## Answerkey

## Answer any 3 questions from 1 to 5. Each carries 1 score

| 1 | Adiabatic |
| :--- | ---: |
| 2 | PV $=$ nRT |
| 3 | c) a $=-5 x$ |
| 4 | a) Decrease |
| 5 | Perpendicular |

## Answer any 5 questions from 6 to 13. Each carries 2 score

| 6 | According to the first law of thermodynamics, the amount of heat $\Delta \mathrm{Q}$ absorbed by a system capable of doing mechanical work is equal to the sum of the increase in internal energy $\Delta U$ of the system and the external work $\Delta \mathrm{W}$ done by the system. Mathematically, $d Q=d U+d W=d U+P d V$. | 2 |
| :---: | :---: | :---: |
| 7 | If they intersect, then at the point of intersection, the volume and pressure of the gas will be the same at two different temperatures which is not possible. | 2 |
| 8 | The temperature is on account of the translational molecular motion. At absolute zero, this molecular motion completely stops. Obviously, a temperature less than absolute zero is not possible. | 2 |
| 9 | Mean free path of a molecule in a gas is the average distance travelled by the molecule between two successive collisions. <br> The mean free path depends on the number of gas molecules in unit volume (number density) and size (diameter) of the molecules. | 2 |
| 10 | i. The restoring force is always proportional to the displacement from the mean position. ii. The restoring force is always directed towards the mean position. |  |
| 11 |  | 2 |
| 12 | $\begin{aligned} & \mathrm{v} \propto \sqrt{ } \mathrm{~T} \\ & \text { Case (I) }=\mathrm{T}=\mathrm{T}_{0}=0^{\circ} \mathrm{C}=273 \mathrm{~K} \\ & v \propto \sqrt{273}-------(1) \\ & \text { Case (II) } \mathrm{T}=\text { ? Velocity }=2 \mathrm{v} \end{aligned}$ |  |


|  | $\begin{gather*} 2 v \propto \sqrt{T}  \tag{2}\\ \frac{2 v}{v}=\sqrt{\frac{T}{273}} \\ 2=\sqrt{\frac{T}{273}} \\ 4=\frac{T}{273} \\ \mathrm{~T}=4 \times 273 \mathrm{~K} \end{gather*}$ |  |  | 2 |
| :---: | :---: | :---: | :---: | :---: |
| 13 | $\begin{aligned} & \mathrm{V}_{\text {(sound in gas) }}=\sqrt{\frac{\gamma P}{\rho}} \\ & \text { Where } \quad \gamma \rightarrow \text { specific heat ratio } \end{aligned}$ | $\mathrm{P} \rightarrow$ pressure | $\rho \rightarrow$ Density of medium | 2 |

## Answer any 3 questions from 14 to 17. Each carries 3 score

14
Suppose 1 gm mole of an ideal gas enclosed in a cylinder of conducting walls. Let $\mathrm{P}_{1}, \mathrm{~V}_{1}, \mathrm{~T}$ be initial pressure, volume, and temperature. Let gas expand to volume $\mathrm{V}_{2}$ where pressure reduces to $\mathrm{P}_{2}$ and temperature remains constant.

If $A$ is the area of piston
$F=P \times A$
$\mathrm{dW}=\mathrm{F} \times \mathrm{dx}$
$=P \times A \times d x$
$W=\int_{v_{1}}^{v_{2}} P d V \quad[\because A d x=d v]$
But, $P V=R T$
$\mathrm{W}=\int_{v_{1}}^{v_{2}} \frac{R T}{V} \mathrm{dV}$
$\mathrm{W}=\mathrm{RT}\left[\log _{\mathrm{e}} \mathrm{V}\right]_{v_{1}}^{v_{2}}$
$W=R T\left[\log _{e} V_{2}-\log _{e} V_{1}\right]$
$\mathrm{W}=2.303 \mathrm{RT} \log _{10} \frac{v_{2}}{v_{1}}$

15 1. A gas consists of a very large number of molecules which are perfectly identical elastic spheres. These molecules are in a state of continuous random motion in all directions with all possible velocities.
2. The size of each molecule is very small as compared to the distance between any two of them. Hence the volume occupied by all the molecules is negligible in comparison to the total volume of the gas.

