NARAYANA IIT ACADEMY - INDIA

IIT – JEE (2012) PAPER I QUESTION & SOLUTIONS (CODE 1)

INSTRUCTIONS :

A. General

1. This booklet is your Question Paper. Do not break the seals of this booklet before being instructed to do so by the invigilators.
2. The question paper CODE is printed on the right hand top of this page and on the back page (Page No. 28) of this booklet.
3. Blank spaces and blank pages are provided in this booklet for your rough work. No additional sheets will be provided for rough work.
4. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers, and electronic gadgets are NOT allowed inside the examination hall.
5. Answers to the questions and personal details are to be filled on a two-part carbon-less paper, which is provided separately. You should not separated these parts. The invigilator will separate them at the end of examination. The upper sheet is a machine-gradable Objective Response Sheet (ORS) which will be taken back by the invigilator. You will be allowed to take away the bottom sheet at the end of the examination.
6. Using a black ball point pen, darken the bubbles on the upper original sheet. Apply sufficient pressure so that the impression is created on the bottom sheet.
7. **DO NOT TAMPER WITH/MUTILATE THE ORS OR THE BOOKLET.**
8. On breaking the seals of the booklet check that it contains 28 pages and all the 60 questions and corresponding answer choices are legible. Read carefully the instructions printed at the beginning of each section.

B. Filling the Right Part of the ORS

9. The ORS has CODES printed on its left and right parts.
10. Check that same CODE is printed on the ORS and on this booklet. IF IT IS NOT THEN ASK FOR A CHANGE OF THE BOOKLET. Sign at the place provided on the ORS affirming that you have verified that all the codes are same.
11. Write your Name, Registration Number and the same of examination centre and sign with pen in the boxes provided on the right part of the ORS. Do not write any of this information anywhere else. Darken the appropriate bubble UNDER each digit of your Registration Number in such away that the impression is created on the bottom sheet. Also darken the paper CODE given on the right side of ORS (R4)

C. Question Paper Format

The question paper consists of **3 parts** (Physics, Chemistry and Mathematics). Each part consists of three sections.

12. Section I contains 10 Multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.
13. Section II contains 5 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** are correct.
14. Section III contains 5 questions. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

D. Marking Scheme

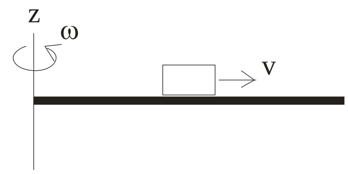
15. For each question in Section I, you will be awarded **3 marks** if you darken the bubble corresponding to the correct answer **ONLY** and zero marks if no bubbles are darkened. In all other cases, minus one **(–1) mark** will be awarded in this section.
16. For each question in Section II, you will be awarded **4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY**. In all other cases zero (0) marks will be awarded. No negative marks will be awarded for incorrect answers in this section.
17. For each question in Section III, you will be awarded **4 marks** if you darken the bubble corresponding to the correct answer **ONLY**. In all other cases zero (0) marks will be awarded. **No negative marks** will be awarded for incorrect answers in this section.

Part I : Physics

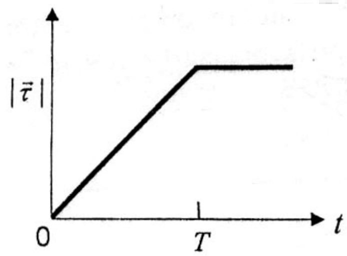
Section I : Straight Correct Answer Type

This section contains 10 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

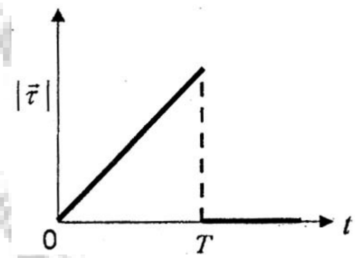
1 A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time t = 0, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at t = T and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $(|\vec{\tau}|)$ on the system about O, as a function of time is best represented by which plot?



(A)



(B)



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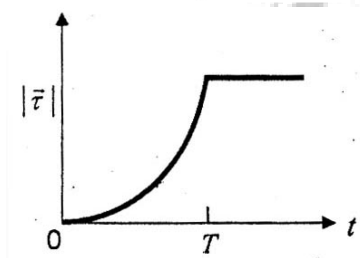
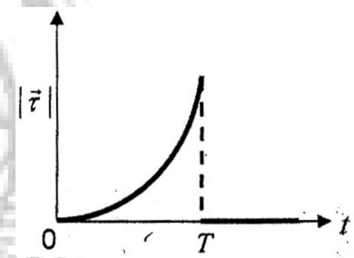


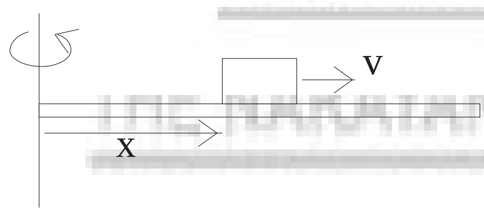
(D)

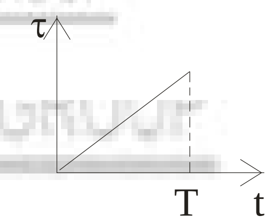


Sol. (B)

$$L = \left[\frac{Ml^2}{3} + mx^2\right]\omega$$

$$\tau = \frac{dL}{dt} = 2m\omega x \times \frac{dx}{dt}$$





$$\tau = 2m\omega v \times x$$

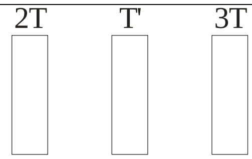
$$x = vt$$

$$\Rightarrow \quad \tau = 2m\omega v^2 t$$

2. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperature 2T and 3T respectively. The temperature of the middle (i.e. second) plate under steady state condition is

(A) $\left(\frac{65}{2}\right)^{\frac{1}{4}}T$ (B) $\left(\frac{97}{4}\right)^{\frac{1}{4}}T$ (C) $\left(\frac{97}{2}\right)^{\frac{1}{4}}T$ (D) $(97)^{\frac{1}{4}}T$

Sol. (C)

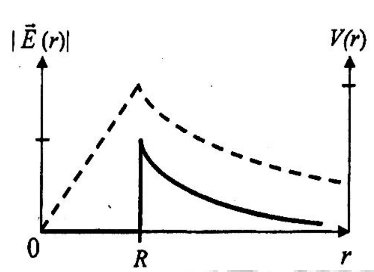


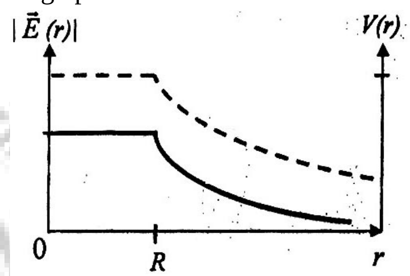
$$\sigma A(2T)^4 + \sigma A(3T)^4 = \sigma(2A)(T')^4$$

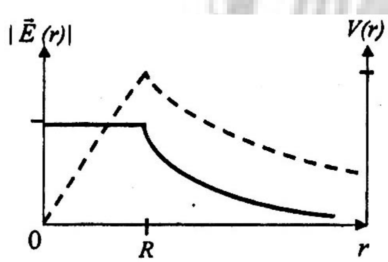
$$16T^4 + 81T^4 = 2(T')^4$$

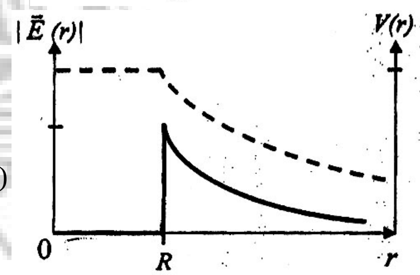
$$97T^4 = 2T'^4 \quad \Rightarrow \quad T' = \left(\frac{97}{2}\right)^{\frac{1}{4}} T$$

3. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $\left|\vec{E}(r)\right|$ and the electric potential V(r) with the distance r from the centre, is best represented by which graph?

(A)


(B)


(C)


(D)


Sol. (D)

Field inside a uniform spherical surface charge distribution is zero, and potential is constant.

At outside points $E \propto \frac{1}{r^2}$ and $v \propto \frac{1}{r}$

4 In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi l d^2}\right)$ by using Searle's method, a wire of length L = 2 m and diameter d = 0.5 mm is used. For a load M = 2.5 kg, an extension l=0.25 mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of division on their circular scale is 100. The contributions of the maximum probable error of the Y measurement

(A) due to the errors in the measurement of d and l are the same

(B) due to the error in the measurement of d is twice that due to the error in the measurement of l

(C) due to the error in the measurement of l is twice that due to the error in the measurement of d

(D) due to the error in the measurement of d is four times that due to the error in the measurement of l.

Sol. (A)

$$\frac{\Delta Y}{Y} = \frac{\Delta l}{l} + \frac{2\Delta d}{d}$$

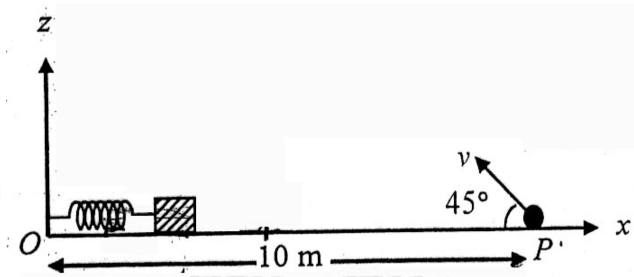
$\Delta l = \Delta d$

l = 0.25 mm

d = 0.50 mm

$\Rightarrow \quad \frac{\Delta l}{l} = \frac{2\Delta d}{d} \quad \Rightarrow$ Contribution of error in 'd' and 'l' is same.

5. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3} \text{rad/s}$. Simultaneously at t = 0, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at t = 1 s, the value of v is (take g = 10 m/s^2)



(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s (C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ m/s

Sol. (A)

For projectile

$$t = \frac{2v\sin 45^0}{g}$$

$$\Rightarrow \quad 1 = \frac{2V}{10\sqrt{2}} \Rightarrow v = 5\sqrt{2}$$

$$\Rightarrow \quad v = \sqrt{50} \text{m/s}$$

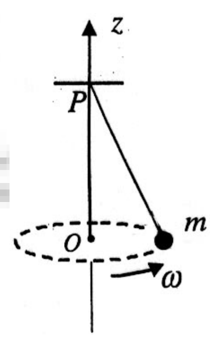
6. Young's double slit experiment is carried out by using green, red and blue light, one color at a time. The fringe widths records are β_G, β_R and β_B respectively. Then,

(A) $\beta_G > \beta_B > \beta_R$ (B) $\beta_B > \beta_G > \beta_R$ (C) $\beta_R > \beta_B > \beta_G$ (D) $\beta_R > \beta_G > \beta_B$

Sol. (D)

$\beta \propto \lambda$

7. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P and denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



(A) $\vec{L}_O$ and $\vec{L}_P$ **do not** vary with time

(B) $\vec{L}_O$ varies with the time while $\vec{L}_P$ remains constant

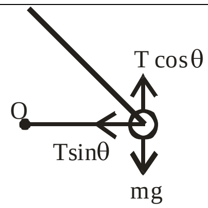
(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time

(D) $\vec{L}_O$ and $\vec{L}_P$ both vary with time

Sol. (C)

About point P torque due to tension is zero, while torque due to mg is there.

So L_P will change with time.



About point O

$$\text{Torque due to } T\cos\theta = \text{Torque due to mg}$$

Torque due to $T\sin\theta$ is zero.

8. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speed $\left(\frac{v_{rms}(\text{helium})}{v_{rms}(\text{arg on})}\right)$ is

(A) 0.32 (B) 0.45 (C) 2.24 (D) 3.16

Sol. (D)

In mixture of gases in average translational kinetic energy of each molecule is same.

$$\therefore \frac{V_{rms}(\text{Helium})}{V_{rms}(\text{Argon})} = \sqrt{\frac{m_{\text{Argon}}}{m_{\text{Helium}}}} = \sqrt{\frac{40}{4}}$$

$$\Rightarrow \quad \sqrt{10} = 3.16$$

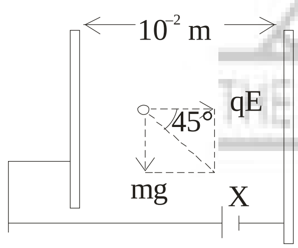
9. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical JUST after release. Then X is nearly

(A) 1×10^{-5} V (B) 1×10^{-7} V (C) 1×10^{-9} V (D) 1×10^{-10} V

Sol. (C)

q E = mg

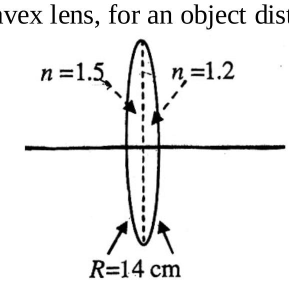
$$E = \frac{X}{10^{-2}}$$



$$\Rightarrow \quad 1.6\times10^{-19}\times\frac{X}{10^{-2}} = 1.6\times10^{-27}\times10$$

$$\Rightarrow \quad X = 1\times10^{-9}\ \text{volt}$$

10. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens s 1.2 . Both the curved surfaces are of the same radius of curvature R = 14 cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be



(A) –280.0 cm (B) 40.0 cm (C) 21.5 cm (D) 13.3 cm

Sol. (B)

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$=(\mu_1-1)\left[\frac{1}{R}-\frac{1}{\infty}\right]+(\mu_2-1)\left[\frac{1}{\infty}-\frac{1}{-R}\right]$$

$$=\frac{(1.5-1)}{R}+\frac{(1.2-1)}{R}$$

$\Rightarrow$ $f=20\,\text{cm}$

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$u = –40$

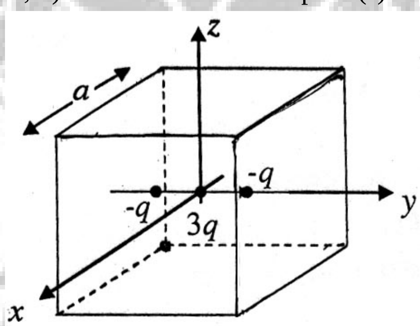
$$\frac{1}{v}-\frac{1}{-40}=\frac{1}{20}$$

$\Rightarrow$ $v=40\,\text{cm}$

Section II: Multiple Correct Answer (s) Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

11. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, –q at (0, –a/4, 0), +3q at (0, 0, 0) and –q at (0, +a/4, 0). Choose the correct option(s).



(A) The net electric flux crossing the plane x = +a/2 is equal to the net electric flux crossing the plane x = –a/2

(B) The net electric flux crossing the plane y = +a/2 is more than the net electric flux crossing the plane y = –a/2

(C) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$

(D) The net electric flux crossing the plane z = +a/2 is equal to the net electric flux crossing the plane x = +a/2

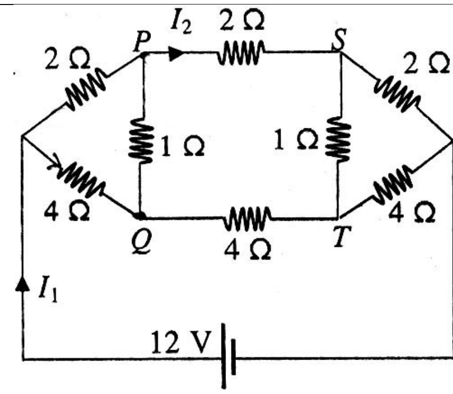
Sol. (A, C, D)

(A) Charges are symmetrically placed about x = +a/2 and x = –a/2 planes so electrical flux through them is same.

(C) $$\text{Net flux}=\frac{\text{charge enclosed}}{\varepsilon_0}=\frac{(3q-q-q)}{\varepsilon_0}=\frac{q}{\varepsilon_0}$$

(D) Charges are symmetrically placed about z = +a/2 and x = +a/2 planes so electric flux through them is same.

12. For the resistance network shown in the figure, choose the correct option(s)



(A) The current through PQ is zero

(B) $I_1 = 3A$

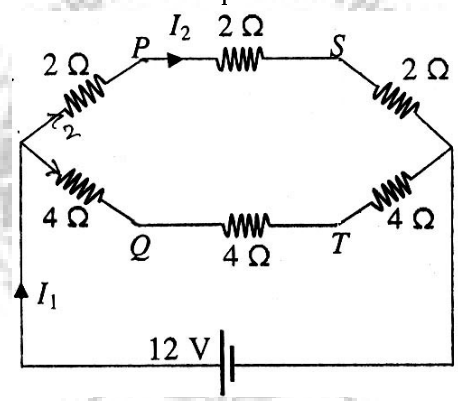
(C) The potential at S is less than that at Q

(D) $I_2 = 2A$

Sol. (A, B, C, D)

Points P and Q are at same potential

Also S and T are at same potential so circuit is equivalent to

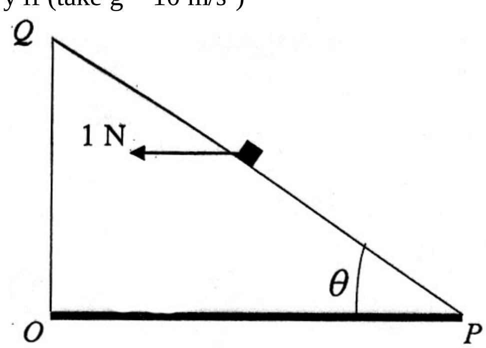


Equivalent resistance of circuit is 4Ω

So $I_1 = \frac{12}{4} = 3A$

$I_2 = \frac{2}{3} \times 3A = 2A$

13. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal . A horizontal force of 1 N acts on the block through its centre of mass as shown in the figure. The block remains stationary if (take g = 10 m/s^2)



(A) $\theta = 45^\circ$

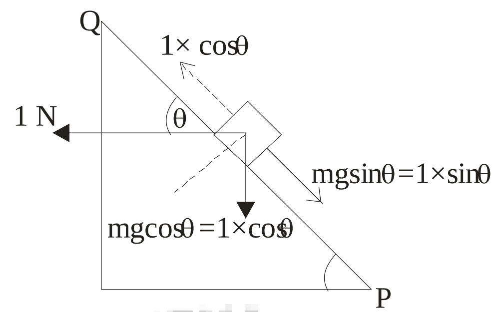
(B) $\theta > 45^\circ$ and a frictional force acts on the block towards P

(C) $\theta > 45^\circ$ and a frictional force acts on the block towards Q

(D) $\theta < 45^\circ$ and a frictional force acts on the block towards Q

Sol. (A, C)

M = 0.1 kg



(A) $\theta = 45^\circ$

$1\times\cos\theta = mg\sin\theta$

(B) $\theta > 45^\circ$

$mg\sin\theta > 1\times\cos\theta$

So block will tend to slip down the plane, so friction will act towards point Q.

When $\theta < 45^\circ$, then $1\times\cos\theta > mg\sin\theta$

So block will have tendency to slip up the plane so friction will at towards point P.

14. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time t = 0, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option (s) is (are) correct for time t > 0?

