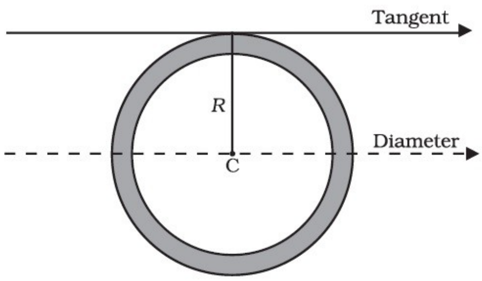
1 Moment of inertia about a diameter of a ring is $I_d = \frac{MR^2}{2}$

a) Name the theorem that helps to find the moment of inertia about a tangent parallel to the diameter.

b) Draw a diagram and find the moment of inertia about a tangent, parallel to the diameter of the ring.

c) The rotational analogue of mass is ....

Ans: (a) Parallel axes theorem.



(b)We have $I_{diameter} = \frac{MR^2}{2}$

By Parallel axis theorem

$$I_{tangent} = I_{diameter} + MR^2$$

$$I_{tangent} = \frac{MR^2}{2} + MR^2 = \frac{3\,MR^2}{2}$$

(c) Moment of inertia.

2 The rotational analogue of force is moment of force, also called torque.

a) The turning effect of force is maximum when the angle between $\vec{r}$ and $\vec{F}$ is.............

b) A wheel starting from rest acquires an angular velocity of 10 rad/s in two seconds. The moment of inertia of the wheel is 0.4 kg m ^{2} . Calculate the torque acting on it.

c) The possibility of falling backward with the ladder is more when you are high up on the ladder than

when you just begin to climb. Explain why.

Ans: (a) 90^0.

(b) Torque $\tau = I\,\alpha = I\,\frac{\omega}{t}$ $\tau = 0.4\, x \frac{10}{2} = 2\,J$

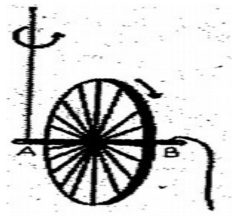
(c)When a person is high up on the ladder, than **a large torque is produced due to his weight about the point of contact between the ladder and the floor**. Whereas when he starts climbing up, the torque is small. Due to this reason, the ladder is more apt to slip, when one is high up on it.

3 Moment of inertia is the analogue of mass in rotational motion. But unlike mass; it is not a fixed quantity.

a) Moment of inertia can be regarded as a measure of rotational inertia. Why?

b) Write any two factors on which the moment of inertia of a rigid body depends.

c) The moments of inertia of two rotating bodies A and B are I_A and I_B ($I_A > I_B$) and their

	angular momentum are equal. Which one has a greater kinetic energy? Explain. Ans: (a) The moment of interia about a given axis resists a change in its rotational motion. Thus it can be regarded as a measure of rotational inertia of the body. (b) (i)The mass of the body, (ii) Its shape and size; (c) We have $KE=\frac{L^2}{2I}$ Here L, the angular momentum is a constant. Therefore $KE\,\alpha\frac{1}{I}$ Given $I_A > I_B$ Therefore $KE_B > KE_A$
4	In an experiment with a bicycle rim, keeping the ring in the vertical position with both the strings in one hand, put the wheel in fast rotation (see fig). When string B is released, the rim keeps rotating in a vertical plane and the plane of rotation turns around the string A.  a) Mention the law that explains the above result. b) Explain the practical example (shown in the fig) based on the law mentioned in (a) (c) How will you distinguish a hard-boiled egg and a raw egg by spinning each on a table top?  (d)A solid cylinder of mass 20kg rotates about its axis with an angular speed of 100 rad s^{-1} . The radius of the cylinder is 0.25m. What is the magnitude of angular momentum of the cylinder about its axis? Ans: (a) Law of conservation of angular momentum. (b)Angular momentum L =Iω is a constant . Thus when she stretches her hands moment of inertia 'I' increases and hence angular velocity 'ω' decreases. (c) To distinguish between a hard boiled egg and a raw egg, we spin each on a table top. The egg which spins at a slower rate **shall be a raw egg**. This is because in a raw egg, liquid matter inside tries to get away from the axis of rotation. Therefore, its moment of inertia I increases and hence angular speed decreases. Whereas the hard boiled egg continues to spin. (d) Angular momentum L =Iω $L=\frac{MR^2}{2}\omega$ Therefore $L=\frac{20\,x\,0.25^2}{2}100=62.5\,Js$