|  | 3. There is no force of attraction or repulsion between the molecules and the walls of the <br> container. <br> 4. The collisions of the molecules amoung themselves and with the walls of the container <br> are perfectly elastic. Therefore, momentum and kinetic energy of the molecules are <br> conserved during collisions. |
| :--- | :--- |
| 16 | The centre of gravity is originally at the centre. When mercury flows out the centre of <br> gravity gets lowered, reaches the lowermost point and then rises to the original place when <br> all the mercury flows out. Therefore $l$ will first increase, reach a maximum and then <br> decrease to the original value. Therefore period will first increase, reach a maximum and <br> then decrease to the original value. |
| 17a) Travelling <br> b) $\frac{\pi}{4}$ <br> c) Amplitude of the wave, A $=3 \mathrm{~cm}$ <br> and frequency, $\quad \omega=2 \pi f$ <br> $\mathrm{f}=\frac{\omega}{2 \pi}=\frac{36}{2 \pi}=5.73 \mathrm{~Hz}$ | 3 |

Answer any 2 questions from 18 to 20. Each carries 4 score

| a) Second law of thermodynamics | 1 |
| :--- | :--- |

b) Carnot's cycle
The Carnot cycle consists of two isothermal processes and two adiabatic processes.

Let the working substance in Carnot's engine be the ideal gas.
Step 1 : The gas absorbs heat $\mathrm{Q}_{1}$ from hot reservoir at T , and undergoes isothermal expansion from ( $\mathrm{P}_{1}, \mathrm{~V}_{1}, \mathrm{~T}_{1}$ ) to $\left(\mathrm{P}_{2}, \mathrm{~V}_{2}, \mathrm{~T}_{1}\right)$.
Step 2 : Gas undergoes adiabatic expansion from $\left(\mathrm{P}_{2}, \mathrm{~V}_{2}, \mathrm{~T}_{1}\right)$ to $\left(\mathrm{P}_{3}, \mathrm{~V}_{3}, \mathrm{~T}_{2}\right)$
Step 3 : The gas release heat $Q_{2}$ to cold reservoir at $T_{2}$, by isothermal compression from $\left(\mathrm{P}_{3}, \mathrm{~V}_{3}, \mathrm{~T}_{2}\right) \operatorname{to}\left(\mathrm{P}_{4}, \mathrm{~V}_{4}, \mathrm{~T}_{2}\right)$.
Step 4: To take gas into initial state, work is done on gas adiabatically $\left(\mathrm{P}_{4}, \mathrm{~V}_{4}, \mathrm{~T}_{2}\right)$ to $\left(\mathrm{P}_{1}\right.$,
$\mathrm{V}_{1}, \mathrm{~T}_{1}$ )
Efficiency of Carnot's engine

$$
\eta=\frac{\mathrm{W}}{\mathrm{Q}_{1}}=\frac{\mathrm{Q}_{1}-\mathrm{Q}_{2}}{\mathrm{Q}_{1}}=1-\frac{\mathrm{Q}_{2}}{\mathrm{Q}_{1}}
$$


a)Simple pendulum consists of a bob of mass ' $m$ ', suspended from one end of an inextensible string of length ' $L$ '. The other end is fixed to a rigid support.
The length of the pendulum is the distance between the rigid support and the centre of the bob.

When the bob is pulled to one side and released the pendulum executes oscillations.
At any instant ' $\theta$ ' be the angular displacement.
The weight of the bob ' mg ' can be resolved into two components,
mgsin $\theta \rightarrow$ directed towards mean position,
mgcos $\theta \rightarrow$ in the direction of string.
Here, 'mgsin $\theta$ ' gives the restoring force.

$$
\begin{aligned}
& \text { ie } \quad F=-m g \sin \theta=-m g \theta \quad(\text { as } \theta \ll) \\
& \text { But } \quad \theta=\frac{x}{L} \\
& \therefore \quad F=-\left(\frac{m g}{L}\right) x
\end{aligned}
$$

Thus for small amplitude oscillations, the force is proportional to the displacement and directed towards mean position. Hence oscillations of simple pendulum is SHM.