(A) If $\theta = 0^\circ$, the charge moves in a circular path in the x-z plane

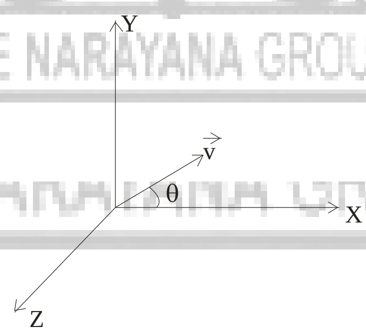
(B) If $\theta = 0^\circ$, the charge undergoes helical motion with constant pitch along the y-axis

(C) If $\theta = 10^\circ$, the charge undergoes helical motion with its pitch increasing with time,along the y-axis

(D) If $\theta = 90^\circ$, the charge undergoes linear but accelerated motion along the y-axis

Sol. (C, D)

$$T = \frac{2\pi m}{qB}$$



P = V_Y T

V_Y is variable due to electric field along y axis, so pitch will vary.

15. A person blows into open end of a long pipe. As a result, a high pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe,

(A) a high pressure pulse starts traveling up the pipe, if the other end of the pipe is open

(B) a low pressure pulse starts traveling up the pipe, if the other end of the pipe is open

(C) a low pressure pulse starts traveling up the pipe, if the other end of the pipe is closed

(D) a high pressure pulse starts traveling up the pipe, if the other end of the pipe is closed

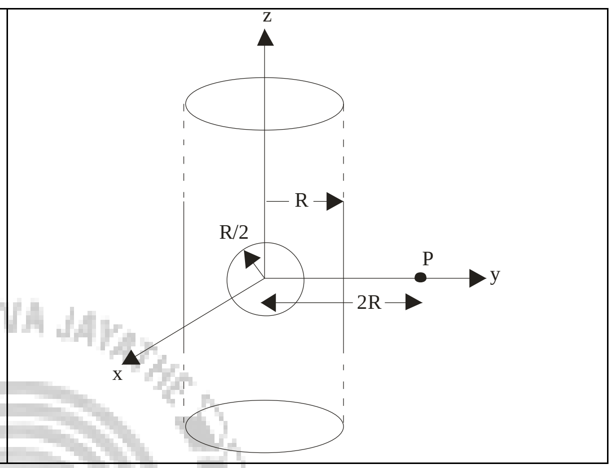
Sol. (B, D)

For pressure waves open end acts as a node, so high pressure pulse gets reflected as low pressure pulse, while closed end behaves as antinode, so high pressure pulse gets reflected again as a high pressure pulse.

Section III : Integer Answer Type

This section contains 5 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive)

16. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius R/2 with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electrical field at the point P, which is at a distance 2R from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is



Ans. (6)

Sol. Apply superposition principle

E due to cylinder is

$$E_1 = \frac{\rho R}{4\varepsilon_o}$$

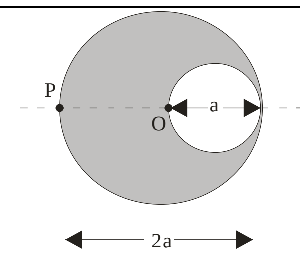
E due to spherical cavity is

$$E_2 = \frac{\rho R}{96\varepsilon_0}$$

Hence net field $= E_1 - E_2 = \frac{23\rho R}{96\varepsilon_o}$

$$\therefore K = 6$$

17. A cylindrical cavity of diameter a exists inside a cylinder of diameter 2a as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is



Ans. (5)

Sol. Apply superposition principle :

B due to complete cylinder is

$$B_1 = \frac{\mu_o Ja}{2}$$

B due to cylindrical cavity is

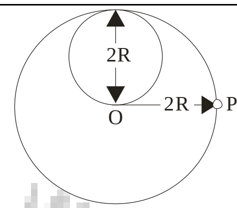
$$B_2 = \frac{\mu_o J a}{12}$$

Hence net magnetic field is

$$= B_1 - B_2 = \frac{5}{12}\mu_o a J$$

$$\therefore N = 5$$

18. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_O and I_P, respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_O}$ to the nearest integer is



Ans. (3)

Sol. Let mass of whole disc is (4m) then mass of removed part is (m). Then M.I. of disc with cavity about O is

$$I_o = \frac{1}{2}(4m)(4R^2) - \left[\frac{1}{2}mR^2 + mR^2\right]$$

$$= \frac{13}{2}mR^2$$

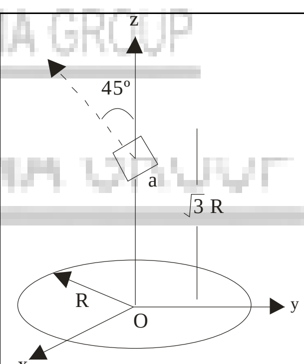
M.I. of disc with cavity about P is

$$I_P = \frac{3}{2}(4m)(4R^2) - \left[\frac{1}{2}mR^2 + m\left(\sqrt{5}R\right)^2\right]$$

$$= \frac{37}{2}mR^2$$

Now $\frac{I_P}{I_o} = \frac{37}{13} \simeq 3(\text{nearest integer})$

19. A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop of side a(a << R) having two turns is placed with its center at $z = \sqrt{3}R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2}R}$, then the value of p is



Ans. (7)

Sol. B at the centre of square loop due to circular loop is

$$B = \frac{\mu_0 i R^2}{2\left(R^2 + 3R^2\right)^{3/2}} = \frac{\mu_0 i}{16R}$$

$$\therefore M = \frac{\phi}{i} = \frac{2BA\cos 45^{o}}{i} = \frac{2\mu_0}{16R}.a^2 \times \frac{1}{\sqrt{2}}$$

$$= \frac{\mu_0 a^2}{(2)^{7/2} R}$$

$$\therefore P = 7$$

20. A proton is fired from very far away towards a nucleus with charge Q = 120 e, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is: (take the proton mass, $m_p = (5/3)\times 10^{-27}$ kg ; $h/e = 4.2\times 10^{-15}$ J.s / C; $\frac{1}{4\pi\varepsilon_0} = 9\times 10^{9}$ m / F ; 1 fm = 10^{-15} m)

Ans. (7)

Sol. Using conservation of energy we have

$$\frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r} = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

$$\therefore \lambda = \sqrt{\frac{4\pi\varepsilon_o r.h^2}{q_1 q_2 (2m)}}$$

$$= \sqrt{\frac{10\times 10^{-10}\times h^2 \times 3}{9\times 10^{9}\times 120e^2 \times 2\times 5\times 10^{-27}}} = 7\,\text{fm}$$

Part II : Chemistry

Section I : Straight Correct Answer Type

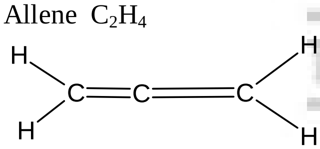
This section contains 10 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

21. In allene (C_3H_4), the type (s) of hybridization of the carbon atoms is (are)

(A) sp and sp^3 (B) sp and sp^2 (C) only sp^2 (D) sp^2 and sp^3

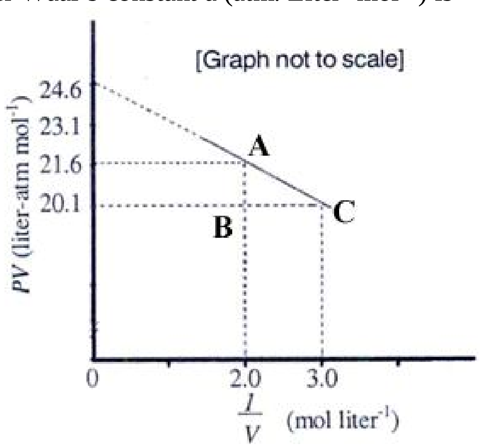
Ans. (B)

Sol. Allene C_2H_4



terminal carbon's are sp^2 hybridized while central carbon is sp hybridized

22. For one mole of a vander Waal's gas when b = 0 and T = 300 K, the PV vs. 1/V plot is shown below. The value of vander Waal's constant a (atm. Liter^2 mol^{-2}) is



(A) 1.0 (B) 4.5 (C) 1.5 (D) 3.0

Ans. (C)

Sol. b = 0 , T = 300 K

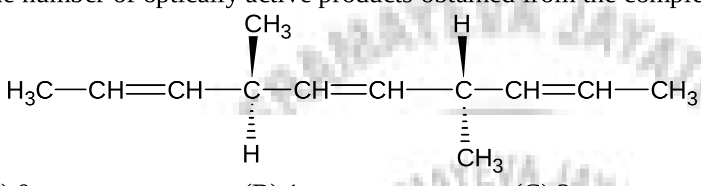
$$\left(P+\frac{a}{V^2}\right)(V-b)=RT \text{ for 1 mole of gas}$$

$$b = 0 \quad \therefore \quad PV+\frac{a}{V}=RT$$

$$PV=RT-\frac{a}{V}\left[Y=mx+c \text{ from}\right]$$

$$\text{Slope} = a = \frac{AB}{BC}=\frac{1.5}{1}=1.5$$

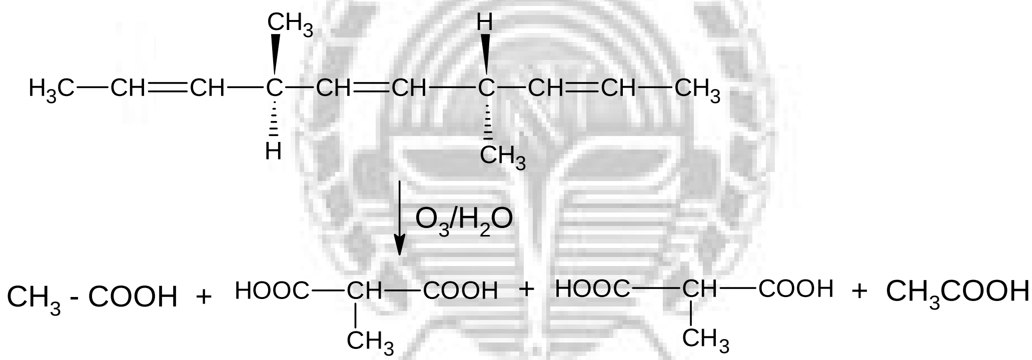
23. The number of optically active products obtained from the complete ozonolysis of the given compound is



(A) 0 (B) 1 (C) 2 (D) 4

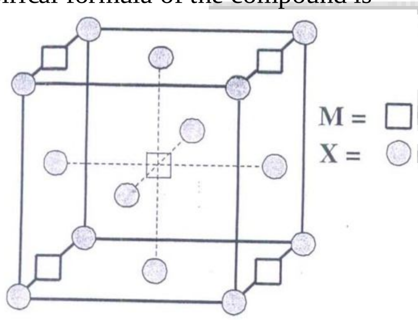
Ans. (A)

Sol.



i.e. 2 moles of CH_3COOH and 2 moles of $CH_3-\underset{\underset{COOH}{|}}{CH}-COOH$ and both of them do contain plane of symmetry and hence optically inactive.

24. A compound M_pX_q has cubic close packing (ccp) arrangement of X. Its unit cell structure is shown below. The empirical formula of the compound is



(A) MX (B) MX_2 (C) M_2X (D) M_5X_{14}

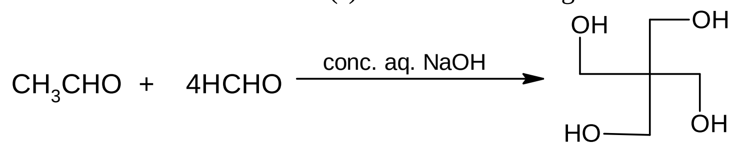
Ans. (B)

Sol. Number of atoms of M = $4\times\frac{1}{4}+1\times1=2$

number of atoms of X = $8\times\frac{1}{8}+6\times\frac{1}{2}=4$

Therefore, empirical formula = MX_2

25. The number of aldol reaction (s) that occurs in the given transformation is

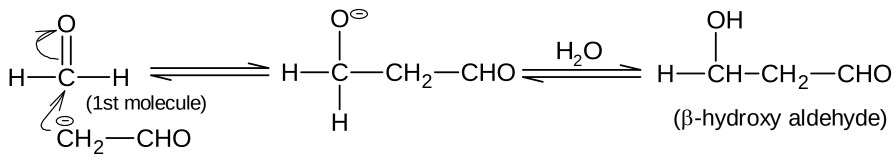


(A) 1 (B) 2 (C) 3 (D) 4

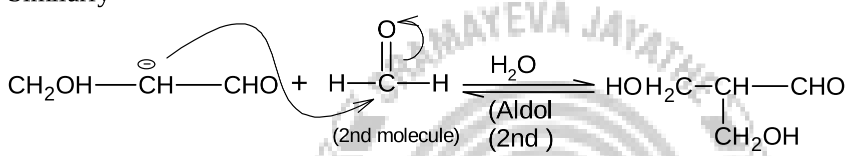
Ans. (C)

Sol. This is an example of base catalysed aldol condensation, followed by cannizaro reaction to finally form penta erythrol.

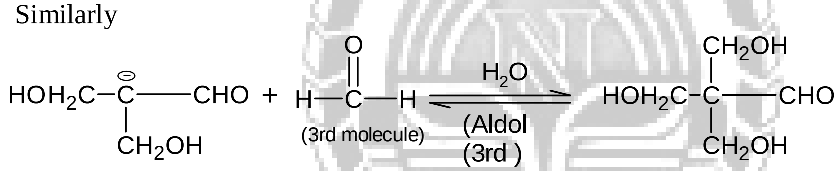
$$CH_3CHO + OH^- \xrightleftharpoons{Aldol(1st)} \bar{C}H_2 - CHO \longleftrightarrow CH_2 = \overset{O^-}{\overset{|}{C}} - H$$



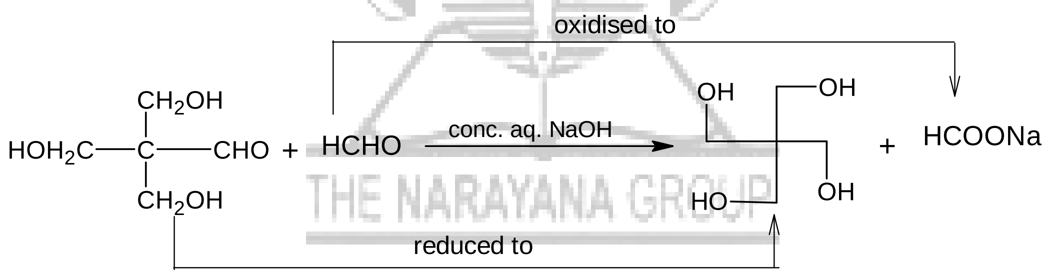
Similarly



Similarly



Now after three aldol condesnation the product aldehyde, does not contain any $\alpha - \mathrm{H}$, therefore it undergoes crossed cannizzaro reaction i.e.



26. The colour of light absorbed by an aqueous solution of $CuSO_4$ is

(A) orange-red (B) blue –green (C) yellow (D) violet

Ans. (A)

Sol. This is colorimetry $CuSO_4$ aqueous solution is blue in colour because of the absorption of orange- red light absorbed from the visible wavelength of light. Higher the concentration of Cu^{+2} ion deeper is the colour.

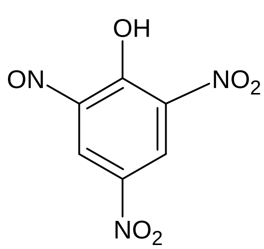
27. The carboxyl functional group (-COOH) is present in

(A) picric acid (B) barbituric acid (D) ascorbic acid (D) aspirin

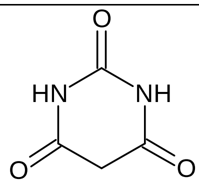
Ans. (D)

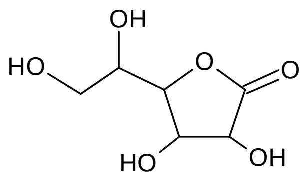
Sol.

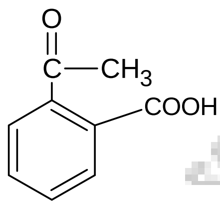
(A)



2, 4, 6 trinitro phenol is called as picric acid.

(B)  barbituric acid

(C)  ascorbic acid or vitamin C

(D)  Acetyl salicylic acid or aspirin

28. The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is [a_0 is Bohr radius]

(A) $\frac{h^2}{4\pi^2 m a_0^2}$ (B) $\frac{h^2}{16\pi^2 m a_0^2}$ (C) $\frac{h^2}{32\pi^2 m a_0^2}$ (D) $\frac{h^2}{64\pi^2 m a_0^2}$

Ans. (C)

Sol. $K.E. = \frac{1}{2}mv^2$ . . . .(i)

$mvr = \frac{nh}{2\pi}$ (Bohr's model)

$mv = \frac{nh}{2\pi r}$

$(mv)^2 = \left(\frac{nh}{2\pi r}\right)^2$

$mv^2 = \left(\frac{nh}{2\pi r}\right)^2 \frac{1}{m}$ . . . . .(ii)

Putting the value of (ii) in equation (i)

$K.E. = \frac{1}{2}mv^2 = \frac{1}{2} \times \frac{n^2h^2}{4\pi^2 r^2 .m}$ . . . . . (iii)

Now n = 2

$r_1 = a_0$

$r_2 = a_0 \times 2^2 = 4a_0$

Put the value of r in equation (iii)

$$K.E. = \frac{1}{2} \times \frac{n^2h^2}{4\pi^2 (4a_0)^2 .m}$$

$$K.E. = \frac{4h^2}{2 \times 4\pi^2 \times 16{a_0}^2 .m}$$

$$K.E. = \frac{h^2}{32\pi^2 m a_0^2}$$

29. Which ordering of compounds is according to the decreasing order of the oxidation state of nitrogen ?

(A) HNO_3, NO, NH_4Cl, N_2
(B) HNO_3, NO, N_2, NH_4Cl
(C) HNO_3, NH_4Cl, NO, N_2
(D) NO, HNO_3, NH_4Cl, N_2

Ans. (B)

Sol. Oxidation state of nitrogen in
HNO_3 has $(+1) + x(1) + 3(-2) = 6$
$\therefore \quad x = +5$
NO $\quad x(1) + (-2)1 = 0 \;\; \therefore \;\; x = +2$
$N_2 \quad x(2) = 0 \;\; \therefore x = 0$
$NH_4Cl \quad x(1) + 1(4) + (-1)\,1 = 0 \;\; \therefore x = -3$

30. As per IUPAC nomenclature, the name of the complex $[Co(H_2O)_4(NH_3)_2]Cl_3$ is
(A) Tetraaquadiaminecobalt (III) chloride (B) Tetraaquadiamminecobalt (III) chloride
(C) Diaminetetraaquacobalt (III) chloride (D) Diamminetetraaquacobalt (III) chloride

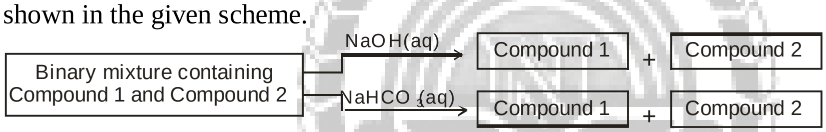
Ans. (D)

Sol. $[Co(H_2O)_4(NH_3)_2]Cl_3$
According to IUPAC nomenclature rule it is
Diamminetetraaquacoblat (III) chloride
i.e. NH_3 is neutral ligand and it is named as ammine and H_2O (aqua)

Section II: Multiple Correct Answer (s) Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

31. Identify the binary mixture (s) that can be separated into individual compounds by differential extraction, as shown in the given scheme.