## Period of oscillation of a simple pendulum:

For a simple pendulum,

$$
\begin{aligned}
& F=-\left(\frac{m g}{L}\right) x \quad \text { and } \\
& F=m a \\
& \therefore \quad m a=-\left(\frac{m g}{L}\right) x \\
& a=-\frac{g x}{L} \\
& \text { But } \quad a=-\omega^{2} x \\
& \therefore \quad-\omega^{2} x=-\frac{g x}{L} \\
& \omega^{2}=\frac{g}{L} \\
& \omega=\sqrt{\frac{g}{L}} \\
& \frac{2 \pi}{T}=\sqrt{\frac{g}{L}} \\
& T=2 \pi \sqrt{\frac{L}{g}}
\end{aligned}
$$

This is the period of oscillation of a simple pendulum.
b)The length of a seconds pendulum (which ticks seconds) $\mathrm{L}=1 \mathrm{~m}$.

Or

## Kinetic energy of SHM:

We know, velocity of a particle executing SHM ,

$$
\begin{aligned}
& v=\omega \sqrt{A^{2}-x^{2}} \\
& \therefore K . E=\frac{1}{2} m v^{2} \\
& \quad=\frac{1}{2} m \omega^{2}\left(A^{2}-x^{2}\right)
\end{aligned}
$$

Case 1: At mean position, $\mathrm{x}=0$.

$$
\therefore \quad K \cdot E_{\max }=\frac{1}{2} m \omega^{2} A^{2}
$$

Case 2: At extreme position, $x=+$ A or $-A$,

$$
\therefore K . E_{\min }=0
$$

## Potential Energy of SHM:

Consider a particle executing SHM. Let ' $x$ ' be the displacement at any instant ' $t$ '.
The work done for a displacement ' $d x$ ' is given by $\mathrm{dW}=-\mathrm{F} . \mathrm{dx}$
But F=ma

$$
\left.=-m \omega^{2} x \quad \text { (because } a=-\omega^{2} x\right)
$$

Thus, $\mathrm{dW}=\mathrm{m} \omega^{2} \mathrm{x} d \mathrm{x}$
Therefore, total work done for a displacement of the particle from $x=0$, to $x=x$ is given by,

$$
\begin{aligned}
& \mathrm{W}=\int_{0}^{x} \mathrm{dW} \\
& W=\int_{0}^{x} m \omega^{2} x d x \\
& W=\frac{1}{2} m \omega^{2} x^{2}
\end{aligned}
$$

This work done is stored as the potential energy,

$$
\therefore P E=\frac{1}{2} m \omega^{2} x^{2} \quad \text { or } \quad \frac{1}{2} k x^{2}
$$

Case1: At the mean position, $x=0$.
$\therefore P E_{\text {min }}=0$
Case2: At the extreme position, $x=+A$ or $-A$
$\therefore P E_{\max }=\frac{1}{2} m \omega^{2} A^{2}$
20 The air column in a closed pipe can vibrate in different modes. In all modes the closed end is a node and the open end is an antinode. In between there may or may not be nodes and antinodes depending on the mode of vibration.

1st mode,
Fundamental


$$
\begin{aligned}
& \frac{\lambda_{1}}{4}=\ell \\
& \lambda_{1}=4 l
\end{aligned}
$$

Fundamental frequency $v_{1}=\frac{v}{\lambda_{1}}=\frac{v}{4 l}$

2nd mode,
1st overtone
3rd harmonic


$$
\begin{aligned}
& \frac{3 \lambda_{2}}{4}=l \\
& \lambda_{2}=\frac{4 l}{3}
\end{aligned}
$$

Frequency of $2^{\text {nd }}$ mode $v_{2}=\frac{v}{\lambda_{2}}$

$$
=\frac{v}{\left(\frac{4 l}{3}\right)}=3\left(\frac{v}{4 l}\right)=3 v_{1}
$$

$$
\frac{5 \lambda_{3}}{4}=l \quad \lambda_{3}=\frac{4 l}{5}
$$

Frequency of $3{ }^{\text {rd }}$ mode

$$
v_{3}=\frac{v}{\lambda_{3}}=\frac{v}{\left(\frac{4 l}{5}\right)}=5\left(\frac{v}{4 l}\right)=5 v_{1}
$$