(A) C_6H_5OH and C_6H_5COOH (B) C_6H_5COOH and $C_6H_5CH_2OH$
(C) $C_6H_5CH_2OH$ and C_6H_5OH (D) $C_6H_5CH_2OH$ and $C_6H_5CH_2COOH$

Ans. (BD)

Sol. In differential extraction one of the compound of binary mixture should be dissolved in given solvent.
(B) $C_6H_5CH_2OH$ is insoluble in both whereas C_6H_5COOH dissolves in both NaOH and $NaHCO_3$.

$$C_6H_5COOH + NaOH \longrightarrow C_6H_5COONa + H_2O \ (dissolved)$$

$$C_6H_5CH_2OH + NaOH \longrightarrow No\ reaction\ (remains\ undissolved)$$

(D) $C_6H_5CH_2OH$ is insoluble in both whereas $C_6H_5CH_2COOH$ dissolves in both NaOH and $NaHCO_3$.

$$C_6H_5COOH + NaHCO_3 \longrightarrow C_6H_5COONa + H_2CO_3$$

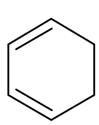
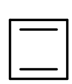
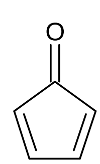
$$C_6H_5CH_2OH + NaHCO_3 \longrightarrow No\ reaction$$

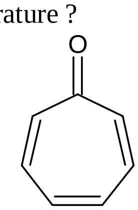
32. Choose the correct reason(s) for the stability of the lyophobic colloidal particles.
(A) Preferential adsorption of ions on their surface from the solution
(B) Preferential adsorption of solvent on their surface from the solution
(C) Attraction between different particles having opposite charges on their surface
(D) Potential difference between the fixed layer and the diffused layer of opposite charges around the colloidal particles.

Ans. (AD)

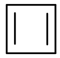
Sol. Colloidal particle are surrounded by particular ions present in solution therefore, a potential difference developed between the fixed layer and the diffused layer (Zeta potential)

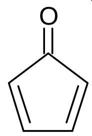
33. Which of the following molecules, in pure form, is(are) unstable at room temperature ?

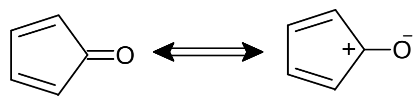
(A)  (B)  (C) 

(D) 

Ans. (BC)

Sol. (B)  cyclobutadiene follows Huckel's criteria for antiaromaticity hence unstable.

(C)  follows Huckel's criteria for antiaromaticity hence unstable.



34. Which of the following hydrogen halides react(s) with $AgNO_3$ (aq) to give a precipitate that dissolves in $Na_2S_2O_3$(aq) ?

(A) HCl (B) HF (C) HBr (D) HI

Ans. (ACD)

Sol. $HBr + AgNO_3(aq) \longrightarrow AgBr(s) + HNO_3$

$AgBr(s) + Na_2S_2O_3 \longrightarrow 2NaBr + Ag_2S_2O_3 \xrightarrow{Na_2S_2O_3} \underset{\text{SolubleComplex}}{Na_5[Ag(S_2O_3)_3]}$

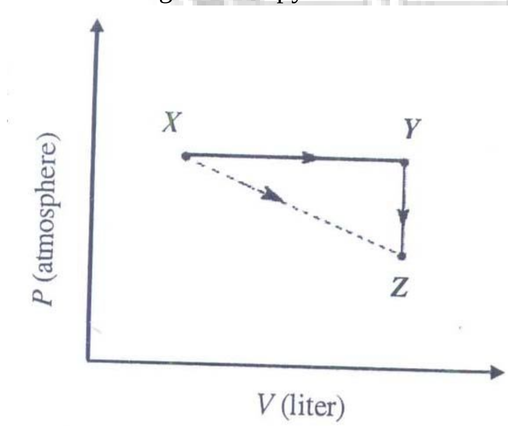
$HCl + AgNO_3(aq) \longrightarrow AgCl(s) + HNO_3$

$AgCl(s) + Na_2S_2O_3 \longrightarrow 2NaCl + Ag_2S_2O_3 \xrightarrow{Na_2S_2O_3} \underset{\text{solublecomplex}}{Na_5[Ag(S_2O_3)_3]}$

$HI + AgNO_3(aq) \longrightarrow AgI(s) + HNO_3$

$AgI(s) + Na_2S_2O_3 \longrightarrow 2NaI + Ag_2S_2O_3 \xrightarrow{Na_2S_2O_3} \underset{\text{solublecomplex}}{Na_5[Ag(S_2O_3)_3]}$

35. For a ideal gas, consider only P-V work in going from an initial state X to the final state Z. The final state Z can be reached by either of the two paths shown in the figure. Which of the following choice (s) is (are) correct ? [Take ΔS as change in entropy and w as work done]



(A) $\Delta S_{x\to z} = \Delta S_{x\to y} + \Delta S_{y\to z}$ (B) $w_{x\to z} = w_{x\to y} + w_{y\to z}$

(C) $w_{x\to y\to z} = w_{x\to y}$ (D) $\Delta S_{x\to y\to z} = \Delta S_{x\to y}$

Ans. (AC)

Sol. Entropy is a state function so does not depends on path where as work is path dependent

ΔS change for x $\to$ y and y $\to$ z is x $\to$ z

so $\Delta S_{x\to z} = \Delta S_{x\to y} + \Delta S_{y\to z}$

work done in y $\to$ z path is zero due to constant volume

$\therefore\ w_{x\to y\to z} = w_{x\to y}$

Section III : Integer Answer Type

This section contains 5 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive)

36. The substituents $\mathbf{R_1}$ and $\mathbf{R_2}$ for nine peptides are listed in the table given below. How many of these peptides are positively charged at pH = 7.0 ?

$$H_3\overset{\oplus}{N}-\underset{\underset{H}{|}}{CH}-CO-NH-\underset{\underset{\mathbf{R_1}}{|}}{CH}-CO-NH-\underset{\underset{\mathbf{R_2}}{|}}{CH}-CO-NH-\underset{\underset{H}{|}}{CH}-CO\overset{\ominus}{O}$$

Peptide	$\mathbf{R_1}$	$\mathbf{R_2}$
I	H	H
II	H	CH_3
III	CH_2COOH	H
IV	CH_2CONH_2	$(CH_2)_4NH_2$
V	CH_2CONH_2	CH_2CONH_2
VI	$(CH_2)_4NH_2$	$(CH_2)_4NH_2$
VII	CH_2COOH	CH_2CONH_2
VIII	CH_2OH	$(CH_2)_4NH_2$
IX	$(CH_2)_4NH_2$	CH_3

Ans: (4)

Sol: Basic amino acids are positively charged at pH = 7. If R_1 or R_2 contains $-NH_2$ (amine) group and not an acidic counterpart, then it is a basic amino acid

I	No
II	No
III	No
IV	Yes
V	N
VI	Yes
VII	No
VIII	Yes
IX	Yes

37. The periodic table consists of 18 groups. An isotope of copper, on bombardment with protons, undergoes a nuclear reaction yielding element X as shown below. To which group, element X belogs in the periodic table?

$${}^{63}_{29}Cu + {}^{1}_{1}H \rightarrow 6\,{}^{1}_{0}n + \alpha + 2\,{}^{1}_{1}H + X$$

Ans: (8)

Sol: $${}_{29}Cu^{63} + {}_{1}H^{1} \rightarrow 6\,{}_{0}n^{1} + {}_{2}He^{4} + 2\,{}_{1}H^{1} + {}_{26}X^{52}$$

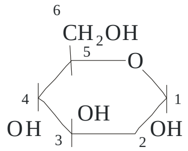
X is Iron. Which is group 8 element.

38. When the following aldohexoes exists in its **D-**configuration, the total number of stereoisomers in its pyranose form is

$$\begin{array}{l} CHO \\ | \\ CH_2 \\ | \\ CHOH \\ | \\ CHOH \\ | \\ CHOH \\ | \\ CH_2OH \end{array}$$

Ans: (8)

Sol:



The pyranose form of this compound contains 4 chiral carbons. Among them C_5 as per-assigned D configuration. So number of steroisomers would be = $2^3 = 8$

39. 29.2% (w/w) HCl stock solution has a density of 1.25 g mL^{-1}. The molecular weight of HCl is 36.5 g mol^{-1}. The volume (mL) of stock solution required to prepare a 200 mL solution of 0.4 M HCl is

Ans: (8)

Sol: Mass of HCl in 1 mL solution

$$= 1.25 \times \frac{29.2}{100} = 0.365\text{g}$$

Mass of HCl in 200 mL 0.4 M solution

$$= 0.2 \times 0.4 \times 36.5 \text{ g}$$
$$= 0.08 \times 36.5 \text{ g}$$

$\therefore$ Vol of HCl required

$$= \frac{0.08 \times 36.5}{0.365} = 8 \text{ mL}$$

40. An organic compound undergoes first- order decomposition. The time taken for its decomposition to 1/8 and 1/10 of its initial concentration are $t_{1/8}$ and $t_{1/10}$ respectively.

What is the value of $\frac{[t_{1/8}]}{[t_{1/10}]} \times 10$? (take $\log_{10}2 = 0.3$)

Ans: (9)

Sol: $$t_{1/8} = \frac{2.303}{K} \log \frac{C_0}{C_0/8}$$

$$t_{1/10} = \frac{2.303}{K} \log \frac{C_0}{C_0/10}$$

$$\therefore \quad \frac{[t_{1/8}]}{[t_{1/10}]} \times 10 = \frac{\log 8}{\log 10} \times 10$$
$$= 3 \times \log 2 \times 10$$
$$= 3 \times 0.3 \times 10$$
$$= 9$$

Part III : Mathematics

Section I : Straight Correct Answer Type

This section contains 10 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

41. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

(A) 75 (B) 150
(C) 210 (D) 243

Ans: (B)

Sol: Possible distributions are either (2, 2, 1) or (3, 1, 1)

Now no. of ways are $\left(\frac{5!}{(2!)^2(1!)2!} + \frac{5!}{3!(1!)^2.2!}\right)3! = 150$

42. Let $f(x)\begin{cases} x^2\left|\cos\frac{\pi}{x}\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$, $x \in IR$, then f is

(A) differentiable both at x = 0 and at x = 2
(B) differentiable at x = 0 but not differentiable at x = 2
(C) not differentiable at x = 0 but differentiable at x = 2
(D) differentiable neither at x = 0 nor at x = 2

Ans: (B)

Sol: Left hand derivative at x=0

$$= \lim_{h\to 0}\frac{f(-h)-f(0)}{-h}$$

$$= \lim_{h\to 0}\frac{h^2\left|\cos\left(\frac{\pi}{-h}\right)\right|-0}{-h} = 0$$

Right hand derivative at 0,

$$= \lim_{h\to 0}\frac{f(h)-f(0)}{h}$$

$$= \lim_{h\to 0}\frac{h^2\left|\cos\left(\frac{\pi}{h}\right)\right|-0}{h} = 0$$

Also left hand derivative at 2,

$$= \lim_{h\to 0}\frac{f(2-h)-f(2)}{-h}$$

$$= \lim_{h\to 0}\frac{-(2-h)^2\cos\frac{\pi}{(2-h)}-0}{-h}$$

$$= \lim_{h\to 0}\frac{-(2-h)^2\sin\left(\frac{\pi}{2}-\frac{\pi}{2-h}\right)}{-h}$$

$$= \lim_{h\to 0}\frac{-(2-h)^2\sin\left(\frac{-\pi h}{2(2-h)}\right)\frac{\pi}{2(2-h)}}{-\frac{h\times\pi}{2(2-h)}} = -\pi$$

Right hand derivative at 2

$$= \lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$= \frac{(2+h)^2\cos\left(\frac{\pi}{2+h}\right)-0}{h}$$

$$= \frac{(2+h)^2\sin\left(\frac{\pi}{2}-\frac{\pi}{2+h}\right)}{h}$$

$$= \operatorname{Lim}_{h\to 0}\frac{(2+h)^2\sin\left(\frac{\pi h}{2(2+h)}\right)\frac{\pi}{2(2+h)}}{\frac{\pi h}{2(2+h)}} = \pi$$

Finally continuous and differentiable at 0 and continuous but not differentiable at 2.

43. The function $f:[0,3]\to[1,29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is

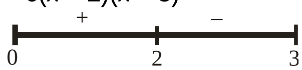
(A) one–one and onto
(B) onto but not one–one
(C) one–one but not onto
(D) neither one–one nor onto

Ans: (B)

Sol: $f(x) = 2x^3 - 15x^2 + 36x + 1$

$f'(x) = 6x^2 - 30x + 36$

$= 6(x-2)(x-3)$



$\Rightarrow$ f(x) is non–monotonic

$\Rightarrow$ f(x) is not one–one

f(0) = 1

f(2) = 29

f(3) = 28

$\Rightarrow$ range of f(x) is [1, 29] = codomain

$\Rightarrow$ f(x) is onto.

44. If $\lim\limits_{x\to\infty}\left(\dfrac{x^2+x+1}{x+1}-ax-b\right)=4,$ then

(A) a = 1, b = 4
(B) a = 1, b = –4
(C) a = 2, b = –3
(D) a = 2, b = 3

Ans: (B)

Sol: $\lim\limits_{x\to\infty}\left(\dfrac{x^2+x+1}{x+1}-ax-b\right)=4$

$\lim\limits_{x\to\infty}\left(\dfrac{x^2+x+1-ax^2-bx-ax-b}{x+1}\right)=4$

Coefficient of x^2 must be zero

$1 - a = 0 \Rightarrow a = 1$

$\lim\limits_{x\to\infty}\dfrac{x(1-b-a)+(1-b)}{x+1}=4$

Putting value of

$\lim\limits_{x\to\infty}\dfrac{-bx+1-b}{x+1}=4$

$\Rightarrow$ $b = -4$

45. Let z be a complex number such that the imaginary part of z is non–zero and a = z^2 + z + 1 is real. Then a **cannot** take the value

(A) – 1
(B) $\dfrac{1}{3}$
(C) $\dfrac{1}{2}$
(D) $\dfrac{3}{4}$

Ans: (D)

Sol: $a = z^2 + z + 1$ ..(i)

Take conjugate on both side

$a=\bar{z}^2+\bar{z}+1$ …(ii) $a \in R$

(i) – (ii) $\Rightarrow$ $(z-\bar{z})(z+\bar{z})+(z-\bar{z})=0$

$(z-\bar{z})(z+\bar{z}+1)=0$

As $a\in R$ $z-\bar{z}\neq 0$

$\Rightarrow$ $z+\bar{z}=-1$ …(iii)

$\Rightarrow$ $\mathrm{Re}(z)=-\dfrac{1}{2}$

(i) + (ii) $\Rightarrow$ $2a=(z+\bar{z})^2-2z\bar{z}+z+\bar{z}+2$

$=1-2z\bar{z}-1+2$

$=2-2z\bar{z}=2-2|z|^2$

Let $z=-\dfrac{1}{2}+iy,\ y\neq 0$

$|z|^2=\dfrac{1}{4}+y^2$

$$2a = 2 - 2\left(\frac{1}{4} + y^2\right)$$

$$= \frac{3}{2} - 2y^2$$

$$a = \frac{3}{4} - y^2 < \frac{3}{4} \text{ as } y \neq 0$$

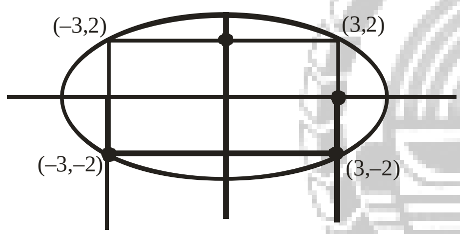
46. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Ans: (C)

Sol: $E_1 = \frac{x^2}{9} + \frac{y^2}{4} = 1$



Let Equation of ellipse E_2 is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Passes through (0, 4)

$$0 + \frac{16}{b^2} = 1$$

$$b^2 = 16$$

It also passes through point (3, 2)

$$\frac{9}{a^2} + \frac{4}{b^2} = 1$$

$$\frac{9}{a^2} + \frac{4}{16} = 1 \Rightarrow a^2 = 12$$

Eccentricity : $a^2 = b^2(1 - e^2)$

$$\frac{12}{16} = 1 - e^2$$

$$e^2 = 1 - \frac{3}{4},\ e^2 = \frac{1}{4} \Rightarrow e = \frac{1}{2}$$

47. Let P = [aij] be a 3 × 3 matrix and let Q = $[b_{ij}]$, where $b_{ij} = 2^{i+j}a_{ij}$ $1 \le i, j \le 3$. If the determinant of P is 2, then the determinant of the matrix Q is

(A) 2^{10} (B) 2^{11}

(C) 2^{12} (D) 2^{13}

Ans: (D)

Sol: $P(x) = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ $Q = [b_{ij}]$

$$Q = \begin{vmatrix} 2^2 a_{11} & 2^3 a_{12} & 2^4 a_{13} \\ 2^3 a_{21} & 2^4 a_{22} & 2^5 a_{23} \\ 2^4 a_{31} & 2^5 a_{32} & 2^6 a_{33} \end{vmatrix}$$

$$= 2^9 \begin{vmatrix} a_{11} & 2a_{12} & 2^2 a_{13} \\ a_{21} & 2a_{22} & 2^2 a_{23} \\ a_{31} & 2a_{32} & 2^2 a_{33} \end{vmatrix}$$

$$= 2^{12}|P$$
$$= 2^{13}$$

48. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$ equals (for some arbitrary constant K)

(A) $-\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(B) $\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(C) $-\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(D) $\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

Ans: (C)

Sol: $I = \int \frac{\sec x(\sec x(\sec x + \tan x)}{(\sec x + \tan x)^{11/2}} dx$

$I = -\frac{2}{9}\frac{\sec x}{(\sec x + \tan x)^{9/2}} + \frac{2}{9}\int \frac{\sec x + \tan x}{(\sec x + \tan x)^{9/2}}$

$\frac{9}{2}I + I = \frac{-\sec x}{(\sec x + \tan x)^{9/2}} + \int \frac{\sec x(\sec x + \tan x)}{(\sec x + \tan x)^{9/2}}$

$\frac{11}{2}I = -\frac{\sec x}{(\sec x + \tan x)^{9/2}} - \frac{2}{7}(\sec x + \tan x)^{-7/2}$

$= -\frac{1}{(\sec x + \tan x)^{9/2}}\left[\frac{7\sec x + 2\sec x + 2\tan x}{7}\right]$

$I = -\frac{1}{(\sec x + \tan x)^{9/2}}\left[\frac{18\sec x + 4\tan x}{77}\right]$

$= -\frac{1}{(\sec x + \tan x)^{11/2}}\left[\frac{(18\sec x + 4\tan x)(\sec x + \tan x}{77}\right]$

$= -\frac{1}{(\sec x + \tan x)^{11/2}}\left[\frac{18\sec^2 x + 4\tan^2 x + 22\sec x \tan x}{77}\right]$

$= -\frac{1}{(\sec x + \tan x)^{11/2}}\left[\frac{7(\sec^2 x - \tan^2 x) + 11(\sec x + \tan x)}{77}\right]^2$

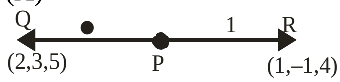
$I = -\frac{1}{(\sec x + \tan x)^{11/2}}\left[\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right] + C$

49. The point P is the intersection of the straight line joining the points Q(2, 3, 5) and R(1, –1, 4) with the plane 5x – 4y – z = 1. If S is the foot of the perpendicular drawn from the point T(2, 1, 4) to QR, then the length of the line segment PS is

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$

(C) 2 (D) $2\sqrt{2}$

Ans: (A)



Sol:

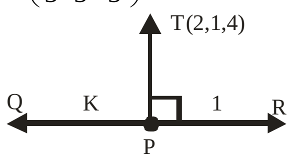
Co-ordinates of P are $\left(\frac{\lambda+2}{\lambda+1}, \frac{-\lambda+3}{\lambda+1}, \frac{4\lambda+5}{\lambda+1}\right)$

Which satisfies 5x – 4y – z = 1

$$5\left(\frac{\lambda+2}{\lambda+1}\right) - 4\left(\frac{-\lambda+3}{\lambda+1}\right) - \left(\frac{4\lambda+5}{\lambda+1}\right) = 1$$

$\Rightarrow$ $\lambda = 2$

So, $P\left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right)$



Now

Co–ordinates of S are $\left(\frac{k+2}{k+1}, \frac{-k+3}{k+1}, \frac{4k+5}{k+1}\right)$

Now direction ratio of TS are $\left(-\frac{k}{k+1}, \frac{-2k+2}{k+1}, \frac{1}{k+1}\right)$

Also direction ratio of QR are (1, 4, 1)

Now two line segments are mutually perpendicular

So, $\left(-\frac{k}{k+1}\right) + 4\left(\frac{-2k+2}{k+1}\right) + 1\left(\frac{1}{k+1}\right) = 0$

$\Rightarrow$ $k = 1$

so, $S\left(\frac{3}{2}, 1, \frac{9}{2}\right)$

Finally length of $PS = \sqrt{\left(\frac{4}{3}-\frac{3}{2}\right)^2 + \left(\frac{1}{3}-1\right)^2 + \left(\frac{13}{3}-\frac{9}{2}\right)^2} = \frac{1}{\sqrt{2}}$

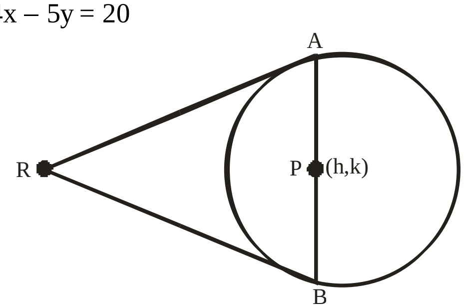
50. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is

(A) $20(x^2 + y^2) - 36x + 45y = 0$
(B) $20(x^2 + y^2) + 36x - 45y = 0$
(C) $36(x^2 + y^2) - 20x + 45y = 0$
(D) $36(x^2 + y^2) + 20x - 45y = 0$

Ans: (A)

Sol: Let P(h, k) is mid point of AB equation of chord AB with mid point P is $T = S_1$

Let Point R lies on the line $4x - 5y = 20$



$$hx + ky - 9 = h^2 + k^2 - 9$$
$$hx + ky = h^2 + k^2$$
$$hx + ky - h^2 - k^2 = 0 \qquad \ldots(i)$$

Let point R lies on the line $4x - 5y = 20$

$\therefore$ $R\left(\alpha, \frac{4\alpha - 20}{5}\right)$

Equation of chord of contact AB is

$$\alpha x+\left(\frac{4\alpha-20}{5}\right)y=9$$

$$\alpha x+\frac{4\alpha-20}{5}y-9=0 \qquad \ldots(ii)$$

(i) & (ii) are identical

$$\frac{h}{\alpha}=\frac{k}{\frac{4\alpha-20}{5}}=\frac{f(h^2+k^2)}{+9}$$

$$\frac{9h}{h^2+k^2}=\alpha,\ \frac{45k}{h^2+k^2}=4\alpha-20$$

Equating values of α $\frac{45k}{h^2+k^2}+20=4\alpha$

$$\frac{45k}{h^2+k^2}+20=4\times\frac{9h}{h^2+k^2}$$

$45k + 20h^2 + 20k^2 = 36h$

$$20x^2+20y^2-36x+45y=0$$

Section II: Multiple Correct Answer (s) Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

51. Let θ, $\phi \in [0, 2\pi]$ be such that $2\cos\theta(1-\sin\phi)=\sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\phi-1$,

$\tan(2\pi-\theta)>0$ and $-1<\sin\theta<-\frac{\sqrt{3}}{2}$. Then ϕ **cannot** satisfy

(A) $0<\phi<\frac{\pi}{2}$ (B) $\frac{\pi}{2}<\phi<\frac{4\pi}{3}$

(C) $\frac{4\pi}{3}<\phi<\frac{3\pi}{2}$ (D) $\frac{3\pi}{2}<\phi<2\pi$

Ans: (ACD)

Sol: $2\cos\theta(1-\sin\phi)=\sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\phi-1$

From given condition, θ will be lie in the interval of $\frac{3\pi}{2}$ and $\frac{5\pi}{3}$

$\because$ θ is in the 4h quadrant

$\therefore$ LHS will be +ve

$\therefore$ RHS must be +ve

i.e. $2\sin\theta\cos\phi - 1 > 0$

$$\cos\phi<\frac{1}{2\sin\theta}$$

$$\cos\phi<-\frac{1}{2}$$

$\therefore$ $\phi\in\left(\frac{2\pi}{3}.\frac{4\pi}{3}\right)$

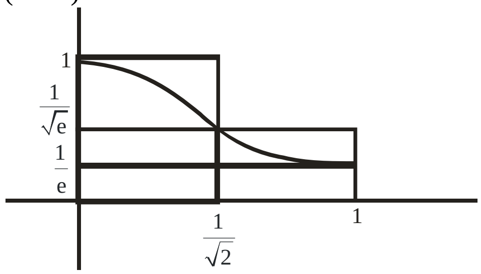
52. Let S be the area of the region enclosed by $y=e^{-x^2}$, y = 0, x = 0, and x = 1. Then

(A) $S\geq\frac{1}{e}$ (B) $S\geq1-\frac{1}{e}$

(C) $S \le \frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$ (D) $S \le \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$

Ans: (ABD)

Sol:



From graph $x \in (0,1)$ $S \ge \frac{1}{e}$

$$e^{-x} < e^{-x^2}$$

$$\int_0^1 e^{-x}dx \le \int_0^1 e^{-x^2}dx$$

$$\left(1-\frac{1}{e}\right) \le S$$

$$S \ge \left(1-\frac{1}{e}\right)$$

Required area $S \le \left(1-\frac{1}{\sqrt{2}}\right)\frac{1}{\sqrt{e}}+\frac{1}{\sqrt{2}}\times 1$

53. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true?

(A) $P\left[X_1^C \mid X\right] = \frac{3}{16}$

(B) P[Exactly two engines of the ship are functioning |X] = $\frac{7}{8}$

(C) $P\left[X \mid X_2\right] = \frac{5}{16}$

(D) $P\left[X \mid X_1\right] = \frac{7}{16}$

Ans: (BD)

Sol: $P(X_1) = \frac{1}{2}$

$$P(X_2) = \frac{1}{4}$$

$$P(X_3) = \frac{1}{4}$$

$$P(X) = \frac{1}{32}+\frac{3}{32}+\frac{3}{32}+\frac{1}{32} = \frac{1}{4}$$

(A) $P(X_1^c / X) = \frac{P(X_1^c \cap X)}{P(X)} = \frac{1/32}{1/4} = \frac{1}{8}$

(B) P(exactly two engines of ship are function)/x)

$$\frac{P(X_1X_2X_3^c)+P(X_1X_2^cX_3)+P(X_1^cX_2X_3)}{P(X)}$$

$$=\frac{7/32}{1/4}=\frac{7}{8}$$

(C) $P(X/X_2)=\dfrac{P(X\cap X_2)}{P(X_2)}=\dfrac{5/32}{1/4}=\dfrac{5}{8}$

(D) $P(X/X_1)=\dfrac{P(X\cap X_1)}{P(X_1)}=\dfrac{7/32}{1/2}=\dfrac{7}{16}$

54. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, parallel to the straight line 2x − y = 1. The points of contact of the tangents on the hyperbola are

(A) $\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$

(C) $\left(3\sqrt{3},-2\sqrt{2}\right)$ (D) $\left(-3\sqrt{3},2\sqrt{2}\right)$

Ans: (AB)

Sol: Equation of tangent on hyperbola

$$\frac{x^2}{9}-\frac{y^2}{4}=1 \text{ is}$$

$$y=mx\pm\sqrt{9m^2-4}$$

Where m = 2

$$y=2x\pm\sqrt{36-4}$$

$$y=2x\pm\sqrt{32} \qquad \ldots(I)$$

Equation of tangent at (x_1 y_1) of $\frac{x^2}{9}-\frac{y^2}{4}=1$ is

$$\frac{xx_1}{9}-\frac{yy_1}{4}=1$$

$$\Rightarrow \quad \frac{yy_1}{4}=\frac{xx_1}{9}-1 \qquad \ldots(ii)$$

Comparing (i) & (ii)

$$\frac{y_1}{4}=\frac{x_1/9}{2}=\frac{-1}{\pm\sqrt{32}}$$

$$x_1=\mp\frac{18}{4\sqrt{2}},\ y_1=\mp\frac{4}{4\sqrt{2}}$$

So, points of contact are

$$\left(-\frac{9}{2\sqrt{2}},-\frac{1}{\sqrt{2}}\right),\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$$

55. If y(x) satisfies the differential equation y′ − y tanx = 2x sec x and y(0) = 0, then

(A) $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8\sqrt{2}}$ (B) $y'\left(\frac{\pi}{4}\right)=\frac{\pi^2}{18}$

(C) $y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{9}$ (D) $y'\left(\frac{\pi}{3}\right)=\frac{4\pi}{3}+\frac{2\pi^2}{3\sqrt{3}}$

Ans: (AD)

Sol: Given : y′ − ytanx = 2x secx

$$\frac{dy}{dx}=\frac{y\sin x+2x}{\cos x}$$

$$(y\sin x+2x)dx-\cos x\,dy=0$$

Here $M=y\sin x+2x$ .. M(x, y)

$N = -\cos x$ ...$N(x, y)$

So $\dfrac{\partial M}{\partial Y} = \sin x$

$\dfrac{\partial N}{\partial X} = \sin x$

Since $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$

$\therefore$ differential equation is exact.

$\therefore$ Solution of differential equation will be

$\int (y \sin x + 2x)dx + \int 0 dy = C$

$\Rightarrow$ $-y\cos x + x^2 = c$ ...(i)

At x = 0; y = 0

$\therefore$ c = 0

$\therefore$ $y = \dfrac{x^2}{\cos x}$

$$\frac{dy}{dx} = \frac{\cos x \times 2x - x^2 \times (-\sin x)}{\cos^2 x}$$

$$\frac{dy}{dx} = \frac{2x\cos x + x^2 \sin x}{\cos^2 x}$$

For A

y at $\dfrac{\pi}{4}$

$$= \frac{(\pi/4)^2}{\cos\frac{\pi}{4}} = \frac{\pi^2}{16 \times \frac{1}{\sqrt{2}}} = \frac{\pi^2}{8\sqrt{2}}$$

For B

y'at $x = \dfrac{\pi}{4}$

$$= \frac{2 \times \frac{\pi}{4} \times \cos\frac{\pi}{4} + \left(\frac{\pi}{4}\right)^2 \sin\frac{\pi}{4}}{\cos^2\frac{\pi}{4}}$$

$$= \frac{2\frac{\pi}{4} \times \frac{1}{\sqrt{2}} + \frac{\pi^2}{16} \times \frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \frac{\frac{2\pi}{4} + \frac{\pi^2}{16}}{\frac{1}{\sqrt{2}}} = \frac{\frac{8\pi + \pi^2}{16}}{\frac{1}{\sqrt{2}}}$$

For C

y at $x = \dfrac{\pi}{3}$

$$= \frac{\pi^2}{9 \times \cos\frac{\pi}{3}} = \frac{2\pi^2}{9}$$

For D

$$y' \text{ at } x = \frac{\pi}{3}$$

$$= \frac{2\times\frac{\pi}{3}\cos\frac{\pi}{3} + \frac{\pi^2}{9}\sin\frac{\pi}{3}}{\cos^2\frac{\pi}{3}}$$

$$= \frac{2\times\frac{\pi}{3}\times\frac{1}{2} + \frac{\pi^2}{9}\times\frac{\sqrt{3}}{2}}{\left(\frac{1}{2}\right)^2}$$

$$= \frac{\frac{2\pi}{3} + \frac{\pi^2}{3\sqrt{3}}}{\frac{1}{2}} = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$$

Section III : Integer Answer Type

This section contains 5 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive)

56. Let f : IR $\to$ IR be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is

Ans: (5)

Sol: $f(x) = |x| + |x^2 - 1|$

$$f(x) = \begin{cases} x^2 - x - 1, & x < -1 \\ 1 - x - x^2, & -1 \le x < 0 \\ x + 1 - x^2, & 0 \le x < 1 \\ x + x^2 - 1, & x \ge 1 \end{cases}$$

$$f'(x) = \begin{cases} 2x - 1, & x < -1 \\ -1 - 2x, & -1 < x < 0 \\ 1 - 2x, & 0 < x < 1 \\ 1 + 2x, & x > 1 \end{cases}$$

$$\begin{array}{cccccccccccc} - & & + & & - & & + & & - & & + \\ & -1 & & -1/2 & & 0 & & 1/2 & & 1 & \end{array}$$

f(x) has local maxima/minima at

$$x = -1, -\frac{1}{2}, 0, \frac{1}{2}, 1$$

57. The value of $6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}....}}}\right)$ is

Ans: (4)

Sol: Let, $x = \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}......}}}$

$\Rightarrow$ $x = \frac{1}{3\sqrt{2}}\sqrt{4 - x}$

$\Rightarrow$ $18x^2 + x - 4 = 0$ $\Rightarrow$ $x = \frac{4}{9}$ or $-\frac{1}{2}$

$\therefore$ Required expression = $6 + \log_{3/2} x$

$\because$ $x > 0$ $\Rightarrow$ $x = \frac{4}{9}$

$\therefore \quad 6+\log_{3/2}\frac{4}{9}=6+\log_{3/2}\left(\frac{3}{2}\right)^{-2} = 6 - 2 = 4.$

58. Let p(x) be a real polynomial of least degree which has a local maximum at x = 1 and a local minimum at x = 3. If p(1) = 6 and p(3) = 2, then p′(0) is

Ans:

Sol: (9)

p(x) has local maxima at x = 1 and local minima at x = 3 and p(x) is of least degree

$\Rightarrow \quad p'(x)=a(x-1)(x-3)$

$p(x)=a\left(\frac{x^3}{3}-2x^2+3x\right)+d$

$p(1)=6 \quad \Rightarrow \quad \frac{4a}{3}+d=6 \quad \ldots(i)$

$p(3)=2 \quad \Rightarrow \quad d=2 \quad ,,,(ii)$

From (ii) in (i)

$\frac{4a}{3}=4 \quad \Rightarrow \quad a=3$

$p'(0)=3a=3\times3=9$

$p'(0)=9$

59. If $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors satisfying $\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2=9,$ then $\left|2\vec{a}+5\vec{b}+5\vec{c}\right|$ is

Ans: (3)

Sol: $\left|\vec{a}-\vec{b}\right|^2+\left|b-c\right|^2+\left|\vec{c}-a\right|^2=9$

$\Rightarrow$ Angle between $\vec{a}$ and $\vec{b}$ $=\frac{2\pi}{3}$

Angle between $\vec{b}$ and $\vec{c}$ $=\frac{2\pi}{3}$

Angle between $\vec{a}$ and $\vec{c}$ $=\frac{2\pi}{3}$

$\left|2\vec{a}+5\vec{b}+5\vec{c}\right|^2=(2\vec{a}+5\vec{b}+5\vec{c}).(2\vec{a}+5\vec{b}+5\vec{c})$

$= 4\left|\vec{a}\right|^2+25\left|\vec{b}\right|^2+25\left|\vec{c}\right|^2+2(10\vec{a}.\vec{b}+25\vec{b}.\vec{c}+10\vec{a}.\vec{c})$

$= 4+25+25+2\left(10\times\frac{-1}{2}+25\times\frac{-1}{2}+10\times\frac{-1}{2}\right)$

$= 54 - 45$

$\left|2\vec{a}+5\vec{b}+5\vec{c}\right|=3$

60 Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is

Ans: (4)

Sol: Focus of $y^2 = 8x$ is (2, 0)

Circle $x^2 + y^2 - 2x - 4y = 0$ and parabola $y^2 = 8x$ intersect at (0, 0) and (2, 4).

Hence the co-ordinate of S(2, 0), P(0, 0) and Q(2, 4)

Area of $\Delta PQS=\frac{1}{2}\begin{vmatrix}1 & 0 & 0\\1 & 2 & 0\\1 & 2 & 4\end{vmatrix} = \frac{1}{2}\times8=4$ sq. units

KEY

PHYSICS		CHEMISTRY			
Q.No.	Answer	Q.No.	Answer Key	Q.No.	Answer Key
1.	B	2 1.	B	41.	B
2.	C	22.	C	42.	B
3.	D	23.	A	43.	B
4.	A	24.	B	44.	B
5.	A	25.	C	45.	D
6.	D	26.	A	46.	C
7.	C	27.	D	47.	D
8.	D	28.	C	48.	C
9.	C	29.	B	49.	A
10.	B	30.	D	50.	A
11.	A, C, D	31.	BD	51.	ACD
12.	A, B, C, D	32.	AD	52.	ABD
13.	A, C	33.	BC	53.	BD
14.	C, D	34.	ACD	54.	AB
15.	B, D	35.	AC	55.	AD
16.	6	36.	4	56.	5
17.	5	37.	8	57.	4
18.	3	38.	8	58.	9
19.	7	39.	8	59.	3
20.	7	40.	9	60.	4